



1 Restratification at a California Current Upwelling Front, Part 1:

2 Observations

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ABSTRACT

16 A coordinated survey between a subsurface Lagrangian float and a ship
17 towed Triaxus profiler obtained detailed measurements of a restratifying sur-
18 face intensified front (above 30 m) within the California Current System. The
19 survey began as down-front winds incited mixing in the boundary layer. As
20 winds relaxed and mixing subsided, the system entered a different dynamical
21 regime as the front developed an overturning circulation with large vertical
22 velocities that tilted isopycnals and stratified the upper ocean within a day.
23 The horizontal buoyancy gradient was $1.5 \times 10^{-6} \text{ s}^{-2}$ and associated with
24 vorticity, divergence and strain that approached the Coriolis frequency. Es-
25 timates of vertical velocity from the Lagrangian float reached $1.2 \times 10^{-3} \text{ m}$
26 s^{-1} . These horizontal gradients and vertical velocities were consistent with
27 submesoscale dynamics that are distinct from the classic quasi-geostrophic
28 framework used to describe larger-scale flows. Vertical and horizontal gradi-
29 ents of velocity and buoyancy in the vicinity of the float revealed that sheared
30 currents differentially advected the horizontal buoyancy gradient to increase
31 vertical stratification. This was supported by analyses of temperature and
32 salinity gradients that comprised the horizontal and vertical stratification. Po-
33 tential vorticity was conserved during restratification at 16 m, consistent with
34 adiabatic processes. Conversely, potential vorticity near the surface (8 m)
35 increased, highlighting the role of friction in modulating near surface strat-
36 ification. The observed increase in stratification due to these submesoscale
37 processes was equivalent to a heat flux of 2000 W m^{-2} , an order of magni-
38 tude larger than the average observed surface heat flux of 100 W m^{-2} .

³⁹ **1. Introduction**

⁴⁰ The upper ocean contains rich variations in temperature (T), salinity (S), and therefore density
⁴¹ (ρ) that change over mesoscale (100 km) and submesoscale (0.1–10 km) distances. Features
⁴² associated with submesoscale density gradients can contain large horizontal velocity shears that
⁴³ induce vorticity, ζ , divergence, δ , and strain, α which are as large as the Coriolis parameter, f
⁴⁴ (Shcherbina et al. 2013). This implies Rossby numbers, $\text{Ro} = \zeta/f \sim 1$, and dynamics that separate
⁴⁵ submesoscale flows from the quasi-geostrophic (QG) framework used to describe mesoscale and
⁴⁶ large-scale flows. Submesoscale features in the upper ocean have small length-scales yet strong
⁴⁷ horizontal gradients in the presence of low stratification, and therefore can undergo instabilities
⁴⁸ or interact with inertia-gravity waves (IGW) and boundary layer turbulence on timescales that
⁴⁹ are faster than mesoscale flows (Boccaletti et al. 2007; Thomas 2012, 2005; McWilliams et al.
⁵⁰ 2015). Many of the dynamics associated with submesoscale flows withdraw available potential
⁵¹ energy stored at the front and induce large ageostrophic velocities that convert horizontal buoy-
⁵² ancy gradients into vertical gradients, increasing vertical stratification on timescales that compete
⁵³ with surface radiative forcing, pointing to the importance of submesoscale fronts on upper ocean
⁵⁴ stratification. As such, the effects of subgrid scale submesoscale frontal restratification has been
⁵⁵ parameterized for course resolution models (Fox-Kemper et al. 2011), though a full understand-
⁵⁶ ing of these phenomena is incomplete due to the challenges in obtaining observations that capture
⁵⁷ frontal slumping.

⁵⁸ Studies have identified an abundance of submesoscale features in influencing the upper ocean
⁵⁹ buoyancy budget (e.g. Rudnick (1999); Mahadevan et al. (2012); Hosegood et al. (2006)). Ob-
⁶⁰ taining detailed observations of submesoscale processes is inherently difficult due to the need for
⁶¹ high-resolution scalar and velocity fields (0.1–1 km) over a large spatial domain (10–100 km)

within short (superinertial, i.e. less than the inertial period, $T_i = 2\pi/f$) timescales. Additionally, submesoscale flows have spatial and temporal scales comparable to unbalanced IGW, such that surveys designed to focus on submesoscale temporal and spatial scales alias wave motions. Larger mesoscale surveys of fronts are particularly designed to smooth out aliased IGW (i.e., Rudnick (1996)) and not resolve submesoscale variability. As such, a common approach is to evaluate regions with many sharp gradients within a small domain in a statistical sense (Shcherbina et al. 2013; Mahadevan et al. 2012; Thompson et al. 2016; Buckingham et al. 2016). This manuscript presents data from a Lagrangian survey that captured the evolution of a single submesoscale surface intensified front as strong ageostrophic flows, with large vertical shears and vertical velocities, tilt the front over and stratify the mixed layer (ML) within one day.

2. Data Collection

The data were collected in the California Current System (CCS) 4 – 5 August 2006, yearday (yd) 216–217, as part of the ONR Assessing the Effects of Submesoscale Ocean Parameterizations (AESOP) program. On 30 July 2006 (yd 212), northerly wind stress increased off the coast near Monterey Bay from near zero to 0.5 N m^{-2} over the course of two days. The along shore winds set up an Ekman transport offshore with an associated upwelling index of 150 (typical values range 100-200 during upwelling, <https://www.pfeg.noaa.gov/products/las.html>), and sea surface temperature (SST) that revealed cold water upwelling from the deep along the coast (Fig. 1).

An energized mesoscale field associated with the southward California Current stirred the upwelled waters with the warmer fresher surface waters offshore to create multiple smaller fronts and filaments. The front sitting at the edge of upwelled waters became the target of coordinated surveys that captured different phases of the frontal evolution. The first phase was 1–3 August

85 2006 (yd 213–215) as northerly winds aligned down-front continuously homogenized the upper
86 30 m (not discussed here). The second phase, 4–5 August 2006 (yd 216–217), occurred as winds
87 decreased rapidly and the upper 30 m stratified. This restratification phase is the focus of this
88 study.

89 During each phase, the front was surveyed by two ships simultaneously. *R/V Wecoma* performed
90 a mesoscale survey of zonal transects set 11 km apart while towing a SeaSoar profiling vehicle
91 (i.e., mesoscale survey, Fig. 1). Details of the mesoscale survey can be found at Pallàs-Sanz et al.
92 (2010a,b) and Johnston et al. (2011), which characterize the vertical velocity and turbulence of
93 the front on scales of 10–40 km. Starting 30 hr later, *R/V Melville* surveyed around a drifting
94 Lagrangian float (D'Asaro 2003) using a Triaxus profiler, conducting a Lagrangian survey on a
95 scale of 5 km. The mesoscale structure of the upwelling region evolved considerably between the
96 two phases, with the front changing from a north-south orientation in phase one that developed
97 cyclonic curvature in phase two. This was coincident with several mesoscale eddies in the sur-
98 rounding regions of phase two that acted to squeeze the front together (Pallàs-Sanz et al. 2010b).

99 The Lagrangian float was equipped with two Seabird sensors 1.4 m apart on the top and bottom
100 of the hull that collected measurements of pressure (P), temperature (T), and salinity (S) every 30 s
101 which allowed for an estimation of density (ρ) and buoyancy, $b = -g\rho/\rho_o$, where g is gravitational
102 acceleration and ρ_o is a reference density of 1024 kg m^{-3} . When neutrally buoyant, the float was
103 designed to follow the average three-dimensional motion of the water immediately surrounding it.

104 The flow-through system on *R/V Melville* collected T and S at $\sim 5 \text{ m}$ depth every 30 s providing
105 a horizontal resolution of roughly 100 m. Shipboard meteorological measurements were used to
106 estimate air-sea fluxes based on the COARE 3.5 bulk formula.

107 Triaxus was equipped with temperature and conductivity sensors, chlorophyll fluorometer, trans-
108 missometer, dissolved oxygen sensor, as well as 300 kHz (down-looking) and 1200 kHz (up-

¹⁰⁹ looking) RDI ADCPs. Triaxus profiled between 4 and 140 m depth with a vertical speed of 1 m
¹¹⁰ s⁻¹ and horizontal speed of roughly 3 m s⁻¹, providing horizontal resolution of 800 m near the top
¹¹¹ and bottom of the profiles and 400 m in the middle of the profiles. Shear from Triaxus ADCP was
¹¹² estimated using a technique similar to the inverse method for processing measurements collected
¹¹³ with lowered ADCPs (Visbeck 2002).

¹¹⁴ Satellite SST was used to locate the front followed by an initial Triaxus transect that identified
¹¹⁵ the cross-frontal structure in depth. The Lagrangian float was placed in the center of the front tar-
¹¹⁶ geting the 23.8 kg m⁻³ isopycnal. The float's position was tracked acoustically as it was advected
¹¹⁷ downstream by the frontal flow using a TrackPoint II USB system operating at 15 kHz, allowing
¹¹⁸ the ship to survey around the float while towing the Triaxus profiler. The survey lasted 30 hr as the
¹¹⁹ float traveled roughly 50 km along the front. During this time, the ship circled the float 31 times,
¹²⁰ taking about one hour to complete loops 3–5 km in diameter (Fig. 1).

¹²¹ The circular sampling pattern was well suited for calculating means and first derivatives (but not
¹²² higher) of tracers and vector fields. The objective of the data processing was to project the frontal
¹²³ structure from Triaxus onto a transect traced by the float trajectory. Tracer and velocity data were
¹²⁴ averaged into 4 m vertical bins. Cross-frontal sections were defined by density extrema at 4 m (e.g.
¹²⁵ density maximum sets an eastern edge and density minimum sets the western edge) for a total of
¹²⁶ 62 cross-front transects. Loops were defined by two consecutive sections, and each section was
¹²⁷ included in two loops for a total of 61 loops, therefore reducing bias that may result from choice
¹²⁸ in loop definition (Fig. 2). Data in each loop and each vertical level were applied to a plane-fit,
¹²⁹ $\mathbf{y} = \mathbf{Ax}_1 + \mathbf{Bx}_2 + C$ using a least squares estimate (Deep 2005)

$$\mathbf{F} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \quad (1)$$

130 where $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2)$, denotes geographic position with \mathbf{x}_1 and \mathbf{x}_2 being the meridional and zonal
131 distances, respectively. The variable to be fit is \mathbf{y} and \mathbf{F} contains the gradients (A, B) and averages
132 (C). A 95% confidence interval (ε) associated with the least squares estimation is

$$\varepsilon = c \sqrt{(\mathbf{X}^T \mathbf{X})^{-1} \left(\frac{1}{m} \sum_{i=1}^m (\mathbf{y} - \mathbf{x}\mathbf{F})_i^2 \right)} \quad (2)$$

133 where m is the number of data points in each loop to be fit and c is the t-test critical value for m .
134 This provides an estimate of error for A and B (i.e. gradients). An example (Fig. 2b) shows a
135 clear slope in the density field that was captured by the least squares plane fit. The results were
136 smoothed further by averaging gradients and means for three consecutive loops ($n-1, n, n+1$),
137 that spanned two hours of data. (Fig. 2 a).

138 Gradients were used to calculate vorticity, $\zeta = \partial v / \partial x - \partial u / \partial y$, divergence, $\delta = \partial u / \partial x + \partial v / \partial y$,
139 and strain $\alpha = \sqrt{(\partial u / \partial x - \partial v / \partial y)^2 + (\partial v / \partial x + \partial u / \partial y)^2}$ (along with propagated errors), which
140 were essential for characterizing the submesoscale. This provided a depth vs. time (or along-front
141 distance) view of the water surrounding the float as it was advected by the frontal flow. At times,
142 it is more convenient to present results referenced to the frontal orientation. In this case, gradients
143 and velocities were rotated in the direction of $\nabla_h b$ at 4 m, where positive cross-front (xf) implies
144 down gradient and positive along-front (af) is along the direction of geostrophic shear.

145 Objective maps of tracer distributions were produced from the Traixus survey (Le Traon 1990;
146 Bretherton et al. 1976) using a Gaussian covariance. Traditionally, anisotropic length-scales are
147 chosen for mapping frontal systems. This approach was not adopted here, instead correlation
148 length-scales were set to the approximate loop size ($L_x = L_y = 5$ km) to minimize along front
149 changes due to temporal evolution. For example, as the wind stops and the float turns eastward,
150 a 5 km swath may contain several loops and up to five hours of data, highlighting the poten-
151 tial influence of time-space aliasing inherent in spatially smoothing such rapidly evolving fronts.

¹⁵² Nonetheless, objective maps reveal essential qualitative information about the frontal structure.

¹⁵³ Results presented here use the loop method outlined above, unless noted otherwise.

