# Week-5 Questions

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1. Let a latent variable $z \sim \text{Bernoulli}(\pi)$, and the conditional distribution of $x$ given $z$ be:

$$p(x \mid z) = \begin{cases} \mathcal{N}(x \mid \mu_1, \sigma^2), & z = 1 \\ \mathcal{N}(x \mid \mu_0, \sigma^2), & z = 0 \end{cases}$$

   What is the expected value $\mathbb{E}[x]$?

   A. $\mu_0 + \pi(\mu_1 - \mu_0)$
   B. $\pi\mu_1 + (1 - \pi)\mu_0$
   C. $\frac{\mu_1 + \mu_0}{2}$
   D. $\pi(\mu_1 - \mu_0)^2$

   **Correct Answer: B**

2. Consider a 2-component GMM:

$$p(x) = \pi \cdot \mathcal{N}(x \mid \mu_1, \sigma^2) + (1 - \pi) \cdot \mathcal{N}(x \mid \mu_2, \sigma^2)$$

   What is the expression for $\log p(x)$?

   A. $\log \left[ \pi \cdot \exp\left(-\frac{(x-\mu_1)^2}{2\sigma^2}\right) + (1 - \pi) \cdot \exp\left(-\frac{(x-\mu_2)^2}{2\sigma^2}\right) \right]$
   B. $-\frac{(x-\mu_1)^2}{2\sigma^2} + \log \pi + \log(1 - \pi)$
   C. $\pi \log \mathcal{N}(x \mid \mu_1, \sigma^2) + (1 - \pi) \log \mathcal{N}(x \mid \mu_2, \sigma^2)$
   D. $\log \left[ \pi \cdot (x - \mu_1)^2 + (1 - \pi) \cdot (x - \mu_2)^2 \right]$

   **Correct Answer: A**

3. In the E-step of EM for a 2-component GMM, the posterior probability that a point $x$ comes from component 1 is:

$$\gamma_1(x) = \frac{\pi_1 \cdot \mathcal{N}(x \mid \mu_1, \sigma^2)}{\pi_1 \cdot \mathcal{N}(x \mid \mu_1, \sigma^2) + \pi_2 \cdot \mathcal{N}(x \mid \mu_2, \sigma^2)}$$

   Let $\pi_1 = 0.6$, $\pi_2 = 0.4$, $\mu_1 = 0$, $\mu_2 = 3$, $\sigma^2 = 1$, and $x = 1$. What is $\gamma_1(x)$ approximately?

A. $\approx 0.80$

B. $\approx 0.63$

C. $\approx 0.50$

D. $\approx 0.35$

**Correct Answer: B**

4. Let $P = (0.5, 0.5)$ and $Q = (0.9, 0.1)$. Compute $D_{\mathrm{KL}}(P \| Q)$.

A. $\approx 0.510$

B. $\approx 0.222$

C. $\approx 0.737$

D. $\approx 0.161$

**Correct Answer: C**

5. Let $f(x) = \log(x)$, which is a concave function. Let $X$ be a random variable such that $\mathbb{E}[X] = 2$. Which of the following is **necessarily true**?

A. $\log(\mathbb{E}[X]) > \mathbb{E}[\log(X)]$

B. $\log(\mathbb{E}[X]) = \mathbb{E}[\log(X)]$

C. $\log(\mathbb{E}[X]) < \mathbb{E}[\log(X)]$

D. Nothing can be said without knowing the distribution of $X$

6. Let $f(x) = x^2$, and consider a discrete random variable $X$ with values 1 and 3 each with probability $\frac{1}{2}$. Compute and compare:

$$f(\mathbb{E}[X]) \text{ and } \mathbb{E}[f(X)]$$

A. $f(\mathbb{E}[X]) = \mathbb{E}[f(X)]$

B. $f(\mathbb{E}[X]) > \mathbb{E}[f(X)]$

C. $f(\mathbb{E}[X]) < \mathbb{E}[f(X)]$

D. None of the above

**Solution:**

$$\mathbb{E}[X] = \frac{1+3}{2} = 2 \quad \Rightarrow \quad f(\mathbb{E}[X]) = 2^2 = 4$$

$$\mathbb{E}[f(X)] = \frac{1^2 + 3^2}{2} = \frac{1+9}{2} = 5$$

**Correct Answer: C**

7. Consider a 1D Gaussian Mixture Model with 2 components:

$$p(x) = \pi_1 \mathcal{N}(x \mid \mu_1, \sigma^2) + \pi_2 \mathcal{N}(x \mid \mu_2, \sigma^2)$$

Given: $\pi_1 = 0.6, \pi_2 = 0.4$, $\mu_1 = 0, \mu_2 = 2$, $\sigma^2 = 1$. Let $x = 1$. What is the responsibility $\gamma(z_1) = p(z = 1 \mid x)$?

   A. $\approx 0.5$
   B. $\approx 0.62$
   C. $\approx 0.71$
   D. $\approx 0.38$

   **Correct Answer: C**

8. Let the log-likelihood at iteration $t$ be $\mathcal{L}^{(t)}$, and at iteration $t+1$ be $\mathcal{L}^{(t+1)}$. Which of the following is always true for the EM algorithm?

   A. $\mathcal{L}^{(t+1)} > \mathcal{L}^{(t)}$
   B. $\mathcal{L}^{(t+1)} \geq \mathcal{L}^{(t)}$
   C. $\mathcal{L}^{(t+1)} < \mathcal{L}^{(t)}$
   D. $\mathcal{L}^{(t+1)} \leq \mathcal{L}^{(t)}$

   **Correct Answer: B**

9. Given a generative model $p_\theta(x, z) = p_\theta(x \mid z)p(z)$, and an approximate posterior $q_\phi(z \mid x)$, which of the following correctly expresses the ELBO $\mathcal{L}(\theta, \phi; x)$?

   A. $\mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x, z) - \log q_\phi(z \mid x)]$
   B. $\log p_\theta(x) - D_{\mathrm{KL}}(q_\phi(z \mid x) \parallel p_\theta(z \mid x))$
   C. $-D_{\mathrm{KL}}(q_\phi(z \mid x) \parallel p(z)) + \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x \mid z)]$
   D. All of the above

   **Correct Answer: D**

10. What is the main idea behind the Variational Autoencoder (VAE) framework in generative modeling?

   A. To maximize the exact log-likelihood of data using a deterministic encoder-decoder structure.
   B. To learn a deterministic mapping from data to a low-dimensional manifold for clustering tasks.
   C. To approximate the intractable posterior over latent variables using a tractable distribution, and optimize a lower bound on the data likelihood.
   D. To model the joint distribution $p(x, z)$ using purely discriminative objectives.

   **Correct Answer: C**