2CS503 – Design and Analysis of Algorithms

Introduction

References

1. Charles E. Leiserson, Thomas H. Cormen, Ronald L. Rivest, Clifford Stein - Introduction to Algorithms, PHI

2. Ellis Horowitz, SartajSahni, Sanguthevar Rajasekharan, Fundamentals of Computer Algorithms, Galgotia.

3. Jean-Paul Tremblay and Paul G. Sorenson, An Introduction to Data Structures with Applications, Tata McGraw Hill

4. Karumanchi, Narasimha, Data Structures and Algorithms Made Easy, CareerMonk Publications.

Syllabus

Syllabus:	**Teaching Hours**
Unit I	**2**
Elementary Algorithmic: Efficiency of Algorithms, Average & worst-case analysis, Elementary Operation	
Unit II	**4**
Analysis Techniques: Empirical, mathematical, Asymptotic analysis and related unconditional and conditional notations	
Analysis of Algorithms: Analysing control structures: sequencing, “For” loops, Recursive calls, “While” and “repeat” loops, Amortized analysis	
Unit III	**4**
Solving Recurrences: Intelligent guesswork, Homogeneous recurrences, Inhomogeneous Recurrences, Change of variable, Range transformations, Master Theorem, Recurrence Tree.	
Unit IV	**7**
Data Structures: Heaps, Binomial heaps, Disjoint set structures.	
Greedy Algorithms: Graphs: Minimum spanning trees-Kruskal’s algorithm, Prim’s algorithm, Graphs: Shortest path algorithms.	
Unit V	**8**
Divide-and-Conquer: Multiplying large integers, Binary search, sorting: sorting by merging, quick sort, finding the median, Matrix multiplication, Exponentiation, approaches using recursion, Memory functions.	
Dynamic Programming: Principles of optimality, Various applications using dynamic programming.	
Unit VI	**5**
Branch and Bound, Backtracking: Design of some classical problems using branch and bound and Backtracking approaches.	
Randomized and Approximation Algorithms: Design of some classical problems using Randomized and Approximation Algorithms.	

Resources

All resources will be available on Moodle.

Teaching & Evaluation Scheme

Teaching Scheme:

Theory	Tutorial	Practical	Credits
2	1	2	4

	LPW	SEE	CE
Exam Duration	Continuous Evaluation + 2 Hrs. End Semester Exam	3.0 Hrs.	Continuous Evaluation
Component Weightage	0.2 (0.75+0.25)	0.4	0.4 (0.35+0.35+0.3)

Assignments in CE:

MOOC

Introduction

- What is an algorithm?
  - An algorithm is any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into the output.

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  - An algorithmic problem is specified by describing the complete set of instances it must work on and what desired properties the output must have.
  - For example, we might need to sort a sequence of numbers into nondecreasing order.

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- What is an algorithm?
  - Here is how we formally define the sorting problem:

Input: A sequence of n numbers $\langle a_1, a_2, \ldots, a_n \rangle$.

Output: A permutation (reordering) $\langle a'_1, a'_2, \ldots, a'_n \rangle$ of the input sequence such that $a'_1 \le a'_2 \le \cdots \le a'_n$.

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- Instance of a Problem
  - For example, given the input sequence (31, 41, 59, 26, 41, 58), a sorting algorithm returns as output the sequence (26, 31, 41, 41, 58, 59). Such an input sequence is called an instance of the sorting problem.

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- Instance of a Problem
  - For example, given the input sequence (31, 41, 59, 26, 41, 58), a sorting algorithm returns as output the sequence (26, 31, 41, 41, 58, 59). Such an input sequence is called an instance of the sorting problem.
  - In general, an instance of a problem consists of the input (satisfying whatever constraints are imposed in the problem statement) needed to compute a solution to the problem.

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  - Contrary to what you might expect, incorrect algorithms can sometimes be useful, if we can control their error rate.
  - Ordinarily, however, we shall be concerned only with correct algorithms.

