CS0351: COMPUTER GRAPHICS

ASSIGNMENT-5

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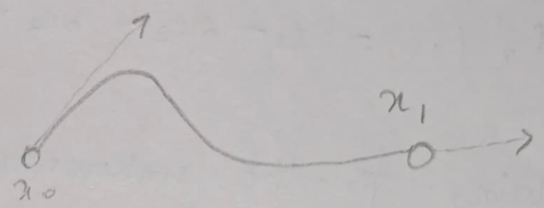
1. Hermite Curves

- A Hermite curve is a curve for which the user provides:

– The endpoints of the curve.
– The parametric derivatives of the curve at the endpoints (tangents with length) (df_x/dt, df_y/dt); where f_x and f_y are functions of t.



> Mathematical Representation –

- For x, we have constraints:

– The curve must pass through x_0 when $t=0$.
– The derivative must be x_0' when $t=0$.
– The curve must pass through x_1 when $t=1$.
– The derivative must be x_1' when $t=1$.

$$f_x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\frac{df_x(t)}{dt} = f_x'(t) = c_1 + 2c_2 t + 3c_3 t^2$$

c_i are unknown, $t = 0 \sim 1$

The curve must pass through x_0 when $t = 0$

$\Rightarrow f_x(0) = c_0 = x_0$

The derivative must be x_0' when $t = 0$

$\Rightarrow f_x'(0) = c_1 = x_0'$

The curve must pass through x_1 when $t = 1$

$\Rightarrow f_x(1) = c_0 + c_1 + c_2 + c_3 = x_1$

The derivative must be x_1' when $t = 1$

$\Rightarrow f_x'(1) = c_1 + 2c_2 + 3c_3 = x_1'$

- Solving for the unknowns:

$c_0 = x_0$

$c_1 = x_0'$

So, from eqn obtained above.

$x_0 + x_0' + c_2 + c_3 = x_1$

$x_0' + 2c_2 + 3c_3 = x_1'$

$\Rightarrow x_0' + 3c_3 - 2x_0 - 2x_0' - 2c_3 = x_1' - 2x_1$

$\Rightarrow c_3 = 2x_0 + x_0' - 2x_1 + x_1'$

$\Rightarrow \quad c_2 = x_1 - x_0 - x_0' - 2x_0 - x_0' + 2x_1 - x_1'$

$\Rightarrow \quad c_2 = -3x_0 - 2x_0' + 3x_1 - x_1'$

So, $\quad f_x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

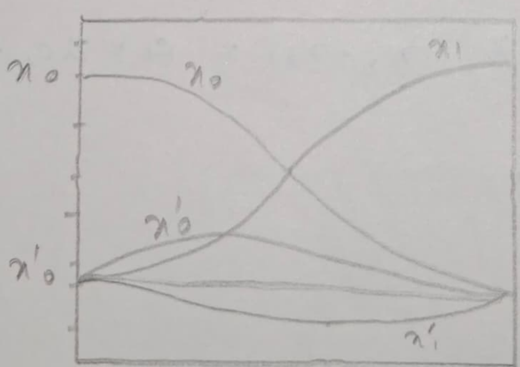
$$\Rightarrow \quad x = x_0 + x_0' t + (3x_1 - 3x_0 - x_1' - 2x_0')t^2 + (-2x_1 + 2x_0 + x_1' + x_0')t^3$$

$$\Rightarrow \quad x = x_1(-2t^3 + 3t^2) + x_0(2t^3 - 3t^2 + 1) + x_1'(t^3 - t^2) + x_0'(t^3 - 2t^2 + t)$$

$$\Rightarrow \quad x = \begin{bmatrix} x_0 & x_1 & x_0' & x_1' \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

- Extending to 3D

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 & x_1 & x_0' & x_1' \\ y_0 & y_1 & y_0' & y_1' \\ z_0 & z_1 & z_0' & z_1' \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$



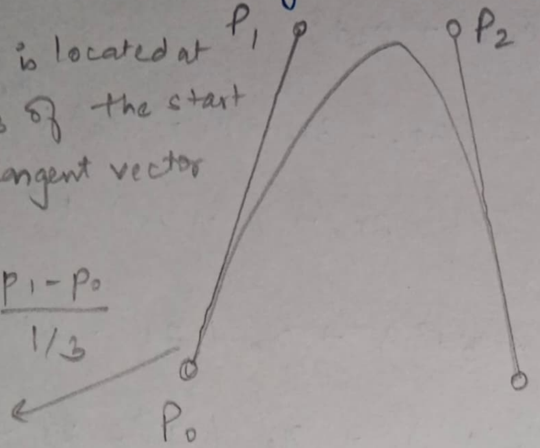
The blending weights

20) Bezier Curves :

- Two control points define endpoints, and two points control the tangents.