¹⁵⁴ The float-following reference frame allow for a Lagrangian analysis of the front, where measured rates of change can be interpreted as Lagrangian rates of change. This assumption was evaluated by estimating the change in density due to advection as $\Delta\rho_{ADV} = \int_{t_o}^{t_i} (u - u_{tri}) (\partial\rho/\partial x) + (v - v_{tri}) (\partial\rho/\partial y) dt$, where t_o is the beginning of the survey (yd 216.0), u_{tri} and v_{tri} are velocities of the survey calculated from the mean location and time of each loop, and u and v are the measured velocities of the flow at 4 m. During the survey, $\Delta\rho_{ADV}$ oscillated between ± 0.1 kg m⁻³, with an average $\Delta\rho_{ADV} = 0.04$ kg m⁻³. This can be compared to the $\Delta\rho$ spanned in each loop of 0.5 kg m⁻³. Oscillations in $\Delta\rho_{ADV}$ could be attributed to the position of loop relative to a Lagrangian parcel, but on average this does not contribute significantly to the material derivative.

¹⁶³ The assumption of Lagrangian rates of change verified above is true only in layers that move in the advective frame of the float, an assumption that may not be valid in depth as the front evolves. The ability to assume Lagrangian rates of change at different depths was assessed by integrating shear in depth and time at each vertical bin, such that $d^{xf} = \int_{t_o}^{t_i} \int_{z_b}^{z_t} (\partial u^{xf}/\partial z) dz dt$, where t_o is the beginning of the survey (yd 216.0), z_t is the upper bin of Triaxus data (4 m), and z_b is the depth to be considered (Fig. 3). Therefore, d^{xf} is the distance a parcel of water at depth (z_b) traveled relative to 4 m, the closest resolved depth to the float during the time of frontal evolution (see section 4) and is a test of whether the deformation of the initially-surveyed volume is beyond the subsequent survey. For example, at yd = 216.7, a parcel of water at 20 m cannot be approximated by Lagrangian rates of change, and advective terms cannot be ignored. Similarly, d_{af} can be estimated from along-front shear (not shown), and is less than d^{xf} (consistent with section 4b). Flows near the surface (i.e., above 12 m) were approximated as Lagrangian rates of change throughout the survey.

¹⁷⁶ **3. Scale Resolution**

¹⁷⁷ Submesoscale motions are energized near the surface (Callies and Ferrari 2013; Shcherbina
¹⁷⁸ et al. 2013; Thompson et al. 2016) and characterized by small, sharp gradients of buoyancy and
¹⁷⁹ velocity with typical length scales of 0.1–10 km and $\zeta \approx f$ that evolve on inertial timescales (T_i
¹⁸⁰ = $2\pi/f = 20.3$ hr). Resolving these space and time scales present an observational challenge,
¹⁸¹ yet are essential for characterizing the structure of the upper ocean. The influence of observa-
¹⁸² tion resolution can be readily seen by comparing tracers, velocities, and their respective gradients
¹⁸³ resolved by AVISO (Archiving, Validation and Interpretation of Satellite Oceanographic Data,
¹⁸⁴ <http://www.marine.copernicus.eu>; >100 km), the mesoscale survey (12 km) and the La-
¹⁸⁵ grangian survey (5 km) (Table 1).

¹⁸⁶ Objective maps of surface density from the mesoscale and Lagrangian surveys exhibit differ-
¹⁸⁷ ences in intensity, structure, and position of the same front observed within 30 hours of each other
¹⁸⁸ (Fig. 4). The surveys aligned initially, yet deviate later as contours of the front between the two
¹⁸⁹ surveys diverged and wavelike features (here referred to as meanders) resolved by the Lagrangian
¹⁹⁰ survey were smoothed by the mesoscale survey. The ~10 km wavelength meanders in the objec-
¹⁹¹ tive maps were also apparent in the raw data (not shown), and resolved in less than 10 hr by the
¹⁹² Lagrangian survey, faster than the local inertial period, $T_i = 20.3$ hr. This suggests the meanders
¹⁹³ were either small scale physical features or superinertial motions and not associated with aliased
¹⁹⁴ tides or inertial motions. Additionally, the spatial scale of along-front variability was smaller
¹⁹⁵ than the objective map correlation length scales (10 - 50 km) often used for mesoscale surveys
¹⁹⁶ (Pallàs-Sanz et al. 2010a; Rudnick 1996).

¹⁹⁷ Tracer and velocity gradients increased with higher spatial resolution (Table 1). This is seen
¹⁹⁸ qualitatively as isopycnals in the Lagrangian survey squeezed together (Fig. 4) compared with the

mesoscale survey, consistent with a factor of two difference in $\nabla_h b$ between the surveys (Table 1). The sharper front in the Lagrangian survey was consistent with larger ζ , δ , and α , of $O(f)$ (Table 1). The fields observed by AVISO and the mesoscale survey catalog a larger-scale flow described by classic QG. This framework predicts a rapid decay in energy and vorticity in the submesoscale, associated with a velocity spectral slope of k^{-3} . The increase in gradients at smaller scales observed by the Lagrangian survey is more consistent with strong stirring and frontogenesis that sharpens lateral buoyancy gradients near the surface. This results in a shallower velocity spectral slope of k^{-2} , as previously theorized (Blumen 1978; Klein et al. 2008; Kunze 2019) and observed (Shcherbina et al. 2013; Callies and Ferrari 2013). As such, lower estimates of ζ , δ , α at larger spatial scales are not simply a result of the smoothed submesoscale field, but are ultimately associated with different dynamics. For example, strain estimated from AVISO were purely geostrophic and resulted from the mesoscale eddy field that acted to stir gradients at the surface and squeeze this front together, an essential ingredient for the submesoscale. On top of this background flow was a submesoscale α implying local processes acting to strain the front (see section 4b).

Finally, the deviation between the two surveys illustrates time space aliasing challenges of observing rapidly evolving submesoscale features and need to be considered when interpreting such data. Here, the Lagrangian survey clearly illustrates the importance of resolving small scales as the sharp gradients observed here are an important feature of submesoscale flows.

218 **4. Frontal Evolution**

219 *a. Three stages of evolution*

220 The initial Triaxus transect revealed the vertical structure of the surface intensified submesoscale
221 front above a pycnocline of 30 m (Fig. 5). The entire front was broad with horizontal changes
222 in density of 0.9 kg m^{-3} over 20 km, with evidence of sloping isopycnals deep into the interior
223 down to 150 m. Embedded in the broad buoyancy gradient was a sharper front with a poten-
224 tial density anomaly difference $\Delta\sigma$ of 0.44 kg m^{-3} over 4 km between the $24-24.4 \text{ kg m}^{-3}$ σ
225 isopycnals with a $|\nabla_h b| = 1 \times 10^{-6} \text{ s}^{-2}$. This sharper portion of the front became the target of the
226 Lagrangian survey. The entire frontal extent was not captured by the 3–5 km loop sampling pat-
227 tern aimed to focus on the sharpest part of the front. The evolution of stratification can be divided
228 into three stages. Stage 1: down-front winds, turbulent mixing, and a homogeneous boundary
229 layer (BL). Stage 2: Low winds, diurnal warming, frontal slumping and increased stratification.
230 Stage 3: Night-time surface cooling, increased winds, rapid near surface restratification, and float
231 subduction (Fig. 6).

232 Stage 1 (yd 216–216.3): Northerly winds that began five days prior had peaked at 0.5 N m^{-2}
233 18 hr before the start of the survey. Stage 1 began with a 0.23 N m^{-2} down front wind stress that
234 decreased to 0.04 N m^{-2} within 6 hr. The float was placed slightly dense (east) of the front at
235 the 24.3 kg m^{-3} isopycnal and began traveling west towards the light side of the front. During
236 this time, the sharpest part of the front was only partially resolved. Isopycnals in the upper 30
237 m were steep as the upper ocean was vertically homogeneous with strong horizontal gradients of
238 buoyancy (Fig. 7a). For simplicity, the region above 30 m will be referred to as the mixed layer
239 (ML), though this region was not well mixed throughout the survey.

240 Stage 2 (yd 216.3–216.8): The heat flux changed from cooling to warming and the wind re-
241 mained less than 0.02 N m^{-2} . The float was trapped between 1–2 m such that the float’s antennae
242 was just below the surface, suggesting a decrease of turbulent mixing (Fig. 6 a, b). The float trajec-
243 tory slowed down and began to veer shore-ward (east, Fig. 4). At this time, and for the remainder
244 of the survey, the shipboard survey resolved the sharpest part of the front between 24–24.2 kg
245 m^{-3} . During this stage, the frontal flow increased and the vertical isopycnals that defined the front
246 squeezed closer together (Fig. 4) and began to tilt, stratifying the waters above the pycnocline
247 (Fig. 6 b, Fig. 7, Fig. 8).

248 Stage 3 (yd 216.8–217.3): The heat flux changed from net warming to cooling, and the wind
249 stress increased to 0.09 N m^{-2} and rotated to an upfront orientation (Fig. 6 c). Along-front
250 wavelike meanders appeared in the survey and the float downwelled along isopycnals. In the
251 classic 1-D view, night-time cooling and winds would erode the day-time stratification (e.g. Price
252 et al. (1986)). Here, stratification in the near surface layer strengthened as warm fresh water slid
253 over the cold salty side of the front (Fig. 7). The remainder of this manuscript aims to detail the
254 frontal evolution. It is shown that ageostrophic circulation, associated with strong vertical shear
255 and large vertical velocity, contributes to ML stratification.

256 *b. Horizontal buoyancy gradient and ageostrophic shear*

257 Horizontal buoyancy gradient was estimated using the loop method outlined in section 2 (Fig. 2)
258 as well as using the ship underway system assuming $|\nabla b_s| = |\Delta b / \Delta s|$, where Δb and Δs are changes
259 along the ship-track (Fig. 10). Lateral buoyancy gradients, $|\nabla b_s|$, were larger in magnitude than
260 estimated by loop method using Triaxus at 4 m by a factor of 1.7, as gradients revealed by the flow
261 through system ($\Delta s \sim 100 \text{ m}$) were not fully resolved by Triaxus with $\sim 800 \text{ m}$ resolution. Lateral
262 gradients of buoyancy from Triaxus were strongest at the surface, decreased with depth and were

almost non-existent below the pycnocline (100–140 m), consistent with an increasingly surface intensified front (Fig. 9). Never throughout the survey did the front become sub-resolution (i.e., smaller than 100 m), and generally maintained a frontal width of 600 m, smaller than the mixed layer Rossby radius of deformation $L_D = NH/|f|$ of 5 km, assuming $H = 30$ m and N^2 averaged over stages 2–3.

Thermal wind balance was evaluated by separating the vertical shear into geostrophic $\partial \mathbf{u}^g / \partial z = \hat{\mathbf{z}} \times (f^{-1} \nabla_h b)$ and ageostrophic ($\partial \mathbf{u}^a / \partial z = \partial \mathbf{u} / \partial z - \partial \mathbf{u}^g / \partial z$) components (Fig. 11). Here, vertical shear was rotated to the along-front (af) and across-front (xf) direction (referenced at 4 m, see section 2).

The front was only partially resolved during stage 1, yet was completely resolved by the start of stage 2. After this time, the front continued to strengthen by a factor of 2, with $|\nabla b_s|$ exceeding $2 \times 10^{-6} \text{ s}^{-2}$ at the surface (underway along track) within 12 hr. Throughout stages 2 and 3, the frontal structure resolved by the ship flow-through fluctuated from tight and organized to broad, and sometimes fragmented with multiple jumps in buoyancy gradient (Fig. 10). The increase in horizontal buoyancy gradient at the surface was not coincident with an increase in shear as along-front shear at 8 m remained close to zero until stage 3, when it began to approach geostrophic balance. This inhibition of total shear implies strong ageostrophic shear in the near surface that acted to oppose the frontal flow. Along-front shear at 16 m fluctuated with geostrophic shear, as ageostrophic shear, of about 0.005 s^{-1} , acted to oppose along-frontal flow. Cross-frontal shear at 16 m and 8 m behaved similarly, increasing before the onset of stage 2 and decreasing towards the end of the survey. It will be shown that this ageostrophic shear was responsible for increasing stratification at the front.

285 *c. Stratification*

286 Stratification in the near surface layer began to increase as turbulent mixing ceased and was co-
287 incident with day-time warming (see section 4a). Yet the evolution and distribution of stratification
288 throughout the mixed layer points to the importance of lateral processes through frontal slumping.
289 This is seen in the different cross-frontal structures of salinity between the beginning and end of
290 the survey (Fig. 7) and the horizontal spreading of the 24.4 isopycnal at different depths as the
291 front tilted over (Fig. 8). The distribution of the stratification was not uniform as deeper layers
292 began to stratify earlier than the surface layers (Fig. 6 b, Fig. 8 a).

