Royal Institute of Technology

Project 2A

SF2812 Applied Linear Optimization

Varme

Author
Dhruv Haldar 951019-6174
haldar@kth.se
William Grunert 930218-4412
wgrunert@kth.se

Supervisor
Gianpiero Canessa

February 10, 2020

1 Project description and background

The assignment was to minimize the cost of a power plant called Varme for a 24 hour cycle. The power plant consisted of 3 power units and the cycle was divided in 5 different periods. The expected power demand varied between the periods, see Table 1.

Time Period	Expected demand [MW]
00 - 05	50
05 - 10	60
10 - 15	80
15 - 20	70
20 - 24	60

Table 1: Expected demand for each period.

The units had an initial cost and a running cost. The initial cost depended on how long the unit had been turned off. If the unit had been turned off more than one period it had to warm up and it would therefore cost 50% more, see Table 2. The units could not run for more than three consecutive periods before they needed to rest.

Unit	Initial cost (1 period rest) [kkr]	Initial cost (2 periods rest) [kkr]	Running cost [kkr/(MW·hour)]
1	10	15	2.5
2	13	19.5	2.5
3	16	24	2.4

Table 2: Cost for each unit.

Furthermore were there a minimum and maximum production level for each unit which can be seen in Table 3.

Unit	Minimum level [MW]	Maximum level [MW]
1	10	50
2	12	45
3	15	55

Table 3: Upper and lower production level for the units.

2 Mathematical formulation

The variables of the mixed integer problem were part of the sets power plant units, i, and time periods, j.

- $a_{i,u}$: upper production capacity for each unit
- $a_{i,l}$: lower production capacity for each unit
- b_j: demand for each period
- c_i: running cost for each unit
- p_i: initial cost

- t_j: hours per period
- $x_{i,j}$: power produced each time period by each unit
- $u_{i,j}$: binary variable to express if a unit is running
- $v_{i,j}$: binary variable to express if a unit has started to run
- $w_{i,j}$: binary variable to express if a unit has been turned off for more than one time period

The cost could be minimized with the following formulation.

$$\text{minimize} \qquad \sum_{i=1}^{3}\sum_{j=1}^{5}\big(x_{i,j}c_i t_j + p_i(v_{i,j} + 0.5w_{i,j})\big)$$

$$\begin{aligned}
\text{subject to} \qquad & x_{i,j} - a_{i,u}u_{i,j} \le 0, \qquad \forall\, i,j \\
& x_{i,j} - a_{i,l}u_{i,j} \ge 0, \qquad \forall\, i,j \\
& \sum_{i=1}^{3} x_{i,j} \ge b_j, \qquad \forall\, j \\
& \left.\begin{aligned}
u_{i,1} + u_{i,2} + u_{i,3} + u_{i,4} \le 3 \\
u_{i,1} + u_{i,2} + u_{i,3} + u_{i,5} \le 3 \\
u_{i,1} + u_{i,2} + u_{i,4} + u_{i,5} \le 3 \\
u_{i,1} + u_{i,3} + u_{i,4} + u_{i,5} \le 3 \\
u_{i,2} + u_{i,3} + u_{i,4} + u_{i,5} \le 3
\end{aligned}\right. \qquad \forall\, i \\
& v(i,1) \ge u_{i,1} - u_{i,5} \qquad \forall\, i \\
& v(i,j) \ge u_{i,j} - u_{i,j-1} \qquad \forall\, i,\ j = 2, \dots, 5 \\
& w(i,1) \ge u_{i,1} - u_{i,5} - u_{i,4} \qquad \forall\, i \\
& w(i,2) \ge u_{i,2} - u_{i,1} - u_{i,5} \qquad \forall\, i \\
& w(i,j) \ge u_{i,j} - u_{i,j-1} - u_{i,j-2} \qquad \forall\, i\ j = 3, \dots, 5 \\
& t_j = 5, \qquad j = 1, \dots, 4 \\
& t_j = 4, \qquad j = 5 \\
& u_{i,j} \in \{0,1\}, \qquad \forall\, i,j \\
& v_{i,j} \in \{0,1\}, \qquad \forall\, i,j \\
& w_{i,j} \in \{0,1\}, \qquad \forall\, i,j
\end{aligned}$$

The first term in the cost function expresses the running cost for the units during each period. The second term in the cost function expresses the initial cost for a unit if it has been turned of only one time period or more than one time period.