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  - An algorithm can be specified in English, as a computer program, or even as a hardware design.
  - The only requirement is that the specification must provide a precise description of the computational procedure to be followed.

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    - Managing, manipulating & retrieving the voluminous data

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- Public-key cryptography and digital signatures
- Resource scheduling, etc.
  - A political candidate may want to determine where to spend money buying campaign advertising in order to maximize the chances of winning an election.

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    - All of these are examples of problems that can be solved using linear programming

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- The first, known as insertion sort, takes time roughly equal to c_1n^2 to sort n items, where c_1 is a constant that does not depend on n. That is, it takes time roughly proportional to n^2.

- The second, merge sort, takes time roughly equal to c_2 n lg n, where lg n stands for $\log_2$ n and c_2 is another constant that also does not depend on n.

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- Let's write insertion sort's running time as c_1 n n and merge sort's running time as c_2 n lg n.

- Then we see that where insertion sort has a factor of n in its running time, merge sort has a factor of lg n, which is much smaller. (For example, when n = 1000, lg n is approximately 10, and when n equals one million, lg n is approximately only 20.)

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- No matter how much smaller c_1 is than c_2, there will always be a crossover point beyond which merge sort is faster.

Efficiency of an Algorithm – Does it Matter?

- Different algorithms, constants & hardware proficiency
  - Problem: Sort 10 million numbers
  - Computer A: insertion sort, 10bi/s, c_1=2
  - Computer B: merge sort, 10mi/s, c_2=50

  - Computer A: $$\frac{2 \cdot (10^7)^2 \text{ instructions}}{10^{10} \text{ instructions/second}} = 20{,}000 \text{ seconds (more than 5.5 hours)}$$

  - Computer B: $$\frac{50 \cdot 10^7 \lg 10^7 \text{ instructions}}{10^7 \text{ instructions/second}} \approx 1163 \text{ seconds (less than 20 minutes)}$$

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  - Each such instruction takes constant amount of time – Primitive / Elementary Operations

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  - Sometimes, it is more appropriate to describe the size of the input with two numbers rather than one. For instance, if the input to an algorithm is a **graph**, the input size can be described by the numbers of vertices and edges in the graph.

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  - We shall indicate which input size measure is being used with each problem we study.

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  - This viewpoint is in keeping with the RAM model, and it also reflects how the pseudocode would be implemented on most actual computers.

Analyzing Algorithms

- Comparison of Running Time

	lg n	n	n lg n	n^2	n^3	2^n
1	0.0	1.0	0.0	1.0	1.0	2.0
2	1.0	2.0	2.0	4.0	8.0	4.0
5	2.3	5.0	11.6	25.0	125.0	32.0
10	3.3	10.0	33.2	100.0	1000.0	1024.0
15	3.9	15.0	58.6	225.0	3375.0	32768.0
20	4.3	20.0	86.4	400.0	8000.0	1048576.0
30	4.9	30.0	147.2	900.0	27000.0	1073741824.0
40	5.3	40.0	212.9	1600.0	64000.0	1099511627776.0
50	5.6	50.0	282.2	2500.0	125000.0	1125899906842620.0
60	5.9	60.0	354.4	3600.0	216000.0	1152921504606850000.0
70	6.1	70.0	429.0	4900.0	343000.0	1180591620717410000000.0
80	6.3	80.0	505.8	6400.0	512000.0	1208925819614630000000000.0
90	6.5	90.0	584.3	8100.0	729000.0	1237940039285380000000000000.0
100	6.6	100.0	664.4	10000.0	1000000.0	1267650600228230000000000000000.0