293 The lateral slumping of isopycnals was imprinted on the T and S structure of the stratification.
294 Contributions of horizontal (frontal slumping) and vertical (i.e, turbulent mixing, vertical advec-
295 tion) to stratification changes can be decomposed into vertical and horizontal contributions of T
296 and S assuming a linear equation of state, $\rho = \rho_o + \rho_o(-\alpha_T(T - T_o) + \beta(S - S_o))$, such that

$$\Delta N^2 \approx g \left[\alpha_T \frac{\partial T^v}{\partial z} + \alpha_T \frac{\partial T^h}{\partial z} - \beta \frac{\partial S^v}{\partial z} - \beta \frac{\partial S^h}{\partial z} \right], \quad (3)$$

297 where $\alpha_T = 2.0 \times 10^{-4} \text{ K}^{-1}$ is the thermal expansion coefficient for seawater and $\beta = 7.5 \times 10^{-4}$
298 psu^{-1} is the haline contraction coefficient for seawater. Here, $\partial T^v/\partial z$, $\partial S^v/\partial z$ are the contribu-
299 tions from vertical processes, and $\partial T^h/\partial z$, $\partial S^h/\partial z$ are the vertical gradients due to horizontal
300 advection. The contribution from $\partial S/\partial z$ on N^2 was assumed to be from horizontal advection en-
301 tirely (precipitation and evaporation were negligible), and therefore $\partial S^v/\partial z = 0$. Estimating the
302 contribution from $\partial T^v/\partial z$ due to vertical processes and heat flux requires knowledge of small scale
303 turbulence and vertical velocity, and is difficult to calculate here. Yet $\partial T^h/\partial z$ can be estimated
304 using knowledge of the horizontal density structure through the density ratio (R),

$$R = \frac{\alpha_T \Delta T}{\beta \Delta S}. \quad (4)$$

305 During adiabatic slumping of isopycnals, horizontal changes in T and S are converted into vertical
 306 ones (Johnson et al. 2016). Assuming that the gradients of T and S are aligned (as in this case), that
 307 vertical changes in S are a result of horizontal advection (therefore $\partial S^h / \partial z$ is observed entirely),
 308 and R is conserved during this process, then

$$\frac{\partial T^h}{\partial z} = \frac{\beta}{\alpha_T} R^h \frac{\partial S^h}{\partial z} \quad (5)$$

309 where $R^h = \alpha_T \nabla_h T / \beta \nabla_h S$ (Fig. 8b). At 8 m, 80% of the vertical changes in T can be explained
 310 by $\partial T_h / \partial z$ and therefore tilted horizontal gradients, while the remaining 20% can be attributed to
 311 a combination of day-time solar warming during stage 2, vertical advection and turbulent mixing.
 312 Using $\partial T^h / \partial z$ and $\partial S^h / \partial z$ (but omitting $\partial T^v / \partial z$) in (3) provides an estimate of stratification,
 313 N^{2h} , that agrees with observed N^2 (Fig. 8 a) and support the conversion of horizontal gradients
 314 into vertical ones through frontal slumping.

315 Furthermore, changes in the vertical gradients of tracers as a result of differential advection by
 316 vertical shear can be quantified as

$$\frac{DC_z^{ADV}}{Dt} = -\frac{\partial C}{\partial x} \frac{\partial u}{\partial z} - \frac{\partial C}{\partial y} \frac{\partial v}{\partial z} \quad (6)$$

317 for C representing tracers T , S , b . The float provided a Lagrangian reference frame for the Triaxus
 318 data such that estimates of (6) were made with the advective terms contained within the mate-
 319 rial derivative (see section 2). Vertical gradients resulting from horizontal advection, $\partial T^{ADV} / \partial z$,
 320 $\partial S^{ADV} / \partial z$ and N^{2ADV} were calculated using (6) at 8 m, where the survey was considered La-
 321 grangian and where shear could be estimated by centered-finite-difference (Fig. 8). The ability
 322 of (6) to predict the increase in vertical gradients signifies that most changes in N^2 , $\partial T / \partial z$, and
 323 $\partial S / \partial z$ were due to horizontally sheared currents advecting tracers across the front. The contri-
 324 bution from vertical advection, $(\partial C / \partial z)(\partial w / \partial z)$ (calculated assuming continuity), added a 10%
 325 increase in stratification to (6). This value is within error of N^{2ADV} and associated with increased

uncertainty such that it was not included in (6). It is concluded that N^2 estimated from (6) and (3) in conjunction with positive cross-front ageostrophic shear present throughout the survey (Fig. 11) support the role of lateral advection of horizontal gradients for increasing stratification and is a major result of this study.

The increase in vertical stratification was used to estimate an equivalent vertical flux of buoyancy,

$$\mathcal{B}_{eq} = \frac{d}{dt} \int_{-H}^0 \int_{-H}^0 N^2 dz dz \quad (7)$$

Integrating total observed N^2 from $H = 30$ m gives $\mathcal{B}_{eq} = 9.58 \times 10^{-7} \text{ m}^2 \text{ s}^{-3}$ and a heat flux equivalent, $Q_{eq} = c_p \rho_o \mathcal{B}_{eq} / g \alpha_T$, of $Q_{eq} \sim 2000 \text{ W m}^{-2}$, where c_p is the heat capacity of seawater. This was an order of magnitude larger than the average heat fluxed onto the ocean surface during the stratification phases (2 and 3) of $Q_{avg} \sim 100 \text{ W m}^{-2}$.

d. Vorticity, divergence and strain

Vorticity, divergence, and strain were surface intensified and fluctuated throughout the survey (Fig. 9). All approached values of $O(f)$ near the surface, several times greater than values deeper below the pycnocline (100 - 140 m). In the ocean interior, inertia-gravity waves (IGW) dominate fluctuations in vorticity and divergence such that

$$\frac{D\zeta}{Dt} \approx -(f + \zeta) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right). \quad (8)$$

This relationship has also been shown to exist along meanders within larger frontal systems (Bower and Rossby 1989; Thomas 2008). To assess the relationship in (8), ζ and δ were averaged at the surface (4–20 m) and depth (100–140 m) and the right hand side was integrated in time to compare with ζ assuming a lagrangian reference frame. Below the ML, where horizontal buoyancy gradient was much less than at the surface (Fig. 9), these terms oscillated with a correlation of

346 0.60. This oscillatory pattern at depth was decoupled from the surface (Fig. 9), where the corre-
 347 lation at 4–20 m decreased to 0.17. The lack of correlation near the surface indicates that terms
 348 in the vorticity equation omitted in (8) were non-negligible in the observed flow. This can include
 349 tilting of horizontal vorticity or frictional torques, and suggest a complicated relationship between
 350 the sharp front, IGW and boundary layer dynamics.

351 A background strain field estimated from the mesoscale survey to be $0.3f$ (Pallàs-Sanz et al.
 352 (2010b)) was attributed to eddies in the surrounding mesoscale field. On top of this background
 353 strain, $O(f)$ strain was resolved by the Lagrangian survey (Table 1) that was particular to the
 354 local dynamics around the front, and was not captured by the mesoscale survey or AVISO. The
 355 influence of this strain field on ∇b_h is captured by the frontogenetic tendency equation associated
 356 with horizontal advection (Hoskins 1982)

$$F_{hadv} = \frac{1}{2} \frac{\mathrm{D}|\nabla_h b|^2}{\mathrm{D}t} \Big|_{hadv} = \left(-\frac{\partial b}{\partial x} \nabla_h u - \frac{\partial b}{\partial y} \nabla_h v \right) \cdot \nabla_h b \quad (9)$$

357 that includes both the geostrophic and ageostrophic component of the flow (Fig. 12). F_{hadv} was
 358 near zero during stage one. After wind forcing ceased, F_{hadv} fluctuated between frontogenetic
 359 and frontolytic between yd 216.2–216.6. During this time, $\nabla_h b$ steadily increased (Fig. 9). The
 360 largest values of F_{hadv} at the end of stage 2 and beginning of stage 3 were simultaneous with strong
 361 $\nabla_h b$. Although there was consistency between positive frontogenetic tendency and an increase in
 362 frontal strength, the tendency of $\nabla_h b$ cannot be explained by integrating (9) in time, including the
 363 increase at the beginning of stage 2 or the deterioration $\nabla_h b$ after yd 217. Large errors in F_{hadv} are
 364 expected with the multiple derivatives needed to compute (9), and may not represent the true F_{hadv}
 365 of the front. Additionally, turbulence and vertical velocity may induce a frontal response (Gula
 366 et al. 2014), that are not resolved here and can be frontolytic and counteract F_{hadv} .

367 The classic frontogenesis problem of Hoskins and Bretherton (1972) assumes the advection of
368 momentum by the ageostrophic flow is negligible following the semigeostrophic approximation.
369 Here, frontogenesis and frontal sharpening occurred in the presence of strong divergence as well as
370 large ageostrophic, cross front shears. Departures from classic frontogenesis have been explored
371 in context of submesoscale fronts by Shakespeare and Taylor (2013) and Barkan et al. (2019),
372 suggesting a regime of submesoscale frontogenesis in addition to that induced by external strain.
373 In particular, Barkan et al. (2019) explored frontogenesis in the presence of large convergence
374 and found cross frontal flows to have a reinforcing role on the frontogenetic sharpening rate.
375 A complete discussion of this observed front in context of different frontogenetic frameworks
376 would require isolating the relative contributions of the geostrophic and ageostrophic flows in
377 the frontogenetic function, which cannot be done in this data set (see section 4b). Nonetheless,
378 the ageostrophic cross front shears along with the increase in strain and divergence resolved by
379 the Lagrangian survey compared to the mesoscale survey and Aviso (Table 1) are characteristics
380 consistent with submesoscale frontogenesis.

381 The different horizontal gradients resolved by the mesoscale survey and the Lagrangian survey
382 lead to contrasting interpretations of frontogenesis. In particular, the sharpening of the front and
383 positive F_{hadv} observed by the Lagrangian survey was opposite than predicted by the mesoscale
384 survey (estimated using a generalized omega equation, Pallàs-Sanz et al. (2010a)) which deduced a
385 frontolytic circulation resulting from the frontal curvature and associated deformation field. Fronto-
386 togenesis was a key part of the Lagrangian survey as it strengthened the horizontal buoyancy
387 gradient and therefore the amount of stratification from horizontal slumping (i.e. through (6)).

388 *e. Vertical Velocity*

389 The float measured pressure and hence depth every 30 s, allowing for direct measurements of
390 vertical velocity. To minimize high-frequency motions from the float, a LOESS was applied to
391 30 min of the float's vertical position to obtain an estimate of vertical velocity. During stages 1
392 and 2 the float was ballasted buoyant and adjusted again before stage 3. During stage 1, the ver-
393 tical velocity and low stratification were consistent with boundary layer turbulence. After winds
394 decreased and boundary layer mixing subsided (stages 2 and 3), the float observed four down-
395 welling events between stratified layers (I–IV on Fig. 13), where the float trajectory implied a
396 downwelling from the dense side of the front under the lighter side of the front (Fig. 13 c). The
397 largest of these events was III and is discussed in detail.

398 At yd 216.8 the float's horizontal velocity slowed as it began to downwell at 1.3×10^{-3} mm s⁻¹
399 (120 m d⁻¹) across and under the warm side of the front (Fig. 7, Fig. 13 a, c). In the upper 4
400 m, the float traveled through changes in density and stratification, suggesting the initial sinking was
401 neither purely turbulent nor purely adiabatic. Below 4 m, the float's density remained constant as
402 it continued to downwell at $w = 0.7 \times 10^{-3}$ mm s⁻¹ (60 m d⁻¹). During this time, the float was
403 caught in an anticyclonic flow as it wrapped westward (Fig. 13a). Throughout the meander, the
404 float's vertical velocity slowed, nearing zero. At yd 217 the float was automatically set to profile
405 and no longer tracks the vertical velocity of the water.

406 The downwelling of the float in III occurred on the upstream side of cyclonic flow (Fig. 13a),
407 with $\zeta > 0$ and $\delta < 0$. This geometry of downwelling was consistent with frontal subduction
408 observed and modelled previously within larger frontal systems (e.g. Bower and Rossby (1989);
409 Lindstrom et al. (1997); Spall (1997)). Here, the subduction occurred in the presence of large

410 convergences and a cyclonic flow that could be tied to either along-front variability or IGW, both
411 which share similar space and time scales and are therefore complicated to separate.

412 The contributions of IGW and frontal dynamics could be achieved theoretically by solving the
413 Eliassen-Sawyer (ES) equation or the omega equation to obtain the balanced ASC. For exam-
414 ple, Mahadevan and Tandon (2006) used numerical simulation fields to solve the omega equation
415 and obtain the contribution of balanced dynamics to the total vertical velocity determined by the
416 simulation. The residual vertical velocity was then attributed to unbalanced motions. The ES or
417 omega formulation has been implemented in many mesoscale observations to obtain ACS (e.g.
418 ES - Thomas (2008), omega equation - Rudnick (1996), generalized omega equation - Pallàs-Sanz
419 et al. (2010a)). A challenge in this set of observations lies in capturing the nuanced structure of
420 buoyancy and momentum needed to constrain a submesoscale frontal ASC using these techniques.
421 This was made unfeasible as the Triaxus survey resolved a narrow and shallow portion of the front
422 only. The unconstrained boundary conditions influence, and therefore add uncertainty, to the so-
423 lution. Additionally, the along front curvature, which can play an essential role in a deformation
424 field, may be aliased IGW and difficult to interpret. The cross frontal extent of the Lagrangian sur-
425 vey is an example of the trade-off between spatial coverage and temporal aliasing, a balance that
426 is paramount to observations in the submesoscale regime. The inability to obtain a cross frontal
427 structure of buoyancy and velocity on a timescale that minimizes temporal aliasing presents a lim-
428 itation on inversion techniques for submesoscale observations. Any assumptions to approximate
429 these fields would obfuscate the interpretation of the submesoscale ASC.

