

# On the meridional structure of the equatorial mixed layer

Mixed layer Equator Upwelling Couche mélangée Équateur Upwelling

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# **ABSTRACT**

We compare three two-dimensional models of the meridional structure of the mixed layer near the equator: a 1 1/2-layer linear reduced gravity model and two bulk mixed layer models, one with meridional advection and one with an atmospheric feedback. The reduced gravity model is a purely dynamic model; the two bulk mixed layer models include the thermodynamic processes of mixing and heating. The bulk mixed layer models can be tuned to reproduce fairly well the observed meridional structure with a cold and shallow mixed layer at the equator and a warmer and deeper layer away from it. In the models this transition represents the transition from an equilibrium state with upwelling to an equilibrium state without upwelling. The reduced gravity model can reproduce the transition in mixed layer depth but does not predict mixed layer temperature. The differences between the models become most pronounced when their response to changes in the upwelling velocity and the atmospheric forcing fields is considered. Most dramatically, an increase in the windstress leads to a shallowing of the upper layer in the reduced gravity model, to a strong deepening in the advective model and to a weak deepening in the feedback model.

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# RÉSUMÉ

# Structure méridienne de la couche de mélange équatoriale

La structure méridienne de la couche de mélange à proximité de l'équateur est étudiée en comparant trois modèles bidimensionnels : un modèle linéaire de gravité réduite à une couche et demie, et deux modèles globaux de couche de mélange, l'un avec advection méridienne et l'autre avec rétroaction atmosphérique. Le modèle gravitationnel est purement dynamique. Les deux modèles globaux prennent en compte les processus thermodynamiques de mélange et d'échauffement; ces modèles globaux peuvent être adaptés pour reproduire convenablement la structure méridienne observée avec une couche mélangée froide et peu épaisse à l'équateur, et une couche plus chaude et plus profonde à distance de l'équateur. Dans les modèles, la transition représente le passage d'un état d'équilibre avec remontée d'eau à un état d'équilibre sans remontée d'eau. Le modèle gravitationnel peut reproduire la transition dans la profondeur de la couche de mélange, mais il ne prédit pas sa température. Les différences entre les modèles sont plus marquées dans leur réponse aux changements de vitesse de la remontée d'eau et au forçage atmosphérique. En outre, une augmentation de la tension de vent diminue l'épaisseur de la couche superficielle dans le modèle gravitationnel; elle augmente considérablement sa profondeur dans le modèle advectif et l'augmente peu dans le modèle à rétroaction.

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# INTRODUCTION

Meridional temperature sections in equatorial oceans often show that both the mixed layer temperature and mixed layer depth increase toward the north and south of the equator (e.g. Wyrtki, Kilonsky, 1984). This structure is regarded as the response of the mixed layer to a meridional circulation with Ekman outflow in the surface layer, geostrophic inflow below, and upwelling of cold water at the equator (Wyrtki, 1981). While this explanation is generally accepted actual modelling efforts differ quite significantly in their physics, especially in the degree to which they include thermodynamic processes. We will compare three models with different parameterizations of mixing and heating. All these models are capable of reproducing the observed meridional structure of the mixed layer but react quite differently to changes in the upwelling velocity and the atmospheric forcing fields.

Several efforts have been directed towards including the thermodynamic processes of mixing and heating into equatorial models. McCreary (1983) parameterizes mixing in terms of a Newtonian cooling coefficient, Pacanowski and Philander (1981) apply a conductivity coefficient that depends on the Richardson number, Hughes (1980) and Schopf and Cane (1983) use a Kraus-Turner (1967) type mixed layer model. Surface heating is either neglected, prescribed, or coupled to the sea surface temperature. These models calculate both the velocity field and the temperature (density) field in response to atmospheric forcing. Here, we are interested in the sensitivity of the thermal field to the parameterization of the thermodynamic processes and regard both the atmospheric forcing and the interior advection velocity as given.

The three models which we compare are a 1 1/2-layer linear reduced gravity model, a bulk mixed layer model with meridional advection, and a bulk mixed layer model with an atmospheric feedback. A simple analysis of one-dimensional bulk mixed layer models reveals that they have two equilibrium solutions, one with upwelling and one without upwelling. The equilibrium solution with upwelling is independent of the upwelling velocity and hence does not merge smoothly into the equilibrium solution without upwelling. This discontinuity can be removed by either introducing meridional advection (our second model) or by an atmospheric feedback (our third model). We will focus on steady, zonally uniform solutions. The first model is hence the Yoshida jet (Yoshida, 1959) and the third model the one of Hughes (1980). The meridional advection model is our own contribution. The comparison will emphasize the sensitivity of these solutions to changes in the upwelling velocity and atmospheric forcing fields.

