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Tutorial 2:- Asymptotic Notation

Q.1. (1) $f(n) = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

(2) $f(n) = 27n^2 + 16n$

(3) $f(n) = 3 \cdot 2^n + 4n^2 + 5n + 3.$

(1) $f(n) = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

For θ notation $0 \le C_1 g(n) \le f(n) \le C_2 g(n)$

Here $g(n) = n^3$

$\therefore$ $0 \le C_1 n^3 \le \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \le C_2 n^3$

$0 \le C_1 \le \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} \le C_2$

Now as we know that in denominator as the value ↑↑ the overall value decrease so the maximum value will be when $n = 1$ $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$

And as $n > 0$ $\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} \geq \frac{1}{3}$

$\therefore$ $\boxed{C_1 = 1/3 \quad C_2 = 1 \quad n_0 = 1}$

(2) $f(n) = 27n^2 + 16n$

$0 \le C_1 g(n) \le f(n) \le C_2 g(n)$

$\therefore$ $0 \le C_1 n^2 \le 27n^2 + 16n \le C_2 n^2$

$0 \le C_1 \le 27 + \frac{16}{n} \le C_2$

$n_0 = 1$ $f(n)$ is Max $\therefore C_2 = 27 + 16 = 43$

And as $n \geq 1$ and $f(n) = 27 + \frac{16}{n}$ Minimum value would be 27.

$\therefore$ $\boxed{C_1 = 27 \quad C_2 = 43 \quad n = 1}$

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(3) $f(n) = 3\cdot 2^n + 4n^2 + 5n + 3$

$0 \le C_1 g(n) \le 3\cdot 2^n + 4n^2 + 5n + 3 \le C_2 g(n)$

$C_1 2^n \le 3\cdot 2^n + 4n^2 + 5n + 3 \le C_2 2^n$

$\therefore C_1 \le 3 + \frac{4n^2}{2^n} + \frac{5n}{2^n} + \frac{3}{2^n} \le C_2$

Now for C_1 the value of $f(n)$ would be at least 3 $\therefore$ $C_1 = 3$

For C_2 as such we can't get any perfect value b/c their is n in both numerator & denominator we need to check behaviour of $f(n)$ for different n

$n=1$ $\quad f(n) = 3 + \frac{4}{2} + \frac{5}{2} + \frac{3}{2} = 9$

$n=2$ $\quad f(n) = 3 + \frac{16}{4} + \frac{10}{4} + \frac{3}{4} = \frac{41}{4} \approx 10.25$

$n=3$ $\quad f(n) = 3 + \frac{36}{8} + \frac{15}{8} + \frac{3}{8} = \cancel{6.75}\ 9.75$

$n=4$ $\quad f(n) = 3 + \frac{64}{16} + \frac{20}{16} + \frac{3}{16} = 8.4375$

So the the $f(n)$ increases upto 2 then decreases so maximum value of ~~n~~ $f(n)$ when so let's take $C_2 = 11$

$\therefore C_1 = 3 \quad C_2 = 11 \quad \& \ n_0 = 1$

Q.3 Show that $(n+a)^b = \Theta(n^b)$ $\quad b > 0$

$0 \le C_1 n^b \le (n+a)^b \le \cancel{C_2 O(n)^b}\ C_2 n^b$

To proove $(n+a) \le$ something

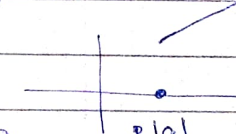
So $n+a \le n+|a|$ For all $\boxed{n \ge |a|}$ $n+a$

$n+a \le 2n$

Similarly to proove $n+a \ge$ something

$n+a \ge n-|a|$ $|a| \le n/2$

$n+a \ge n - n/2$


2|a|

$n+a \ge \frac{n}{2}$ ∴ $2|a| \le n$

$|a| \le n/2$

$0 \le \frac{n}{2} \le n+a \le 2n$ ∴ $|a| \le n/2$

$\frac{1}{2}n \le n+a$

$\left(\frac{n}{2}\right)^b \le (n+a)^b \le (2n)^b$ $\frac{1}{2}^b \times n^b$ c_1

$n+a \le 2n$

∴ $\boxed{c_1 = (1/2)^b \quad c_2 = 2^b}$ $\boxed{n_0 = 2|a|}$ $2^b \times n^b$ c_2