430 In lieu of mesoscale inversion techniques, the divergent flow field was used to estimate vertical
431 velocity (assuming a rigid lid $w = 0$)

$$w_\delta = \int_{-8m}^{0m} \delta dz \quad (10)$$

432 with a bottom limit (8 m) set by the vertical extent of the float. During III, w_δ predicted down-
 433 welling, but greatly underestimated the vertical velocities experienced by the float (Fig. 13 b).
 434 This suggests a highly localized region of downwelling at the front that could not be resolved by
 435 the 5 km distances used to calculate δ . This highly localized vertical velocity is reminiscent of the
 436 increase in ζ , α , and δ at smaller scales presented in Table 1, and is a feature of the submesoscale
 437 in general.

438 Finally, the $T - S$ gradients that comprised the vertical stratification measured by the float were
 439 similar to the $T - S$ gradients of the horizontal stratification measured by Triaxus and the ship
 440 flow-through during the time of subduction (yd 216.8–217, Fig. 13 d), consistent with budgets
 441 in section 4c. The classic paper by Iselin (1939) recognized the relationship between horizontal
 442 water mass changes in the winter ML and vertical water mass changes in the thermocline as an
 443 indicator of wintertime subduction of surface waters into the interior. The horizontal and vertical
 444 $T - S$ relationship observed by the Lagrangian survey captured this same signature of subduction,
 445 yet are a result of different dynamics occurring on smaller length and faster temporal scales.

446 *f. Potential vorticity*

447 Ertel potential vorticity (PV)

$$q = (f\hat{\mathbf{z}} + \nabla \times \mathbf{u}) \cdot \nabla b \quad (11)$$

448 is a dynamically relevant tracer and is conserved following fluid parcels unless subject to non-
 449 conservative forces or diabatic processes (Marshall and Nurser 1992), such that

$$\frac{Dq}{Dt} = 0 \quad (12)$$

450 In the absence of horizontal density gradients, PV conservation implies that the vertical term of
451 PV

$$q_v = (f + \zeta)N^2 \quad (13)$$

452 does not change following a fluid parcel. Neglecting derivatives in vertical velocity, the horizontal
453 term is

$$q_h = \frac{\partial u}{\partial z} \frac{\partial b}{\partial y} - \frac{\partial v}{\partial z} \frac{\partial b}{\partial x} \quad (14)$$

454 Near fronts, the horizontal term becomes leading order and an important contributor to a fluid
455 parcel's PV. If the shear is purely geostrophic, then the horizontal term becomes

$$q_{hg} = -\frac{|\nabla_h b|^2}{f} \quad (15)$$

456 a negative definite quantity in the Northern Hemisphere. The presence of ageostrophic shears
457 and surface forcing, which are often crucial to momentum and buoyancy budgets in the ML, can
458 influence both q_v and q_h . The evolution of PV estimated from this survey (Fig. 14) exhibited two
459 different stories: a deeper layer (16 m) where PV was conserved (~ 0), lying underneath a surface
460 layer of increasing PV (8 m). The components of PV following the float were used to describe this
461 evolution. Thomas (2008) laid out three conditions under which PV at fronts can have near zero

462 PV

463 i. Vertically mixed momentum and buoyancy to create $N^2 = 0$ and $|\mathbf{u}_z| = 0$;

464 ii. Vortically low PV as $\zeta \rightarrow -f$ and $|\nabla_h b| = 0$

465 iii. Baroclinically low PV $q_h \rightarrow -q_v$

466 In the beginning of the survey, BL turbulence homogenized tracers and momentum throughout the
467 ML. This caused a lack of shear and stratification that resulted in close to zero q_v and q_h as in (i).

⁴⁶⁸ Both terms were smaller than the value associated with geostrophic balance and consistent with
⁴⁶⁹ large ageostrophic shears (Fig. 11).

⁴⁷⁰ At the start of stage 2, PV throughout the upper 30 m evolved differently. Deeper in the ML (16
⁴⁷¹ m), isopycnals began to tilt, causing the once homogeneous ML to stratify and q_v to increase. The
⁴⁷² tilting of isopycnals (e.g., Fig. 8a) was accompanied by an increase in both horizontal buoyancy
⁴⁷³ gradient and vertical shear such that q_h compensated q_v as in (iii). This resulted in near zero PV
⁴⁷⁴ through yd 216.7 (at 16 m) after which advective terms may become important and interpretation
⁴⁷⁵ is less clear (see section 2, Fig. 3). At this depth (16 m), changes in q_v and q_h tracked $|\nabla_h b/f|$
⁴⁷⁶ (Fig. 14), demonstrating the balanced state of the front during this time.

⁴⁷⁷ PV conservation was not evident in the near surface layer (8 m). During stage 2, q_v increased
⁴⁷⁸ with stratification. During stage 3, q_v remained level and decreased slightly as the increase in
⁴⁷⁹ stratification at the end of the survey was offset by a decrease in ζ and consistent with the anticy-
⁴⁸⁰ clonic circulation. Unlike the middle of the ML (16 m), changes in q_v at 8 m were not balanced
⁴⁸¹ by q_h . Horizontal buoyancy gradient increased during stage 2, yet strong upfront shear inhibited
⁴⁸² development of q_h such that q_h did not approach q_{hg} . Furthermore, the presence of ageostrophic
⁴⁸³ cross-front shear encouraged frontal tilting (Fig. 8) and increased stratification (and therefore
⁴⁸⁴ q_v), but did not contribute to q_h because the along-front buoyancy gradient was, by definition,
⁴⁸⁵ zero. In summary, cross-front shear resulted in an increase in q_v through N^2 , while along-front
⁴⁸⁶ ageostrophic shear inhibited q_h , such that q_v and q_h did not balance and total q increased. This
⁴⁸⁷ reveals the importance of ageostrophic shear in modulating PV at 8 m.

⁴⁸⁸ The relationship between horizontal and vertical PV is an important part of understanding sub-
⁴⁸⁹ mesoscale frontal dynamics as was captured by the loop method here. This balance was not
⁴⁹⁰ maintained by the objectively mapped fields that underestimated q_h and therefore predicted an in-
⁴⁹¹ crease in total q at 16 m. The difference in PV between the loop method and the objective maps

492 highlights the challenge in estimating and interpreting observed PV at submesoscale fronts, where
493 the horizontal component of PV plays an essential role and therefore needs to be resolved.

494 5. Discussion

495 A Lagrangian survey, processed on spatial scales of 5 km (diameter of the looped survey pattern)
496 and temporal scales of 2 hours (time-span contained in one data point), revealed surface intensified
497 gradients of buoyancy and velocity as well as vertical velocities that were larger than the accom-
498 panying mesoscale survey or estimates from AVISO (Table 1, Fig. 1, Fig. 4). Horizontal gradient
499 magnitudes were largest near the surface and decayed with depth. These patterns are not consis-
500 tent with a classic QG framework, but instead are signatures of the submesoscale range. Flows
501 approaching $\text{Ro} = \zeta/f \sim 1$, are better described by a semi-geostrophic framework and result in
502 shallower velocity spectral slopes of $\sim k^{-2}$ at high wavenumbers as found in model studies that
503 resolve the submesoscale (Capet et al. 2008; Klein et al. 2008). This is also consistent with $\sim k^{-2}$
504 spectral slopes observed near the surface that are not predicted by estimates using satellite alti-
505 try or found deeper below the ML (Shcherbina et al. 2013; Callies and Ferrari 2013). Spectral
506 slopes of $\sim k^{-2}$ result from a velocity field influenced by frontogenesis, instabilities, and large
507 ageostrophic motions, all which manifest signatures at this front. Here, the large values of vor-
508 ticity, divergence and strain may result from a combination of frontal dynamics and IGWs within
509 the ML. These balanced and unbalanced velocities are intertwined in the ML, yet are decoupled
510 from an internal wave field observed at depth as gradients of buoyancy and velocity decay below
511 the pycnocline.

512 The coordinated mesoscale and Lagrangian surveys provided a nested view of this submesoscale
513 front. Yet the two surveys document different phenomena. The mesoscale survey described by
514 Pallàs-Sanz et al. (2010a) and Johnston et al. (2011) spanned 130 km meridionally, 70 km zonally

and 16 - 355 m vertically. In contrast, the Lagrangian survey spanned 5 km across the front and 50 km in the along front direction. The Lagrangian survey was located in the northwest quadrant of the mesoscale survey and overlapped with four Seasor tracks that were set 11 km apart (Fig. 4). Pallàs-Sanz et al. (2010b) and Johnston et al. (2011) used a generalized omega equation and the classic QG omega equation, respectively, to discuss the frontal response to deformation fields and its impact on the turbulence and tracer distribution at the front. These results showed that strong ASC developed as a response to the external deformation field (Pallàs-Sanz et al. 2010a) as surrounding mesoscale eddies strain the front. Additionally, Johnston et al. (2011) mapped a deep chlorophyll maximum around 100 m (a feature seen deep in the Triaxus data as well), consistent with strong downwelling on the edge of the neighboring eddy. An ASC derived from the mesoscale fields using a generalized omega equation approach (Pallàs-Sanz et al. 2010a), predicts frontolysis due to ageostrophic velocities from the frontal curvature in the domain of the Lagrangian survey. Conversely, the Lagrangian survey documented frontal strengthening simultaneous with ageostrophic cross-frontal shear, float subduction and tilting isopycnals, consistent with a restratifying ASC, though not formally quantified here. The frontal curvature deviates between the Lagrangian and the mesoscale survey (Fig. 4), a result of the rapidly evolving and tilting submesoscale front and therefore is not comparable to the curve in the mesoscale survey.

The difference in frontogenesis between the two surveys reiterate the multiple scales of processes that occur in a single region and presents an inconsistency with the near surface frontal dynamics and those happening deeper (i.e. 5-20 m vs. 20-100 m). The mesoscale ASC was calculated using an objectively mapped buoyancy and flow field with decorrelation lengths comparable to the entire extent of the Lagrangian survey and a rigid lid assumption that set $w = 0$ at 16 m. It therefore was not targeted to isolate the large near surface vertical velocities, high shears, or frontal restratification observed in the upper 10 m of the Lagrangian survey. The near

539 surface frontogenesis observed by the Lagrangian survey is reminiscent of the submesoscale fron-
540 togenesis discussed in Shakespeare and Taylor (2013) and Barkan et al. (2019), distinctly different
541 from those explored in the classic or generalized omega equations. The mesoscale and Lagrangian
542 surveys each resolved different frontogenetic regimes, yet neither survey captured the processes
543 occurring on multiple scales simultaneously.

544 Satellite SST (Fig. 1) revealed filaments and meanders along the upwelling front suggest-
545 ing along front variability. Furthermore, the Lagrangian survey observed large horizontal gra-
546 dients, frontogenetic tendency, vertical velocities and possible meandering structures consistent
547 with frontal baroclinic instabilities (mixed layer instabilities, MLI, (Boccaletti et al. 2007)). MLI
548 baroclinic waves grow with length-scales that follow L_D (here, 5 km), and a timescale of days.
549 These waves release available potential energy by converting horizontal stratification into a verti-
550 cal one. While the along-front variability of 5–10 km (Fig. 4) may be consistent with growing
551 baroclinic waves, the rapid stratification of this front (i.e. less than the inertial period, $T_i = 20.3$
552 hr) presents an inconsistency between the observations and MLI theory. Additionally, it was im-
553 possible to isolate the physical along-front variability from temporal IGW. Therefore the role of
554 MLI remains illusive.

555 A characteristic of this front was the non-conservation of PV near the surface as the ageostrophic
556 shear impeded growth of the q_h while stratification (q_v) increased. Surface friction due to wind
557 driven or geostrophic stress can modulate shear and therefore PV. The role of wind driven and
558 geostrophic shear at fronts are usually explored in steady state (Thomas and Lee 2005; Thompson
559 2000; Cronin and Kessler 2009; Wenegrat and McPhaden 2015; McWilliams et al. 2015) and
560 therefore assuming subinertial timescales. Previous observations have isolated ageostrophic shears
561 in the presence of geostrophic currents on timescales of days (Lee and Eriksen 1996) and months
562 (Cronin and Kessler 2009) that satisfy the Ekman relation, rotating right and decreasing with depth.