P_1 is located at $1/3$ of the start tangent vector

P_2 located at $2/3$ of the end tangent vector.



$p'(0) \approx \dfrac{P_1 - P_0}{1/3}$

$p'(1) \approx \dfrac{P_3 - P_2}{1/3}$

Slope $p'(0)$

Slope $p'(1)$

- The end site conditions are same as Hermite curves.

$$f_x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\frac{d\, f_x(t)}{dt} = \frac{f_x'(t)}{dt} = c_1 + 2c_2 t + 3c_3 t^2$$

- $f_x(0) = x_0 = c_0$
- $f_x(1) = x_3 = c_0 + c_1 + c_2 + c_3$

- Approximating derivative conditions.
  - $f'_x(0) = 3(x_1 - x_0) = c_1$
  - $f'_x(1) = 3(x_3 - x_2) = c_1 + 2c_2 + 3c_3$

- Solving equations.

$c_0 = x_0$

$c_1 = 3(x_1 - x_0)$

$$x_0 + 3x_1 - 3x_0 + c_2 + c_3 = x_3$$

$$3x_1 - 3x_0 + 2c_2 + 3c_3 = 3x_3 - 3x_2$$

$$\Rightarrow \quad 3x_1 - 3x_0 + \cancel{2c_2} + 3c_3 - 2x_0 - 6x_1 + 6x_0 - \cancel{2c_2} - 2c_3$$

$$= 3x_3 - 3x_2 - 2x_3$$

$$\Rightarrow \quad c_3 = x_3 - 3x_2 + 3x_1 - x_0$$

Now $\quad c_2 = \cancel{x_3} - \cancel{x_3} + 3x_2 - 3x_1 + \cancel{x_0} - \cancel{x_0} - 3x_1 + 3x_0$

$$\Rightarrow \quad c_2 = 3x_2 - 6x_1 + 3x_0$$

So,

$$c_0 = x_0$$

$$c_1 = 3x_1 - 3x_0$$

$$c_2 = 3x_2 - 6x_1 + 3x_0$$

$$c_3 = x_3 - 3x_2 + 3x_1 - x_0$$

- On replacing the original Hermite Matrix (x_s, x_e, x'_s, x'_e)

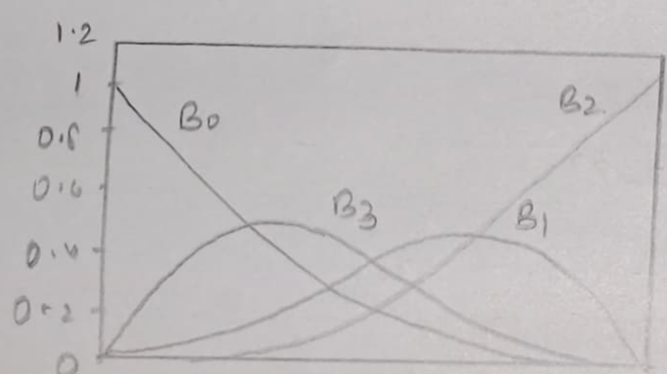
$$\begin{bmatrix} x_s \\ x_e \\ x'_s \\ x'_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_s & x_e & x'_s & x'_e \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

- A Bezier Curve (x value) becomes

$$x(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} (1-t)^3 \\ 3t(1-t)^2 \\ 3t^2(1-t) \\ t^3 \end{bmatrix}^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



The blending weights

Properties of Bezier Curves:

- Not all of the control points are on the line
  - Some just attract it towards themselves.
- Points have "influence" over the course of the line.
- "Influence" (attraction) is calculated from a polynomial expression.
- The first and last control points are interpolated.
- The tangent to the curve at the first control point is along the line joining the first and second control points.
- The tangent at the last control point is along the line joining the second last and last control points.
- The curve lies entirely within the convex hull of its control points
  - The Bernstein polynomials (the basis functions) sum to 1 and are everywhere positive,
- They can be rendered in many ways.
  - E.g: Convert to line segments with a subdivision algorithm.