Analyzing Algorithms

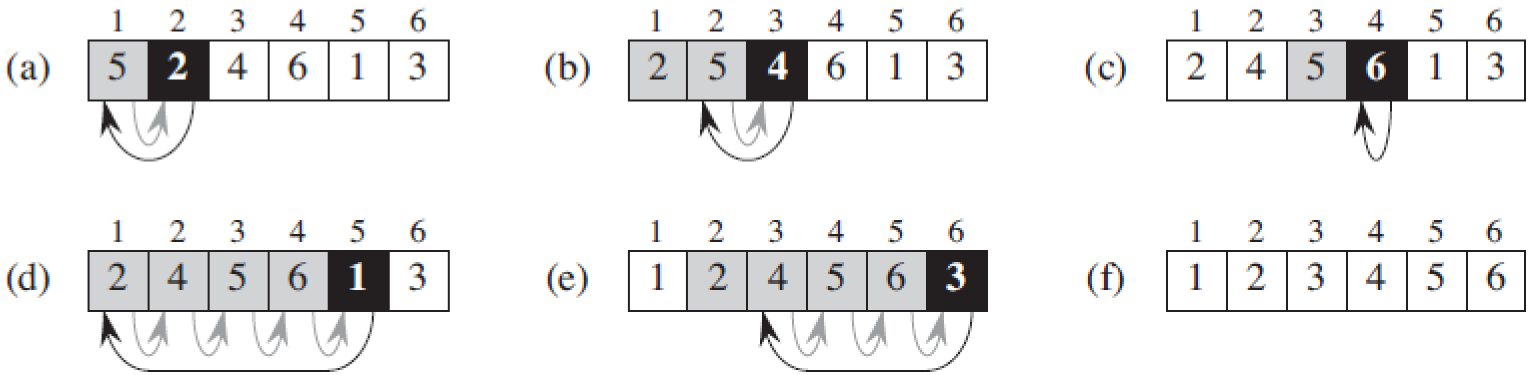
- Space Complexity
    - Space Complexity of an algorithm is total space taken by the algorithm with respect to the input size.

Analyzing Algorithms – Which one is better?

```
largest = a
if b > largest then
   largest = b
end if
if c > largest then
   largest = c
end if
if d > largest then
   largest = d
end if
return largest
```

```
if a > b then
   if a > c then
      if a > d then
         return a
      else
         return d
      end if
   else
      if c > d then
         return c
      else
         return d
      end if
   end if
else
   if b > c then
      if b > d then
         return b
      else
         return d
      end if
   else
      if c > d then
         return c
      else
         return d
      end if
   end if
end if
```

Analyzing Insertion Sort



INSERTION-SORT(A)

```
for j = 2 to A.length
    key = A[j]
    // Insert A[j] into the sorted sequence A[1 .. j - 1].
    i = j - 1
    while i > 0 and A[i] > key
        A[i + 1] = A[i]
        i = i - 1
    A[i + 1] = key
```

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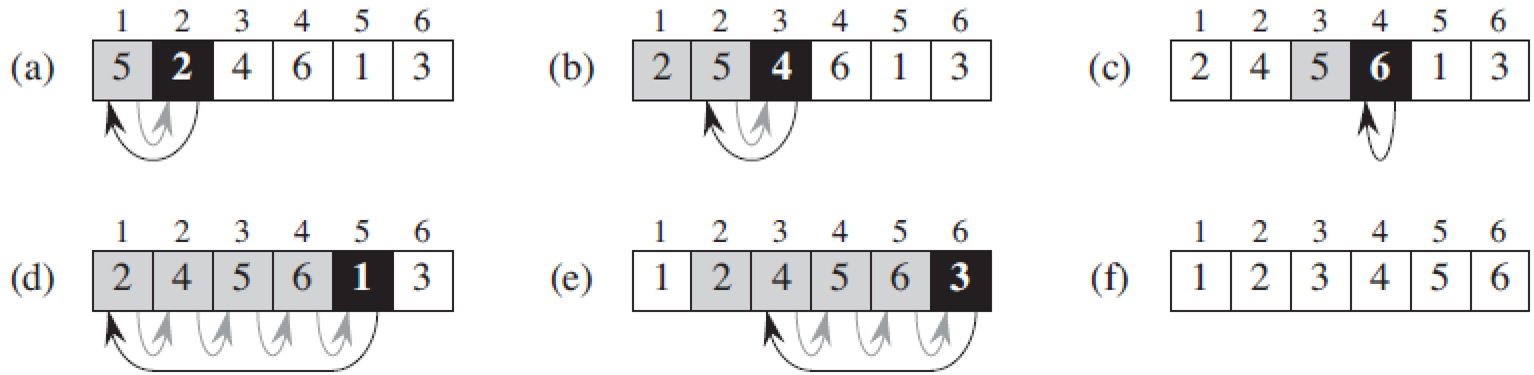