563 Ageostrophic shear averaged throughout this survey reveal a similar rotation profile in depth (not
564 shown). Not surprisingly, this Ekman like pattern is absent in instantaneous profiles. Additionally,
565 using average shear in place of instantaneous shear in (6) underestimates stratification by 60%.
566 These discrepancies highlight the importance of unsteady forcing and superinertial fluctuations in
567 shear for increasing stratification at this front and modulating PV near the surface.

568 6. Conclusion

569 A highly detailed process study captured the restratification of a surface intensified submesoscale
570 front in the California Current System on superinertial timescales. The survey pattern allowed for
571 reliable calculation of vertical and horizontal gradients in a Lagrangian framework and showed
572 that vertical gradients in b , T , and S were a result of differential advection of horizontal gradi-
573 ents by ageostrophic cross front vertical shear. The increase in stratification resulting from frontal
574 slumping was equivalent to a flux of buoyancy of 2000 W m^{-2} , compared to an average heat
575 flux of 100 W m^{-2} during the restratification phases (2 and 3). Strong ageostrophic circulation
576 was accompanied by vertical velocities reaching $1.3 \times 10^{-3} \text{ mm s}^{-1}$ (120 m d^{-1}), as well as ζ , δ
577 and α that approached the Coriolis frequency. These features are a departure from the classic QG
578 framework and are characteristic of a submesoscale regime. Frontogenesis and the strengthening
579 of the horizontal buoyancy gradient played a key role in frontal evolution, transferring energy to
580 smaller scales (through frontal sharpening) and influencing the upper ocean buoyancy budget (by
581 increasing stratification due to horizontal slumping). The increase in stratification was accompa-
582 nied by an increase in the vertical component of PV. In the middle of the ML (16 m), the increase
583 in vertical PV was balanced by decreases in horizontal PV and evidence of PV conservation. This
584 relationship did not exist near the surface (8 m), as vertical PV increased without compensation
585 from the horizontal component. The results presented here point to the importance of near sur-

586 face Ekman dynamics and frontal instabilities, which are explored in a companion manuscript
587 combining these observations with idealized models.

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772 LIST OF TABLES

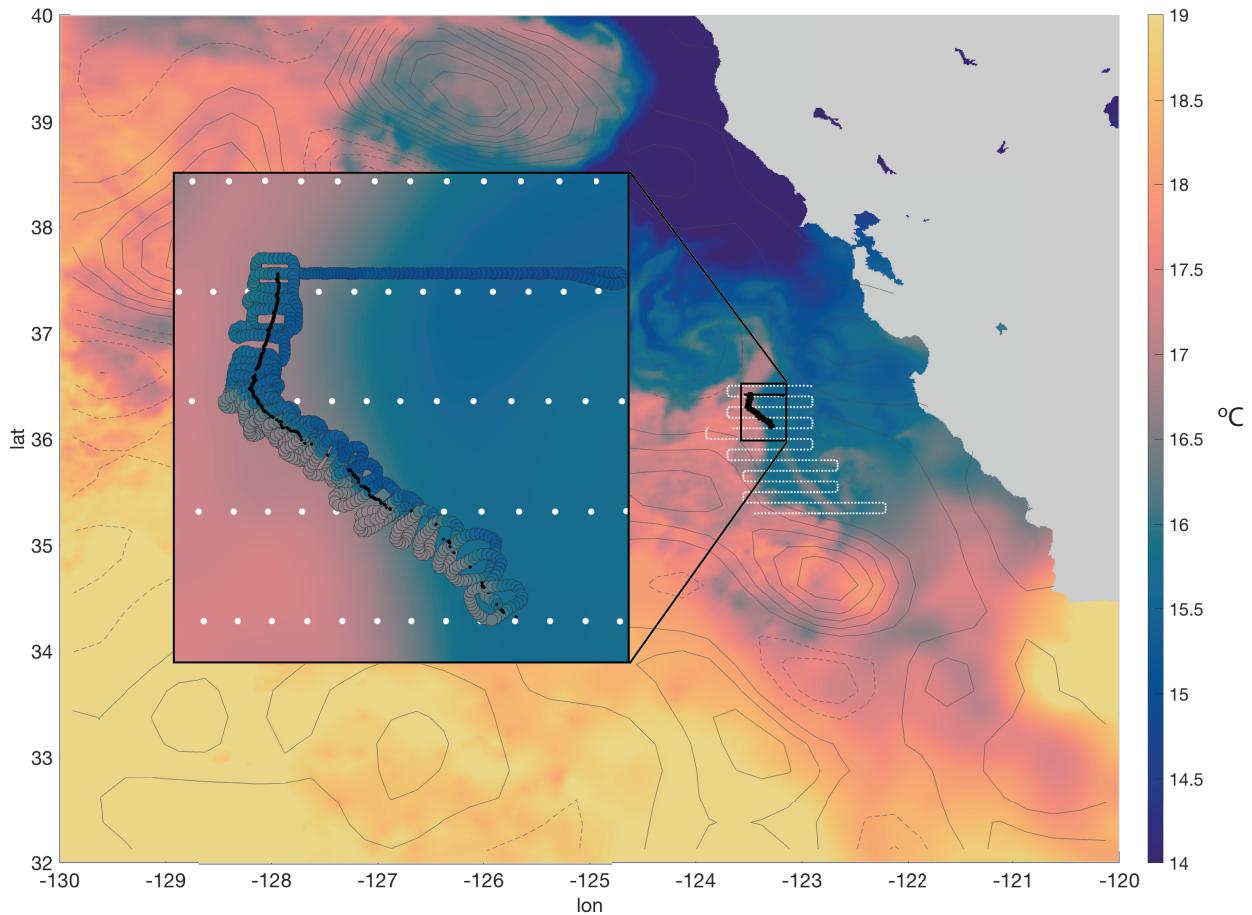
TABLE 1. Values for scalars, velocities, and their gradients resolved at different observational scales.

| | AVISO | SEASOAR | TRIAXUS | FLOAT |
|--------------|---------------------------------|-------------------------------------|-------------------------------------|--------------------------------------|
| SCALES | >100 km | 12 km | 5 km | 0.5 km |
| ζ | $0.03f\text{ s}^{-1}$ | $0.15f\text{ s}^{-1}$ | $0.7f\text{ s}^{-1}$ | — |
| δ | $0.001f\text{ s}^{-1}$ | $0.03f\text{ s}^{-1}$ | $0.7f\text{ s}^{-1}$ | — |
| α | $0.10f\text{ s}^{-1}$ | $0.13f\text{ s}^{-1}$ | $1.2f\text{ s}^{-1}$ | — |
| $\nabla_h b$ | — | $0.32 \times 10^{-6}\text{ s}^{-2}$ | 1.4×10^{-6} | — |
| KE | $0.12\text{ m}^2\text{ s}^{-2}$ | $0.27\text{ m}^2\text{ s}^{-2}$ | $0.27\text{ m}^2\text{ s}^{-2}$ | — |
| w | — | $5 \times 10^{-5}\text{ m s}^{-1}$ | $20 \times 10^{-5}\text{ m s}^{-1}$ | $100 \times 10^{-5}\text{ m s}^{-1}$ |

775 LIST OF FIGURES

- 776 **Fig. 1.** SST off the California Coast on 4 August 2006 from the Group for High Resolution SST
777 (GHRSSST - <https://podaac.jpl.nasa.gov>). Contours are AVISO positive (solid) and nega-
778 tive (dashed) mean sea level anomaly. White dots outline the mesoscale survey ship track
779 described in Pallàs-Sanz et al. (2010b); Johnston et al. (2011). Block dots outline the La-
780 grangian survey ship track. Inset) Detail of sea surface temperature (SST), the mesoscale
781 survey (white), and the Lagrangian float track (black dots). The ship track is colored with
782 ship underway temperature. 41
- 783 **Fig. 2.** a) Example of loops from underway data. A plane was fit to each loop, e.g. n-1 (blue), n
784 (red), and n+1 (green). Values from each plane fit were averaged together to form a single
785 value for n. Each loop contained one hour of data, therefore each value for n contained two
786 hours of data. b) An example plane fit to potential density over one loop of Traixus data at
787 4 m. Black circles are the observed data, and grey circles are projections of the observations
788 onto the plane fit (1). The difference between the grey and black dots were used to estimate
789 95% confidence interval, ϵ in (2). 42
- 790 **Fig. 3.** Lagrangian analysis - Deformation distance at depth from initial volume. x-dist is the cross-
791 frontal distance in the reference frame of the float as a function of time. The dashed line
792 is the ship track, where each zigzag in time represents one loop. Colored lines are d^{xf} as
793 described in section 2 and represent the distance a particle of water at depth z has been
794 advected relative to the float. This can be used to assess the Lagrangian assumption of the
795 survey. For example, at $y_d=216.7$ (dashed line), d^{xf} implies that flow at 20 m is no longer
796 true to the Lagrangian reference set at the beginning of the survey. While flow at 8 m is
797 considered in the Lagrangian reference frame throughout the span of the observations. 43
- 798 **Fig. 4.** Objective maps of potential density for the mesoscale survey at 16 m (background) in both a)
799 and b), and the Lagrangian survey (foreground) at a) 4 m and b) 16 m). Distances are merid-
800 ional (y-axis) and zonal (x-axis). Contours outline isopycnals every 0.1 kg m^{-3} . The float
801 track (grey dots) and SeaSoar transect (white triangles) are included for reference. Yearday
802 at two latitudes show time separation between the mesoscale and Lagrangian surveys. The
803 tightly looped Traixus track has been excluded for clarity, refer to Fig. 1. 44
- 804 **Fig. 5.** Initial transect used to identify the front before placement of the float. a) cross-frontal po-
805 tential density from the ship flow through system. b) cross-frontal potential density from
806 Traixus. Black lines are isopycnal contours of 0.1 kg m^{-3} and the dark black line is the 24.4
807 kg m^{-3} isopycnal. 45
- 808 **Fig. 6.** Scalars and velocity of the front surrounding the float throughout the Lagrangian survey in
809 depth and time. Values are means from the loops (see section 2). a) potential density and
810 float depth (grey dots); b) N^2 ; c) along-front wind stress (green) and cross-front wind stress
811 (purple), Q_{NET} positive into the ocean (grey); d) salinity; e) zonal velocity u ; f) zonal shear
812 u_z ; g) temperature; h) meridional velocity v ; and i) meridional shear v_z . Dashed grey lines
813 denote the three stages outlined in section 4. 46
- 814 **Fig. 7.** Raw Traixus salinity [PSU] a) at the beginning of the survey, stage 1, y_d 216.1 and b) at
815 the end of the survey, stage 3, y_d 216.8. Potential density is contoured every 0.1 kg m^{-3}
816 with the solid contour marking the 24.3 kg m^{-3} isopycnal. Circles denote the position of the
817 float within ± 15 min of the transect and are colored by the average salinity measured by the
818 float's sensors. 47

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|-----|-----------------|--|----|
| 819 | Fig. 8. | a) Objective map of potential density at 4 m. The map has been rotated to follow the float 820 trajectory (orange dots) during stages 2 and 3. The grey scale lines denote the 24.4 kg m^{-3} 821 isopycnal at different depths. b) Vertical gradients discussed in section 4a. top to bottom: 822 N^2 , $\partial T/\partial z$, and $\partial S/\partial z$ at 8 m. Black lines are observations, purple lines are integrated 823 values from (6). Blue dashed lines are the difference between the observations (black) and 824 integrated (purple) values. The red line in the top plot is N^2 calculated from (3). Green line 825 in the middle plot is estimated from (5). Scaled float depth is included for reference. Shaded 826 regions are 95% confidence intervals (ε). | 48 |
| 827 | Fig. 9. | (a) ζ/f , (b) α/f , (c) δ/f , and (d) $\nabla_h b/f^2$ plotted against time at depths 4 through 20 m 828 and averaged between 100-140 m. Float depth scaled by $\times 10^3$ (grey dots) are included for 829 reference. Shaded regions are 95% confidence intervals (ε) at 4 m. | 49 |
| 830 | Fig. 10. | $ \nabla_h b [\text{s}^{-2}]$ calculated along the ship track (color) and float positions (grey dots) rotated 831 along the average trajectory of the float during stage 2 and 3. Inset) example of a cross-front 832 transect of potential density resolved by the underway (purple) and Triaxus at 4 m (blue). | 50 |
| 833 | Fig. 11. | Geostrophic shear, $\widehat{k \times (\nabla_h b f^{-1})}$ (red), ageostrophic shear (purple), and total shear (blue) 834 for a) along-front (u_z^{af}) at 8 m, b) cross-front (u_z^{xf}) at 8 m, c) along-front (u_z^{af}) at 16 m, d) 835 cross-front (u_z^{xf}) at 16 m. All terms have been rotated to align with $\nabla_h b$ at 4 m (section 2.2). 836 Shaded regions are 95% confidence intervals (ε). | 51 |
| 837 | Fig. 12. | Frontogenetic tendency ($F_{hadv} [\text{s}^{-5}]$) as a function of time at $z = 8 \text{ m}$. Shaded regions are 838 95% confidence intervals (ε). | 52 |
| 839 | Fig. 13. | Vertical velocity and float subduction. a) 3-D float subduction. The float's positions (circles) 840 are colored by salinity. The float's trajectory is shown at the surface (black) and pro- 841 jected again at 10 m. Each float location and velocity vector is connected by a dashed grey 842 line. b) Vertical velocity estimated directly from the pressure measured by the float (purple) 843 and using the divergence calculated from Triaxus (green). c) Cross-frontal distance of the 844 float as it downwelled under the front during events I–IV. d) $T - S$ diagram during down- 845 welling events III. The float's two sensors (P1, yellow and P2, gold) and Triaxus (green) 846 captured $T - S$ changes of the vertical stratification during the downwelling event, while the 847 ship underway (purple) provided $T - S$ changes of the horizontal stratification. In all plots, 848 downwelling events are labeled according to section 4e | 53 |
| 849 | Fig. 14. | PV (q , black), the vertical term of PV (q_v , blue) and the horizontal term of PV (q_h , green) at 850 a) 8 m and b) 16 m. Both plots include planetary PV ($N^2 f$, purple) and the horizontal term 851 in PV if the flow were in thermal wind balance (q_{hg} , dashed blue and $-q_{hg}$, dashed green). 852 Shaded regions are 95% confidence intervals (ε). | 54 |