# REDUCED GRAVITY MODEL

The simplest models to study the dynamic response of the equatorial ocean to an applied windstress are linear reduced gravity models. These models have been very successful in reproducing sea level changes during El Niño events (e.g. Busalacchi, O'Brien, 1981). Reduced gravity models do not contain any thermodynamics. The layers have prescribed densi-

ties, and the interfaces between the layers are material surfaces. The specific reduced gravity model we consider is a linear 1 1/2-layer model, which assumes a dynamically active, incompressible, hydrostatic, homogeneous upper layer overlying a quiescent infinitely deep lower layer. The interface is the model pycnocline. On an equatorial beta plane the equations of motion take the form

$$\partial_t u - \beta y v + \frac{1}{\rho_0} \partial_x p = \frac{\tau_x}{\rho_0 H}$$
 (2.1 a)

$$\partial_t v + \beta y u + \frac{1}{\rho_0} \partial_y p = \frac{\tau_y}{\rho_0 H}$$
 (2.1 b)

$$\partial_t p + \rho_0 c^2 (\partial_x u + \partial_v v) = 0 \tag{2.1 c}$$

where u and v are the zonal and meridional velocity components, respectively, p is the perturbation pressure,  $\beta$  the beta parameter,  $c=(g'\,H)^{1/2}$  the baroclinic phase speed, H the equilibrium thickness of the upper layer, and  $g'=g\Delta\rho/\rho_0$  the reduced gravitational acceleration. The windstress  $\tau=(\tau_x,~\tau_y)$  enters as a body force. The actual thickness of the upper layer is  $h=H-\eta+\zeta\approx H-\eta$  where  $\eta$  is the displacement of the interface and  $\zeta\ll\eta$  the sea surface elevation. The displacement of the interface is given by

$$\eta = -\frac{p}{\rho_0 g'} = \int dt w = \int dt H \left(\partial_x u + \partial_y v\right) \qquad (2.2)$$

where w is the vertical velocity. The response of an unbounded ocean to a uniform zonal windstress is given by (Yoshida, 1959; Moore, Philander, 1977)

$$u = t \left( \beta yv + \frac{\tau_{x}}{\rho_{0}H} \right) \tag{2.3 a}$$

$$\eta = tH\partial_{y}v \tag{2.3 b}$$

where v is a solution of

$$R^2 \partial_y \partial_y v - \frac{y^2}{R^2} v = \frac{y}{R} a \tag{2.4}$$

Here  $R = (c/\beta)^{1/2}$  is the equatorial baroclinic radius of deformation and  $a = (\beta \hat{R} \rho_0 H)^{-1} \tau_x$ . This solution describes the so called Yoshida jet and is shown in Figure 1. Far away from the equator v describes a meridional Ekman drift. Near the equator where the Coriolis force is not sufficient to balance the windstress a zonal jet develops which accelerates indefinitely. The jet and the associated meridional circulation are confined to within  $\pm$  R from the equator. For typical values of c = 2 ms<sup>-1</sup>,  $\beta$  = 2 · 10<sup>-11</sup> m<sup>-1</sup> s<sup>-1</sup>,  $\tau_x$  = 5 · 10<sup>-2</sup> Nm<sup>-2</sup>, H = 100 m we find R  $\approx$  300 km and a  $\approx$  10<sup>-1</sup> ms<sup>-1</sup>. After 20 days  $u \approx 1 \text{ ms}^{-1}$  and  $\eta \approx 30 \text{ m}$ , reasonable values. The Yoshida jet continues to grow. Bounded solutions can be obtained by introducing meridional boundaries so that a zonal pressure gradient can develop to balance the windstress. Bounded solutions can also be obtained by including vertical friction and vertical mixing. In the simplest case (McCreary, 1983) one adds the terms -ru (Rayleigh friction) to (2.1 a) and -rp (Newtonian cooling) to (2.1 c). The factor t in (2.3 a) and

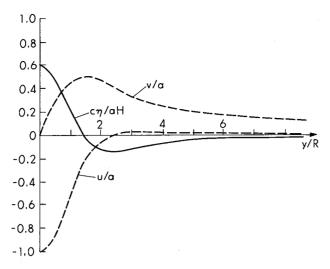


Figure 1 Yoshida jet solution for the linear 1 1/2-layer reduced gravity model. The normalized zonal velocity u/a and the normalized interface displacement  $c\eta/aH$  are plotted at time  $t=(\beta c)^{-1/2}$  or for a Rayleigh friction and Newtonian cooling coefficient of  $r=(\beta c)^{1/2}$  (after Moore, Philander, 1977).

