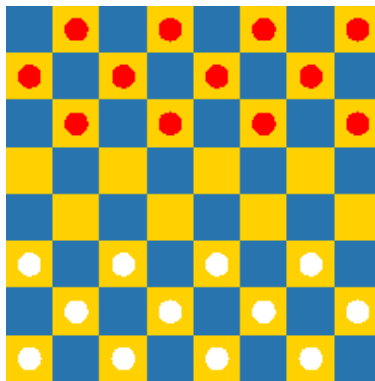


Final Exam
Wednesday, June 12, 2019
180 minutes, 4 questions, 100 points, 5 pages

<p>While we don't expect you will need more space than provided, you may continue on the back of the pages. This exam is closed book and closed notes.</p>
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Do not turn to the next page
until the start of the exam.

1. (25 points) Create an $n \times n \times 3$ numpy array such that displaying it with `pyplot.imshow(...)` shows



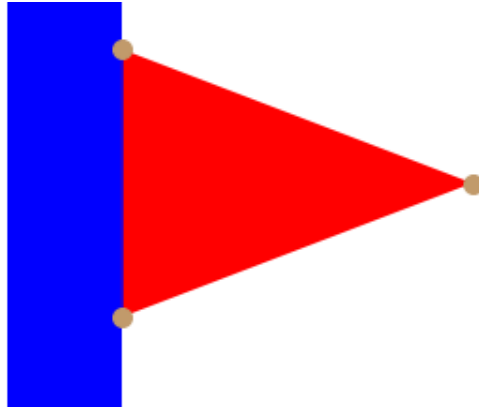
This is an 8×8 checkers board. The radii of the circles are $1/32$ of the length of the sides of the board. The colors are 39 116 174 (blue), 255 209 0 (yellow), 255 0 0 (red), and 255 255 255 (white). Your code should begin with the definition of the variable n and should produce a reasonable image with the sizes $n = 64, 128, 256, 1024, 2048, \dots$

Remark. You may want to use `plt.axis('off')`.

Remark. The inequality defining the region enclosed by a circle of radius r centered at (a, b) is

$$(x - a)^2 + (y - b)^2 \leq r^2$$

2. (25 points) You are by a river. You have three wooden stakes (poles) and one rope of length 1. Using the river as the fourth side, you are to enclose a quadrilateral patch of land with the stakes and the rope. The local authorities have determined that the land you enclose is yours.



Land has a positive value `val_land` per unit area. The riverside has a nonnegative value `val_river` per unit length. Find a configuration that maximizes the value of the enclosed land and the riverside. Write code that visualizes your solution.

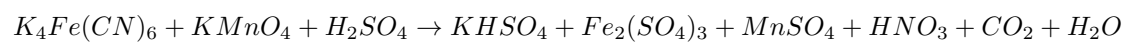
Remark. Assume that the river is straight. Assume that it is possible to stably plant the stake beside the river. (The river will not wash away your stake.)

Remark. This problem is written in reference to the Queen Dido's problem of Greek mythology.

Remark. The list of methods for `scipy.optimize.minimize` is

- Nelder-Mead
- Powell
- CG
- BFGS
- Newton-CG
- L-BFGS-B
- TNC
- COBYLA
- SLSQP
- trust-constr
- dogleg
- trust-ncg
- trust-exact
- trust-krylov

3. (25 points) Balance the chemical equation



4. (25 points) In the 115th United State Congress, from January 3, 2017, to January 3, 2019, 450 representatives voted in the House of Representatives. Although there are at most 435 representatives at any given time, 15 additional representatives filled vacant seats. The votes are labeled by their “rollnumbers”, numbered chronologically from 1 through 1207. The file `house_votes.csv` contains the voting record. A record of 1 means yes, 0 means no, and 0.5 means abstention.

Representative Ted Lieu represents California’s 33rd district, which includes the UCLA campus. Among the 1207 votes, Lieu abstained from 110 of them. Using k -nearest neighbor classification with $k = 10$, predict what Lieu’s votes would have been had he voted. You should run k -nearest neighbor classification 110 times. When performing the nearest neighbor search for a vote Lieu abstained from, exclude representatives who had also abstained from that vote.