S-2 Find the O-notation

(1) $f(n) = 5n^3+n^2+3n+2$

Now $0 \le f(n) \le c g(n)$

$g(n) = n^3$ ∴ $0 \le 5n^3+n^2+3n+2 \le c n^3$

$0 \le 5+\frac{1}{n}+\frac{3}{n^2}+\frac{2}{n^3} \le c$

$c \ge 5+\frac{1}{n}+\frac{3}{n^2}+\frac{2}{n^3}$ for all $n \ge n_0$.

when $n=1$ $c=11$ ∴ we got c for $g(n)=n^3$

∴ $\boxed{O(f(n)) = O(n^3).}$

(2) $f(n) = 3n^3+4n$

$0 \le f(n) \le c g(n)$

$0 \le 3n^3+4n \le c n^3$

$0 \le 3+\frac{4}{n^2} \le c$

$c \ge 7$ & $n \ge 1$ we can find asymptotic eqn

∴ $\boxed{O(f(n)) = n^3}$

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Q-4 Is $2^{n+1} = O(2^n)$, Is $2^{2n} = O(2^n)$,

$0 \le 2^{n+1} \le cg(n)$
$0 \le 2^n \cdot 2^1 \le c(2^n)$
$2^1 \le c$ For any $c \ge 2$ & $n_0 = 1$

we can proove $\boxed{2^{n+1} = O(2^n)}$

$0 \le 2^{2n} \le cg(n)$
$0 \le 2^n \cdot 2^n \le c\,2^n$
$2^n \le c$ $c \ge 2^n$ their is no.
no. for which $c \ge 2^n$ always holds b/c
2^n is strictly increasing func^n.
$\therefore$
$\boxed{2^{2n} \ne O(2^n)}$

Q.5 Proove $(n\log n - 2n + 13) = \Omega(n\log n)$

$0 \le cg(n) \le f(n)$
$0 \le cg(n) \le n\log n - 2n + 13$
$0 \le n\log n - 2n \le n\log n - 2n + 13$

If we proove $c\,n\log n \le n\log n - 2n$
then definatetly $c\,n\log n \le n\log n - 2n + 13$

$\therefore$ $c\,n\log n \le n\log n - 2n$
$c \le 1 - \frac{2}{\log n}$

If we approximate some values then
like $n = 1, 2, 3 \ldots$
For $n = 4$ $c \le 1 - \frac{2}{\log_2 4}$ $c \le 0$ but cannot
be positive constant.

$n_0 \ge 5 \rightarrow$ not perfect value.

$c = 1/3$ for simplicity
we have taken
$1 - \frac{2}{\log_2 2^3} = 1 - \frac{2}{3} = 1/3$

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Q.-6. Proove that $\sum_{i=1}^{n} \log(i) = \theta(n \log n)$

$$0 \le c_1 (n\log n) \le \sum_{i=1}^{n} \log(i) \le c_2\, n \log n$$

$$0 \le c_1 n\log n \le \log(1)+\log(2)+\ldots\log(n) \le c_2\, n\log n$$
$$0 \le c_1 n\log n \le \log(n!) \le c_2\, n\log n$$

$$\log(n!) \le c_2 n\log n)$$
$$\therefore \log(1)+\log(2)\ldots \log(n) \le c_2 n\log n$$
$$\le \log n + \log n \ldots$$
$$\therefore \log(n!) \le 1 (n\log n)$$

$\therefore$ $c_2 = 1$ For c_1

$$c_1 n\log n) \le \log(n!)$$

Let's take only first half term

$\therefore$

$$c_1 \log(n)+\log(n-1)\ldots \cancel{\log(\tfrac{n}{2})} \le \log(n!)$$
c_1 (

we can say $\log(n) \ge \log(n/2)$

$\therefore$ Taking $\log(n/2) \to n/2$ prooves. left equality

$$\log(\cancel{1})+\log(\tfrac{n}{2})\ldots \tfrac{n}{2} \text{ time} \le \cancel{o\log n)} \log(n!)$$

$$c_1 \left(\log\left(\tfrac{n}{2}\right)^{n/2}\right) \le \log(n!)$$

$$\therefore \frac{n}{2}\log\left(\frac{n}{2}\right) \le \log(n!)$$

Thus $c_1 = 1$ $c_2 = 1$