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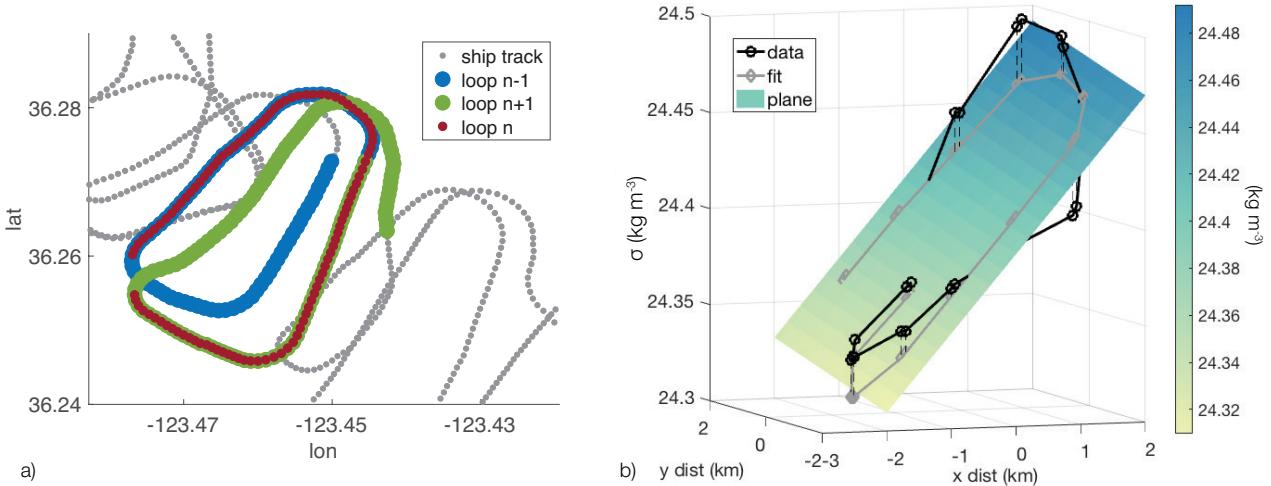
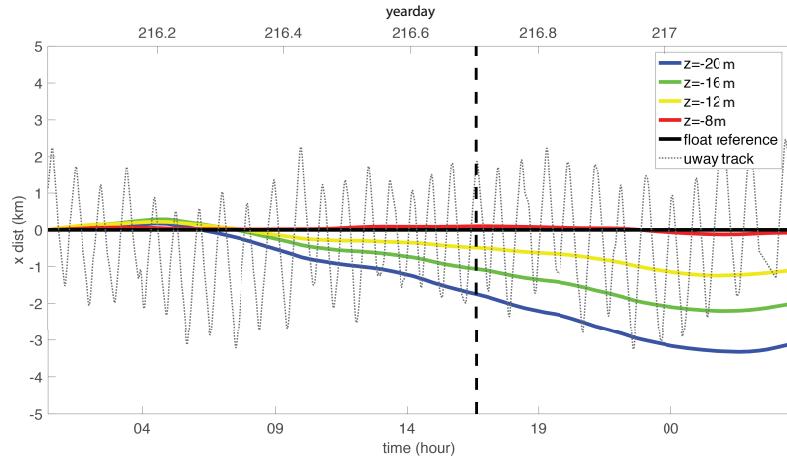


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 868 particle of water at depth z has been advected relative to the float. This can be used to assess the Lagrangian
 869 assumption of the survey. For example, at $yd=216.7$ (dashed line), d^{xf} implies that flow at 20 m is no longer true
 870 to the Lagrangian reference set at the beginning of the survey. While flow at 8 m is considered in the Lagrangian
 871 reference frame throughout the span of the observations.

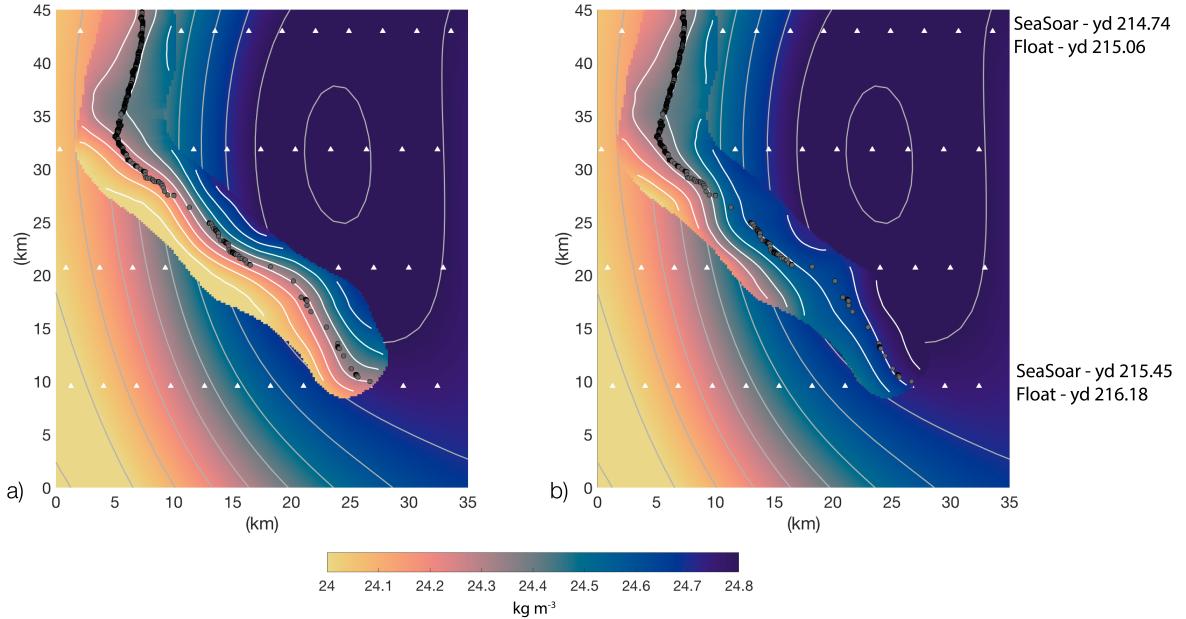


FIG. 4. Objective maps of potential density for the mesoscale survey at 16 m (background) in both a) and b), and the Lagrangian survey (foreground) at a) 4 m and b) 16 m). Distances are meridional (y-axis) and zonal (x-axis). Contours outline isopycnals every 0.1 kg m^{-3} . The float track (grey dots) and SeaSoar transect (white triangles) are included for reference. Yearday at two latitudes show time separation between the mesoscale and Lagrangian surveys. The tightly looped Triaxus track has been excluded for clarity, refer to Fig. 1.

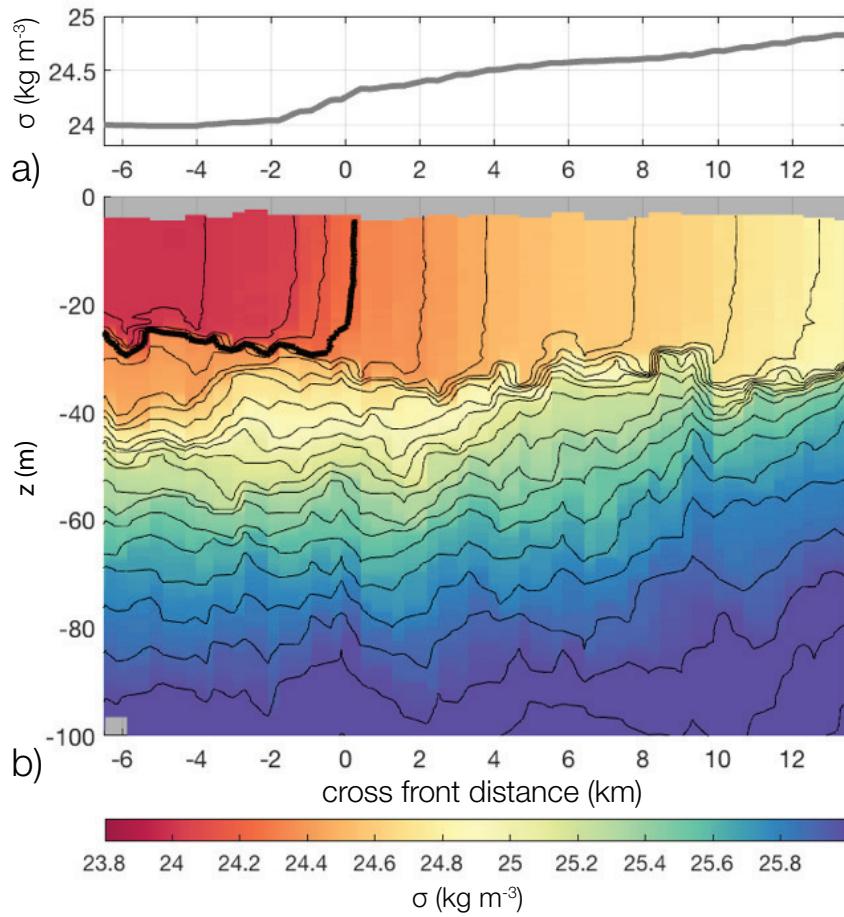


FIG. 5. Initial transect used to identify the front before placement of the float. a) cross-frontal potential density from the ship flow through system. b) cross-frontal potential density from Triaxus. Black lines are isopycnal contours of 0.1 kg m^{-3} and the dark black line is the 24.4 kg m^{-3} isopycnal.

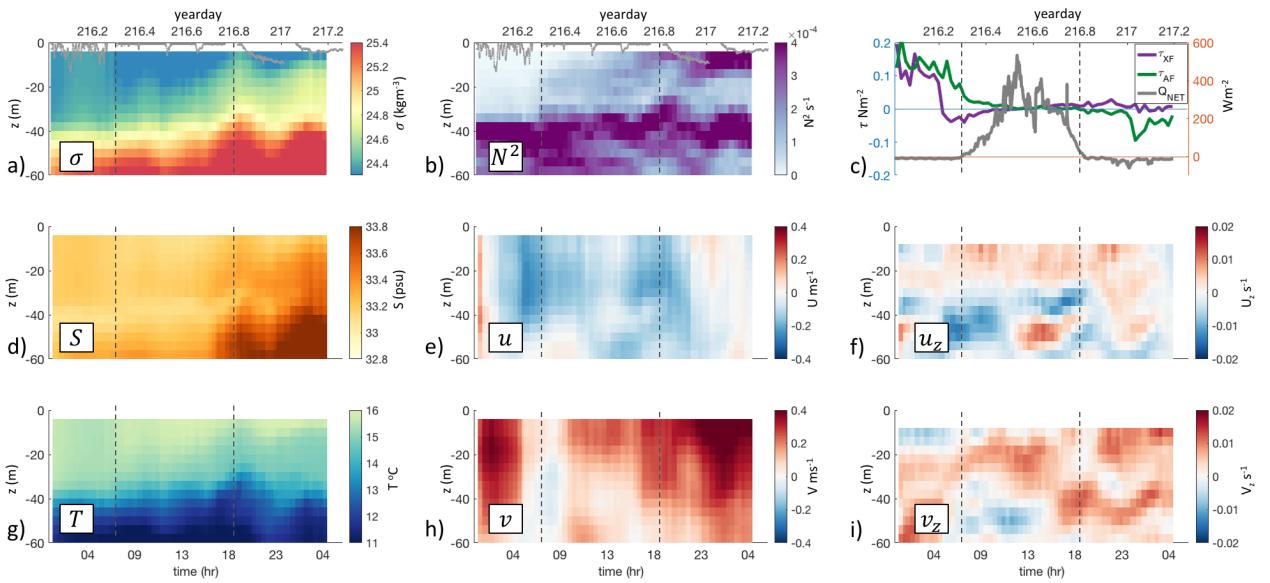
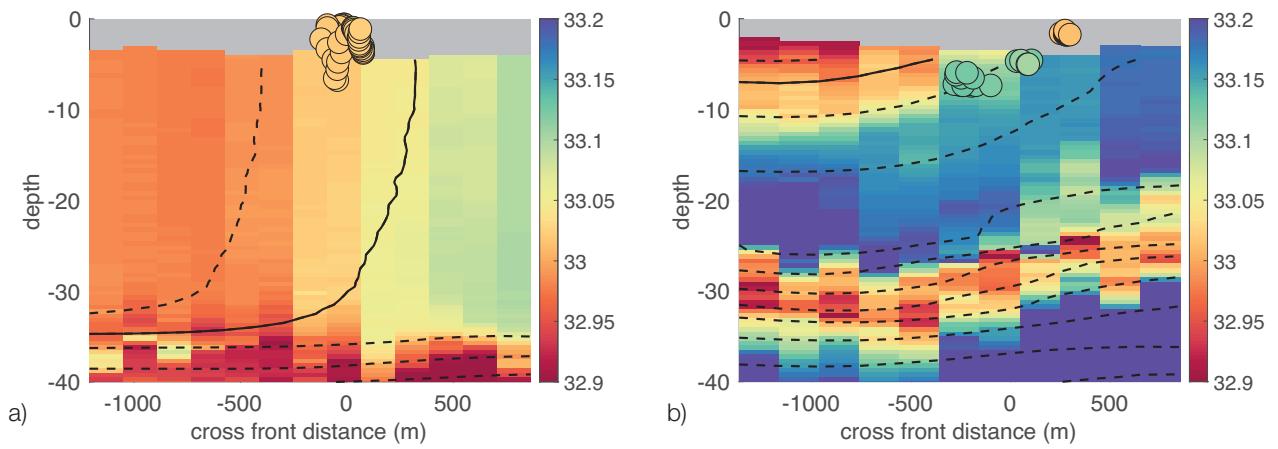


