Homework 2 — P1.2

Thursday, June 1, 2023 9:13 PM

P1.2

$$F = \sum_i \sum_k \pi_{ik} \|x_i - \mu_k\|^2$$

$$\frac{\partial F}{\partial \mu_k} = \sum_{i=1}^{n} \frac{\partial}{\partial \mu_k} \boxed{\left(\sum_{k=1}^{K} \pi_{ik} \|x_i - \mu_k\|^2\right)}$$

assume L

$$L = \sum_{k=1}^{K} \pi_{ik} \|x_i - \mu_k\|^2$$

$$\frac{\partial L}{\partial \mu_k} = \sum_{k=1}^{K} \frac{\partial}{\partial \mu_k}\left(\pi_{ik} \|x_i - \mu_k\|^2\right)$$

$$= \sum_{k=1}^{K} 2\pi_{ik}(x_i - \mu_k)$$

Equating this derivative to zero,

$$\sum_{k=1}^{K} 2\pi_{ik}(x_i - \mu_k) = 0$$

$$\Rightarrow 2\sum_{k=1}^{K} \pi_{ik}(x_i - \mu_k) = 0$$

$$\Rightarrow 2\sum_{k=1}^{K} \pi_{ik} x_i - 2\sum_{k=1}^{K} \pi_{ik}\mu_k = 0$$

$$\Rightarrow \cancel{2}\sum_{k=1}^{K} \pi_{ik} x_i = \cancel{2}\sum_{k=1}^{K} \pi_{ik}\mu_k$$

$$\Rightarrow \mu_k = \frac{\sum_{k=1}^{K} \pi_{ik} x_i}{\sum_{k=1}^{K} \pi_{ik}}$$

This equation shows that the optimum value of μ_k is the weighted average of data points, where the weights are membership probabilities (π_{ik})

By iteratively updating the centroids according to this equation and reassigning data points to nearest centroid, the k-means algorithm converges to a solution where the centroids achieve the minimum objective given current memberships.