Quantifying ocean turbulence using two-point statistics

Structure functions

Dhruv Balwada - 19 May 2021
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Honorable mentions:
Oliver Buhler, & CARTHE (Consortium for Advanced Research on Transport of Hydrocarbon in the Environment)

Big whirls have little whirls
that feed on their velocity,
And little whirls have lesser whirls
and so on to viscosity

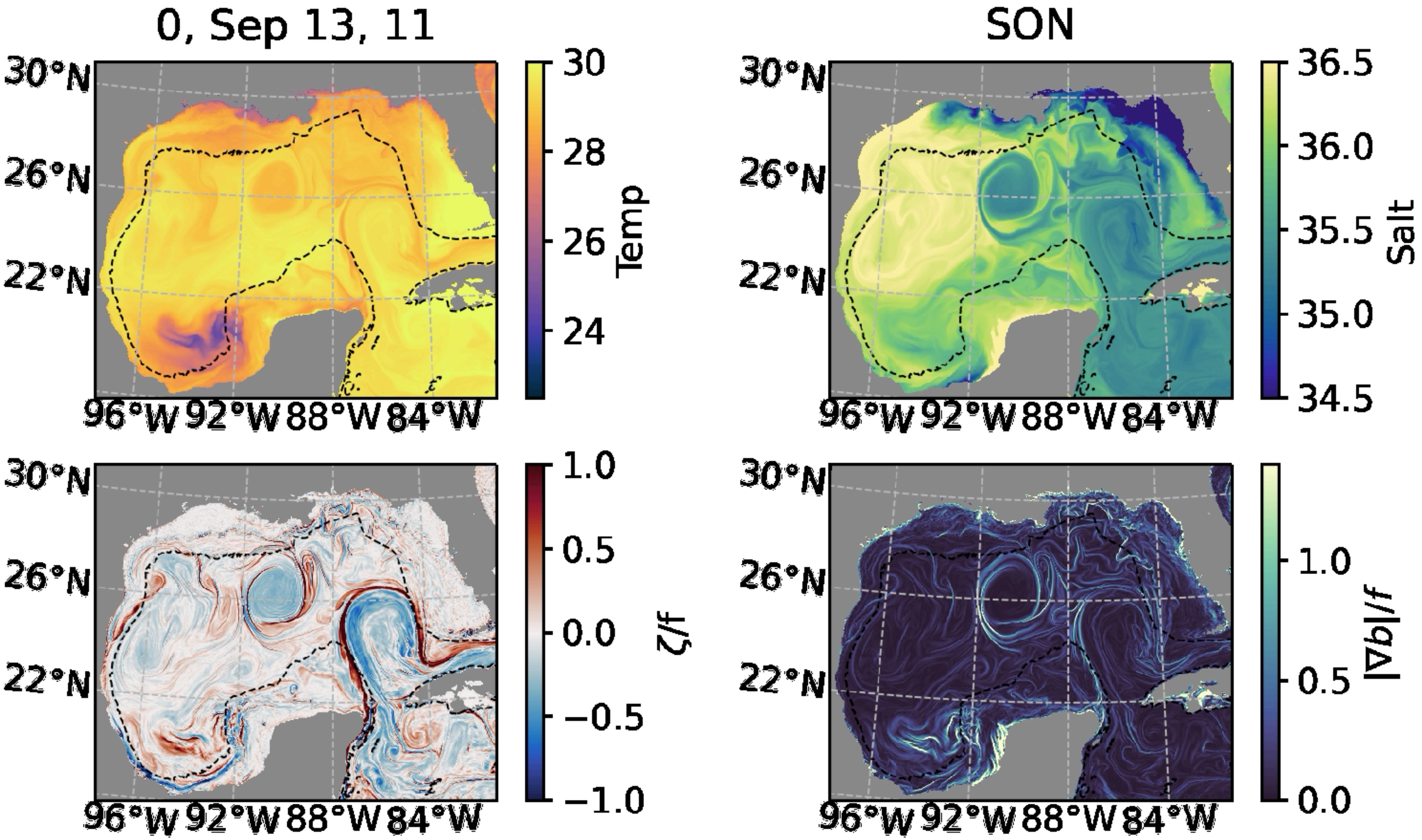
-Lewis F. Richardson, 1922

Overview

- Introduction
- Two-point statistics and relationship to spectral properties
- The GLAD and LASER drifter experiments
  - 2nd order statistics and decompositions
  - 3rd order statistics
- Final summary

References of recent CARTHE drifter based studies:
Poje et al 2014 (PNAS), Beron-Vera & LaCasce 2016 (JPO), Balwada et al 2016 (GRL), Novelli et al 2017 (JTech), Poje et al 2017 (POF), D'Asaro et al 2018 (PNAS), Haza et al 2018 (JTech), Huntley et al 2019 (JGRO), Pearson et al 2019 (JPO), Berta et al 2020 (JGRO), Pearson et al 2020 (JGRO), Wang & Buhler 2021 (JPO)

Surface flow in the Gulf of Mexico

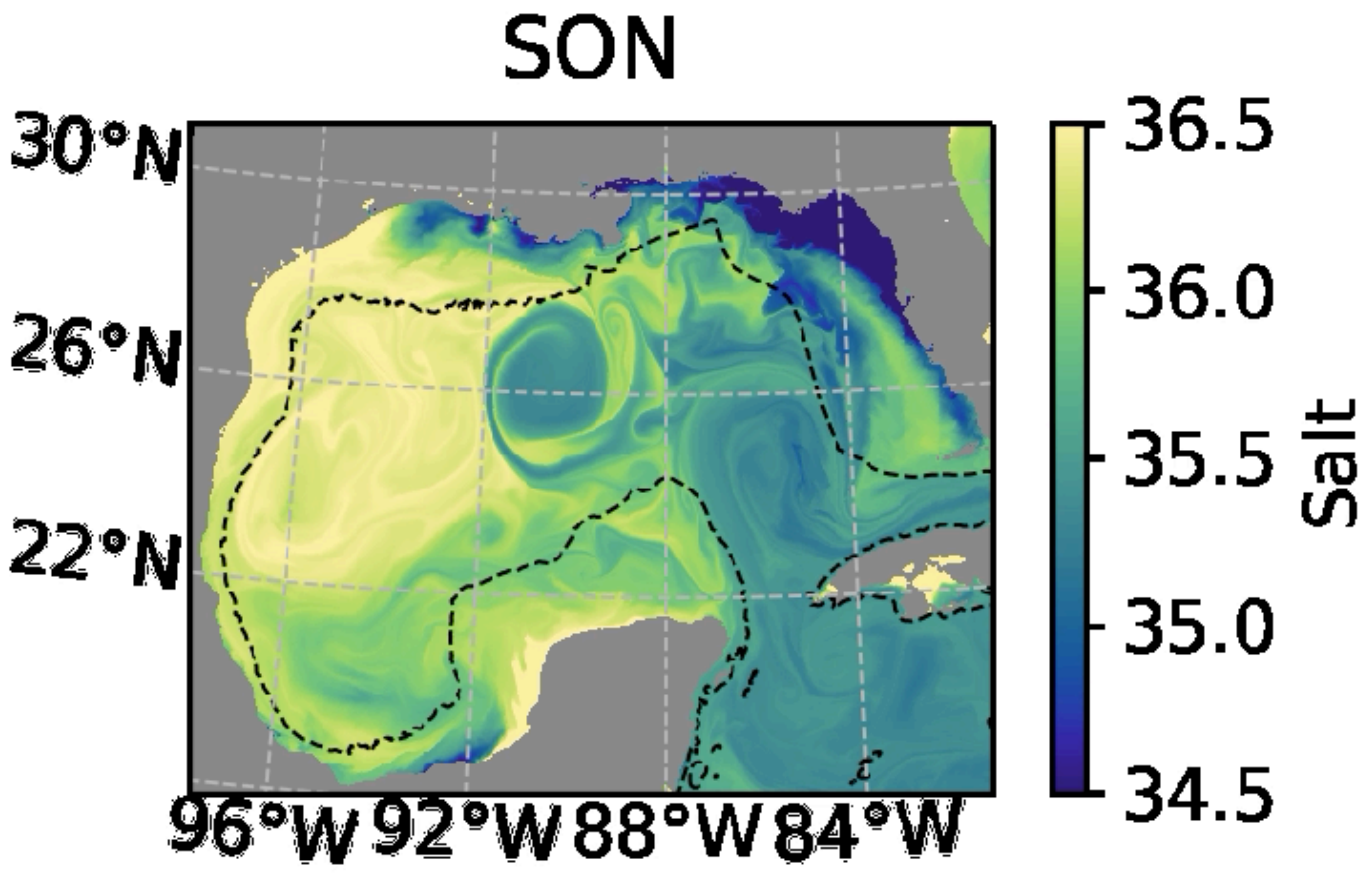


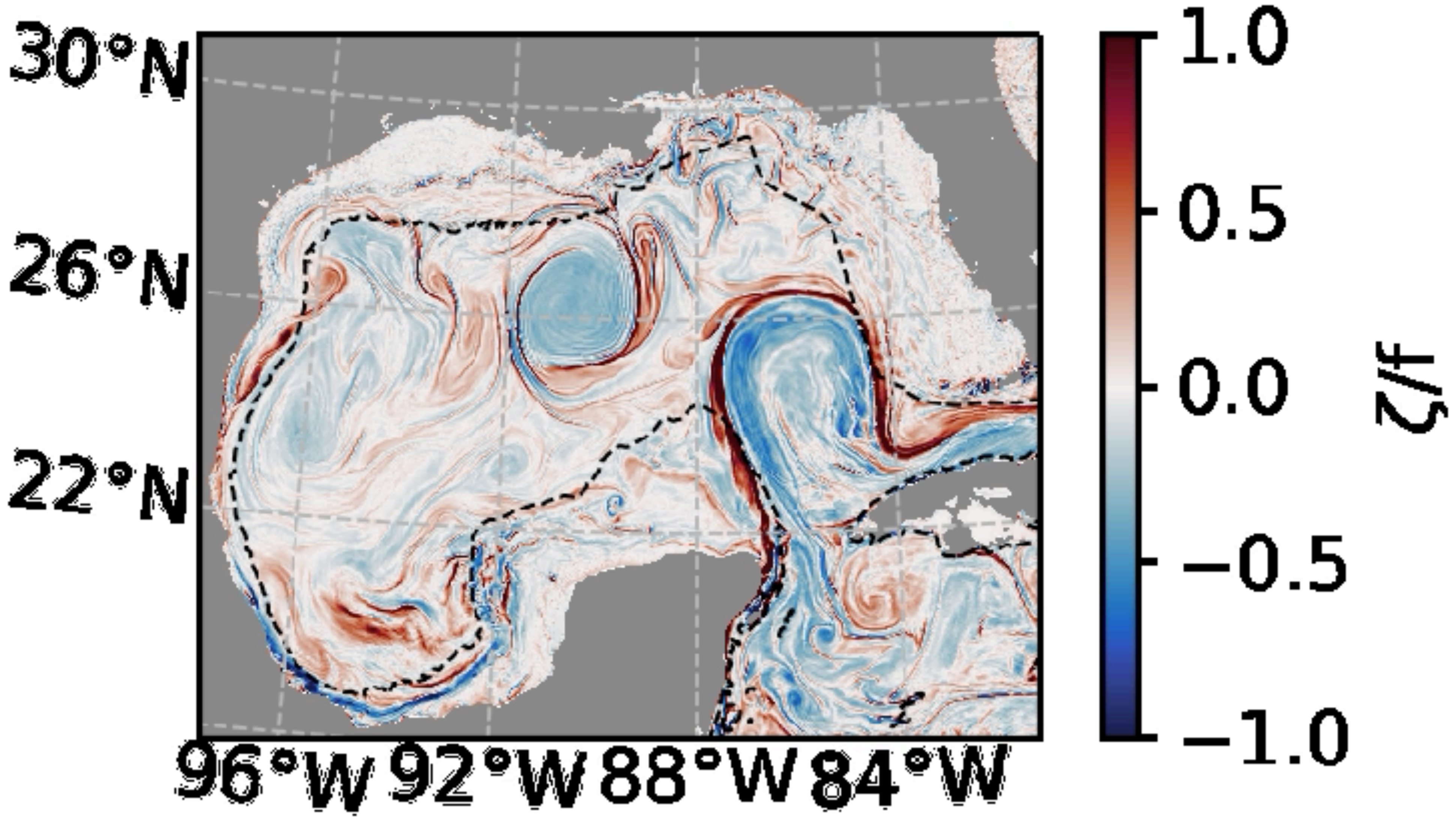
LLC4320:
1/48 degree MITgcm global simulation. Forced with 6hourly ECMWF winds, bulk formulae for surface fluxes, tides.

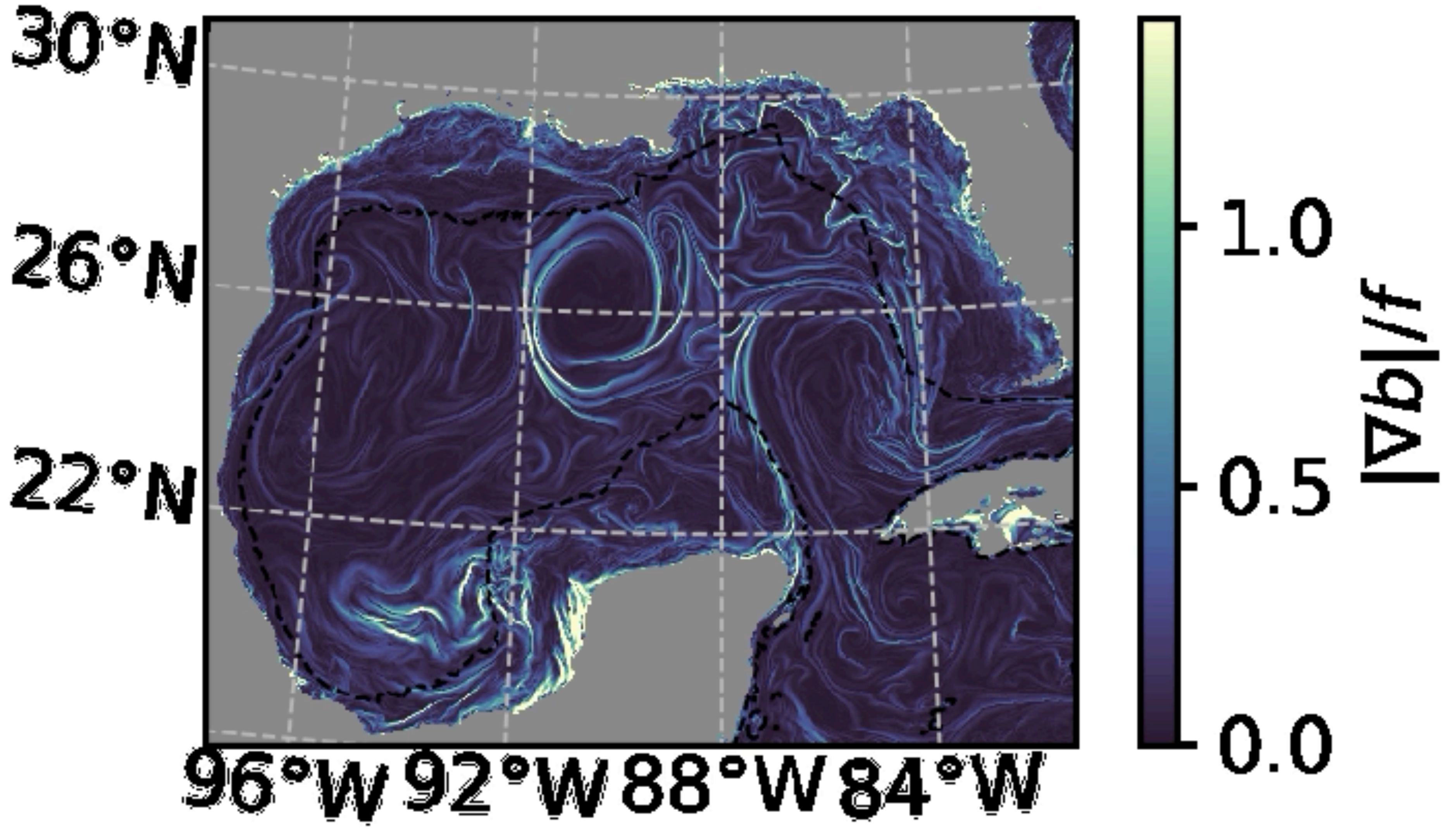
Acknowledgements:
Dimitris Menemenlis for simulation.
Ryan Abernathey for accessibility.

Surface flow in the Gulf of Mexico

Some relevant questions:
- What are the flow dynamics/ dominant physical balances?
  - Storms, rivers, topography, seasonality, resolution
- *How are tracers stirred and transported by the flow?*
- What are the characteristics of the individual flow structures: eddies vs fronts?
- **How is kinetic energy distributed across scales, is the flow ‘smooth’ or ‘rough’? and why?**







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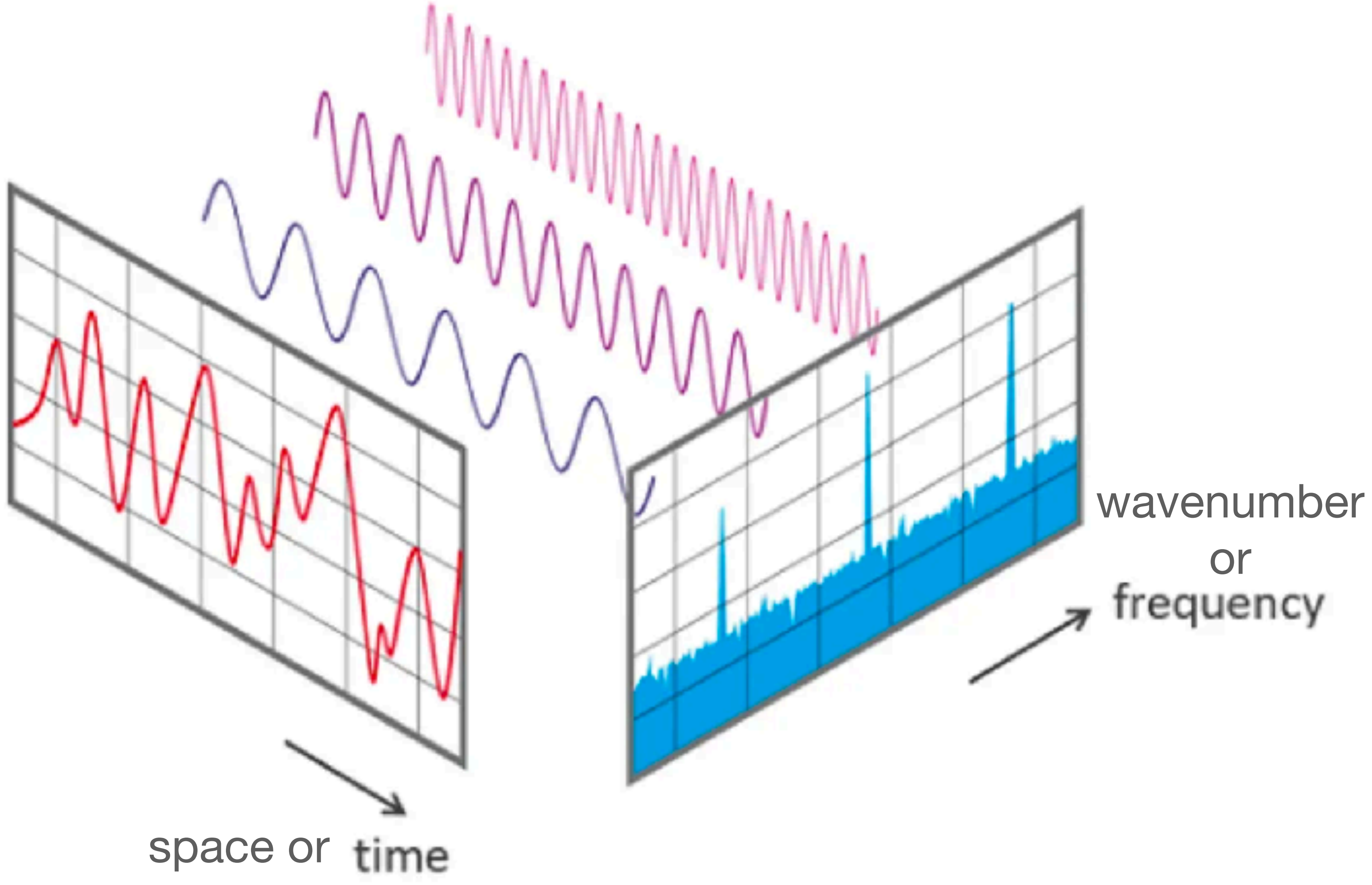
2 ways to quantify the structures

***Assume** homogeneity and isotropy of statistics at zeroth order for this talk.*

Spectral space

Fourier Transform: $u(x) = \int \hat{u}(k) e^{ikx} dk$



Energy or variance can be decomposed as a function of wavelength.