FIG. 6. Scalars and velocity of the front surrounding the float throughout the Lagrangian survey in depth and time. Values are means from the loops (see section 2). a) potential density and float depth (grey dots); b) N^2 ; c) along-front wind stress (green) and cross-front wind stress (purple), Q_{NET} positive into the ocean (grey); d) salinity; e) zonal velocity u ; f) zonal shear u_z ; g) temperature; h) meridional velocity v ; and i) meridional shear v_z . Dashed grey lines denote the three stages outlined in section 4.



885 FIG. 7. Raw Triaxus salinity [PSU] a) at the beginning of the survey, stage 1, yd 216.1 and b) at the end of the
 886 survey, stage 3, yd 216.8. Potential density is contoured every 0.1 kg m^{-3} with the solid contour marking the
 887 24.3 kg m^{-3} isopycnal. Circles denote the position of the float within ± 15 min of the transect and are colored
 888 by the average salinity measured by the float's sensors.

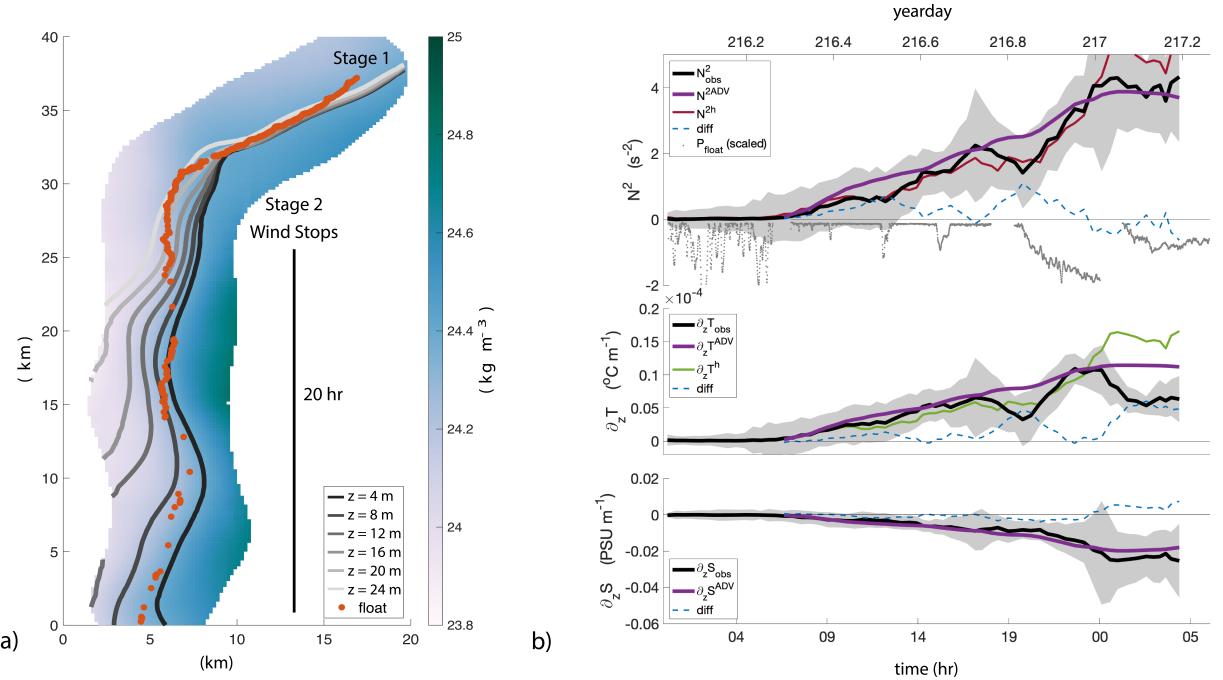


FIG. 8. a) Objective map of potential density at 4 m. The map has been rotated to follow the float trajectory (orange dots) during stages 2 and 3. The grey scale lines denote the 24.4 kg m^{-3} isopycnal at different depths. b) Vertical gradients discussed in section 4a. top to bottom: N^2 , $\partial T / \partial z$, and $\partial S / \partial z$ at 8 m. Black lines are observations, purple lines are integrated values from (6). Blue dashed lines are the difference between the observations (black) and integrated (purple) values. The red line in the top plot is N^2 calculated from (3). Green line in the middle plot is estimated from (5). Scaled float depth is included for reference. Shaded regions are 95% confidence intervals (ϵ).

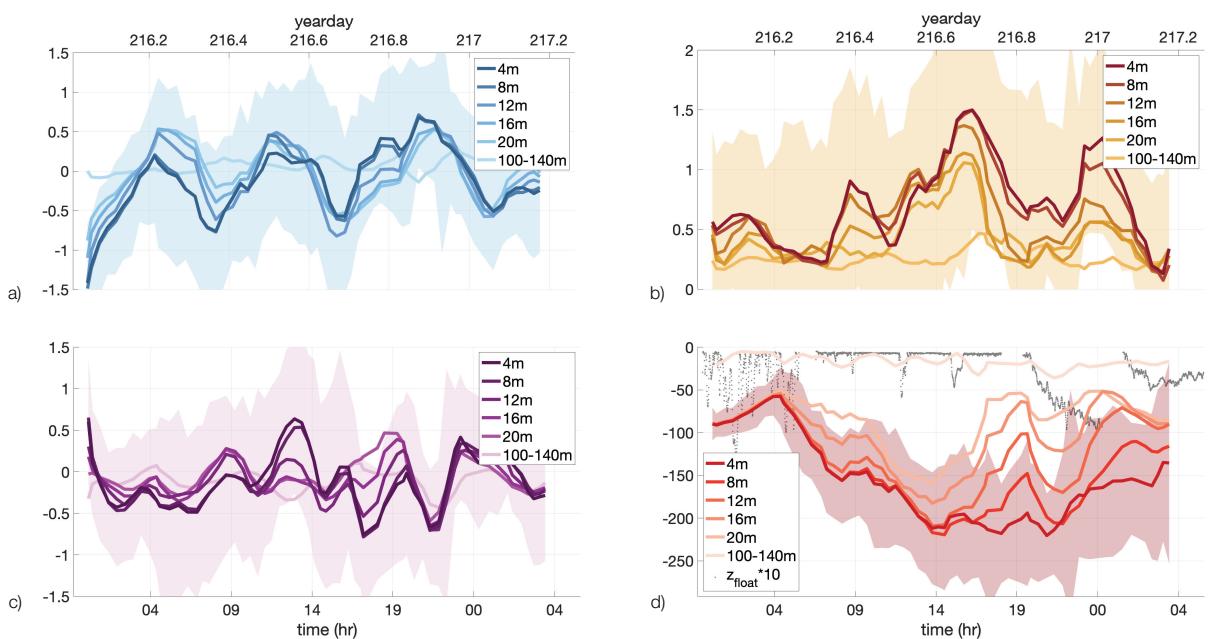
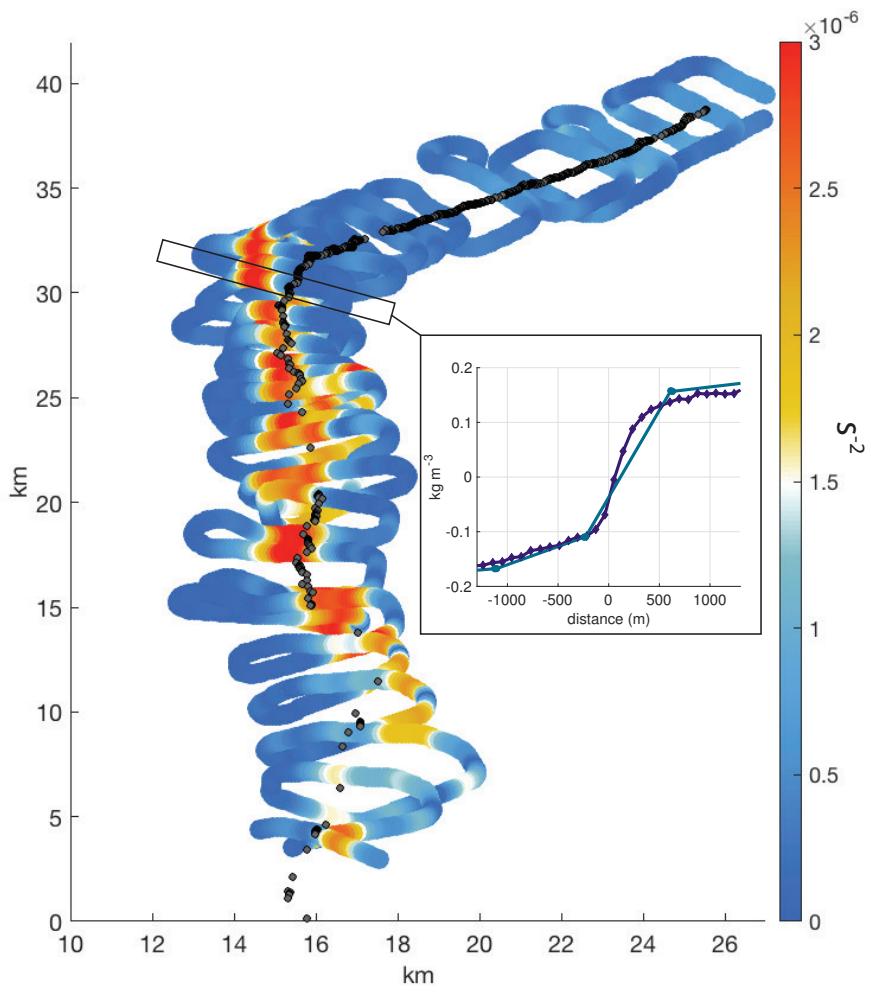
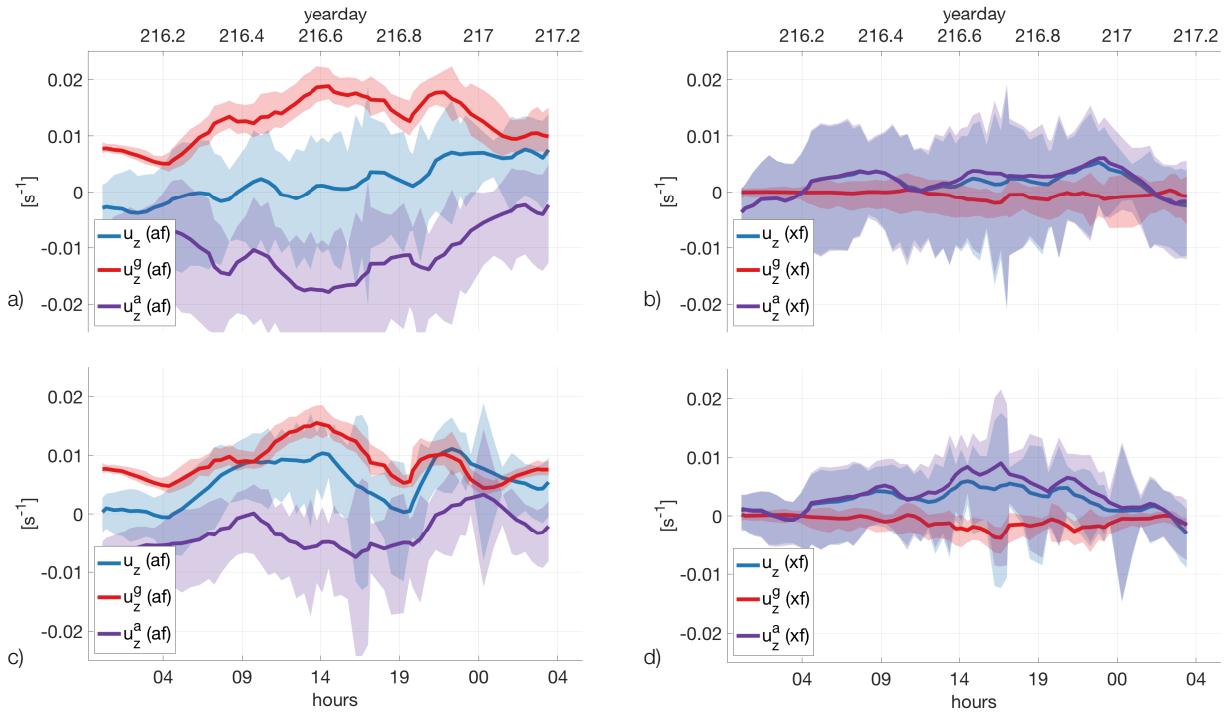