(2.3 b) is then replaced by  $r^{-1}$ . However, this is a fairly simplistic parameterization of the mixing processes.

The main point for our discussion is that the interface between the upper and lower layer represents a material surface in this and other dynamical models. The position of the interface is determined purely by the advective velocity field, in our linear model by the vertical velocity w.

# **BULK MIXED LAYER MODELS**

Most models which include thermodynamic effects regard the upper layer as a well-mixed turbulent boundary layer which exchanges heat with the atmosphere and entrains water from below. The temperature equation takes the form

$$\partial_{t}T + u\partial_{x}T + v\partial_{y}T = -\frac{\Delta T}{h}e\theta(e) + \frac{Q}{\rho_{0}C_{p}h}$$
 (3.1)

where T is the mixed layer temperature,  $\Delta T = T - T_I$  the temperature jump across the interface,  $T_I$  the temperature just below the interface, e the rate of entrainment or detrainment, h the mixed layer depth, Q the surface heat flux (positive downward),  $C_p$  the specific heat at constant pressure, and  $\theta(e)$  the unit step function. The horizontal velocities are depth averaged velocities. The temperature equation describes the balance of storage, advection, entrainment and heating. Note that only entrainment, e > 0, of water from below changes the mixed layer temperature. Detrainment, e < 0, does not affect the temperature since water is not unmixed but simply left behind.

The interface equation takes the form

$$\partial_t h + u \partial_x h + v \partial_v h = e - w$$
 at  $z = -h$  (3.2)

which is the defining equation for e. If e = 0 the interface is a material surface.

Equations (3.1) and (3.2) are prognostic equations for the mixed layer temperature T and mixed layer depth h. Traditional bulk mixed layer models infer the rate of entrainment e from the turbulent kinetic energy budget

$$G - D - 1/2 hB - 1/2 he\Delta b = 0 (3.3)$$

where G is the shear production and D the dissipation of turbulent kinetic energy. The third and fourth term describe the turbulent kinetic energy required to mix heat supplied by surface heating and entrainment throughout the mixed layer. In (3.3)  $B = \alpha g Q/\rho_0 C_p$  is the surface buoyancy flux and  $\Delta b = \alpha g \Delta T$  the buoyancy jump across the interface.

If 
$$G - D - 1/2 hB > 0$$

there is sufficient turbulent kinetic energy to entrain and mix water from below. In this case the entrainment rate is determined from (3.3) and substituted into (3.2) to prognosticate the mixed layer depth h. If there is not enough turbulent kinetic energy to entrain water from below e is set equal to zero in (3.3) and the mixed layer depth is calculated diagnostically from a balance of the remaining terms. The detrainment rate e < 0 is then calculated from the interface equation (3.2). Note that the one-dimensionality of the turbulent kinetic energy budget is a good approximation even if the temperature and interface equation must include horizontal advection.

The closure of the bulk mixed layer equations by the turbulent kinetic energy equation was originally proposed by Kraus and Turner (1967) who also suggested  $G-D\sim u_*^3$  where  $u_*=(\tau/\rho_0)^{1/2}$  is the friction velocity. Alexander and Kim (1976) found that the Kraus-Turner parameterization overestimates the mixed layer depth in mid-latitudes and introduced a background dissipation  $\epsilon_0$  such that

$$G - D = u_*^3 - h\varepsilon_0 \tag{3.4}$$

Different mixed layer physics lead to different specification of  $\varepsilon_0$ . Garwood's (1977) mid-latitude mixed layer model corresponds to  $\varepsilon_0 = \mathrm{fu}_*^2$  where f is the Coriolis parameter. Garwood *et al.*'s (1985 a) recent equatorial mixed layer model corresponds to

$$\varepsilon_0 \approx \rho_0^{-1} \, \Omega_{\rm y} \tau_{\rm x} \tag{3.5}$$

where  $\tau_x$  is the zonal component of the windstress and  $\Omega_y$  the meridional component of the earth's rotation. The rationale behind (3.5) is that the rotation stress term  $\Omega_y\tau_x$  affects vertical mixing by converting horizontal to vertical turbulent kinetic energy and vice versa, although the term does not appear in the total turbulent kinetic energy budget. Mixing is enhanced for easterlies,  $\tau_x<0$ , and suppressed for westerlies,  $\tau_x>0$ . Enhanced mixing formally corresponds to a negative background « dissipation ».