$$E = \int \widehat{E}(k) dk$$

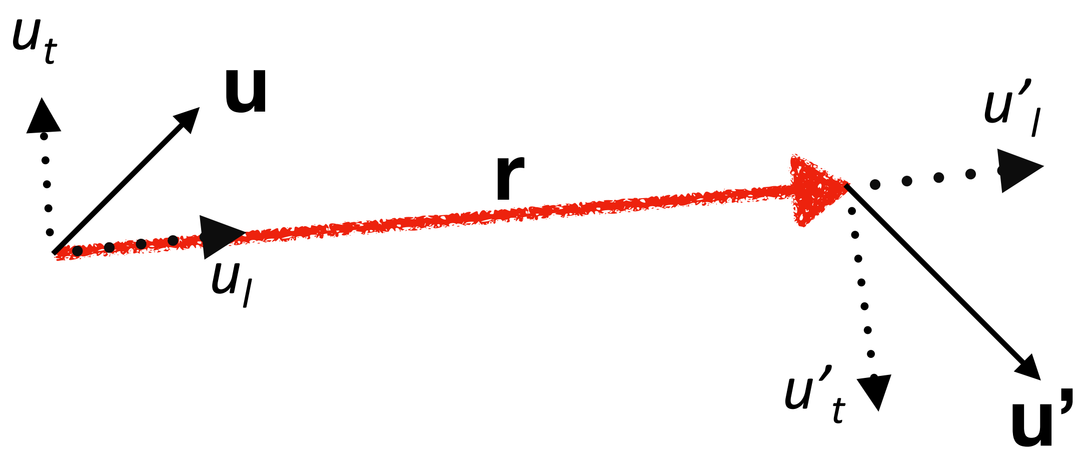
$$E = \frac{\overline{u^2}}{2}$$

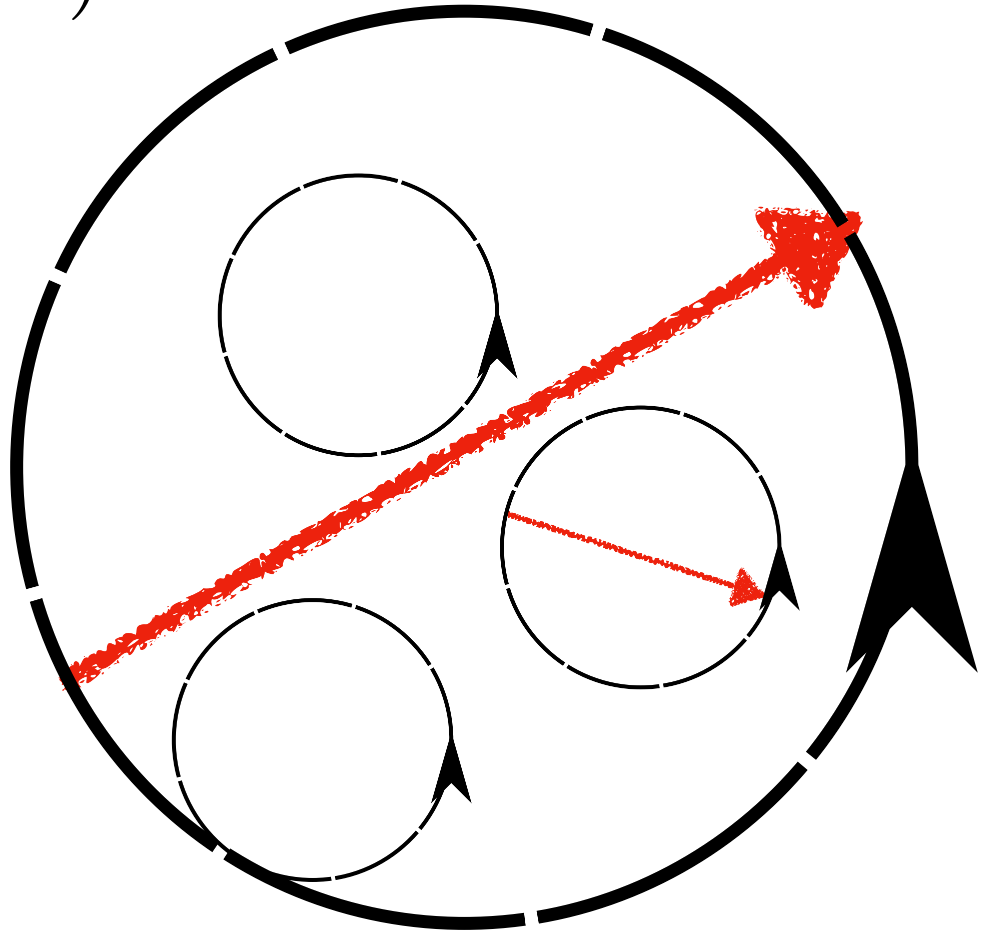
$$\hat{E}(k) = \frac{\hat{u}^2}{2}$$

Real space

Difference: $\delta \mathbf{u}(\mathbf{r}) = \mathbf{u}'(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$

Covariance: $\mathbf{C}(\mathbf{r}) = \mathbf{u}(\mathbf{x})\mathbf{u}'(\mathbf{x}+\mathbf{r})$





Second order structure function (SF_2) has a "meaningful" scale dependence.

$$SF_2(r) = \overline{\delta u(r)^2} = \overline{u^2} + \overline{u'^2} - 2\overline{uu'}(r)$$

$$= 4E - 2C(r)$$

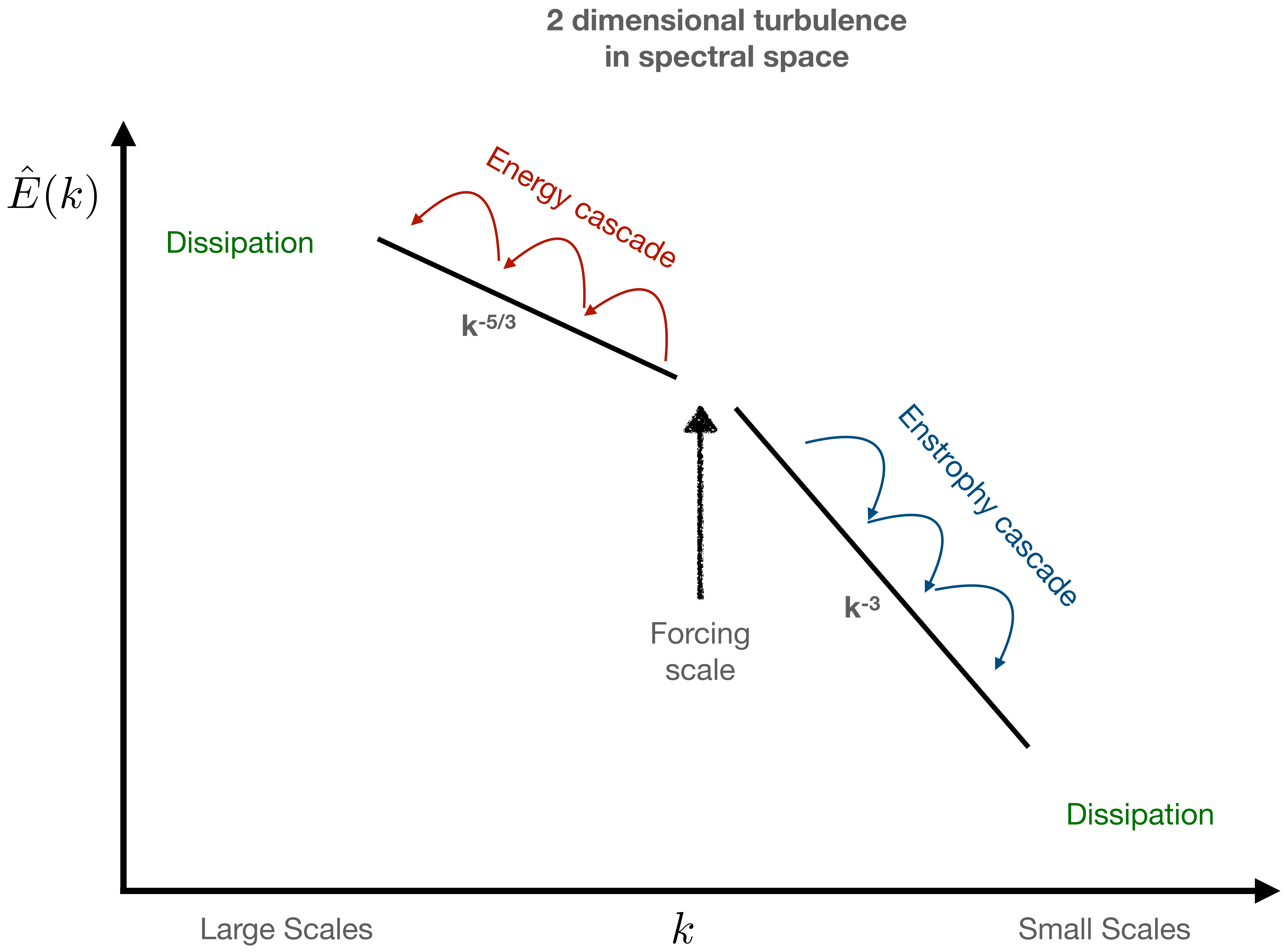
$$C(0) = 2E$$

$$= 0 \text{ as } r \to 0$$

$$= 4E \text{ as } r \to \infty$$

**Formulae are only representative. Exact forms can be found in Davidson 2015 or Frisch 1995.*

Inertial ranges



Transformations

Spectral space $\xleftrightarrow[\textbf{Theorem}]{\textbf{Weiner-Khinchin}}$ **Real space**

$$C(r) = 2\int_{-\infty}^{\infty} \hat{E}(k)e^{ikr}dk$$

Covariance is related to the power spectrum through a Fourier transform.

$$SF_2(r) = 2\int_{0}^{\infty} \hat{E}(k)(1 - J_0(kr))dk$$

Expectations for infinite power laws or long inertial ranges:

$$\hat{E}(k) \sim k^{-\alpha}$$

$$SF_2(r) \sim r^{\alpha-1} \qquad 1 \leq \alpha \leq 3$$

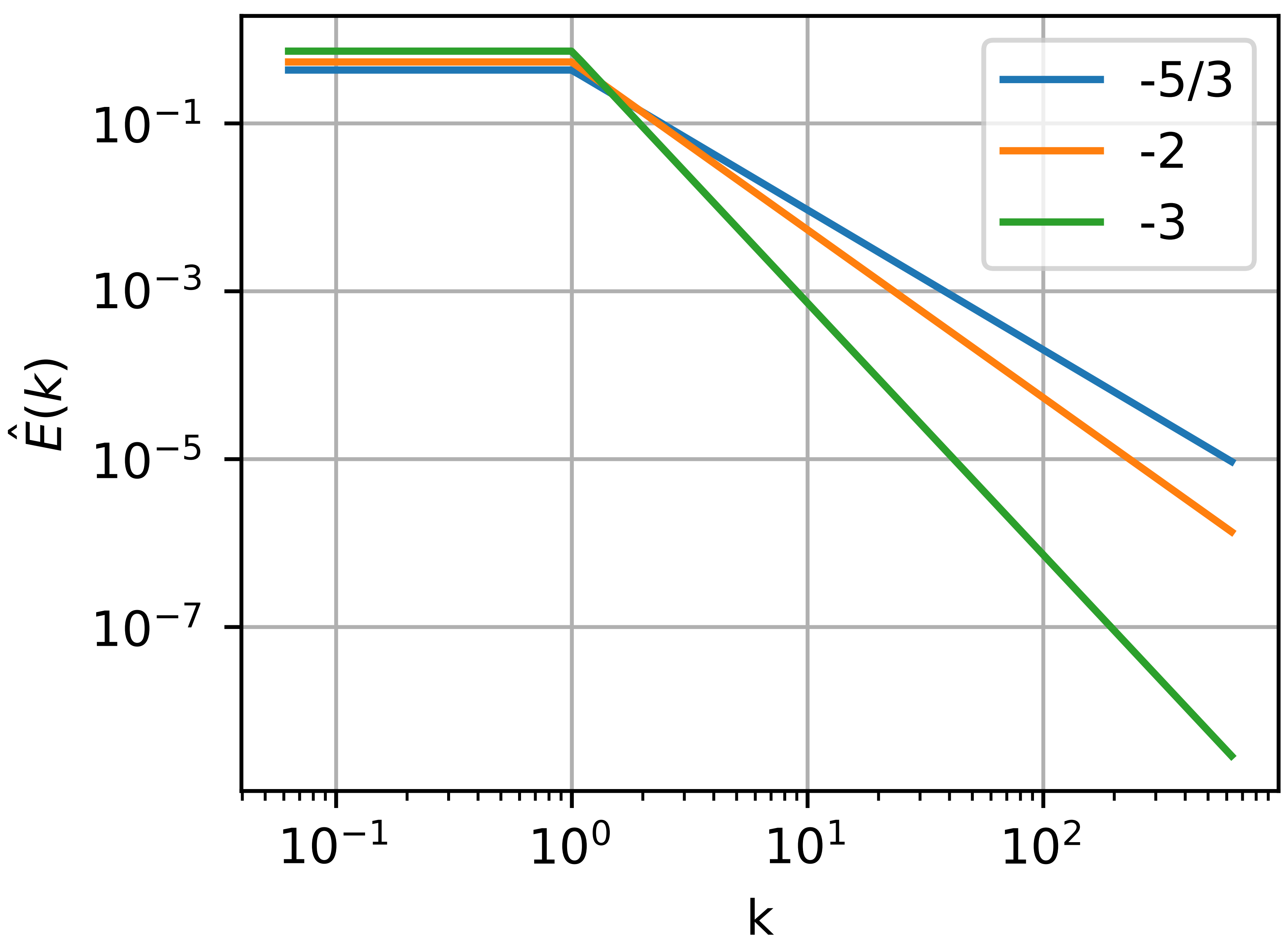
$$\sim r^2 \qquad \alpha > 3$$

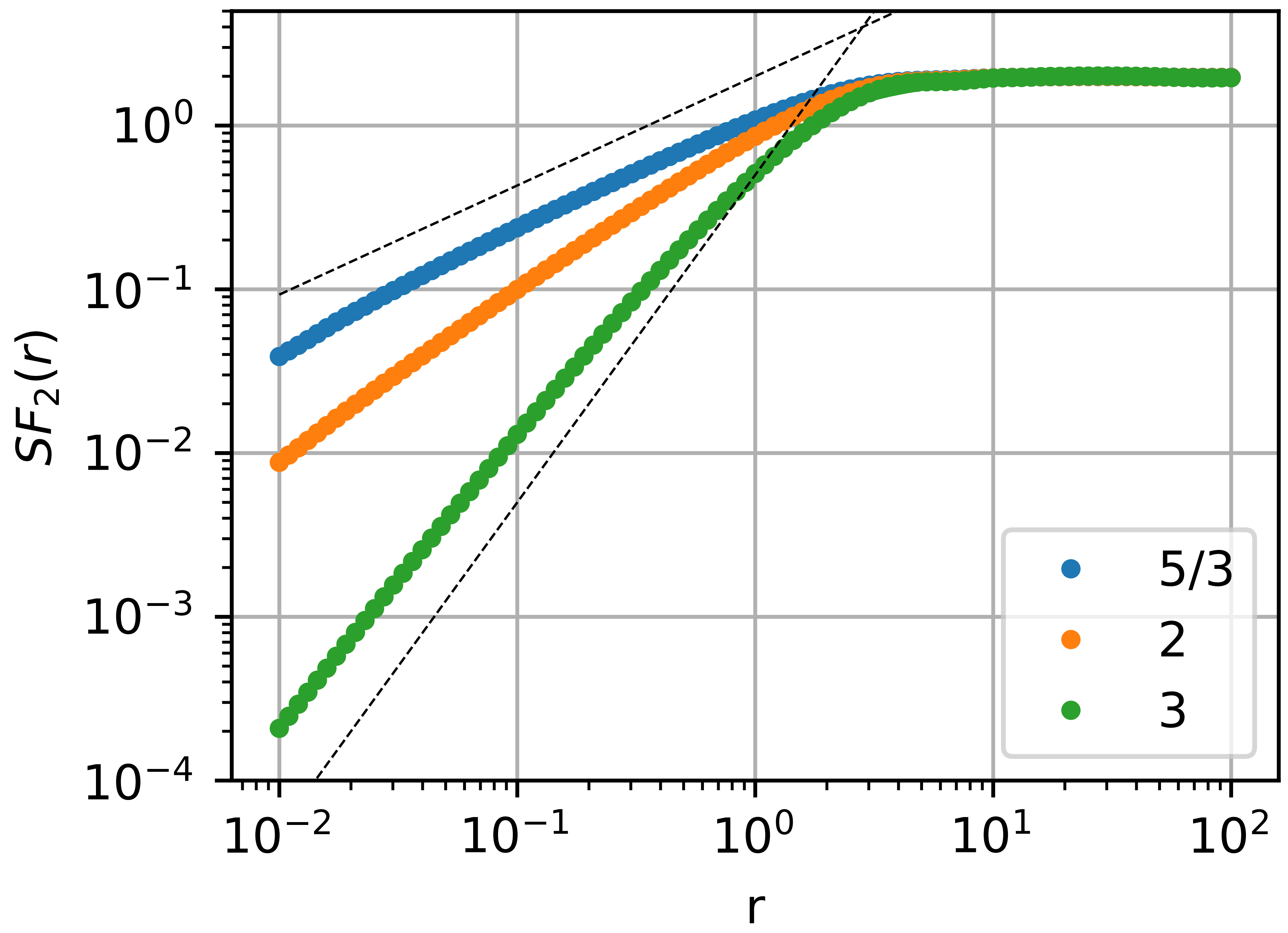
$$k^{-3} \rightarrow r^2 \ ; \ k^{-2} \rightarrow r^1 \ ; \ k^{-5/3} \rightarrow r^{2/3}$$

Transformations

What are 2nd order structure functions measuring?

