Green Destruct AB: Material Destruction Process

Project Assignment 1B, Group 3

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Submitted: February 2020

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Abstract

Companies are effectively searching for efficient allocation of resources to increase their revenues and profits while keeping the costs under control. The purpose of our report is to carry out analysis for Green Destruct AB, a material destruction company. We have come to a solution that implements Linear Programming by using GAMS. The program processes the given data, constraints and variables in question. The problem was split into two cases, the first being the deterministic constraints and the second with the same constraints except the demand which had a normal distribution. The final solution implies an optimal profit and allocation of resources.

1 Problem Description

1.1 Part 1

A wooden floorboard company produces an estimated amount of 31 tons of hazardous waste weekly along with their produced floorboards, spread throughout their eight factories in eight cities as shown in the table below:

Production factory, b_j	Supply in tons
Eskilstuna, b_1	2
Falun, b_2	2
Gävle, b_3	4
Norrköping, b_4	3
Stockholm, b_5	9
Uppsala, b_6	6
Västerås, b_7	3
Örebro, b_8	2

Table 1: Production supply constraints

The floorboard company hires *Green Destruct AB* to pick-up and handle the waste at a cost of 250 kr per kilogram waste. Green Destruct AB has four factories for destruction in four different cities, with the following capacities:

Destruction factory, a_i	Capacity in tons
Arboga, a_1	7
Fagersta, a_2	8
Ludvika, a_3	9
Nyköping, a_4	8

Table 2: Waste demand capacities

Furthermore, Green Destruct AB is responsible for the transportation of the waste from the production factories to the destruction factories. The transportation cost is 1 kr per kilogram per kilometer up to 1800 kg, and 1.45 kr per kilometer for every kilogram above 1800 kg. Additionally, there is a cost of destruction of 120 kr per kilogram for waste up to 1800kg, and an estimated destruction cost of 100 kr for kilogram above 1800 kg as a result of economies of scale. The distances between the production factories and destruction factories are given in the table below.

	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	45	165	195	110	156	129	52	40
Fagersta	95	88	126	187	176	117	66	103
Ludvika	137	63	150	220	222	163	112	112
Nyköping	82	260	257	59	106	168	128	135

Table 3: Distances between factories in km

The objective is to help Green Destruct AB minimize costs by finding the ideal transportation routes and where the waste should be destructed.

1.2 Part 2

Green Destruct AB is about to enter a deal in which they have to handle all the waste produced, i.e. an uncertain amount, rather than the fixed amount as mentioned in section 1.1. The given numbers of supply will be the estimated value of production each week, and the production can be modeled with a normal distribution with a 5% standard deviation. Additionally, the cheaper transportation up to 1800 kg must be the same each week, whereas the transportation with the more costly freight for any weight above 1800 kg can be changed.

Green Destruct AB is not sure if they have the capacity needed for such fluctuations. If necessary, they can increase their capacities at the plants for a cost of 200 kr per kilogram extra capacity. The objective of this part is to help Green Destruct AB analyze the situation and suggest changes for their factories.

2 Mathematical Formulation

The variables used in the mathematical formulation of the problem is defined and explained in the table below.

Definition	Variable	Value
Shipment quantities for less than 1.8 tons (1800 kilos)	x_{low}	Positive Variable
Shipment quantities for more than 1.8 tons (1800 kilos)	x_{high}	Positive Variable
Destruction cost for less than 1.8 tons (1800 kilos)	c_{low}	120000 kr per ton
Destruction cost for more than 1.8 tons (1800 kilos)	c_{high}	100000 kr per ton
Transportation cost for less than 1.8 tons (1800 kilos)	f_{low}	1 kr per kilo per kilometer
Transportation cost for more than 1.8 tons (1800 kilos)	f_{high}	1.45 kr per kilo per kilometer
Supply constraints for factory j	b_j	See Table 1
Demand constraints for factory i	a_i	See Table 2
Distances between factories i and j	$d_{i,j}$	See Table 3

Table 4: Variables used

The constraints for the mathematical formulation is given by the capacity constraints of the destruction factories. The shipment quantities $x_{i,j}$ from production company j can not exceed the capacity constraints of destruction factory i. This gives:

$$\begin{aligned} \sum_{j=1}^{8} x_{1,j} &\leq 7 \\ \sum_{j=1}^{8} x_{2,j} &\leq 8 \\ \sum_{j=1}^{8} x_{3,j} &\leq 9 \\ \sum_{j=1}^{8} x_{4,j} &\leq 8 \\ x_{i,j} &\geq 0 \end{aligned} \tag{1}$$

In addition to the above constraints, an equation which defines the threshold of 1.8 tons (1800 kilos) was defined. Hence, the shipment quantity up to 1.8 tons is allocated to the variable $x_{low_{i,j}}$, and the excess, if any, is allocated to the variable $x_{high_{i,j}}$. The lower limit constraint equation is defined as :

$$x_{low_{i,j}} \leq 1.800$$

For example, for a given capacity of 2 tons, the variable $x_{low_{i,j}}$ would be 1.8 tons and the variable $x_{high_{i,j}}$ would be 0.2 tons. In another case with a given capacity of 1.7 tons, the variable $x_{low_{i,j}}$ would be 1.7 and $x_{high_{i,j}}$ would be 0 tons.

