Python with Applications, PIC16
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Homework 4
Due 5pm, Wednesday, May 29, 2019

Name your file `hw4.py` and submit on CCLE. Comment your code adequately.

Problem 1: You are controlling a rocket in the 2D space. At time $t = 0, \dots, T$ with $T = 16$, the rocket is positioned at (x_t, y_t) and has velocity v_t. (Velocity at a given time is a 2D vector.) At time $t = 0$, the rocket is positioned at $(0, 0)$ and is stationary, i.e., $(x_0, y_0) = (0, 0)$, and $v_0 = (0, 0)$. The rocket is subject to download gravity of magnitude 9.8 (assume the rocket has mass 1). At time $t = 0, \dots, T-1$, is propelled by a thruster with force F_t. (The force at a given time is a 2D vector.) The acceleration at time $t = 0, \dots, T-1$ is $a_t = (F_t - (0, 9.8))$, i.e., the acceleration is the sum of the thruster's force and gravity.

The position and velocity of the rocket is subject to

$$v_k = \sum_{i=0}^{k-1} a_i, \qquad (x_k, y_k) = \sum_{i=1}^{k} v_i$$

for $k = 1, \dots, T$. The summation is the discretization (approximation) of the integral given by the calculus definition of velocity and acceleration

$$\begin{aligned} \frac{d}{dt}(x(t), y(t)) = v(t) \qquad & (x(t), y(t)) = (x(t), y(t)) - (x(0), y(0)) = \int_0^t v(s)\, ds \\ \frac{d}{dt}v(t) = a(t) \qquad & v(t) = v(t) - v(0) = \int_0^t a(s)\, ds. \end{aligned}$$

The fuel use of this rocket is given by

$$\text{total fuel used} = \sum_{t=0}^{T-1} \|F_t\|$$

where $\|(\alpha, \beta)\| = \sqrt{\alpha^2 + \beta^2}$ is the magnitude of a vector.

You control the thruster with the following goals:

(A) At time T, the rocket shall be (approximately) at position $(10, 1)$.

(B) At time T, the rocket shall be (approximately) stationary.

(C) Reduce fuel use.

Find a trajectory that optimizes these goals.

Remark. There are many simplifying assumptions going into this problem.

- Solving a 2D problem. (Actual rocket dynamics is in 3D.)
- Rocket thrust F_t can point in any direction. (Actual rocket thrust can only point in the general forward direction, since fire comes out from the back of the rocket.)
- Mass of rocket stays constant. (Mass reduces as rocket expends fuel.)
- No air resistance.
- Constant gravity. (Gravity weakens as rocket moves away from the earth.)

Problem 2: You now no longer concerned with the fuel use, i.e., forget about goal (C). Rather, you now have a new goal:

(D) The maximum thrust must be less than M.

Clearly, if M is too small, the rocket cannot reach the target position $(10, 1)$. (If $M < 9.8$, then the thurster cannot even counteract gravity.) What is the smallest value of M that allows you to satisfy goals (A), (B), and (D)?

Remark. Imagine you are in an engineering team. Your job is to perform this simulation and tell your mechanical engineer colleague "In order to complete the mission, the thruster needs to deliver at an output of M or more."

Problem 3: The plotting code shown in lecture does not show the target location, and does not clearly indicate the thurster use. Overall, improve the plotting of the rocket dynamics. Your plot should be visually appealing.