$$SF_2(r) = 2\int_0^\infty \hat{E}(k)(1 - J_0(kr))dk$$



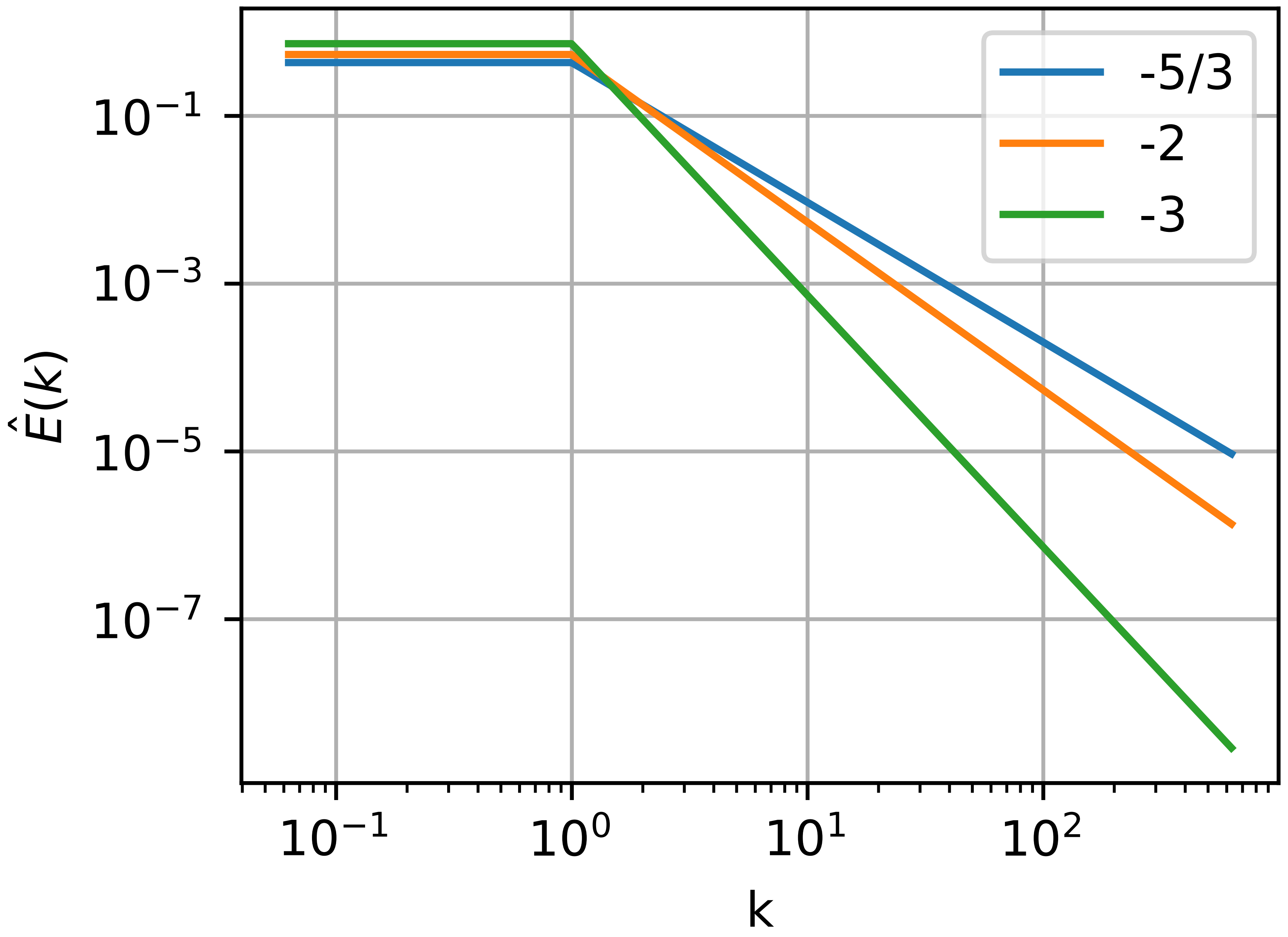


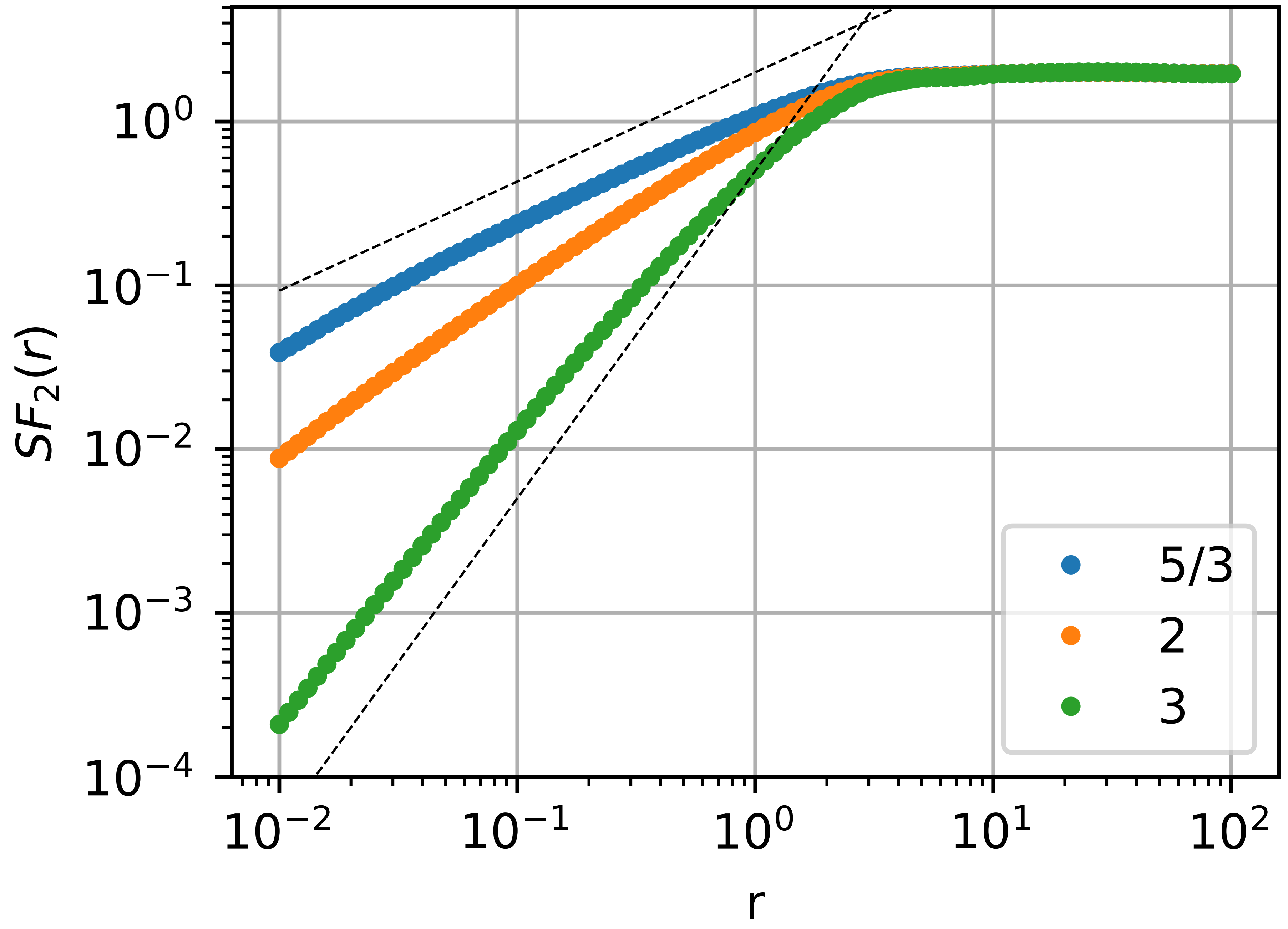
Transformations

What are 2nd order structure functions measuring?

$$SF_2(r) = 2\int_0^\infty \hat{E}(k)(1 - J_0(kr))dk$$
$$\approx \frac{r^2}{2}\int_0^{2/r} k^2\hat{E}(k)dk + 2\int_{2/r}^\infty \hat{E}(k)dk$$

Cumulative enstrophy from largest scales to scales close to r. + **Cumulative energy** from smallest scales to scales close to r.





Transformations

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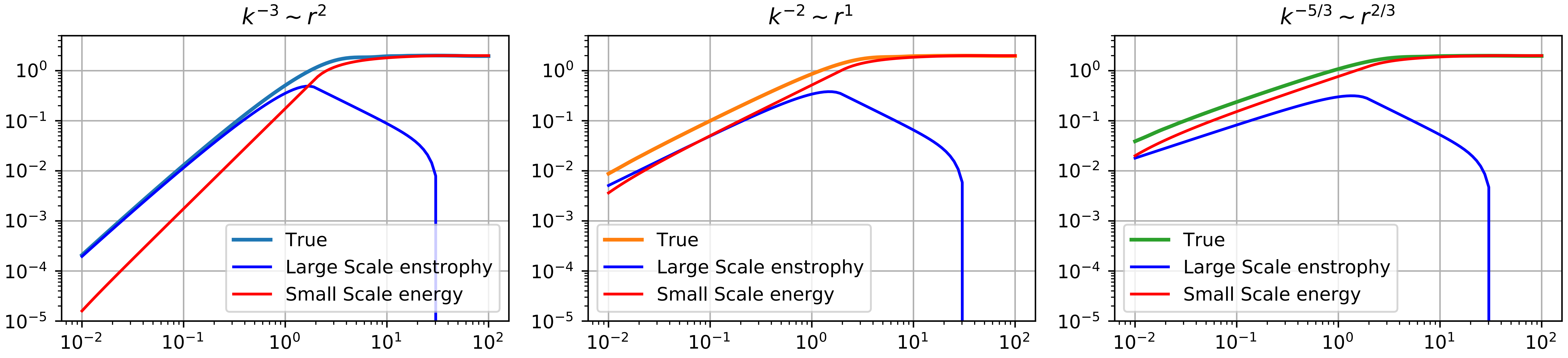


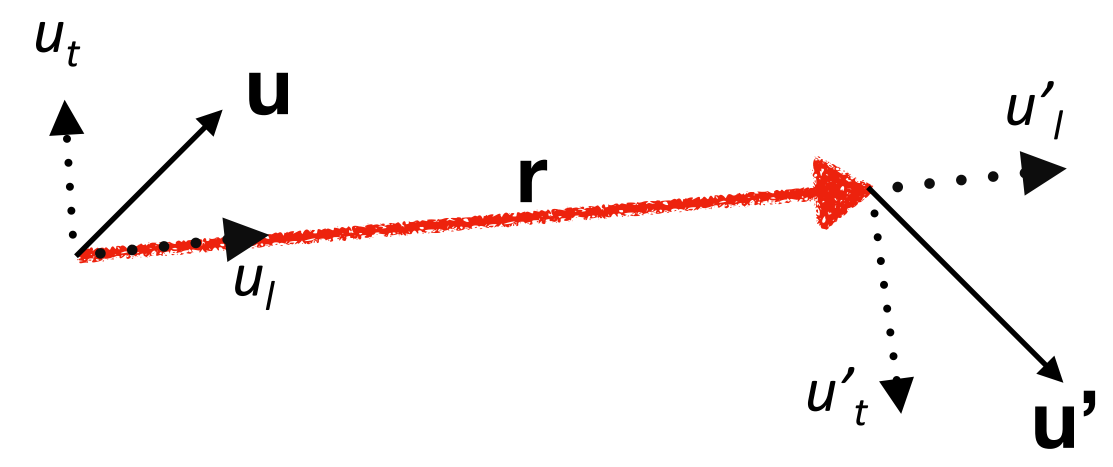
Primarily a signal from enstrophy

Primarily a signal from energy

Decompositions

Helmholtz decomposition

$$\mathbf{u} = -\nabla\phi + \nabla \times \psi$$
$$= \mathbf{u}_{div} + \mathbf{u}_{rot}$$



$$(SF_{2l}, SF_{2t}) \rightarrow (SF_{rot}, SF_{div})$$

Buhler et al 2014, Lindborg 2015

Note: (a) This a decomposition of the statistics of the flow, and not the flow itself.
(b) Waves usually have a divergent part > rotational part.

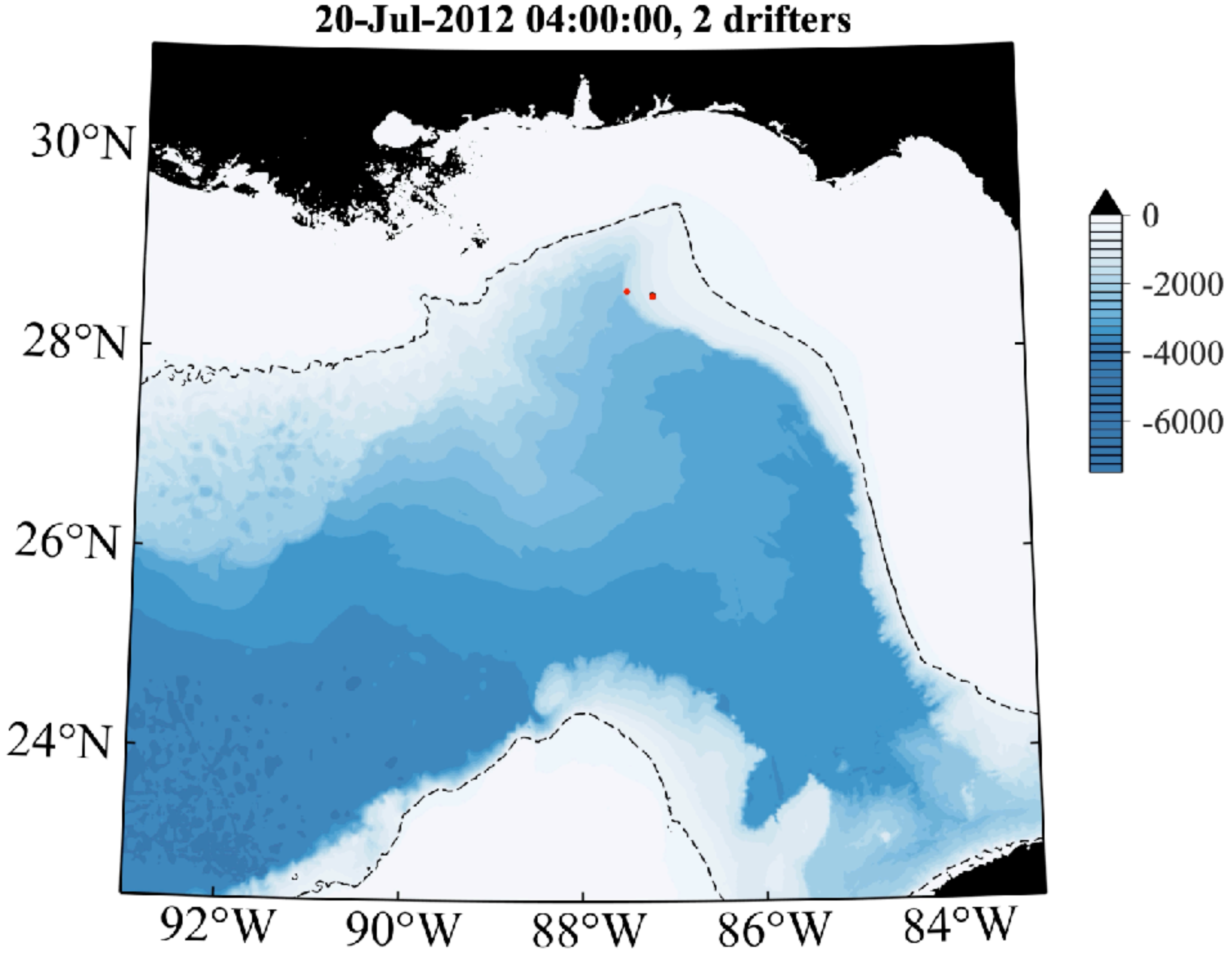
Wave-vortex decomposition

$$\mathbf{u} = \mathbf{u}_{vortex} + \mathbf{u}_{wave}$$

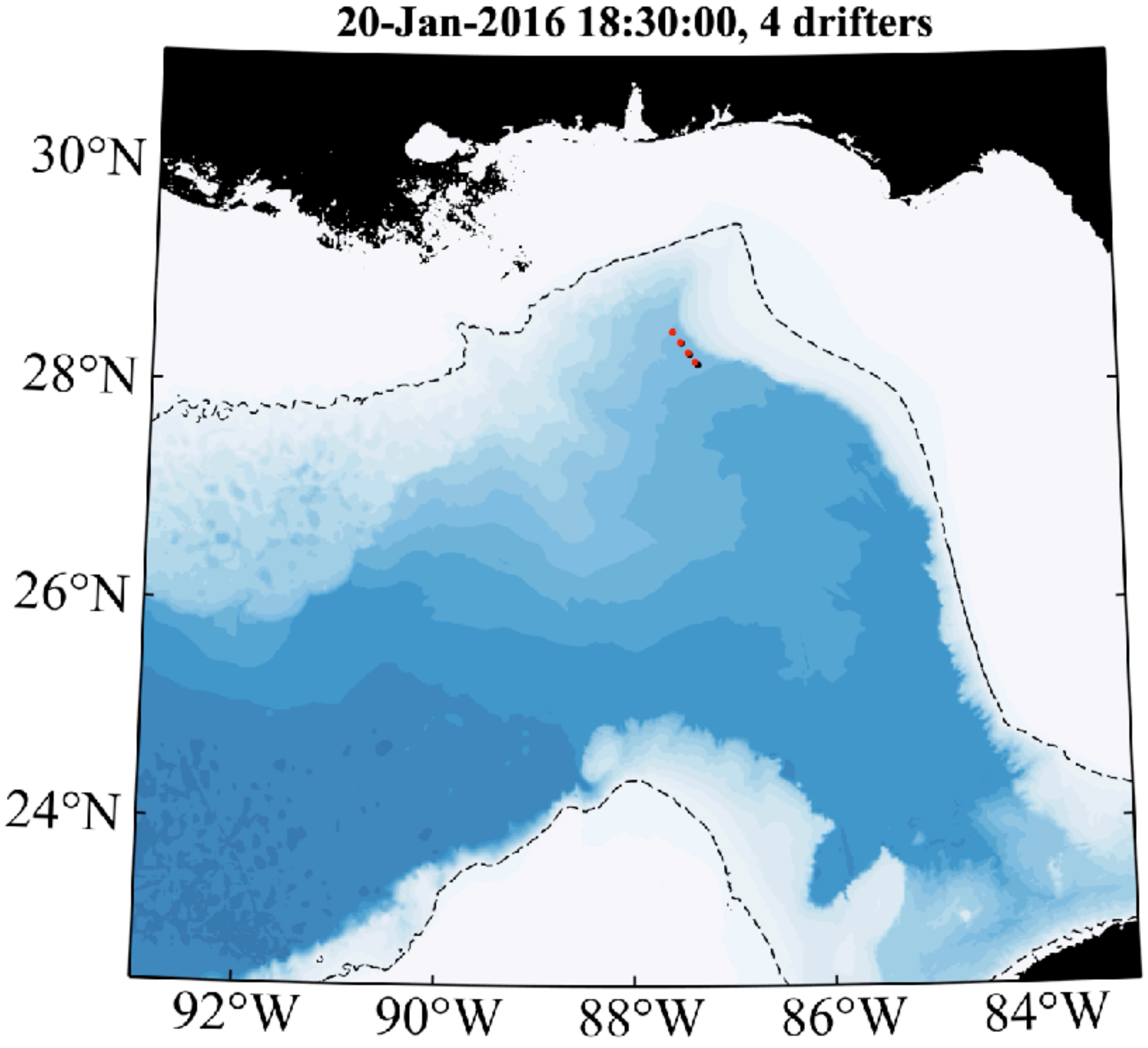
$$(SF_{2l}, SF_{2t}, SF_{2buoyancy}) \rightarrow (SF_{2wave}, SF_{2vortex})$$

Buhler et al 2014

Grand Lagrangian Deployment (GLAD)
Summer 2012



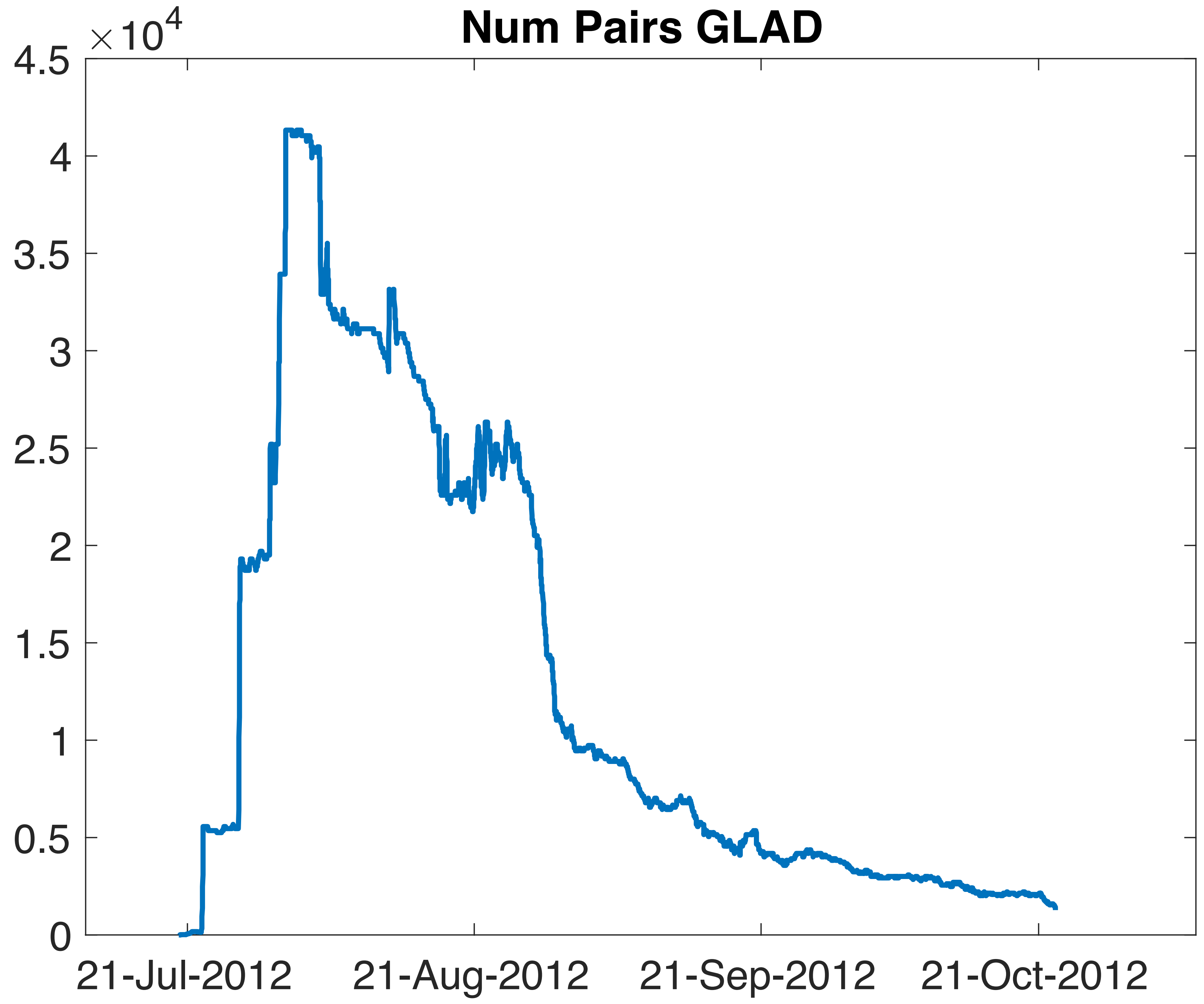
LAgrangian Submesoscale ExpeRiment (LASER)
Winter 2016