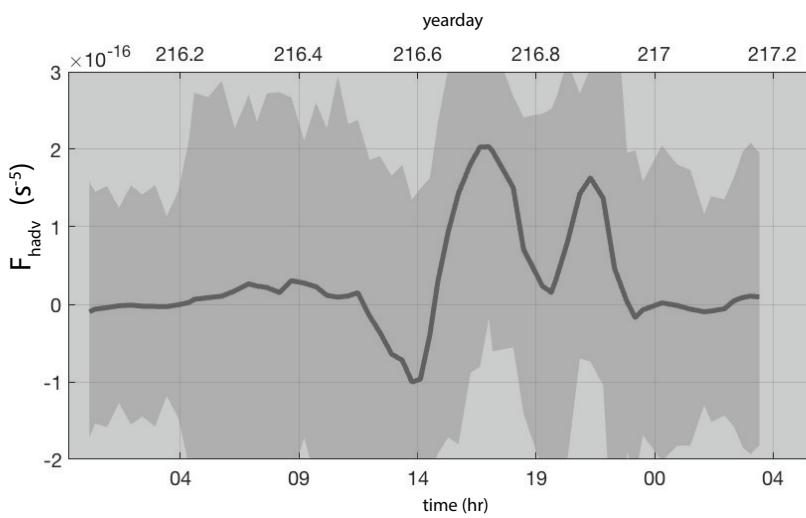
FIG. 9. (a) ζ/f , (b) α/f , (c) δ/f , and (d) $\nabla_h b/f^2$ plotted against time at depths 4 through 20 m and averaged between 100-140 m. Float depth scaled by $\times 10^3$ (grey dots) are included for reference. Shaded regions are 95% confidence intervals (ε) at 4 m.



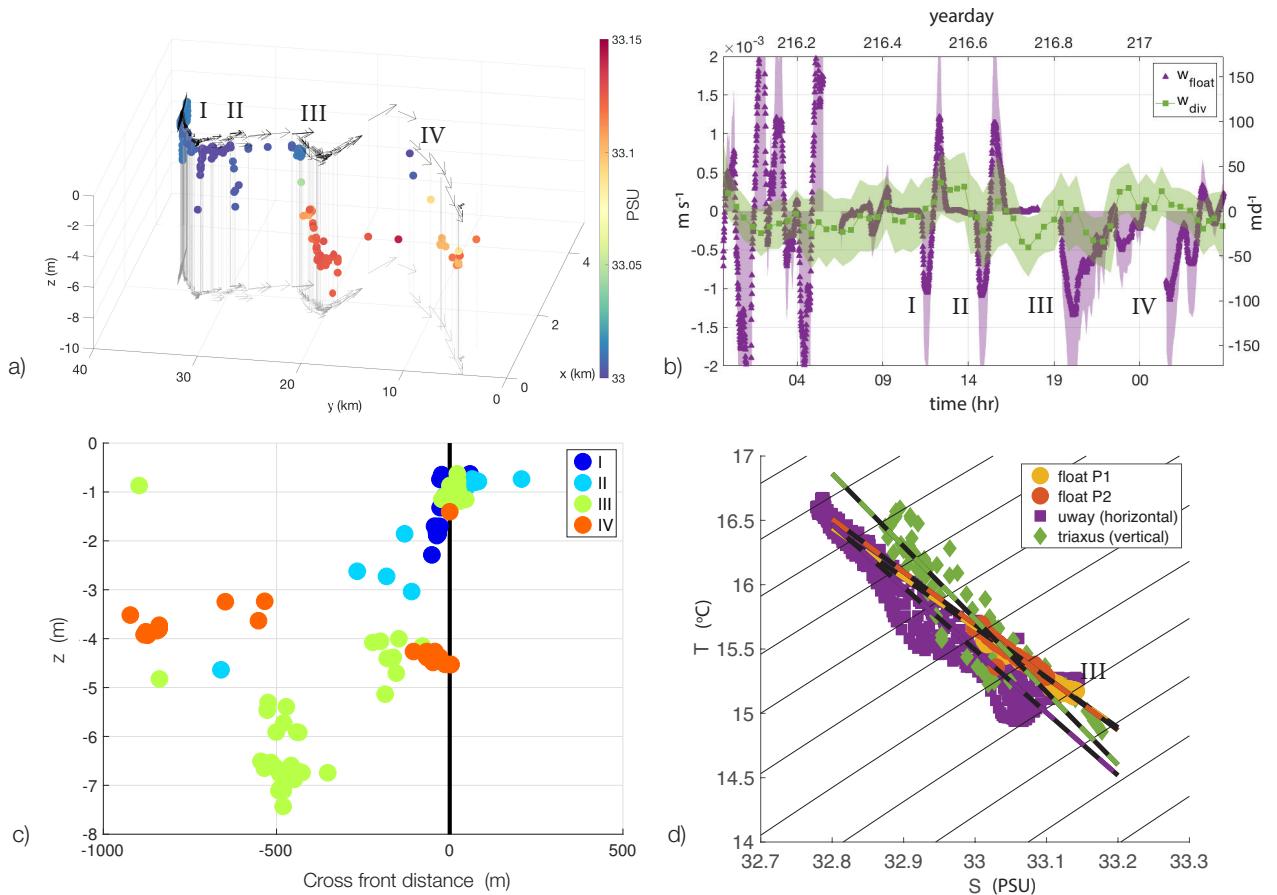
899 FIG. 10. $|\nabla_h b|$ [s^{-2}] calculated along the ship track (color) and float positions (grey dots) rotated along the
 900 average trajectory of the float during stage 2 and 3. Inset) example of a cross-front transect of potential density
 901 resolved by the underway (purple) and Triaxus at 4 m (blue).



902 FIG. 11. Geostrophic shear, $\hat{k} \times (\nabla_h b f^{-1})$ (red), ageostrophic shear (purple), and total shear (blue) for a)
903 along-front (u_z^{af}) at 8 m, b) cross-front (u_z^{xf}) at 8 m, c) along-front (u_z^{af}) at 16 m, d) cross-front (u_z^{xf}) at 16 m.
904 All terms have been rotated to align with $\nabla_h b$ at 4 m (section 2.2). Shaded regions are 95% confidence intervals
905 (ε).



906 FIG. 12. Frontogenetic tendency (F_{hadv} [s^{-5}]) as a function of time at $z = 8$ m. Shaded regions are 95%
907 confidence intervals (ε).



908 FIG. 13. Vertical velocity and float subduction. a) 3-D float subduction. The float's positions (circles) are
 909 colored by salinity. The float's trajectory is shown at the surface (black) and projected again at 10 m. Each
 910 float location and velocity vector is connected by a dashed grey line. b) Vertical velocity estimated directly
 911 from the pressure measured by the float (purple) and using the divergence calculated from Triaxus (green). c)
 912 Cross-frontal distance of the float as it downwelled under the front during events I–IV. d) $T - S$ diagram during
 913 downwelling events III. The float's two sensors (P1, yellow and P2, gold) and Triaxus (green) captured $T - S$
 914 changes of the vertical stratification during the downwelling event, while the ship underway (purple) provided
 915 $T - S$ changes of the horizontal stratification. In all plots, downwelling events are labeled according to section
 916 4e .

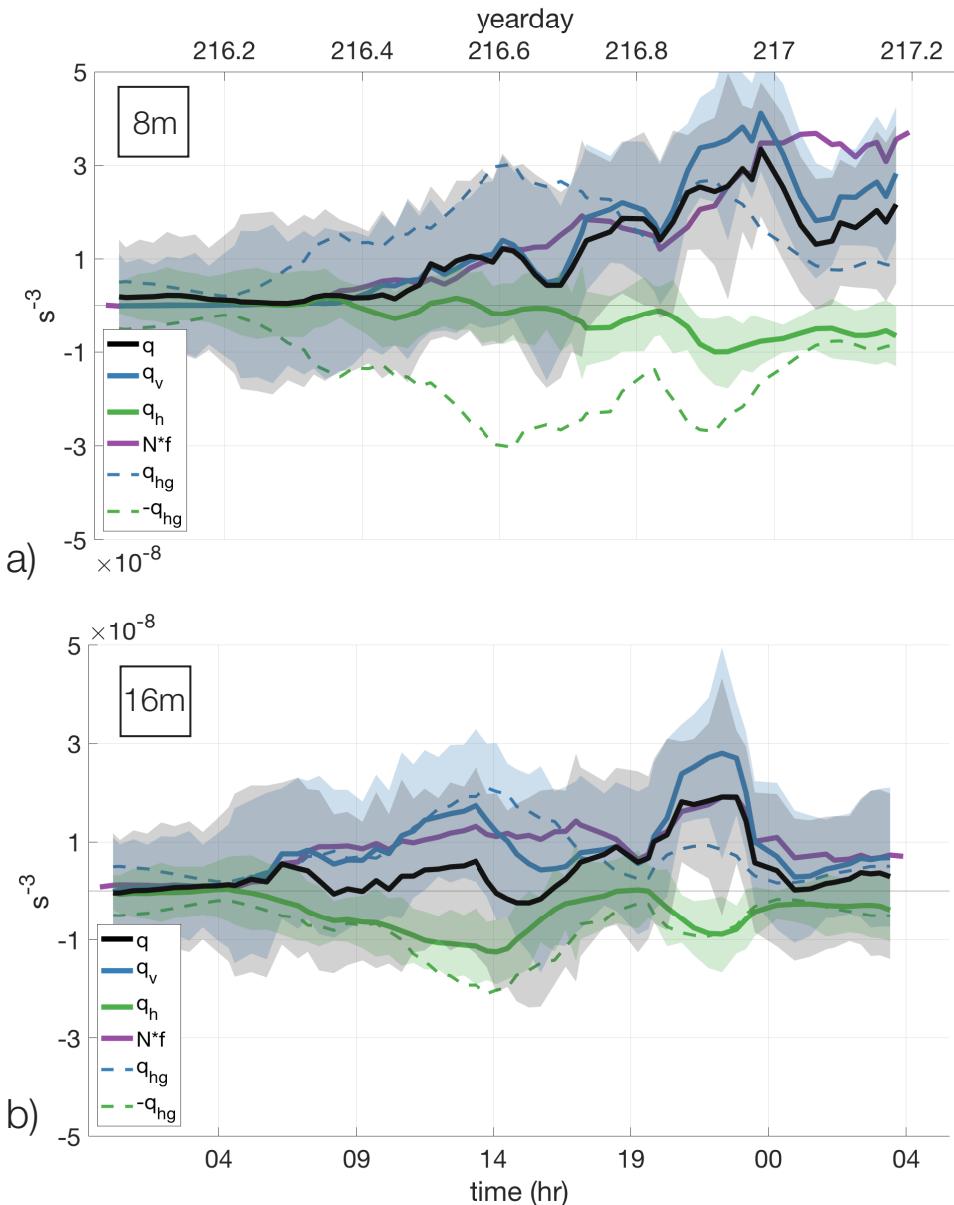


FIG. 14. PV (q , black), the vertical term of PV (q_v , blue) and the horizontal term of PV (q_h , green) at a) 8 m and b) 16 m. Both plots include planetary PV ($N^2 f$, purple) and the horizontal term in PV if the flow were in thermal wind balance (q_{hg} , dashed blue and $-q_{hg}$, dashed green). Shaded regions are 95% confidence intervals (ε).