Clustered Subset Count: A Proposed Combinatorial Function

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Abstract

We introduce a new combinatorial counting function, the *Clustered Subset Count*, which counts the number of subsets of a finite integer set where all elements are mutually within a limited distance from each other. This function captures a novel subset structure not addressed by classical combinatorial functions and opens new pathways for study in pure combinatorics.

1 Introduction

Combinatorics traditionally focuses on counting objects under various constraints, with foundational concepts such as permutations, combinations, and partitions. In this paper, we propose a novel construct: subsets of integers that are *clustered*, meaning each element is within a given distance of another element in the subset. This leads to a new counting function, $C(n, k; d)$, which counts the number of clustered subsets of size k drawn from the set $\{1, 2, \ldots, n\}$.

2 Definition of Clustered Subsets

Given an integer set $S = \{1, 2, \ldots, n\}$, a subset $A \subseteq S$ of size k is said to be *clustered* if for every element $a \in A$, there exists at least one other element $b \in A$ such that $|a - b| \leq d$, where $d \in \mathbb{N}$ is a fixed clustering distance parameter.

3 Clustered Subset Count Function

We define the *Clustered Subset Count Function* $C(n, k; d)$ as follows:

$$C(n, k; d) = \left|\{A \subseteq \{1, \ldots, n\} : |A| = k, \forall a \in A, \exists b \in A \setminus \{a\}, |a - b| \leq d\}\right|. \quad (1)$$

This function captures the idea of locally-dense subsets and provides a new way to examine proximity relationships within a combinatorial framework.

4 Example Computation

Let us compute $C(5, 3; 2)$:

- All 3-element subsets of $\{1, 2, 3, 4, 5\}$ are considered.
- **Clustered subsets**: $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{1, 3, 5\}$, $\{2, 3, 4\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$.
- **Non-clustered subsets**: $\{1, 2, 5\}$ and $\{1, 4, 5\}$.

Hence,

$$C(5, 3; 2) = 8.$$

5 Explanation of Parameters

The parameters used in defining the function are as follows:

- n: The size of the integer set $S = \{1, 2, \ldots, n\}$. It represents the range of integers from which subsets are drawn.
- k: The size of the subset A being considered. It specifies how many elements are included in each subset.
- d: The clustering distance parameter. It determines the maximum allowable distance between any two elements in a clustered subset.

6 Conclusion

The proposed function introduces a new concept to combinatorics: subsets governed by proximity rather than selection alone. This paves the way for richer structural studies in subset dynamics and discrete geometry.