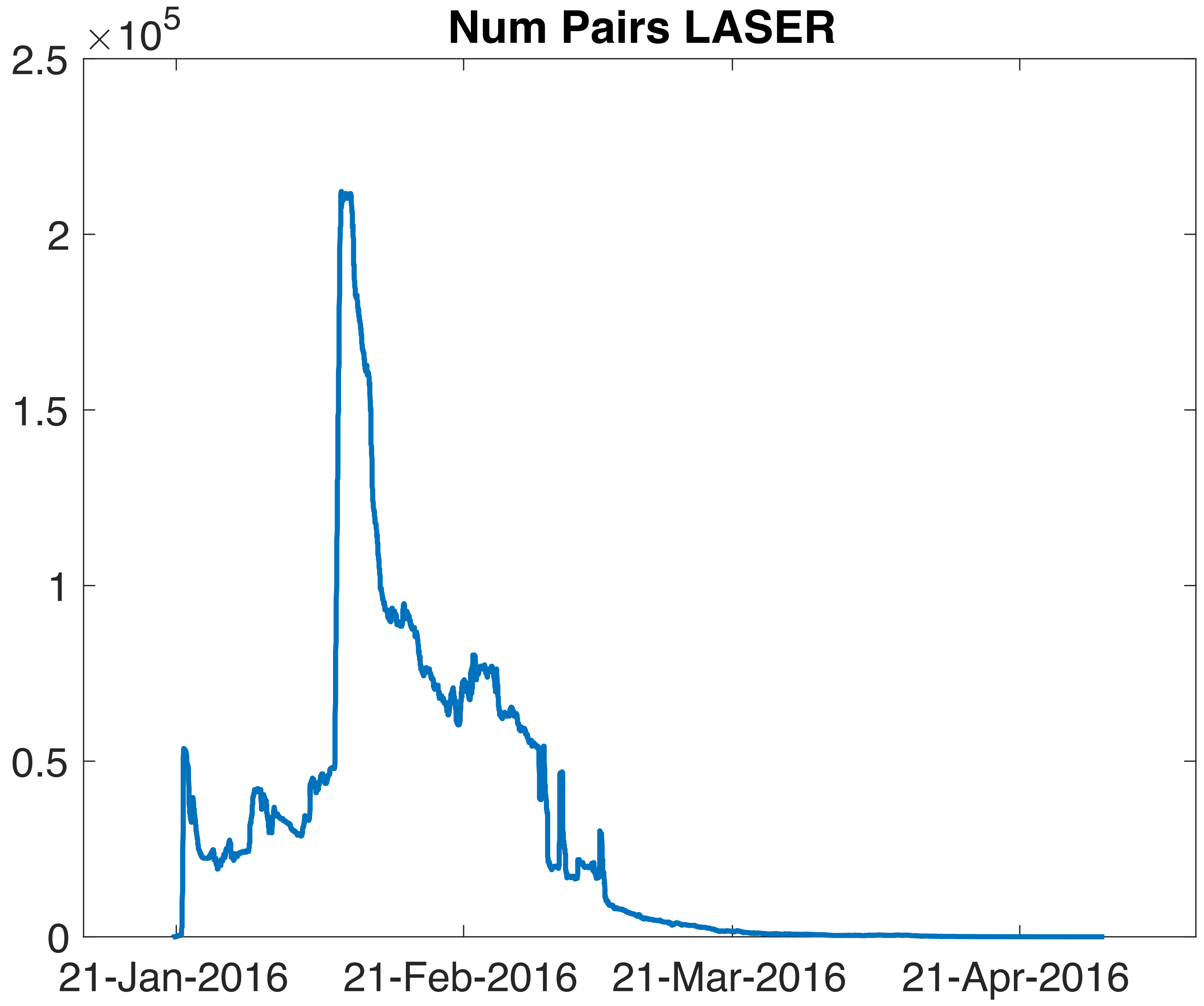


Thanks to: **Consortium for Advanced Research on Transport of Hydrocarbon in the Environment**

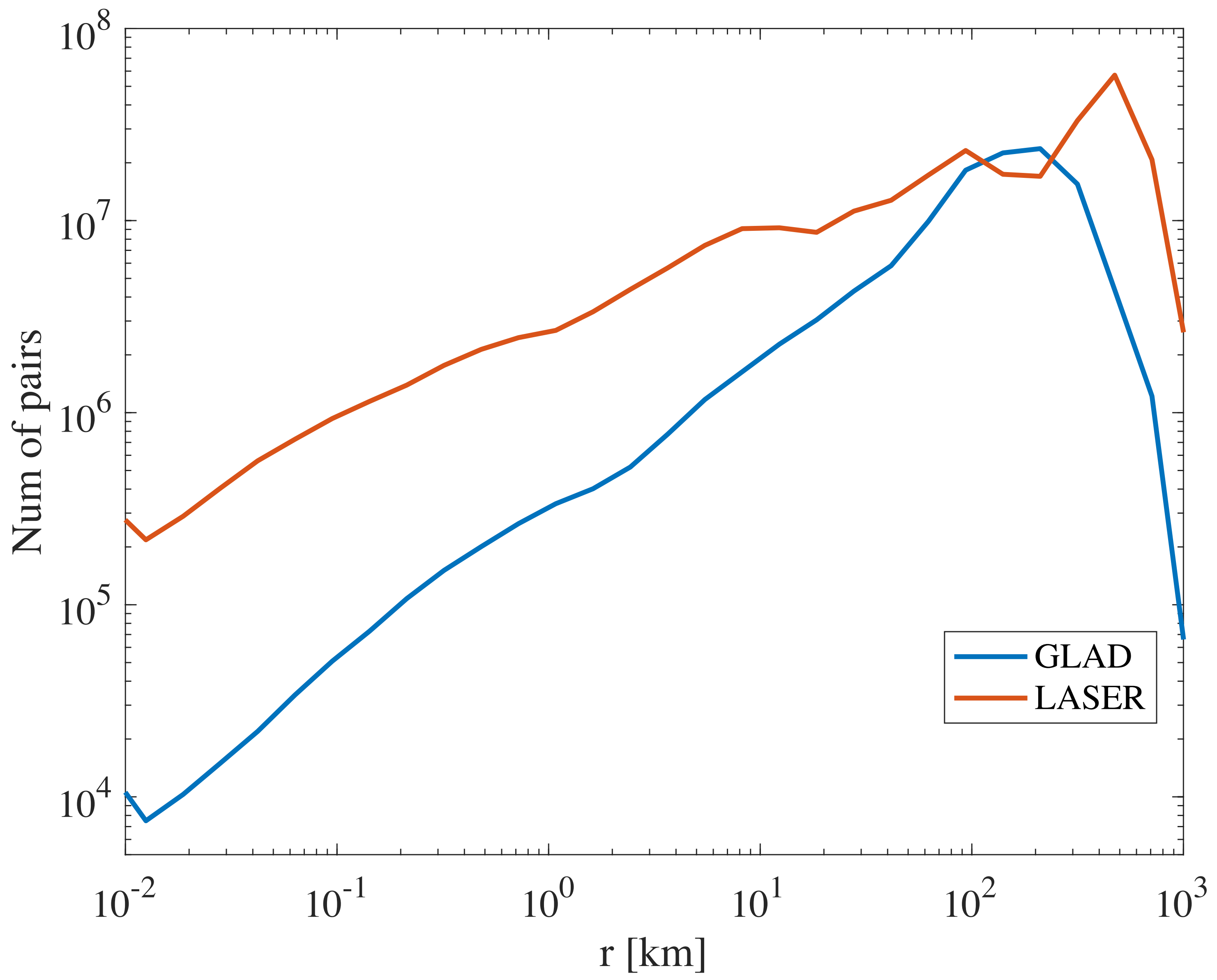
CARTHE

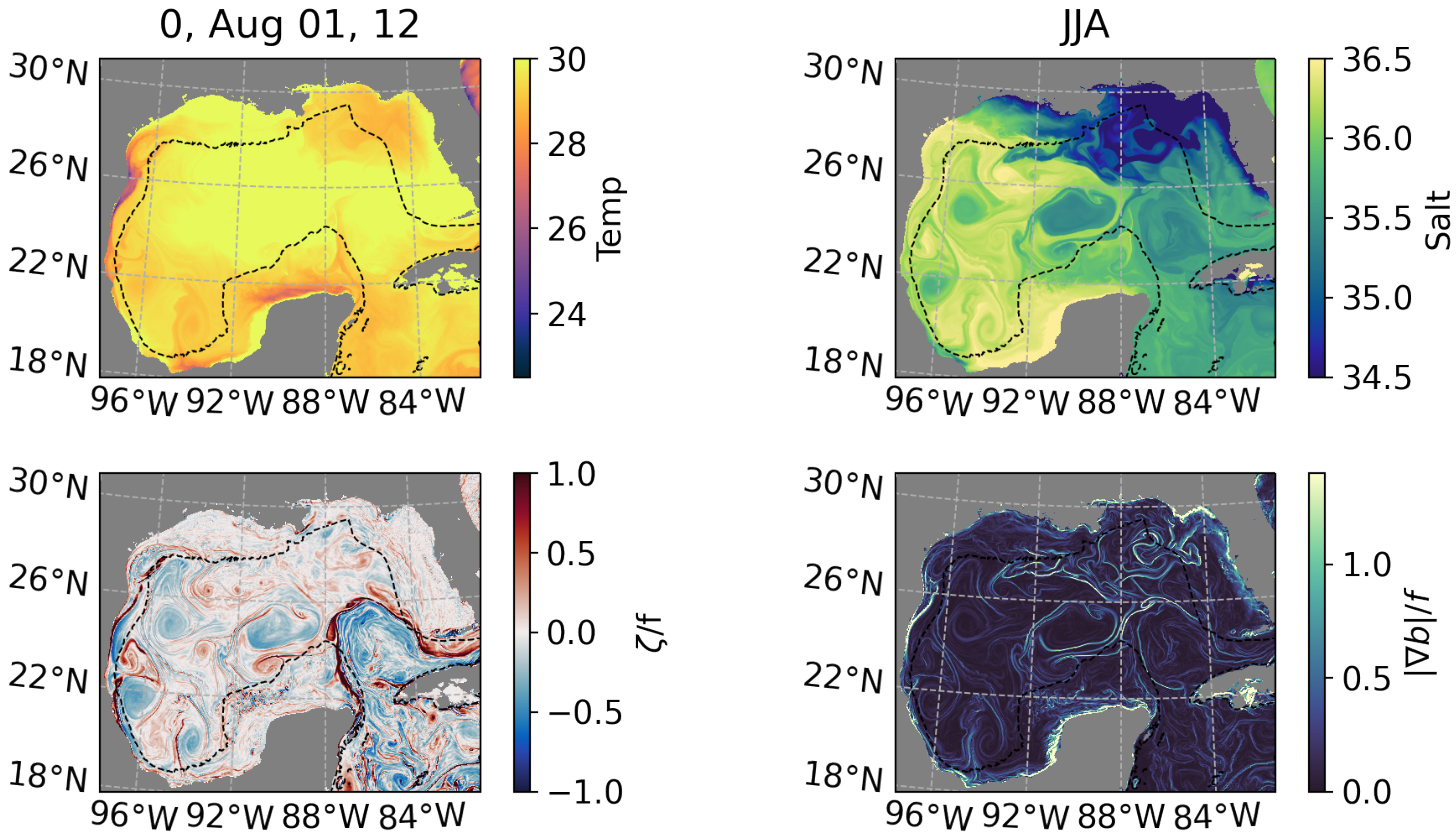
Grand Lagrangian Deployment (GLAD)
Summer 2012
LAgrangian Submesoscale ExpeRiment (LASER)
Winter 2016
30°N
27°N
24°N
21°N
18°N
96°W
92°W
Depth [m]
0
-2000
-4000
-6000
Cyclonic
Anticyclonic
ω^{-2}
$\hat{E}(\omega)$
ω/f
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{0}
10^{1}



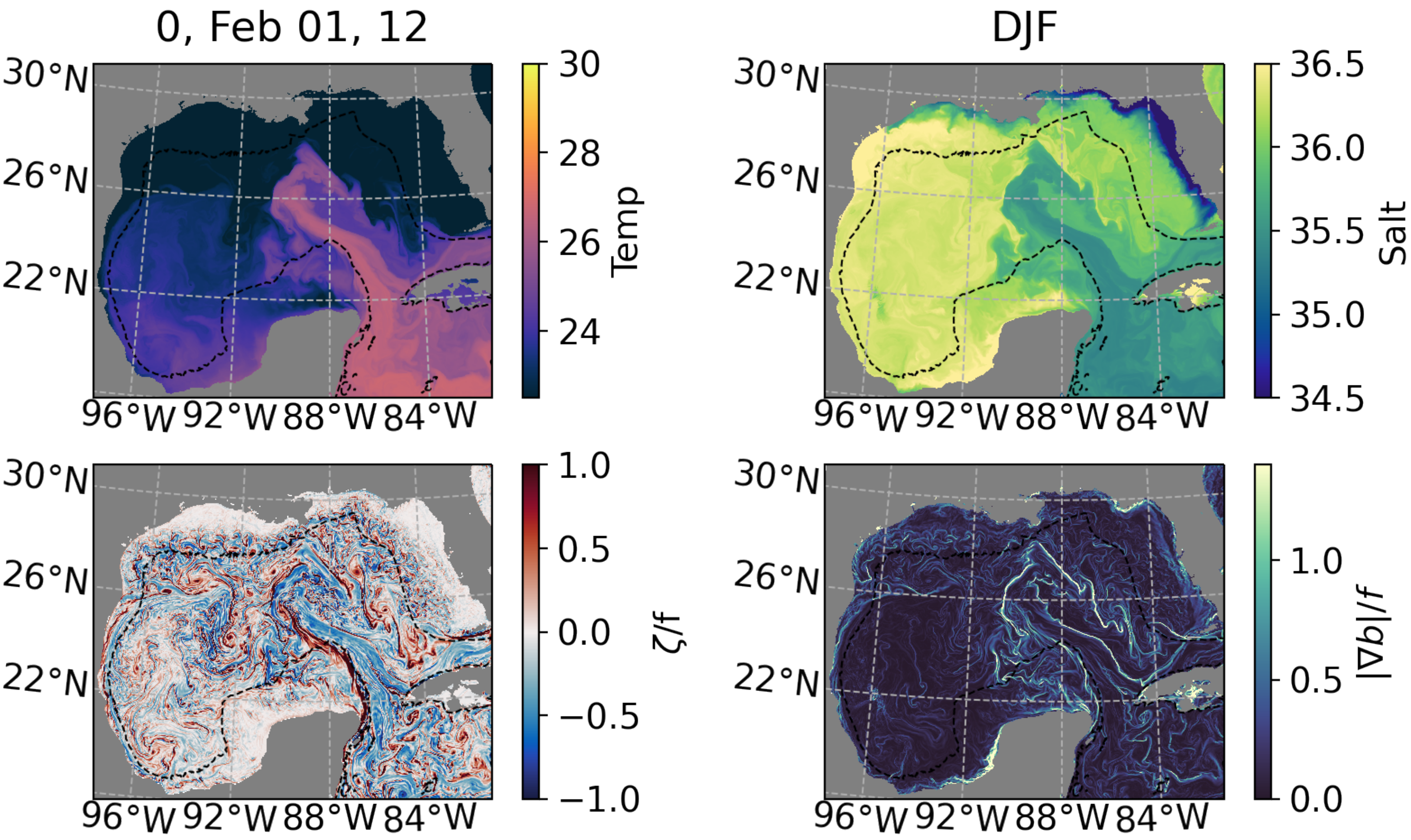


- There is a lot of data.
- We limit analysis to drifters that were in the same region and deeper than 500m.

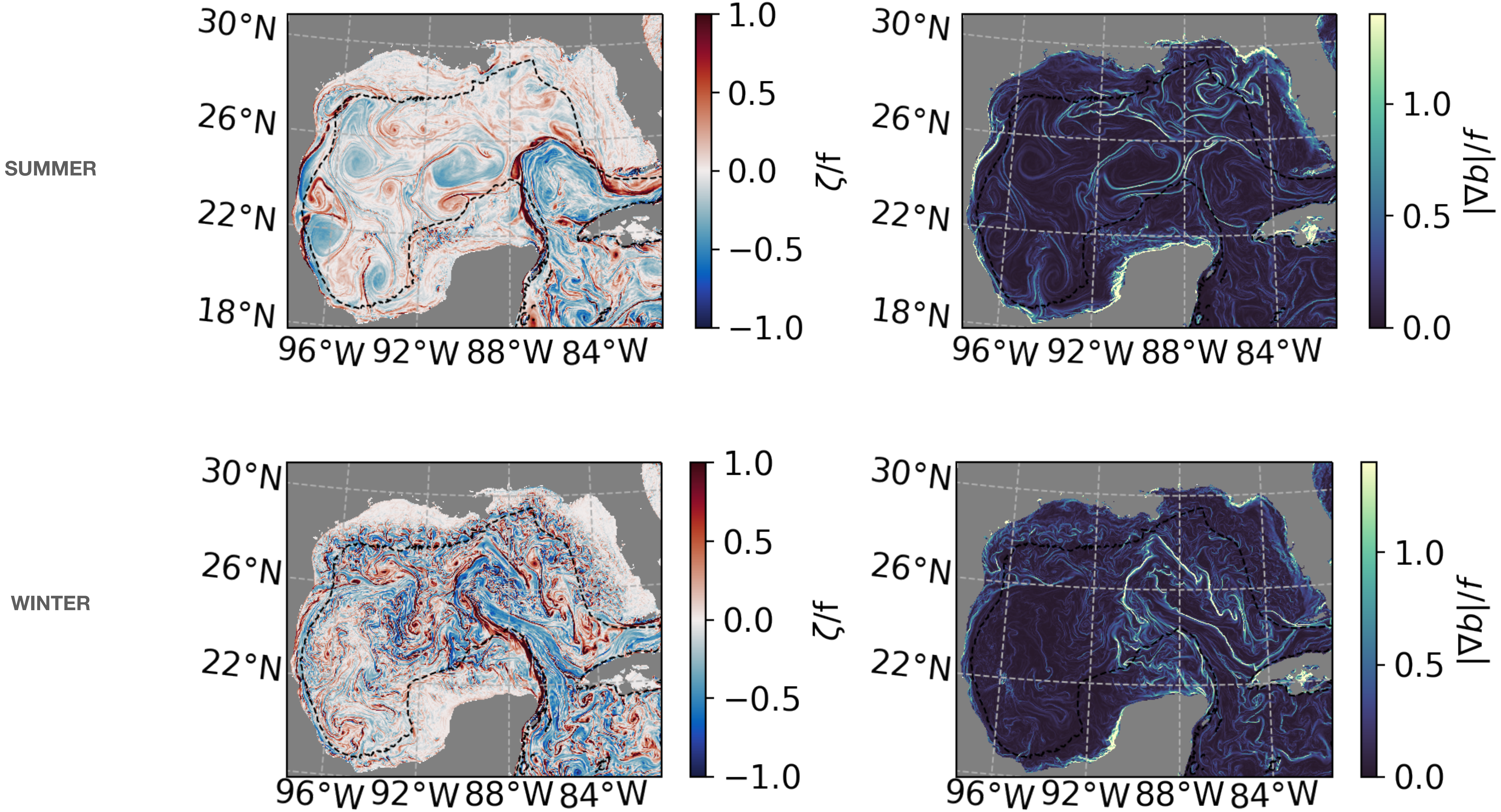




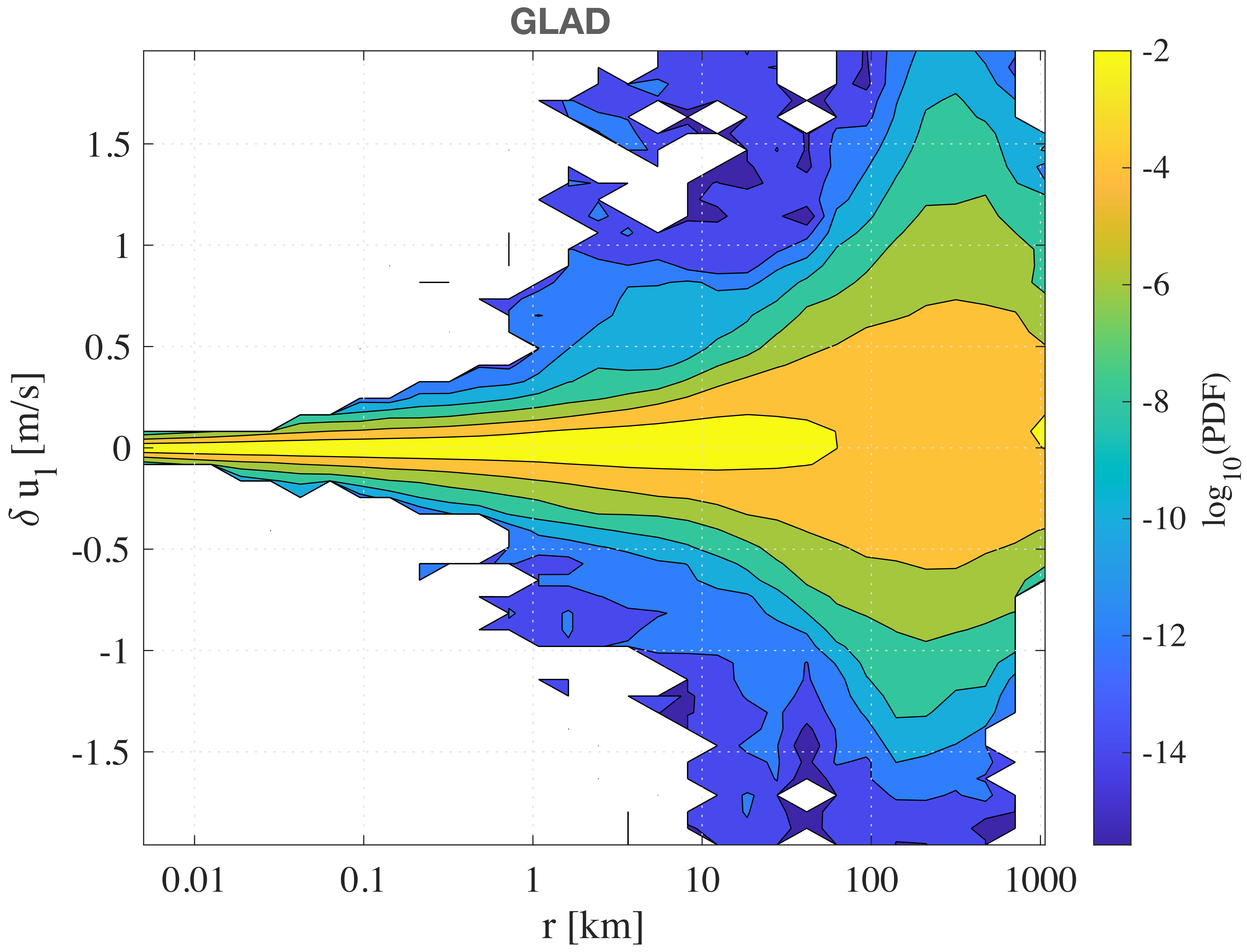
0, Aug 01, 12
JJA
30°N
26°N
22°N
18°N
96°W 92°W 88°W 84°W
30
28
26
24
Temp
36.5
36.0
35.5
35.0
34.5
Salt
1.0
0.5
0.0
−0.5
−1.0
ζ/f
1.0
0.5
0.0
$|\nabla b|/f$