# RESPONSE TO UPWELLING

For given friction velocity  $u_*$  or windstress  $\tau$  and given positive surface heat flux Q > 0 the maximum mixed layer depth is reached when e = 0 in (3.3) and

is given by

$$h = H = \left(\frac{1}{L} + \frac{\varepsilon_0}{u_*^3}\right)^{-1} \tag{4.1}$$

where

$$L = \frac{2u_*^3}{B} \tag{4.2}$$

is the Obukhov length scale. The depth H represents the equilibrium depth of the mixed layer in the case of no upwelling. The dependence of H on  $\epsilon_0$  is shown in Figure 2. For vanishing background dissipation (Kraus-Turner model) we find H=L. For positive background dissipation H<L, and H>L for negative background dissipation. Equation (4.1) with  $\epsilon_0$ 

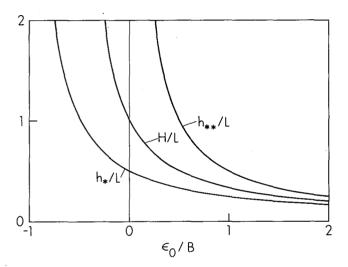


Figure 2 Equilibrium depths of the mixed layer, normalized by the Obukhov length scale L, as a function of the background dissipation  $\varepsilon_0$ , normalized by the surface buoyancy flux. H is the equilibrium depth without upwelling,  $h_{\bullet}$  the equilibrium depth with upwelling and  $h_{\bullet \bullet}$  the equilibrium depth with atmospheric feedback.

given by (3.5) reproduces remarkably well the mixed layer depth in the eastern and central equatorial Pacific, with a shallow mixed layer in the eastern part, a deep mixed layer in the central part, and a sharp transition in between (Garwood et al., 1985 b). Note that the mixed layer temperature does not have an equilibrium value. It continually increases due to the positive surface heat flux.

In the presence of upwelling, w > 0, the one-dimensional mixed layer equations have the steady solution

$$e = w (4.3 a)$$

$$\Delta T = \frac{Q}{\rho_0 C_p} \frac{1}{w} \tag{4.3 b}$$

$$h = h_* = \left(\frac{2}{L} + \frac{\varepsilon_0}{u_*^3}\right)^{-1} \tag{4.3 c}$$

where entrainment balances upwelling, and the surface heat flux balances the entrainment heat flux. The mixed layer temperature is inversely proportional to

the upwelling velocity w. The larger the upwelling velocity the colder the mixed layer. The mixed layer or sea surface temperature is hence a good indicator of the upwelling velocity. The dependence of the mixed layer depth  $h_*$  on the background dissipation is shown in Figure 2. For  $\epsilon_0=0$  the mixed layer depth is half the Obukhov length, a fact also recognized by Schopf and Cane (1983). The most important result is, however, that the mixed layer depth  $h_*$  is determined by the thermodynamic parameters  $u_*$ , B and  $\epsilon_0$ , and independent of the magnitude of the upwelling velocity w. This is in contrast to the results of the dynamic models. The magnitude of w determines, however, the rate at which the equilibrium depth  $h_*$  is approached from arbitrary initial conditions (Müller  $et\ al.$ , 1984).

The case of downwelling, w < 0, is similar to the case of no vertical advection. There is no entrainment. The mixed layer depth is given by H and the mixed layer temperature increases. The only effect of downwelling is to create a thermocline below the mixed layer.

The discontinuous transition from the h=H solution for  $w \le 0$  to the  $h=h_*$  solution for w>0 is a consequence of the asymmetric interaction between vertical velocity and entrainment. In the model of Schopf and Cane (1983) this discontinuity is the primary reason for the formation of a surface front away from the equator, with a cold shallow mixed layer on the upwelling side of the front and a warm and deep layer on the downwelling side. The discontinuity can be smoothed by including additional processes. Time-dependence is discussed in detail in Müller et al. (1984). In this paper we consider meridional advection and atmospheric feedback.