Taking into account the transportation and freight costs up to 1.8 tons and in excess of 1.8 tons and combining them with the distances between factories i and j, we have the following two equations for the total transportation cost:

$$c_{low_{i,j}} = f_{low_{i,j}} \cdot d_{i,j} + 120000 \tag{2}$$

$$c_{high_{i,j}} = f_{high_{i,j}} \cdot d_{i,j} + 100000 \tag{3}$$

The supply equation of destruction factory i is represented as:

$$supply(i) : x_{low_{i,j}} + x_{high_{i,j}} \leq a_i \tag{4}$$

The demand equation of production company j is represented as:

$$demand(j) : x_{low_{i,j}} + x_{high_{i,j}} = b_j \tag{5}$$

The objective cost function to be minimized which allows additional purchases made for more than 1.8 tons is represented as :

$$z = c_{low_{i,j}} \cdot x_{low_{i,j}} + c_{high_{i,j}} \cdot x_{high_{i,j}} - 250000(x_{low_{i,j}} + x_{high_{i,j}}) \tag{6}$$

The mathematical formulation for part two uses the definitions from part one with a few additions. In addition to equations (2) and (3), we have:

$$c_{excess_{i,j}} = f_{high} \cdot d(i,j) + 200000 \tag{7}$$

The supply constraint is the same for both parts but the demand constraint is changed to:

$$demand(j) : x_{low_{i,j}} + x_{high_{i,j}} + x_{excess_{i,j}} = b_j \tag{8}$$

An additional constraint is added to contain x_{high}:

$$x_{high}(i,j) \leq a(i) \tag{9}$$

Taking into account the constraints,the objective function z becomes:

$$z = c_{low_{i,j}} \cdot x_{low_{i,j}} + c_{high_{i,j}} \cdot x_{high_{i,j}} + c_{excess_{i,j}} \cdot x_{excess_{i,j}} \tag{10}$$

3 Results and Analysis

Table 5 describes the optimal transportation routes when no excess is allowed, and demand and supply are set according to tables 1 and 2.

	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	2				1.4	1.6		2
Fagersta			1.8		1.8	2.6	1.8	
Ludvika		2	2.2		0.8	1.8	1.2	
Nyköping				3	5			

Table 5: Optimal transportation routes for question 1.

Table 6 describes the allocation of waste to the destruction plants. Arboga, Fagersta and Nyköping are all at max capacity while Ludvika has one ton to spare. This doesn't affect much if there is excess production in Falun which is only 63 km from Ludvika but if there suddenly is a larger production in Norrköping or Stockholm which are both more than 200 km away it will affect the cost more.

City	Used	Capacity
Arboga	7	7
Fagersta	8	8
Ludvika	8	9
Nyköping	8	8

Table 6: The optimal allocation of waste to the waste plants.

The resulting objective value is: -560 860 kr which means the company makes a profit of 560 860 kr with this allocation of waste.

Taken into account the ability to have excessive production at a plant the optimal allocation changes to the allocation shown in table 7.

City	Used	Capacity
Arboga	7	7
Fagersta	8	8
Ludvika	4	9
Nyköping	12	8

Table 7: The optimal allocation of waste to the waste plants with the option of excess.

Because of the high cost of transportation to Ludvika it is more cost effective to pay the higher destruction cost of exceeding the capacity at Nyköping to destruct the waste there. Therefore it might be a good recommendation to Green Destruct to, if possible, reallocate destruct capacity from Ludvika to Nyköping.

If the company starts managing the waste on a week to week basis and the demand is distributed according to the normal distribution model the amount of waste changes from week to week. Using the known parameters we know that the mean value of the distribution is 31 and the standard deviation is $31 \cdot 0.05 = 1.55$. Using these, the supply limit of 32, and the cumulative distribution we show that the probability of a demand larger than 32 is approximately 26%. Therefore a reasonable recommendation to the company is to increase its destruct capacity.

The objective value of this model varies because of the normal distribution but approximately its values varies around -641 070 kr which corresponds to a profit of 641 070 kr.

The results are reasonable and useful but the model isn't perfect. There are some problems common to all kind of models. First of all the model is a simplification of the real world. The distances between cities can be defined accurately but the transport costs can be hard to model correctly. Especially with this rather big simplification of two fixed costs. In the real case the transportation costs can probably vary a lot. Furthermore, no regard is taken of the possibility that something breaks. Whether the transportation or a factory breaks down it will affect the costs and this should also be modeled to make a recommendation to the company credible. For example, what happens if a waste plant breaks, where should the excess waste be transported then?

4 Summary and Conclusion

Analyzing the data provided by the company a couple of conclusions can be made. First of all, if there are no fluctuations of demand, the site with the excess capacity is Ludvika. If however, excess is allowed at the factories it is more efficient to allocate more of the waste destruction to Nyköping and less to Ludvika. If the demand is modeled as a normal distribution there is a 26% risk that the demand surpasses the supply. Even though the cost for the destruction increases, the company will still make a profit.