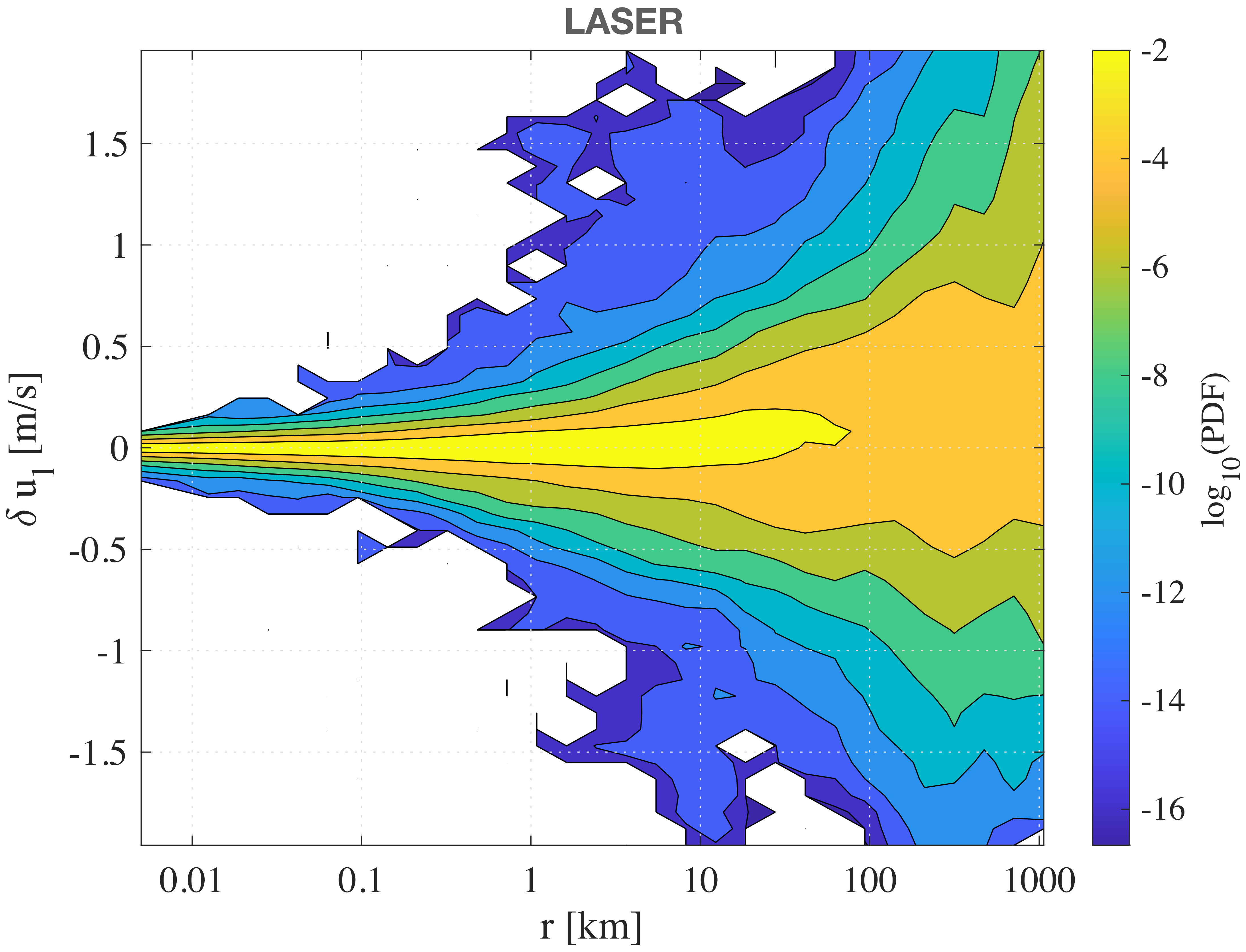


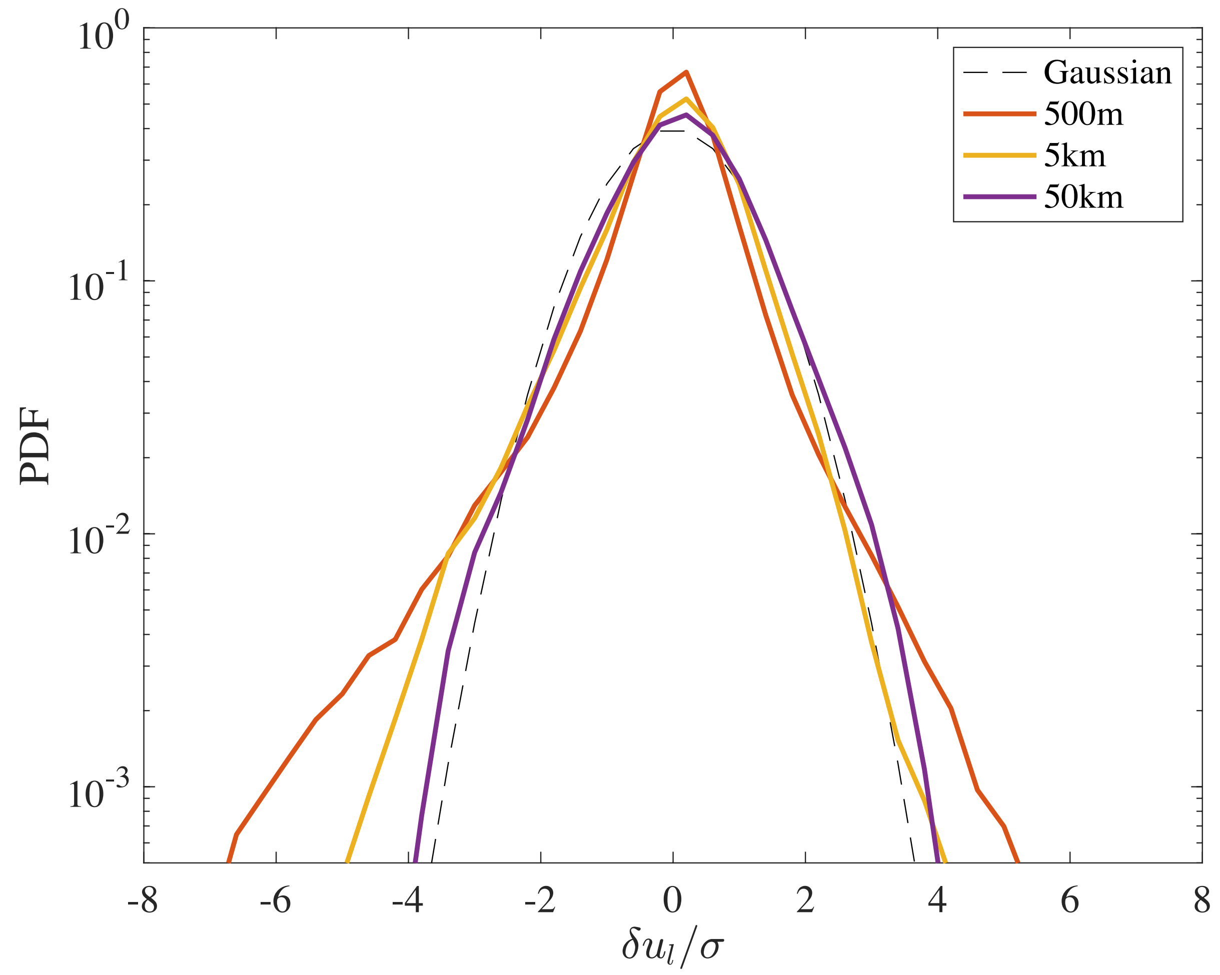
0, Feb 01, 12
DJF
Temp
Salt
ζ/f
|∇b|/f
30°N
26°N
22°N
96°W
92°W
88°W
84°W
30
28
26
24
36.5
36.0
35.5
35.0
34.5
1.0
0.5
0.0
−0.5
−1.0

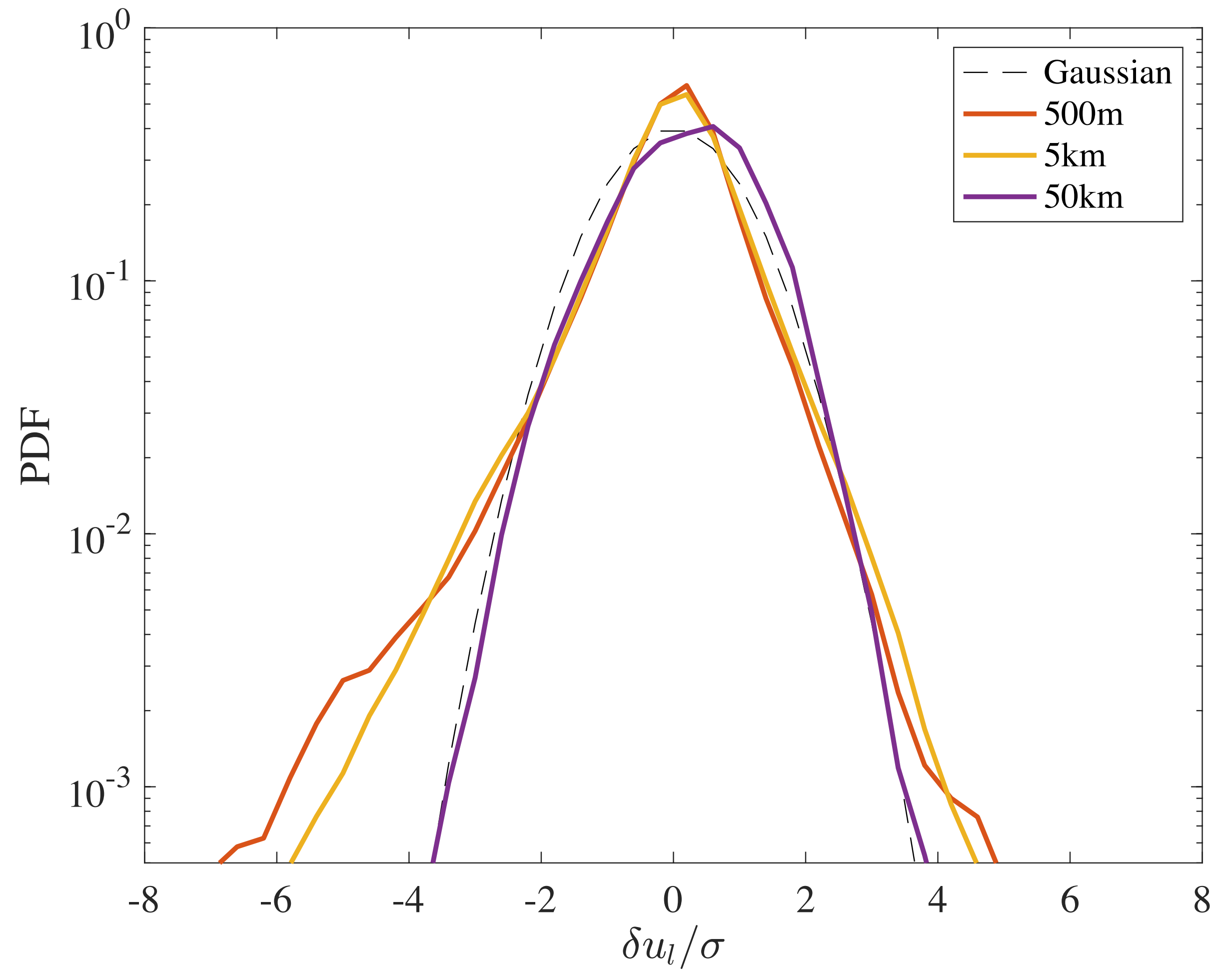

SUMMER
WINTER
30°N
26°N
22°N
18°N
96°W
92°W
88°W
84°W
ζ/f
1.0
0.5
0.0
−0.5
−1.0
$|\nabla b|/f$
1.0
0.5
0.0

Distributions



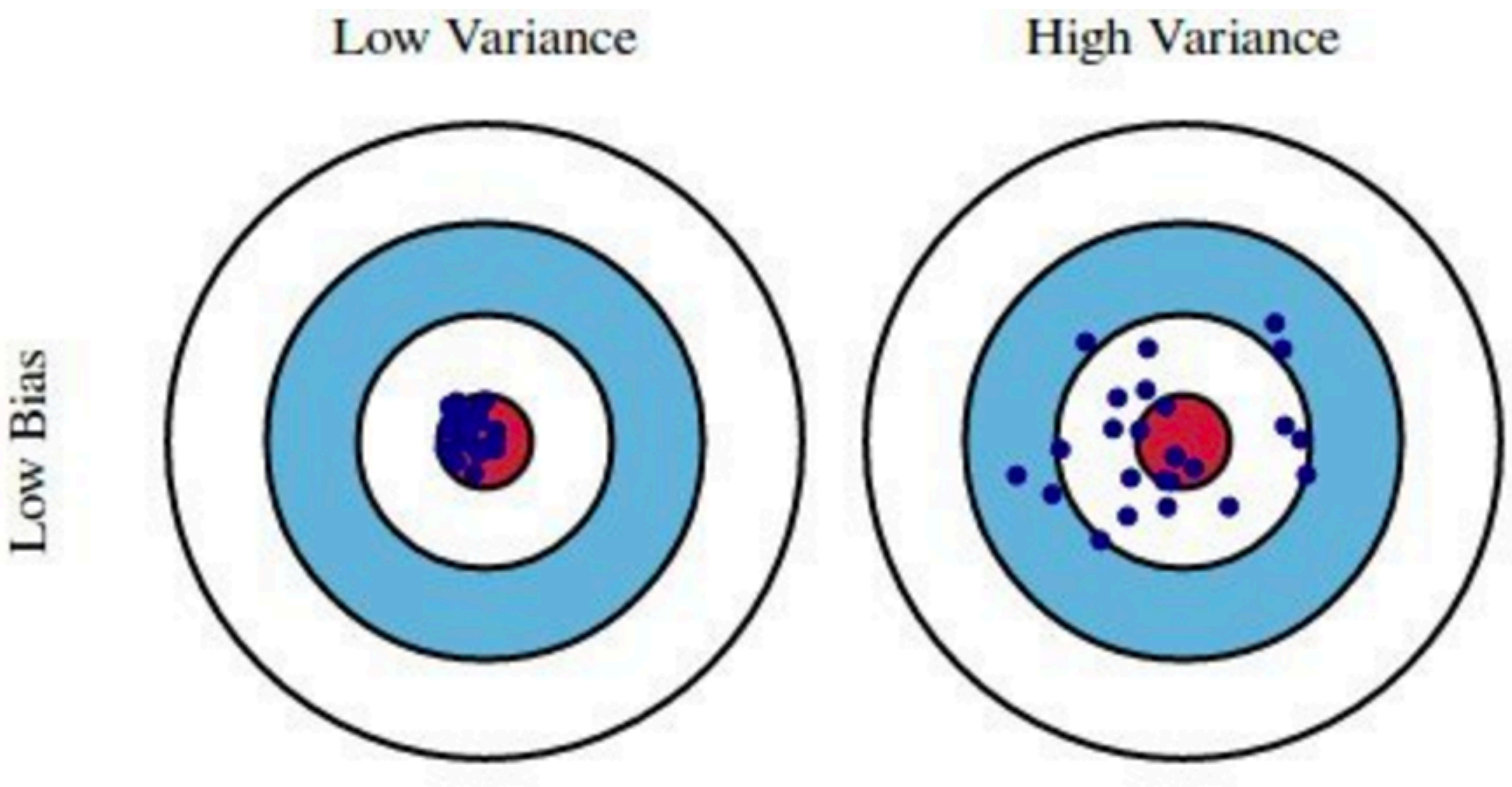






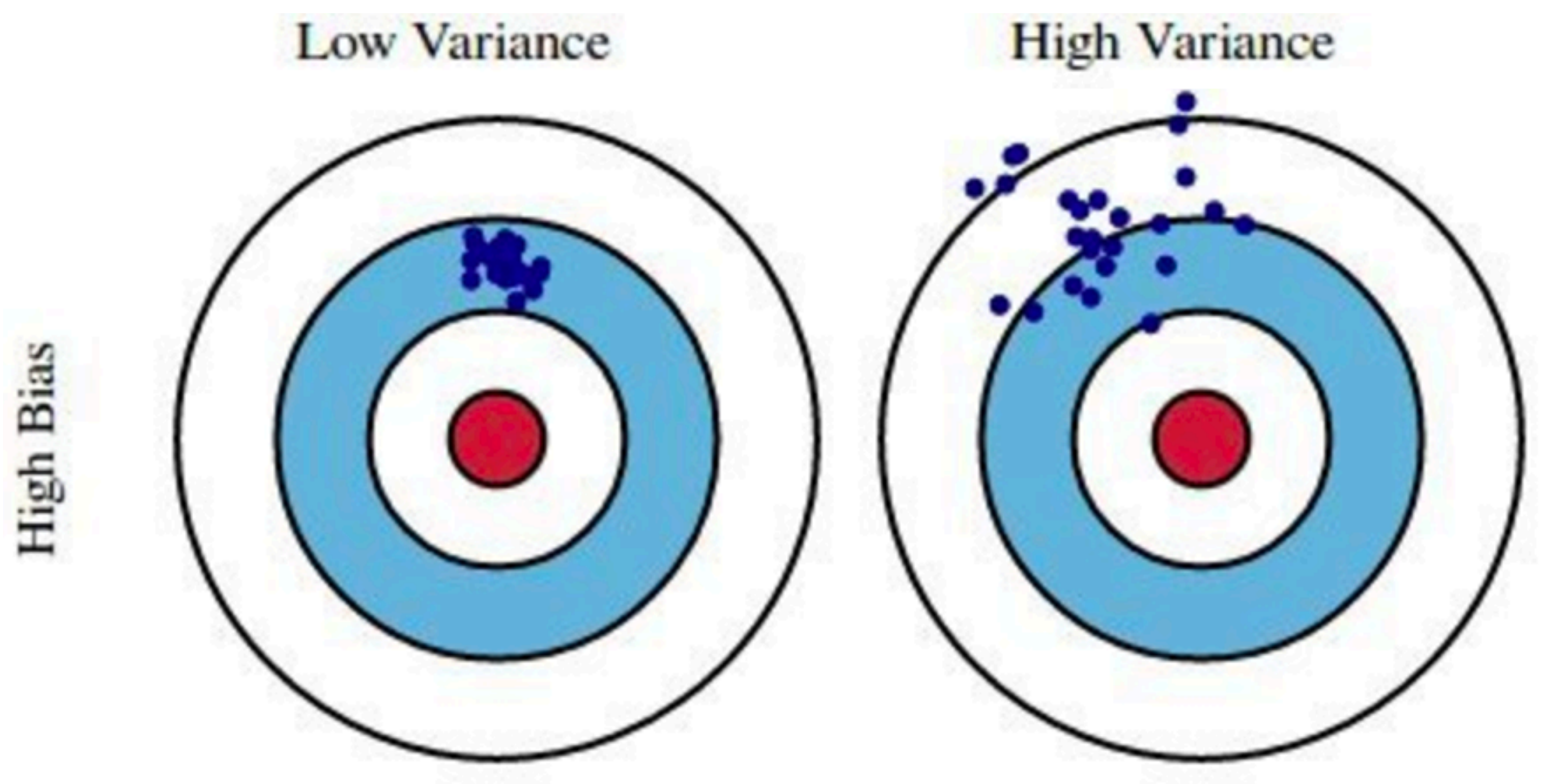
A cautionary note about errors and biases

Random Errors/ Variance/ Noise/ Standard error



What to do about it?
- Sample more. (Not always an option, but CARTHE did it!)
- Logarithmic binning.