2.1 Constraints

Constraint (1) keeps the production for each unit within its maximum production level.

$$x_{i,j} - a_{i,u}u_i \le 0, \qquad \forall\, i,j \tag{1}$$

Constrain (2) ensures that the units are at least run at the minimum level of production.

$$x_{i,j} - a_{i,l}u_i \ge 0, \qquad \forall\, i,j \tag{2}$$

Constraint (3) makes sure that the combined production for all running units meets the demand for each time period.

$$\sum_{i=1}^{3} x_{i,j} \geq b_j, \qquad \forall\, j \tag{3}$$

Constraint (4) limits the number of consecutive time period a unit can run to three.

$$\begin{cases} u_{i,1} + u_{i,2} + u_{i,3} + u_{i,4} & \leq 3 \\ u_{i,1} + u_{i,2} + u_{i,3} + u_{i,5} & \leq 3 \\ u_{i,1} + u_{i,2} + u_{i,4} + u_{i,5} & \leq 3 \\ u_{i,1} + u_{i,3} + u_{i,4} + u_{i,5} & \leq 3 \\ u_{i,2} + u_{i,3} + u_{i,4} + u_{i,5} & \leq 3 \end{cases} \qquad \forall\, i \tag{4}$$

Constraint (5) ensures that the unit is turned on when it has been turned off at least the previous time period.

$$\begin{cases} v(i,1) \geq u_{i,1} - u_{i,5} & \forall\, i \\ v(i,j) \geq u_{i,j} - u_{i,j-1} & \forall\, i,\ j = 2, \ldots, 5 \end{cases} \tag{5}$$

Constraint (6) sets the binary variable w to one when a unit starts to run after more than one time period turned off and zero otherwise.

$$\begin{cases} w(i,1) \geq u_{i,1} - u_{i,5} - u_{i,4} & \forall\, i \\ w(i,2) \geq u_{i,2} - u_{i,1} - u_{i,5} & \forall\, i \\ w(i,j) \geq u_{i,j} - u_{i,j-1} - u_{i,j-2} & \forall\, i\ j = 3, \ldots, 5 \end{cases} \tag{6}$$

Constraint (7) expresses the length of each time period.

$$\begin{cases} t_j = 5, & j = 1, \ldots, 4 \\ t_j = 4, & j = 5 \end{cases} \tag{7}$$

3 Result and Analysis

The optimal solution was calculated in GAMS with mixed-integer programming. Unit 1 initiated its run in period 5, unit 2 in period 3 and unit 3 in period 2. All units ran for three consecutive time periods followed by two periods turned off. The distribution of the produced energy for the time periods can be seen in Table 4.

Unit	Period 1	Period 2	Period 3	Period 4	Period 5
Expected demand	50	60	80	70	60
1	50	10	-	-	48
2	-	-	25	15	12
3	-	50	55	55	-

Table 4: Production per unit per time period [MW].

Since unit 3 had the lowest running cost it ran as close as possible at maximum capacity for its periods. Although, unit 1 had to reach its minimum production level in period 2 which resulted in a lower production for unit 3. Unit 1 was enough to satisfy the demand in period 1. In period 5 unit 2 produces at its minimum production level. This is just an arbitrary solution and does not affect the minimum cost since both unit 1 and unit 2 have the same running cost.
The minimum cost calculated with mixed-integer programming was for Varme 3828.5 kkr.

4 Conclusion

The best possible solution for the given problem comes out to be 3828.150 kkr, which is 0.350 kkr less than the total cost with mixed-integer programming. It is also observed that the cold starts are more than the warm starts, which proposes the notion that the model can be improved by increasing the cycle time period from 24 hours to a longer duration, so that the units are down only for one period i.e. warm-start.
Furthermore is a 24 hour cycle uncertain for the reason that the consumption of energy may differ between seasons. But the consumption could also differ between the days within a season. The cycle could therefore be improved from to a longer one, for instance to a year.
If the demand is known for every hour of the day it might decrease the cost if the 24 hour cycle is divided into more time periods. That way the problem could be optimized with a higher accuracy.