#### MERIDIONAL ADVECTION

Consider the response of the mixed layer to a prescribed steady, zonally uniform, meridional circulation v = v(y,z) and w = w(y,z) which satisfies the continuity equation  $\partial_y v + \partial_z w = 0$ . The equations which must be solved are the turbulent kinetic energy equation (3.3) and

$$\overline{v}\frac{d}{dy}T = -\frac{\Delta T}{h}e\theta(e) + \frac{Q}{\rho_0 C_p}\frac{1}{h}$$
 (5.1 a)

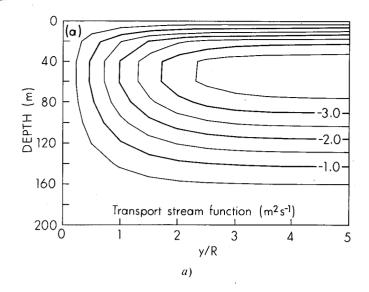
$$v\frac{d}{dy}h = e - w \quad \text{at} \quad z = -h \tag{5.1 b}$$

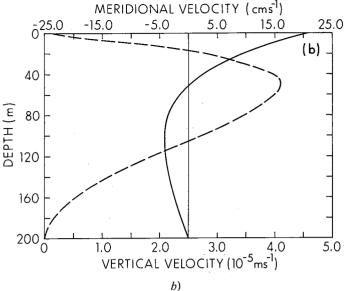
where

$$\overline{v}(y) = \frac{1}{h(y)} \int_{-h(y)}^{0} dz \, v(y, z) \,. \tag{5.2}$$

For simplicity we will assume that the temperature  $T_I$  of the fluid below the mixed layer, the heat flux Q and the friction velocity  $u_*$  are constant. Instead of v and w we can also use  $\bar{v}$  and  $\bar{w} = h \frac{d}{dv} \bar{v}$  in (5.1 b).

To model the equatorial upwelling we consider circulations with poleward surface flows and equatorward subsurface flows, representing the Ekman outflow and the geostrophic inflow, respectively. We will neglect any cross equatorial flux and assume the circulation to be symmetric about the equator. We are





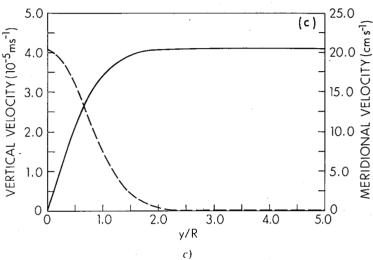


Figure 3
Meridional circulation for the advective mixed layer model: a) contours of the transport streamfunction; b) vertical profile of the upwelling velocity at the equator (dashed line, lower abscissa) and of the meridional velocity at the poleward boundary (solid line, upper abscissa); c) meridional profile of the upwelling velocity at a depth of 50 m (dashed line, left ordinate) and of the meridional velocity at the surface (solid line, right ordinate).

also not concerned about the compensating downwelling far away from the equator and will hence assume that the vertical velocity field decreases monotonically from a maximum value at the equator to zero far away from the equator. The streamline pattern and vertical and meridional profiles of our standard meridional circulation are shown in Figure 3. The circulation represents Ekman outflow above  $z=-50\,\mathrm{m}$  and geostrophic inflow below.

The total transport is  $4 \text{ m}^2\text{s}^{-1}$  and the maximum upwelling velocity  $w = 4 \cdot 10^{-5} \text{ ms}^{-1}$ . Additionally, we assume that the vertical velocity field is positive at z = -H and above. The actual mixed layer depth h will hence be smaller than the maximum depth H and the mixed layer will be in an entraining mode, e > 0. Since  $T_1$  is constant the temperature and interface equation combine to the heat equation

$$\frac{d}{dy}\left(\overline{v}h\Delta T\right) = \frac{Q}{\rho_0 C_p} \tag{5.3}$$

which relates changes in the meridional heat flux to surface heating. This heat equation is easily integrated and yields

$$\Delta T = \frac{Q}{\rho_0 C_p h \overline{v}} y \tag{5.4}$$

where we have used the fact that Q is constant and that there is no heat flux across the equator. The rate of entrainment can then be inferred from the turbulent kinetic energy equation as

$$e = \frac{\overline{v}L}{yH} (H - h) \tag{5.5}$$

and the interface equation takes the form

$$\bar{v}\frac{dh}{dy} = \frac{\bar{v}L}{y\bar{H}}(H-h) - \bar{w}$$
 (5.6)

Note that (5.6) is an implicit equation for h since  $\bar{v}$  and  $\bar{w}$  depend on h.