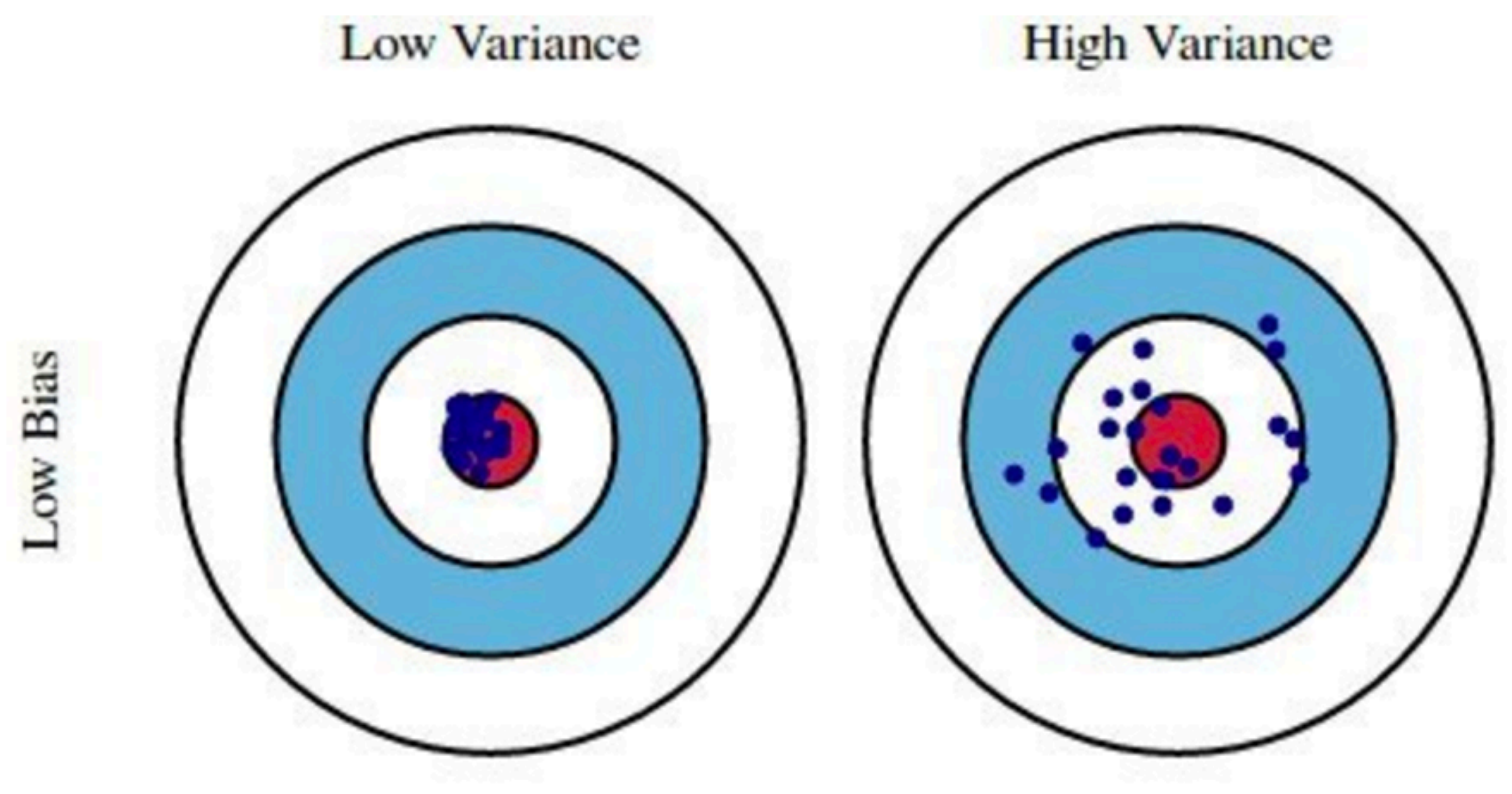
Sampling Errors/ Bias/ Systematic error



What to do about it?
- Throw out the data and work on something else.
- Define a target and try to sample close to that:
  - In this case the target being the Eulerian state.
  - But even this is biased because of the short time scale.
- Acknowledge that there might be bias, and I will keep that at the back of my mind in interpretation.
- Attempt to merge data sets.

A cautionary note about errors and biases

Random Errors/ Variance/ Noise/ Standard error



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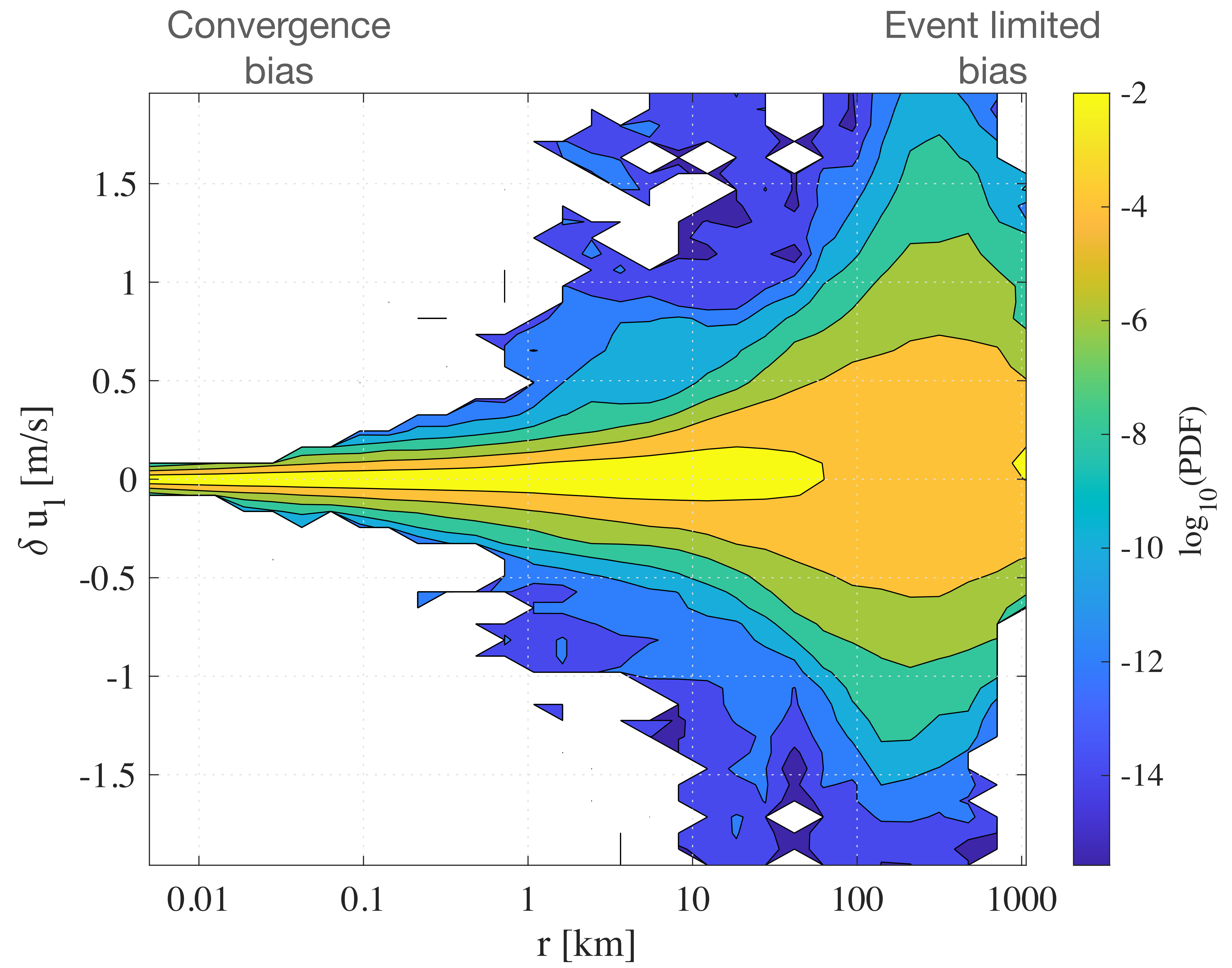

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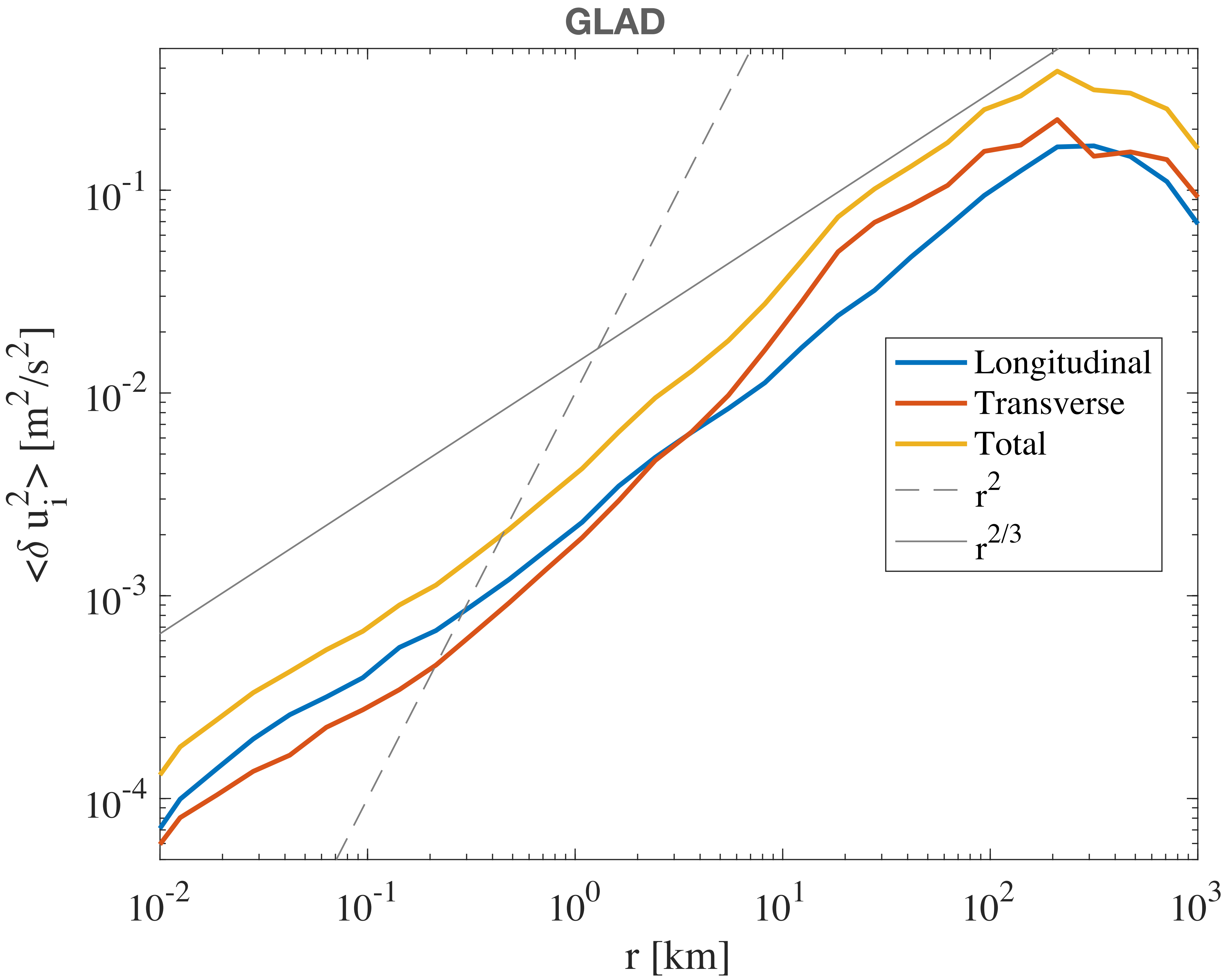
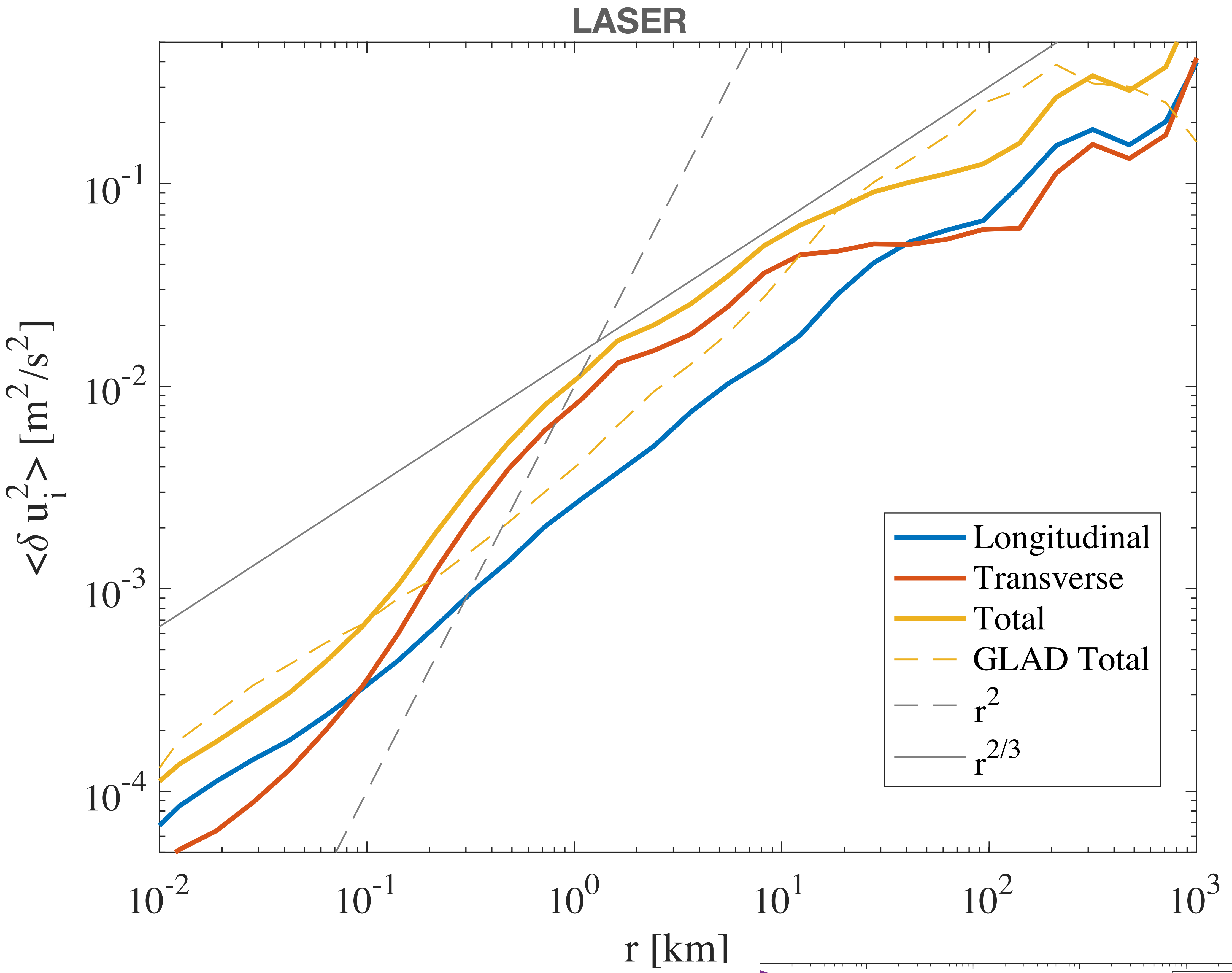
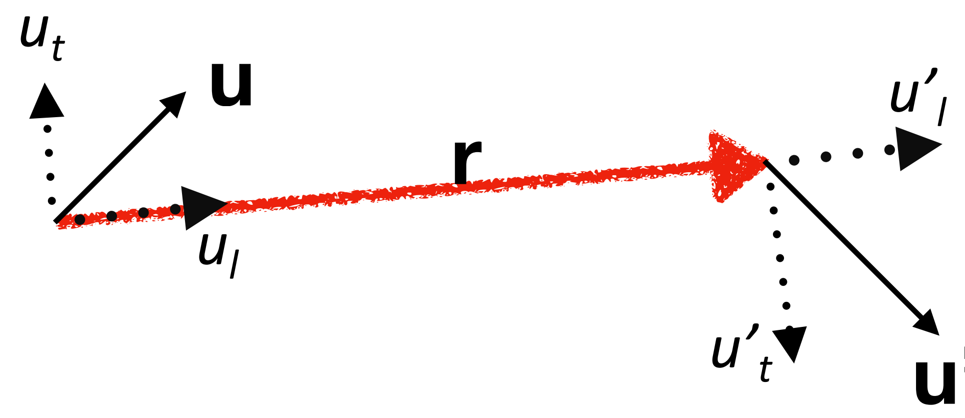
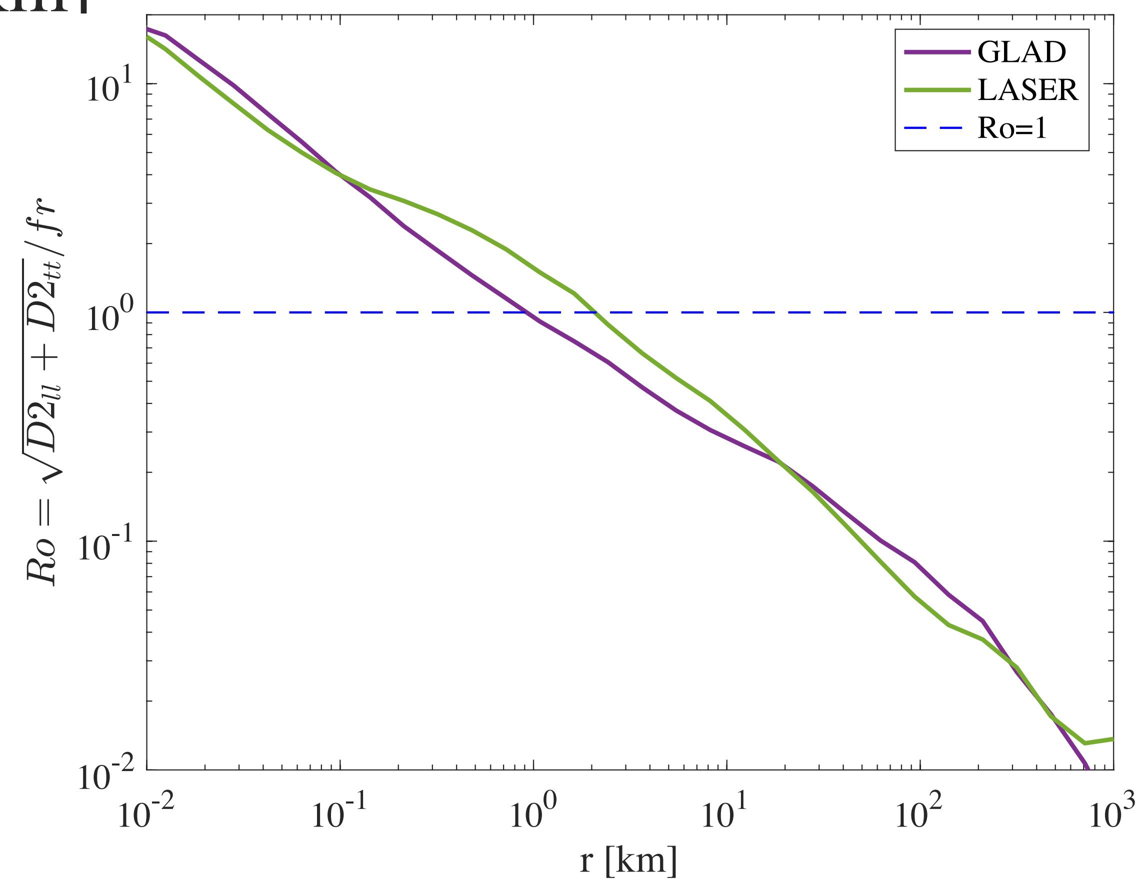
2nd Order Statistics : Longitudinal/Transverse



