

## ADRL - midterm 2023

25 marks, 90 minutes

### Instructions:

- i) Kindly answer in terms of equations wherever possible and refrain from verbose.
- ii) Write your name & SR number on the sheet.
- iii) Answer to the point & be precise. Kindly do not write long descriptions.

Question 1 :

a) Suppose  $x$  is a continuous random variable.

Define the (cumulative) distribution function  $F_x$  of  $x$ .  
(i)

b) If  $y = F_x$ , is  $y$  a valid random variable?  
Justify your answer (i)

c) If  $p_y$  denote the density function of  $y$ , derive  
an expression for  $p_y$  and show that it will be the  
same irrespective of form of  $F_x$ . (2)

Question 2:

- (a) Define a latent variable model  $p_\theta(x)$ , mathematically.  
(i)
- b) Derive a lower bound on the log likelihood of  
the above model in terms of the model parameters  
& an arbitrary density over the latent variable (3)
- c) Derive a condition for the above lower bound  
to be tight (2).

Question 3 :

Suppose  $D = \{x_1, x_2, \dots, x_n\} \sim \text{i.i.d } p_x, x_i \in \mathbb{R}$ .

Let  $p_\theta(x)$  be a two-component mixture of an exponential density.

- a) How many parameters does  $p_\theta(x)$  have? List them  
(1)
- b) Derive an iterative update algorithm to estimate all the parameters of  $p_\theta(x)$  via ELBO optimization  
(4)

Hint: PDF of exponential density :  $\lambda e^{-\lambda x}$ .

#### Question 4 :

- a) Write the expression for the ELBO, that is optimized in a VAE (1).
- b) Which of the terms in the expression of ELBO, contributes to the "Auto-Encoding" part of VAE & why ? (1)
- c) How are each of the terms in the ELBO, realized in a VAE ? (2)
- d) Which part of the VAE is a generative model & why ? (2)

## Question 5 :

- a) What is the issue in using the reverse KL minimization while building a model (1) ?
- b) Show that the forward KL is a special case of f-divergence with an appropriate f-function (1).
- c) Write an expression for minimization of a lower bound on f-divergence & denote the "adversarial" part in it (2).