The solution for our standard circulation is shown in Figure 4. The thermodynamic parameters are  $Q=100 Wm^{-2}, \quad u_*=10^{-2} ms^{-1}$  and  $\epsilon_0=0,$  making H=L=40 m. The interior temperature is  $20^{\circ} C$ . The mixed layer deepens from h(0)=20 m at the equator to  $h(\infty)=40$  m far away from the equator. At the equator the mixed layer temperature is about  $1^{\circ} C$  warmer than the interior temperature. One Rossby radius of deformation away from the equator the mixed layer temperature is about  $3^{\circ} C$  warmer. All these numbers are fairly reasonable for the eastern Pacific. By using the appropriate thermo-

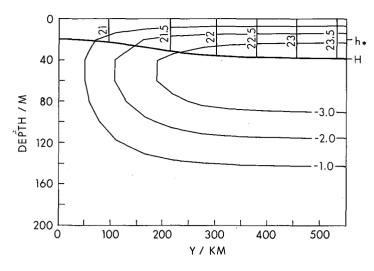


Figure 4 Transport streamfunction (in  $ms^{-2}$ ), and computed mixed layer depth and temperature (in  ${}^{\circ}C$ ) for advective model. The transport streamfunction has a meridional scale of R=125 km. The mixed layer deepens from the equilibrium value with upwelling,  $h_{\bullet}$ , at the equator to the equilibrium value without upwelling, H, far away from the equator. The mixed layer temperature increases away from the equator. The interior temperature in  $20^{\circ}C$ .

dynamic parameters one can also reproduce the mixed layer depth and temperature in the central parts of the equatorial Pacific (Garwood *et al.*, 1985 b).

In general, the solution for the mixed layer depth increases from  $h(0) = h_*$  at the equator to  $h(\infty) = H$ far away from the equator. The mixed layer depth hence changes from the equilibrium solution with upwelling at the equator to the equilibrium solution without upwelling far away from the equator. This transition is independent of the magnitude of the velocity field or total transport. Multiplying the velocity field by a constant factor does not change equation (5.6) for the mixed layer depth. Furthermore, similar velocity fields yield similar solutions for the mixed layer depth in the sense that if v and w are functions of  $\tilde{y} = y/\tilde{R}$  then h is a function of  $\tilde{y}$  only. The meridional scale of the transition from h, to H is hence set by the meridional scale of the circulation, only slightly modified by the thermodynamic parameters (see Fig. 5).

The mixed layer temperature at the equator is given by

$$\Delta T(0) = \frac{Q}{\rho_0 C_p \bar{w}} \tag{5.7}$$

which is the equilibrium solution with upwelling. Far away from the equator the temperature increases with a gradient

$$\frac{d}{dy} \Delta T(\infty) = \frac{Q}{\rho_0 C_p H \, \overline{v}(\infty)} \tag{5.8}$$

which is the equilibrium solution without upwelling. The normalized temperature gradient  $\frac{d}{dy}\Delta T(y)/\frac{d}{dy}\Delta T(\infty)$  is again independent of the

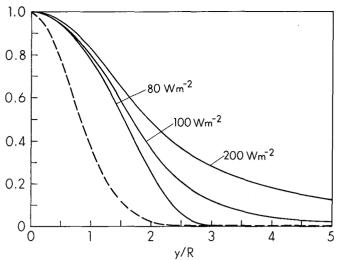


Figure 5 Meridional profile of the normalized interface displacement  $\eta = \frac{H-h}{H-h_{\star}}$  for three different surface heat fluxes (solid lines) and of the vertical velocity normalized by its value at the equator (dashed line).

magnitude of the velocity field and similar for similar velocity fields. The dependence on the thermodynamic parameters is small (see Fig. 6).

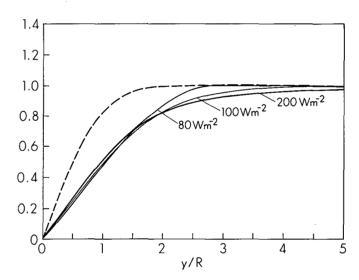


Figure 6
Meridional profile of the temperature gradient normalized by its far field value for three different surface heat fluxes (solid lines) and of the meridional velocity normalized by its far field value (dashed line).