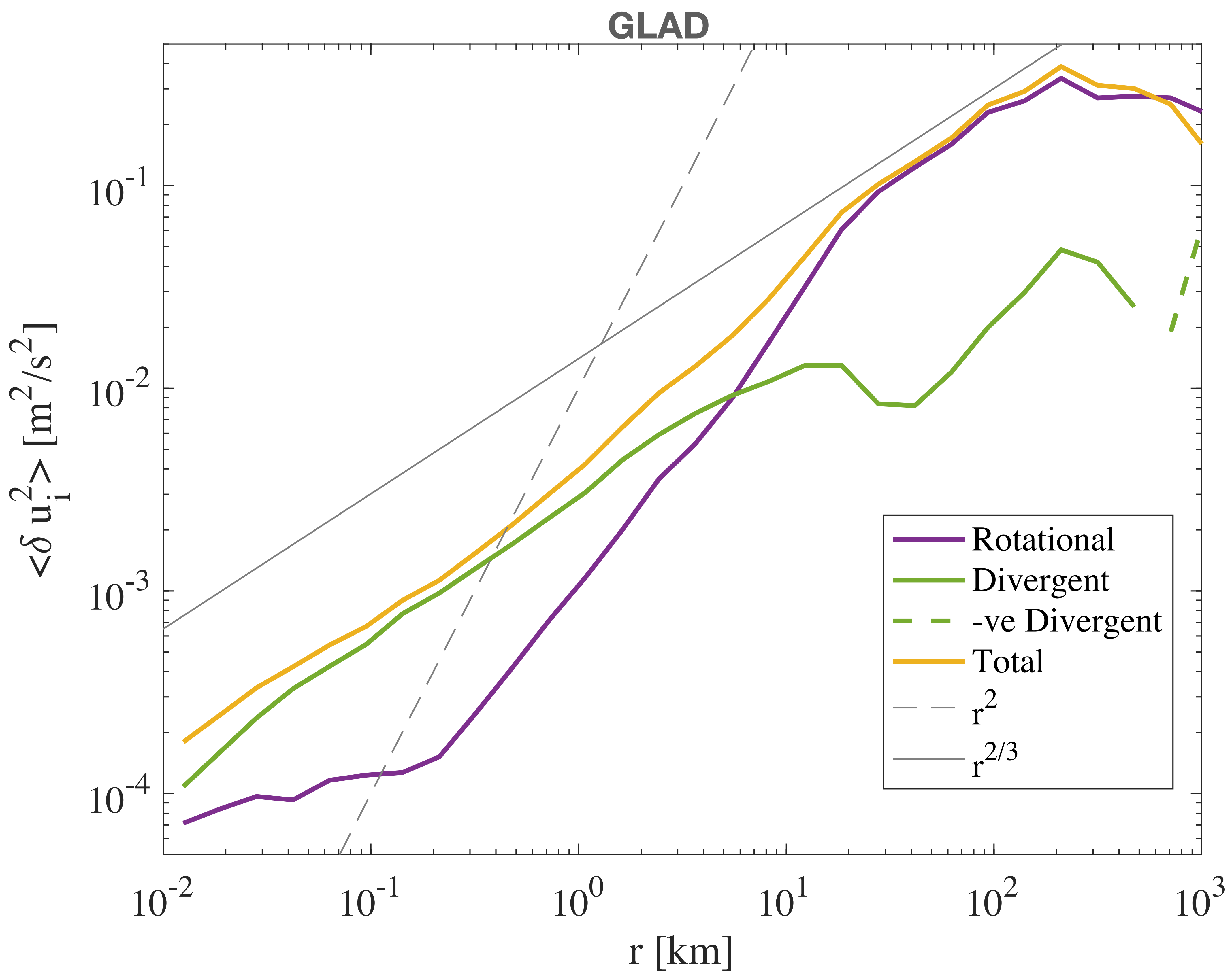


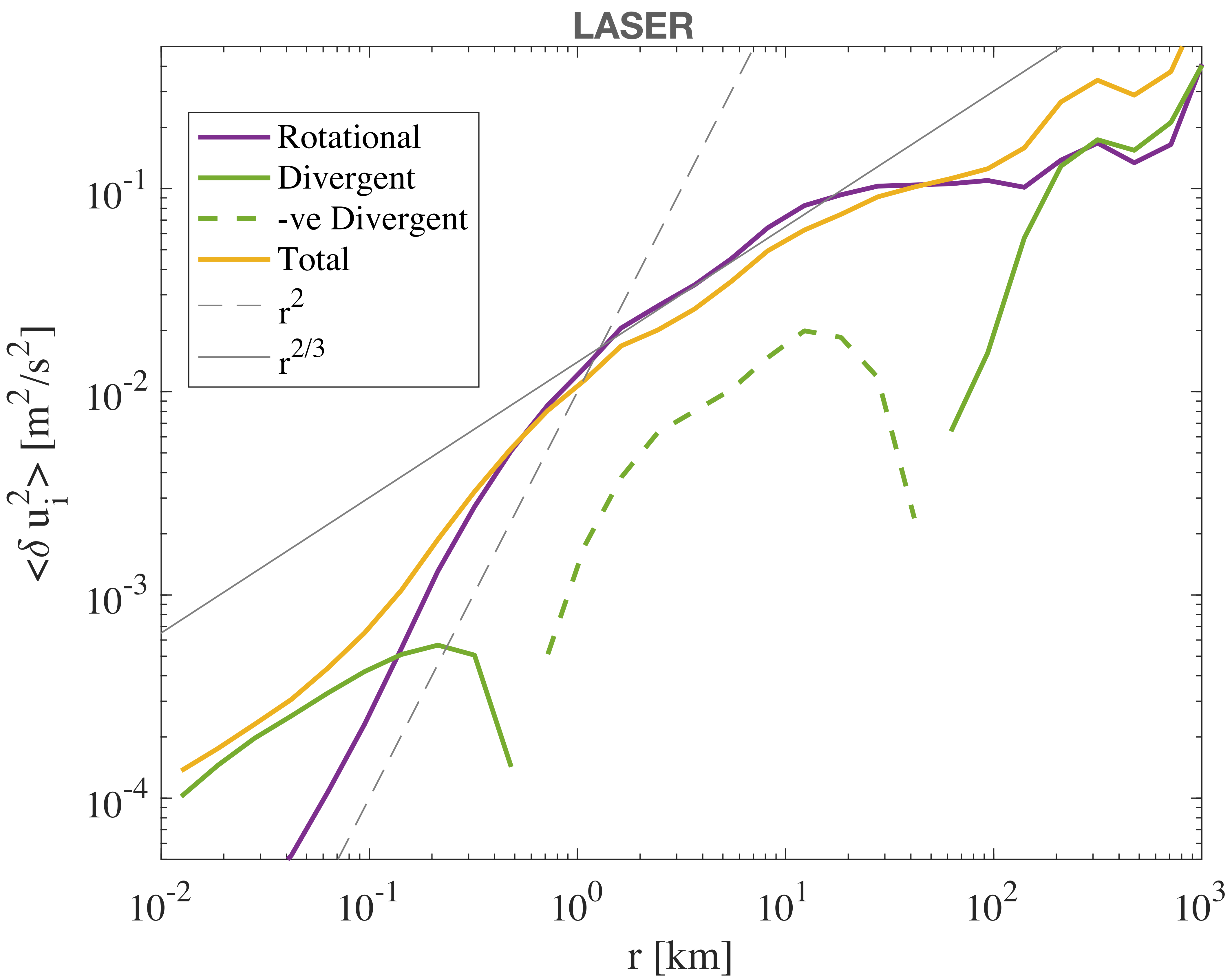


- Velocities start to get uncorrelated at larger scales (>50km).
- Both GLAD and LASER SF2s have power laws broadly close to 2/3.
- Ro>1 at scales <1-5km, and this scale is larger in winter.



2nd Order Statistics : Rotational/Divergent





Rotational: (2D turbulence like)

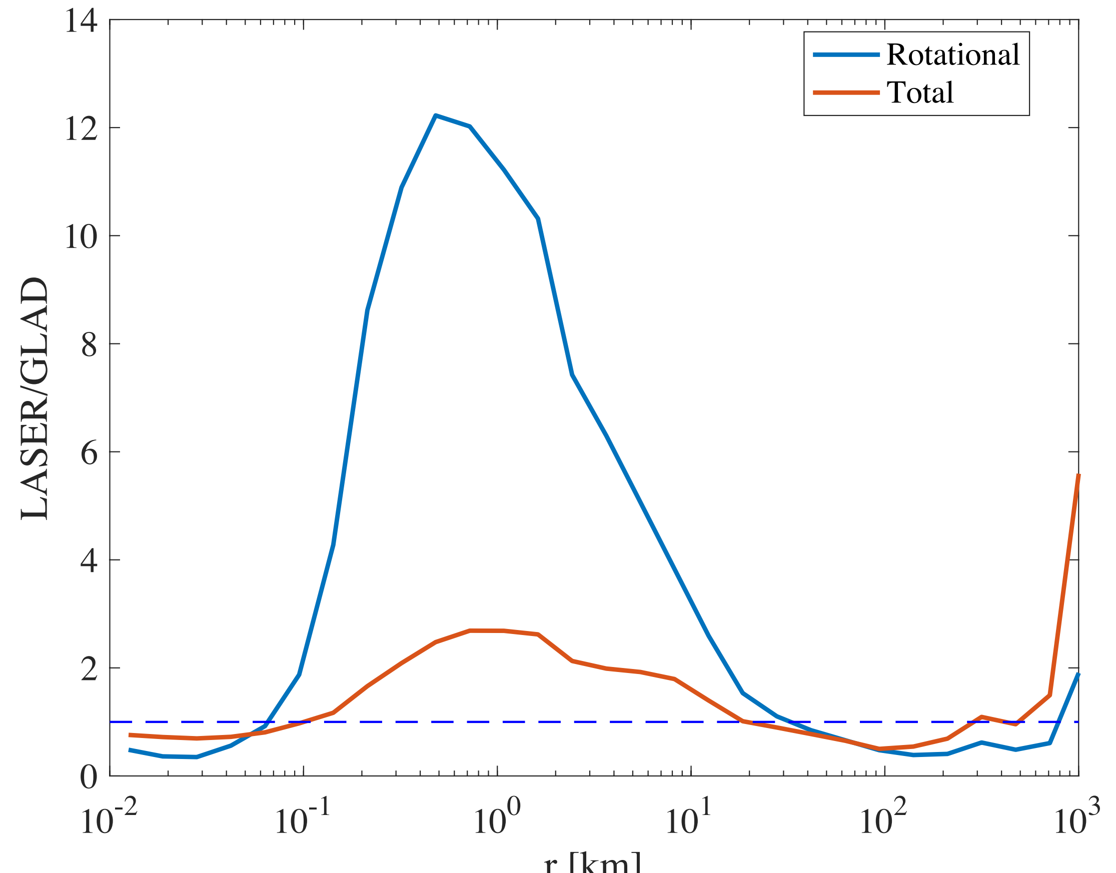
- Summer: inverse energy cascade like signal at scales larger than 20km.
- Winter: inverse energy cascade like signal at scales larger than 1km.
- Enstrophy cascade like signal at smaller scales.

Divergent: (An important part of the signal)

- The flatter power of the total SF is due to presence of divergent motions (~waves).
- Stronger and extending to larger scales in the summer.

Ratios: (Winter shows enhanced energy and enstrophy)

- Steeper part of rotational, which is indicative of enstrophy, shows enhancement by factor of ~10 in winter.
- Total, which is more indicative of energy shows enhancement by factor of ~2-3 in winter.



3rd order structure functions and inertial ranges

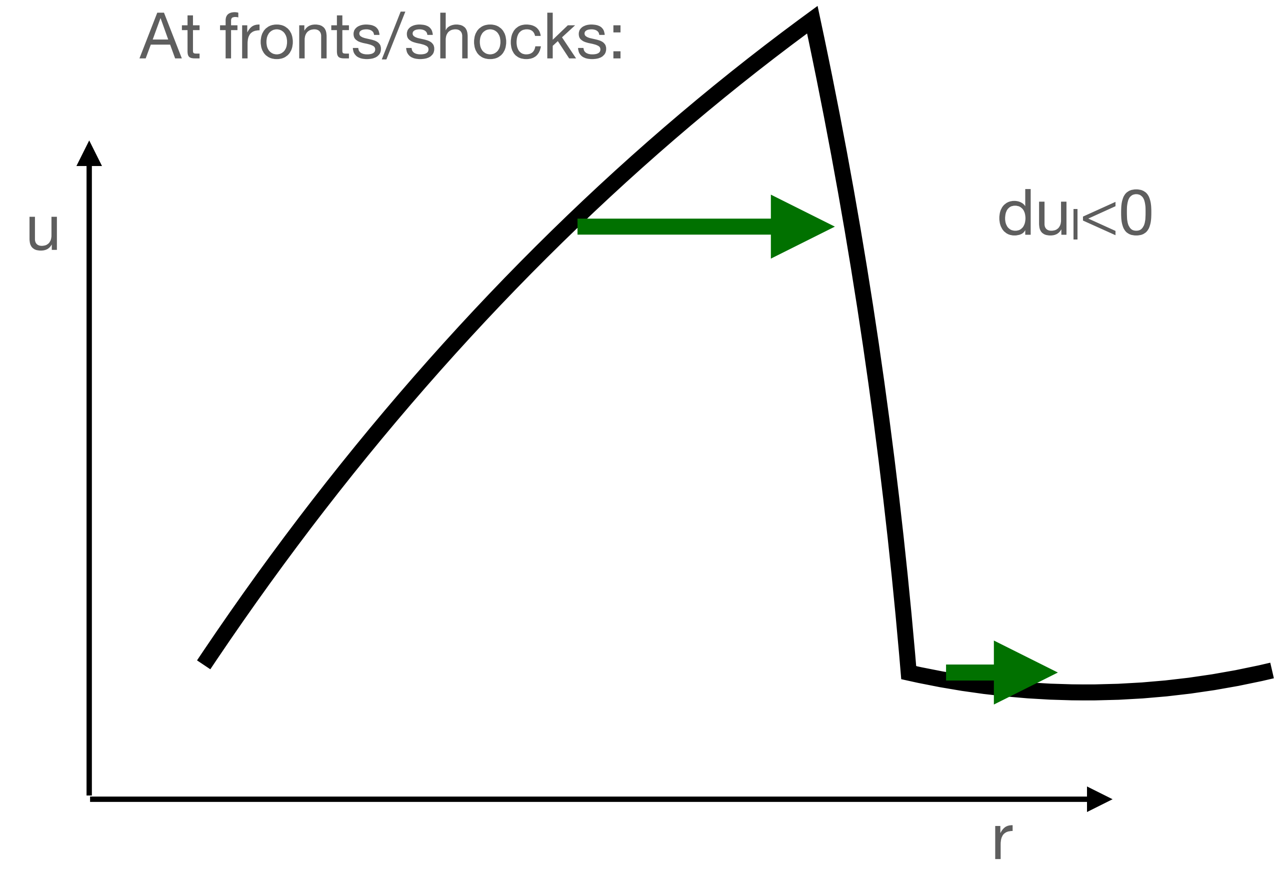
$$SF3(r) = V(r) = \overline{\delta u_l^3} + \overline{\delta u_l \delta u_t^2}$$

An oversimplified cartoon of what might happening at small-scale dissipation regions.

3D Turbulence

Forward energy cascade: $V(r) = -\frac{4}{3}\epsilon r$

At fronts/shocks:



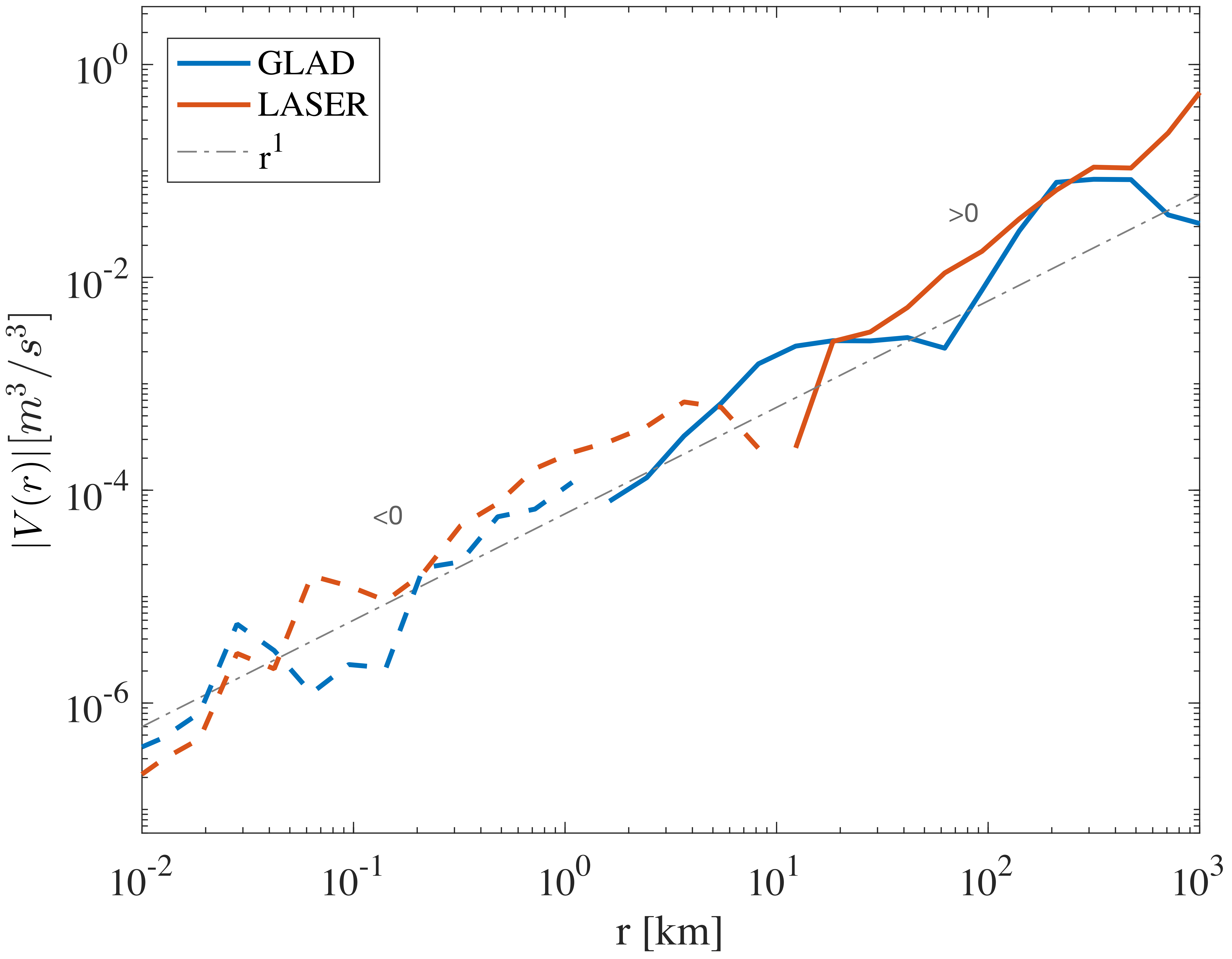
2D Turbulence

Forward enstrophy cascade: $V(r) = \frac{1}{4}\eta r^3$

Inverse energy cascade: $V(r) = 2\epsilon r$

…, Frisch 1995,
Xie and Buhler 2018

3rd Order Structure Functions



- Follows a ~ linear power over a large range of scales.
- Sign change happens in the submesoscale range, with negative values at smaller scales.

