QAM Final Project

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Introduction

Research Objective: Comparative Analysis of Various Allocation Strategies

Inspired by paper: "Optimal Versus Naive Diversification: How Inefficient is the *1/N* Portfolio Strategy?" by DeMiguel *et al*.

Evaluation criteria : expected excess returns, standard deviations, Sharpe ratios, certainty equivalent returns, and turnover rates

Table 1: Strategies

#	Model	Abbreviation
1.	$1/N$ with rebalancing	eq or $1/N$
2.	Value-weighting	vw
3.	Mean-Variance efficient in-sample	mv_{in}
4.	Mean-Variance efficient out-of-sample	mv_{out}
5.	Naïve risk-parity	rp

Sectors replicated & Data

Table 2: List of Data

#	Dataset and source	N	Abbreviation
1.	S & P Sector data Source: S &P Dow Jones Indices	10 + 1	SPX
2.	Ten industry portfolios and the US equity market portfolio Source: Ken French's Web site	10 + 1	Industry
3.	SMB and HML portfolios and the US equity market portfolio Source: Ken French's Web site	2 + 1	MKT/SMB/HML
4.	Twenty size- and book-to-market portfolios and the US equity MKT Source: Ken French's Web site	20 + 1	FF-1-factor
5.	Twenty size- and book-to-market portfolios and the MKT, SMB, HML, and MOM portfolios Source: Ken French's Web site	20 + 4	FF-4-factor

Date Range: September 1989 to December 2019

MVE In sample/ Out Sample & Risk Parity

- Mean Variance Efficient :

$$\min_{\boldsymbol{w}_t} \quad \boldsymbol{w}_t' \Sigma_t \boldsymbol{w}_t$$

$$\text{subject to} \quad \mathbf{1} \cdot \boldsymbol{w}_t = 1.$$

- Naive Risk Parity:

$$w_t^i = \frac{1/\sigma_t^i}{1/\sigma_t^1 + 1/\sigma_t^2 + \ldots + 1/\sigma_t^N}.$$

Excess Returns & Standard Deviations

Table 3: Excess Returns

Strategy	SPX $N = 11$	Industry Portfolios $N = 11$	Mkt/SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$	Mean
$1/N$	0.006584	0.006820879	0.002909	0.007769	0.007105	0.006237576
mve_{in}	0.046232	0.011940283	0.004402	0.025654	0.006833	0.019012257
mve_{out}	0.015140	0.021819592	0.003321	0.023020	0.105233	0.033706718
vw	0.006841	0.006557692	0.006558	0.006558	0.006558	0.006614538
rp	0.006694	0.007302720	0.002368	0.008547	0.007191	0.006420544

Table 4: Standard Deviation of Excess Returns

Strategy	SPX $N = 11$	Industry Portfolios $N = 11$	Mkt/SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$	Mean
$1/N$	0.04014436	0.03930094	0.019409	0.050617	0.044485	0.03879126
mve_{in}	0.10504217	0.15207191	0.025830	0.049555	0.012363	0.068972416
mve_{out}	0.16135713	0.36925550	0.210896	0.161758	1.733706	0.527394526
vw	0.04049606	0.04217485	0.042175	0.042175	0.042175	0.041839182
rp	0.03914803	0.03675082	0.018359	0.050516	0.041462	0.03724717

Sharpe Ratio

The Sharpe ratio of naïve risk-parity produces the largest Sharpe ratio, while *1/N* comes in second. The mean-variance-efficient portfolio out-of-sample produces a very low Sharpe ratio.

Table 5: Sharpe Ratios

Strategy	SPX $N = 11$	Industry Portfolios $N = 11$	Mkt/SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$	Mean
$1/N$	0.164018	0.17355510	0.149891	0.153490	0.159727	0.16013622
mve_{in}	0.440128	0.07851735	0.170433	0.517681	0.552708	0.35189347
mve_{out}	0.093668	0.05909077	0.015748	0.142310	0.060698	0.074302954
vw	0.161893	0.15548821	0.155488	0.155488	0.155488	0.156769042
rp	0.170995	0.19870901	0.128997	0.169190	0.173446	0.178085003

Certainty Equivalent Returns (CEQ)

Formula for CEQ:

$$CEQ_k = \mu_k - \frac{\gamma}{2}(\sigma_k)^2.$$

Table 6: Certainty Equivalent Returns

Strategy	SPX $N = 11$	Industry Portfolios $N = 11$	Mkt/SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$	Mean
$1/N$	0.005778	0.0060485970	0.002721	0.006488	0.006116	0.005980866
mve_{in}	0.024096	0.0003773499	0.004069	0.024426	0.006757	0.013242087
mve_{out}	0.002077	-0.0463552197	-0.018917	0.009937	-1.397635	-0.290178644
vw	0.005948	0.005668333	0.005668	0.005668	0.005668	0.005738083
rp	0.005927	0.0066274083	0.002200	0.007271	0.006332	0.005671482

Turnover Rate

Formula:

$$\frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{i=1}^{N}|w_t^i - w_{t^-}^i| \quad \text{where} \quad w_{t^-}^i = \begin{cases} 0, & t=1 \\ \frac{w_{t-1}^i(1+R_{t-1}^i)}{1+\boldsymbol{w}_{t-1}\cdot R_{t-1}}, & t>1. \end{cases}$$

Table 7: Asset Turn-Over

Strategy	SPX $N=11$	Industry Portfolios $N=11$	Mkt/SMB/HML $N=3$	FF 1-factor $N=21$	FF 4-factor $N=24$	Mean
$1/N$	0.006584	0.02194351	0.023718	0.018814	0.022231	0.018658102
mve_{in}	-	-	-	-	-	-
mve_{out}	26.845508	26.46202826	6.078604	32.830331	46.131542	27.66960265
vw	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
rp	0.071080	0.02829952	1.996711	19.937500	22.927632	8.992244504

Takeaways

The *1/N* was competitive with the other implementable strategies we considered. Though some strategies performed better in some circumstances, there was no clear winner. There seems to be no premium for sophistication.

Thank You

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