# ATMOSPHERIC FEEDBACK

The discontinuous behavior of the one-dimensional mixed layer solution can also be removed by introducing an atmospheric feedback. This approach was first explored by Hughes (1980). He assumes that the heat flux depends on the mixed layer temperature via Haney's (1971) relation

$$\frac{Q}{\rho_0 C_p} = -\gamma \left( T - T_A \right) \tag{6.1}$$

where  $\gamma$  is Haney's constant and  $T_A$  an equilibrium temperature the ocean would possess if it were not exchanging heat with the atmosphere. For tropical regions Haney suggested  $T_A\approx 30~^{\circ}C$  and  $\gamma=10^{-5} \text{ms}^{-1}.$  With this form of Q the one-dimensional mixed layer equations have an equilibrium solution where entrainment balances upwelling (e = w) and

$$\Delta T = \frac{Q}{\rho_0 C_p} \frac{1}{w} = \frac{\gamma}{\gamma + w} (T_A - T_I)$$
 (6.2 a)

$$h = h_* = \frac{u_*^3}{\varepsilon_0 + \frac{w\gamma}{\gamma + w} \alpha g (T_A - T_I)}, \qquad (6.2 b)$$

$$\frac{Q}{\rho_0 C_p} = \frac{w\gamma}{\gamma + w} \left( T_A - T_I \right). \tag{6.2 c}$$

This is the equilibrium solution (4.3) with the temperature  $T_A$  prescribed instead of the heat flux Q. For  $\epsilon_0=0$ , equations (6.2) reduce to Hughes' results. Hughes evaluates his expressions for a vertical velocity w=w(y) which he calculates from a non-advective 1 1/2-layer reduced gravity model with Rayleigh friction. Solutions for different upwelling velocities are shown in Figure 7. They again reproduce reasonably well the observed structure. The values of  $\Delta T$  and h at the equator depend on the upwelling velocity. Far away from the equator the temperature approaches  $T_A$  and the mixed layer depth continues to increase indefinitely. This indefinite increase is due to Hughes' neglect of any background dissipation. If background dissipation is included the mixed layer deepens to a depth

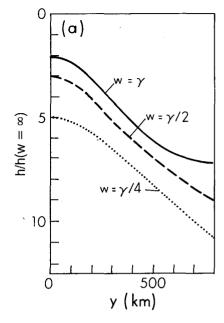
$$h(\infty) = h_{**} = \frac{u_*^3}{\varepsilon_0} \tag{6.3}$$

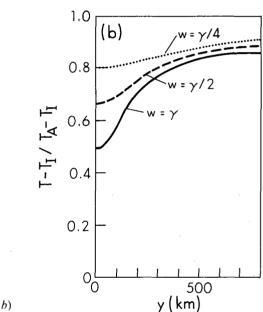
where shear production of turbulent energy is balanced by background dissipation. This limiting depth  $h_{**}$  is also included in Figure 2 which shows that  $h_{**} > H$ .

In general, the solution for the atmospheric feedback model depends on the upwelling velocity because the upwelling velocity determines the heat flux and hence the Obukhov length scale L.

#### **CONCLUSIONS**

We have compared three models of the meridional structure of the mixed layer near the equator. Two of the models are drawn from the literature, the advective model is our own contribution. All of these models neglect zonal gradients and regard the meridional mixed layer structure as the response to local atmospheric forcing and to a locally driven meridional circulation. All models can be tuned to reproduce fairly well the observed structure but differ significantly in their sensitivity to changes in external fields and parameters. One such difference can be seen in Figure 8 which shows the mixed layer depth at the equator as a function of the upwelling velocity. The interface displacement increases linearly with increasing w for the reduced gravity model, is independent of w for the advective model, and increases to a limiting value for the feedback model. For the reduced





a)

Figure 7 Meridional profiles as calculated from the feedback model of Hughes: a) mixed layer depth normalized by  $h(w=\infty)=26~m$ ; b) mixed layer temperature normalized by  $T_A-T_I=5^{\circ}C$ . The three cases correspond to different values of the upwelling velocity at the equator (after Hughes, 1980).

gravity model the dependence on w is a direct one, for the feedback model an indirect one since w determines the heat flux and hence the Obukhov length scale.