INSERTION-SORT(A)	*cost*	*times*
for $j = 2$ **to** $A.length$	c_1	n
$key = A[j]$	c_2	$n - 1$
// Insert $A[j]$ into the sorted sequence $A[1..j-1]$.	0	$n - 1$
$i = j - 1$	c_4	$n - 1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
$A[i+1] = A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i = i - 1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i+1] = key$	c_8	$n - 1$

Analyzing Insertion Sort

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) \\
&\quad + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1) .
\end{aligned}
$$

Best Case Running Time (i.e. Array is already Sorted):

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5(n-1) + c_8(n-1) \\
&= (c_1 + c_2 + c_4 + c_5 + c_8)n - (c_2 + c_4 + c_5 + c_8) .
\end{aligned}
$$

Worst Case Running Time (i.e. Array is Sorted in Reverse Order)

$$
\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1
$$

and

$$
\sum_{j=2}^{n} (j-1) = \frac{n(n-1)}{2}
$$

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \left(\frac{n(n+1)}{2} - 1 \right) \\
&\quad + c_6 \left(\frac{n(n-1)}{2} \right) + c_7 \left(\frac{n(n-1)}{2} \right) + c_8(n-1) \\
&= \left(\frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2} \right) n^2 + \left(c_1 + c_2 + c_4 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2} + c_8 \right) n \\
&\quad - (c_2 + c_4 + c_5 + c_8) .
\end{aligned}
$$

Analyzing Insertion Sort

Average Case Running Time

On average, half the elements in A[1 .. j – 1] are less than A[j], and half the elements are greater. On average, therefore, we check half of the subarray A[1 .. j – 1], and so t_j is about j/2. The resulting average-case running time turns out to be a quadratic function of the input size, just like the worst-case running time.

Analyzing Insertion Sort

- Order of Growth
  - We used some simplifying abstractions to ease our analysis of the INSERTION-SORT procedure.

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  - Then, we observed that even these constants give us more detail than we really need: we expressed the worst-case running time as $an^2 + bn + c$ for some constants a, b, and c that depend on the statement costs c_i .

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  - We thus ignored not only the actual statement costs, but also the abstract costs c_i .

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  - We also ignore the leading term's constant coefficient, since constant factors are less significant than the rate of growth in determining computational efficiency for large inputs.

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  - We therefore consider only the leading term of a formula (e.g., an^2), since the lower-order terms are relatively insignificant for large values of n.
  - We also ignore the leading term's constant coefficient, since constant factors are less significant than the rate of growth in determining computational efficiency for large inputs.
  - For insertion sort, when we ignore the lower-order terms and the leading term's constant coefficient, we are left with the factor of n^2 from the leading term.

Analyzing Insertion Sort

- Order of Growth
  - We write that insertion sort has a worst-case running time of $\theta(n^2)$ (pronounced "theta of n-squared").

  - We usually consider one algorithm to be more efficient than another if its worst-case running time has a lower order of growth.

  - Due to constant factors and lower order terms, an algorithm whose running time has a higher order of growth might take less time for small inputs than an algorithm whose running time has a lower order of growth.

  - But for large enough inputs, $\theta(n^2)$ algorithm, for example, will run more quickly in the worst case than a , $\theta(n^3)$ algorithm.

Disclaimer

- The presentation is not original and its is prepared from various sources for teaching purpose only.