Energy transfers across scales

Spectral space

$$\frac{\partial}{\partial t}\left(\int_{k\leq K}\hat{E}(k)dk\right) + F(K) = D_K + P_K$$

$$\updownarrow$$

Fourier transform +
some integrals

Real space

$$\frac{1}{2}\frac{\partial}{\partial t}C - \frac{1}{4}\nabla\cdot V = D + P$$

$$\boldsymbol{C} = \overline{\boldsymbol{u}\cdot\boldsymbol{u}'},$$
$$\boldsymbol{V} = \overline{\delta\boldsymbol{u}|\delta\boldsymbol{u}|^2}$$

$$C(r) = 2\int_{-\infty}^{\infty}\hat{E}(k)e^{ikr}dk$$

Frisch 1995,
Xie and Buhler 2018

Spectral energy flux is related to the 3rd order structure function.

$$V(r) = -4r\int_0^\infty \frac{1}{K}F(K)\mathrm{J}_2(Kr)\,\mathrm{d}K.$$

$$F(K) = -\frac{K^2}{4}\int_0^\infty V(r)\mathrm{J}_2(Kr)\,\mathrm{d}r,$$

Energy transfers across scales

$$V(r) = -4r \int_0^\infty \frac{1}{K} F(K) \mathrm{J}_2(Kr)\, \mathrm{d}K.$$

One forcing scale:

$$F(K) = -\epsilon_u + \epsilon H(K - k_f)$$

$$V(r) = 2\epsilon_u r - 4\frac{\epsilon}{k_f}\mathrm{J}_1(k_f r),$$

$$V(r) = \begin{cases} \underbrace{-2\epsilon_d r}_{\textit{downscale energy}} + \underbrace{\tfrac{1}{4}\epsilon k_f^2 r^3}_{\textit{'enstrophy'}} + O((k_f r)^5), & \text{when } k_f r \ll 1, \\ \underbrace{2\epsilon_u r}_{\textit{upscale energy}} + O((k_f r)^{-1/2}), & \text{when } k_f r \gg 1. \end{cases}$$

ϵ_u Upscale energy transfer rate

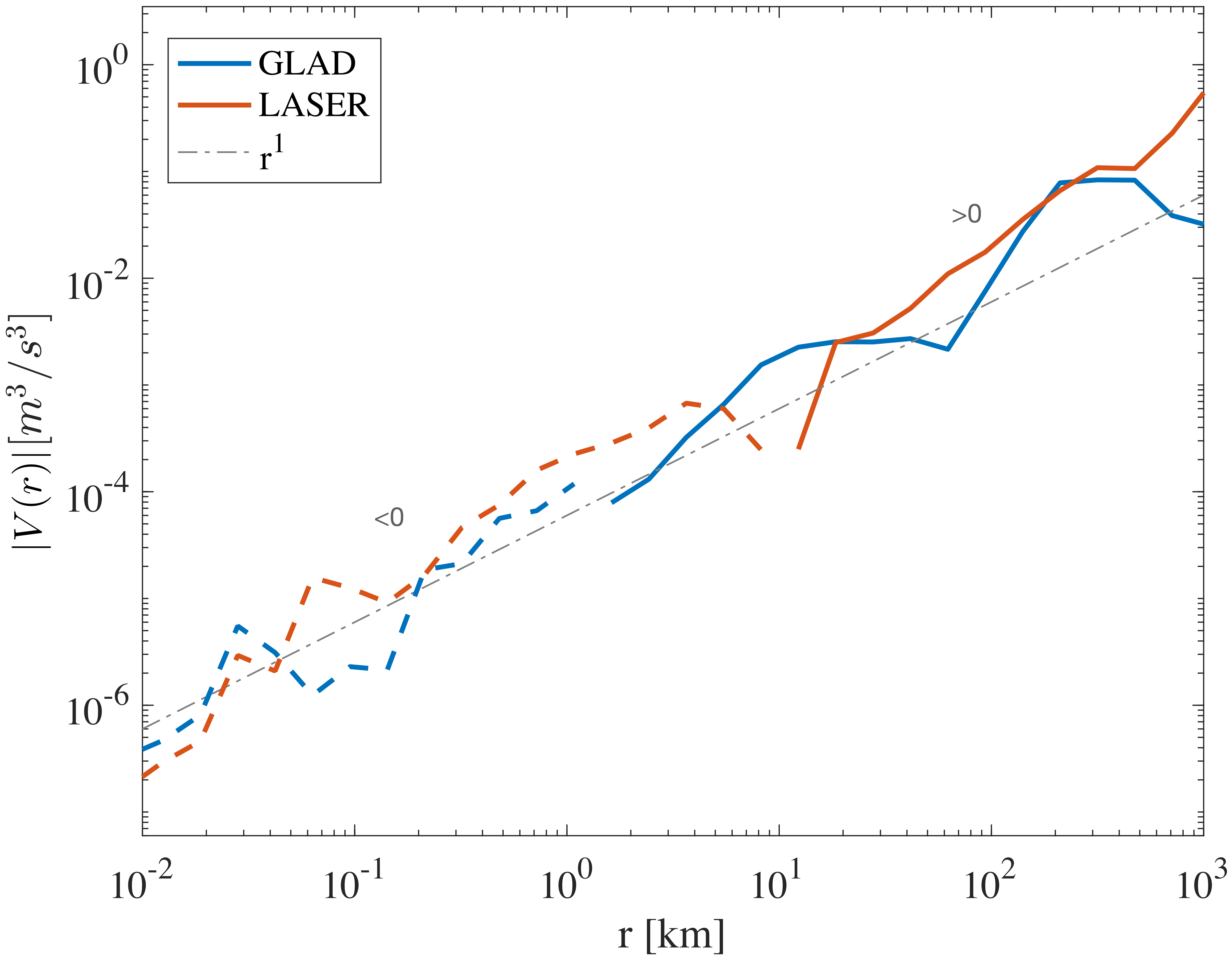
ϵ Energy injection (extraction) rate

Many forcing scales:

$$F(K) = -\epsilon_u + \sum_j \epsilon_j H(K - k_{fj})$$

Xie and Buhler 2019

3rd Order Structure Functions



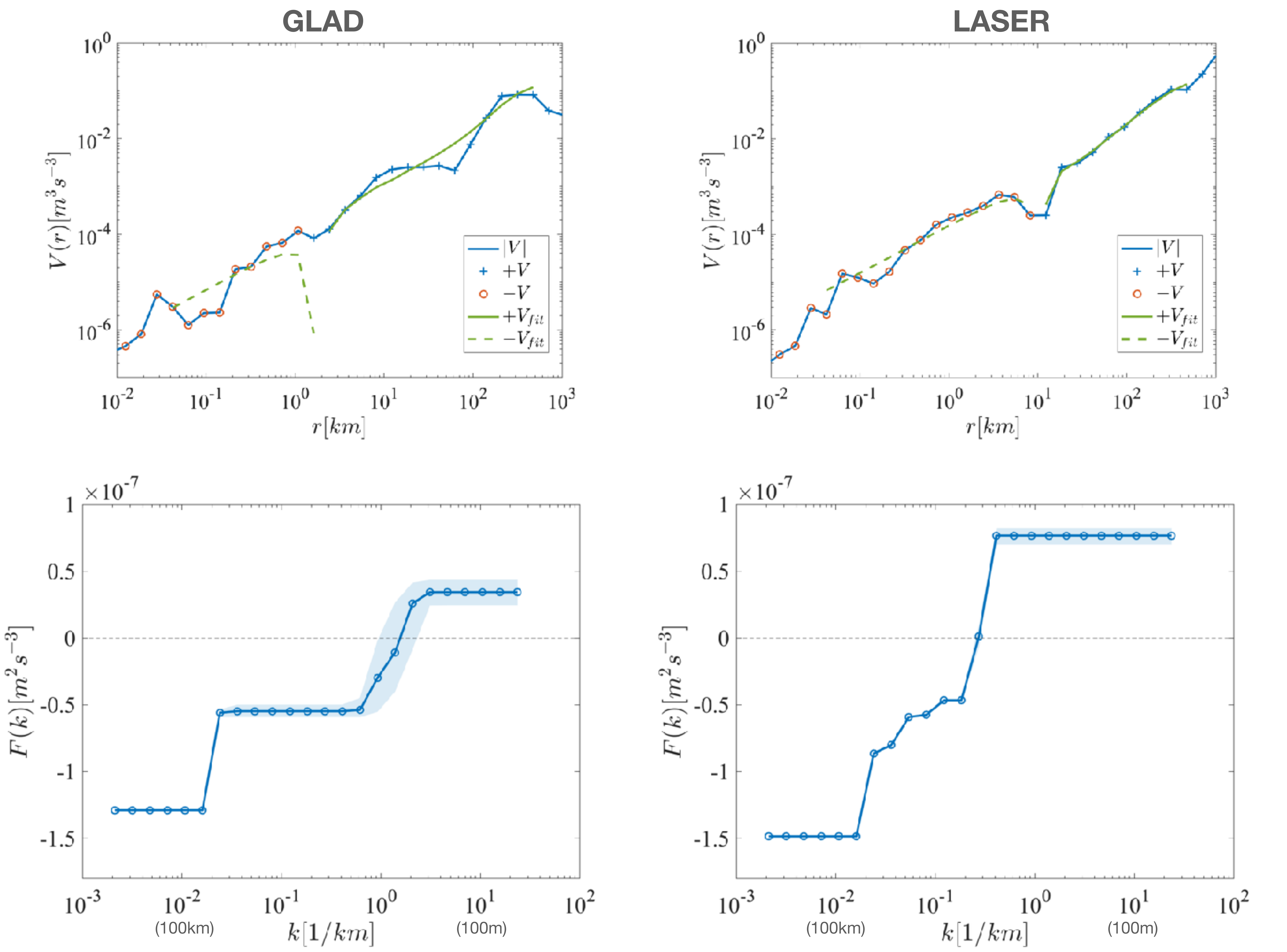
- Follows a ~ linear power over a large range of scales.
- Sign change happens in the submesoscale range, with negative values at smaller scales.

Fit with regularization and find coefficients:

$$V(r) = 2\epsilon_u - \sum_j 4\frac{\epsilon_j}{k_{fj}} J_1(k_{fj} r)$$

$$F(K) = -\epsilon_u + \sum_j \epsilon_j H(K - k_{fj})$$

- Fit works well, but improvements are possible.
- Downward flux at small scales, and upward flux at larger scales.
-

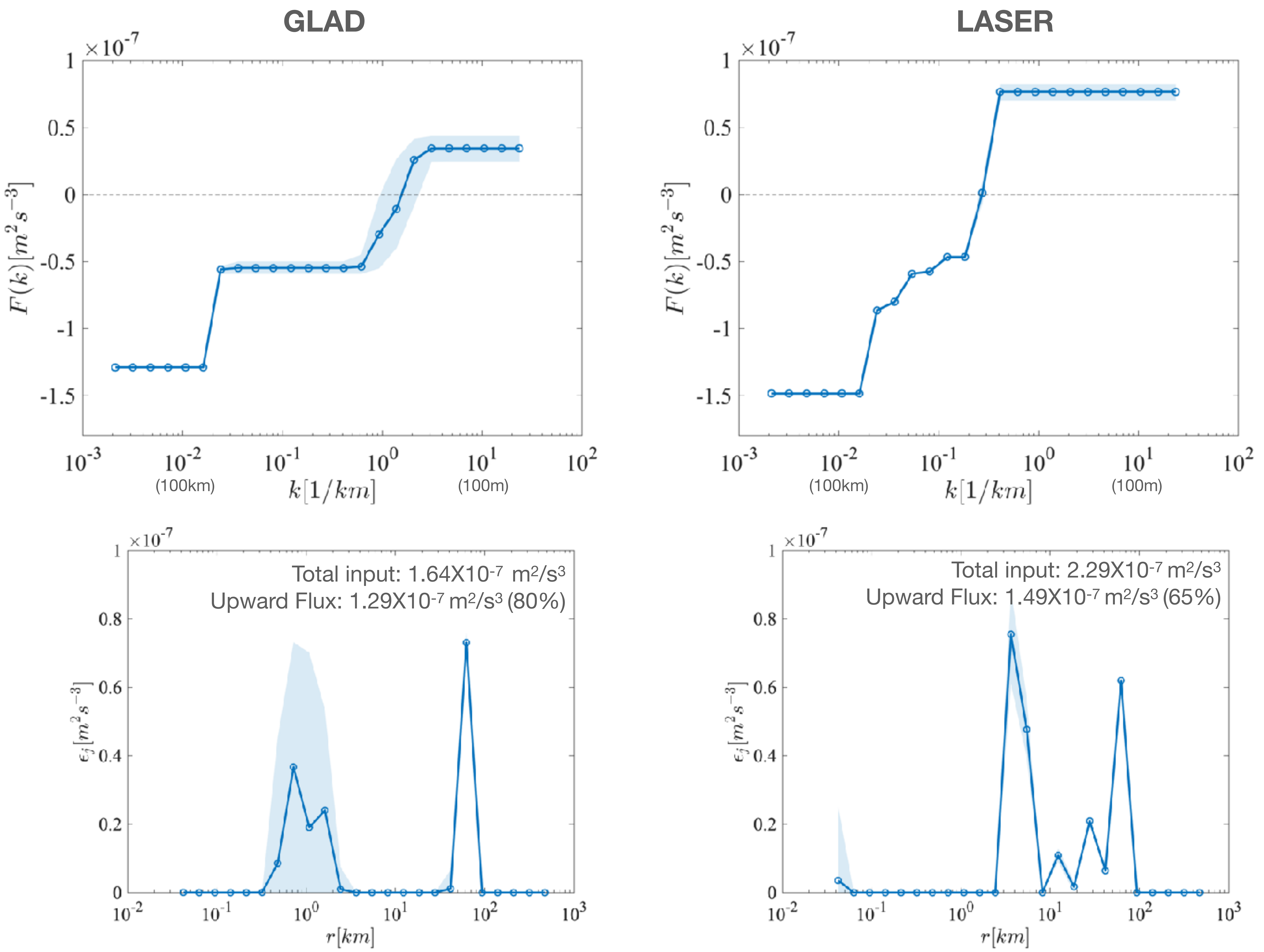


Fit with regularization and find coefficients:

$$V(r) = 2\epsilon_u - \sum_j 4\frac{\epsilon_j}{k_{fj}} J_1(k_{fj} r)$$

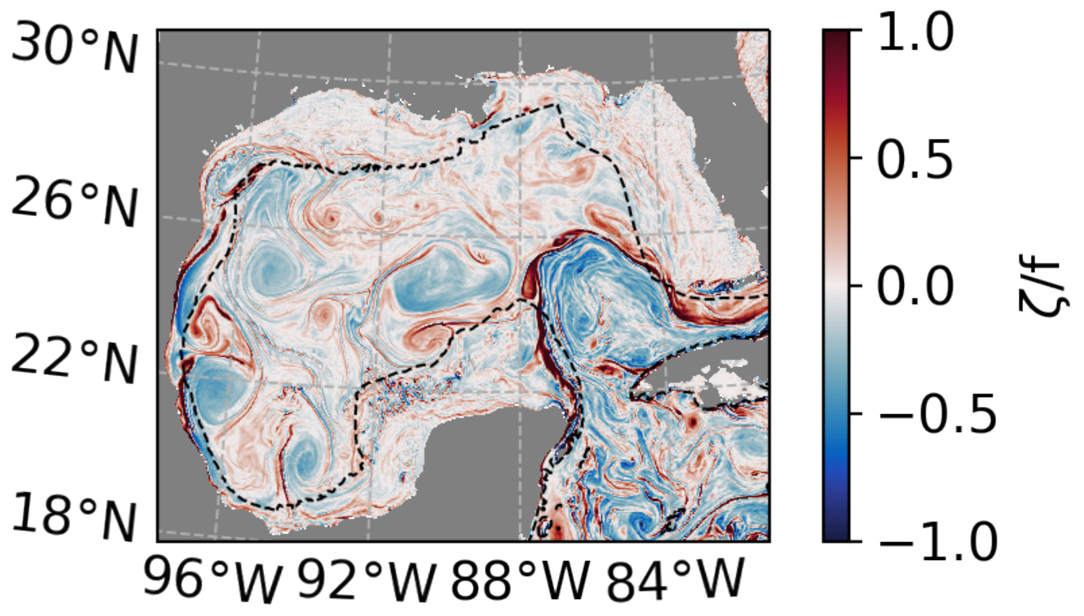
$$F(K) = -\epsilon_u + \sum_j \epsilon_j H(K - k_{fj})$$

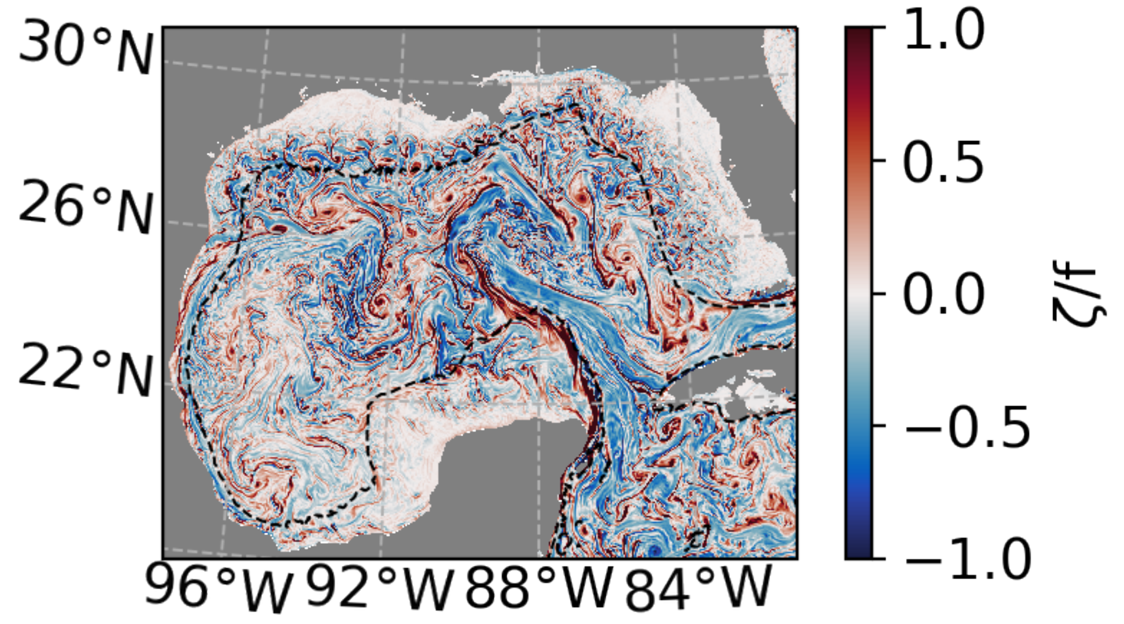
- Fit works well, but improvements are possible.
- Downward flux at small scales, and upward flux at larger scales.
- More energy input in the winter.
- Energy injection at scales of interior deformation radius (~50km) and ML deformation radius (~1-5 km)
- Stronger downscale and upscale cascades in winter.
- Larger fraction of energy injected cascades downscale in winter.

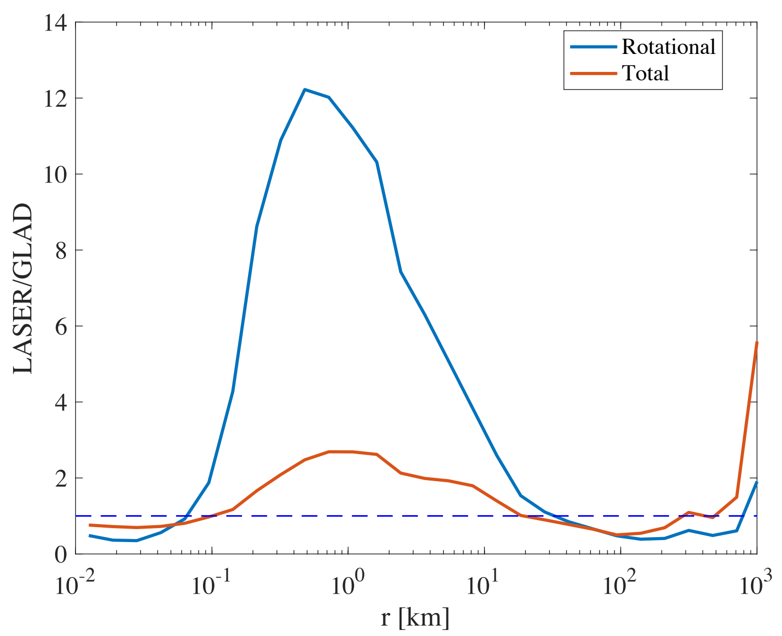


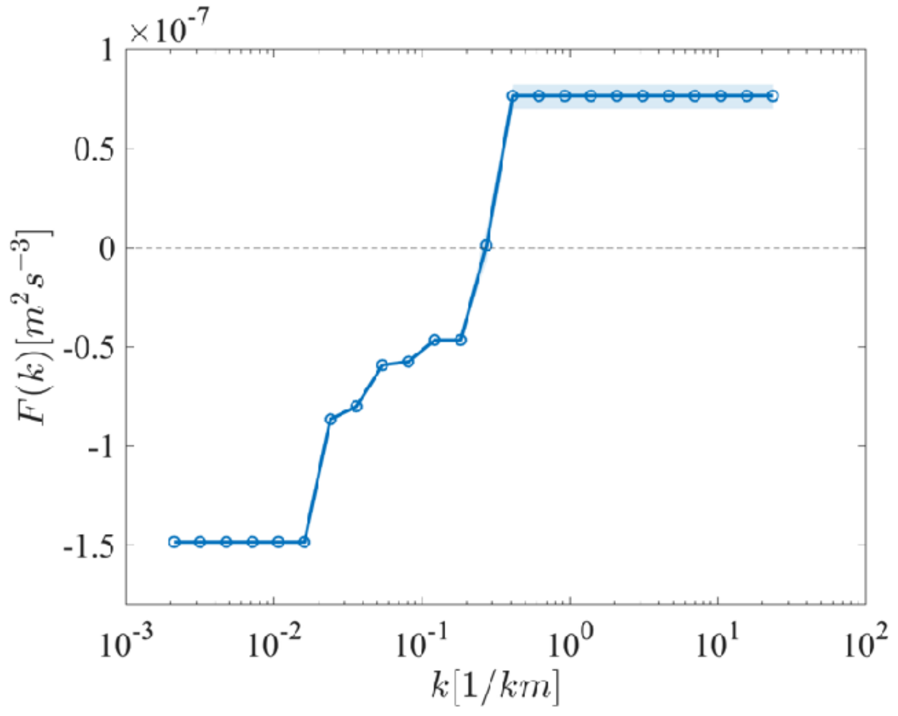
Summary

- Structure functions are useful tools!!!
  - We need more experimental and theoretical work with them.
- Statistical picture of the flow has rough consistency with “2D like turbulence” + “waves”.
- Summer and winter statistics are different, with more waves in summer and a more vigorous vorticity in the winter.
- Spectral fluxes can be estimated:
  - Most of the energy goes upscale.
  - A more vigorous energy injection in winter.









Thanks