The model differences are even more pronounced when the responses to changes in the windstress are considered. An increase in the strength of the easterlies leads to an increase in the friction velocity u<sub>\*</sub> and to an increase in the upwelling velocity w. This leads to a shallowing of the mixed layer in the reduced gravity model, and to a deepening in the advective and feedback model. The deepening is weaker in the feedback model since the effects of increased w counteract the effects of increased u<sub>\*</sub>. The differences in the mixed layer temperatures are less dramatic. By its nature the reduced gravity model does not predict any temperature changes. The feedback model pre-

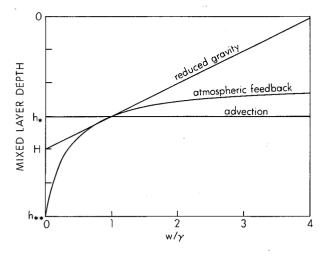


Figure 8 Mixed layer depth at the equator as a function of upwelling velocity for three different models. The reduced gravity model assumes a Newtonian cooling coefficient  $r=4\gamma/H$ , the advective model a buoyancy flux  $B=\varepsilon_0$ , and the feedback model a temperature difference  $\gamma \alpha g(T_A-T_I)=2\varepsilon_0$ . These parameters are chosen so that all models predict the same mixed layer depth for  $w=\gamma$ .

dicts less cooling than the advection model, although this is primarily a consequence of the different formulations of the heat flux, being constant in one case and proportional to T- $T_A$  in the other. The advective and feedback model also differ in their far field solutions. If upwelling ceases the advection model adjusts to a constant temperature gradient (if v becomes a constant) whereas the feedback model adjusts to a constant temperature.

These model differences show that we need to correctly formulate the thermodynamic processes of mixing and heating if we wish to understand and predict the response of the mixed layer to changes in the atmospheric and interior forcing fields.

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#### REFERENCES

Alexander R. C., Kim J. W., 1976. Diagnostic model study of mixed-layer depths in the summer North Pacific, J. Phys. Oceanogr., 6, 293-298.

Busalacchi A., O'Brien J. J., 1981. Interannual variability of the equatorial Pacific in the 1960's, J. Geophys. Res., 86, 10, 901-10, 907.

Garwood R. W., 1977. An oceanic mixed layer model capable of simulating cyclic states, J. Phys. Oceanogr., 7, 455-468.

Garwood R. W. Jr., Gallacher P. C., Muller P., 1985 a. Wind direction and equilibrium mixed layer depth: general theory, J. Phys. Oceanogr., 15, 1325-1331.

Garwood R. W. Jr., Muller P., Gallacher P. C., 1985 b. Wind direction and equilibrium mixed layer depth in the tropical Pacific Ocean, J. Phys. Oceanogr., 15, 1332-1338.

Haney R. L., 1971. Surface thermal boundary condition for ocean circulation models, *J. Phys. Oceanogr.*, 1, 241-248.

Hughes R. L., 1980. On the equatorial mixed layer, *Deep-Sea Res.*, 27A, 1067-1078.

**Kraus E. B., Turner J. S.,** 1967. A one-dimensional model of the seasonal thermocline: II. The general theory and its consequences, *Tellus*, 19, 98-106.

McCreary J. P. Jr., 1983. A model of tropical ocean-atmosphere interaction, *Month. Weath. Rev.*, 111, 370-387.

Moore D. W., Philander S. G. H., 1977. Modelling of the tropical oceanic circulation. Chapter 8, in: The sea: ideas and observations on progress in the study of the seas, edited by E. D. Goldberg, 319-361.

Muller P., Garwood R. W., Jr., Garner J. P., 1984. Effect of vertical advection on the dynamics of the oceanic surface mixed layer, *Annal. Geophys.*, 2, 387-398.

Pacanowski R. C., Philander S. G. H., 1981. Parameterization of vertical mixing in numerical models of tropical oceans, *J. Phys. Oceanogr.*, 11, 1443-1451.

Schopf P. S., Cane M. A., 1983. On equatorial dynamics, mixed layer physics and sea surface temperature, *J. Phys. Oceanogr.*, 13, 917.035

Wyrtki K., 1981. An estimate of equatorial upwelling in the Pacific, J. Phys. Oceanogr., 11, 1205-1214.

Wyrtki K., Kilonsky B., 1984. Mean water and current structure during the Hawaii-to-Tahiti shuttle experiment, J. Phys. Oceanogr., 14, 242-254.

Yoshida K., 1959. A theory of the Cromwell current and of the equatorial upwelling, J. Oceanogr. Soc. Jpn, 15, 154-170.