19BCE248_2CS503_5_1-08-2021_01

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Q.1 (a) $T(n) + 5T(n-1) + 6T(n-2) = 3n^2 - 2n + 1$

so here it is imhomogenous Recurrence relation

∴ solving LHS as homogenous and then adding extra parameter of the for $b^n \cdot f(n)$

$3n^2$

$3^n \, n^d$

$$(x^2 + 5x + 6)(x-3)^3 (x+2)^2 (x-1)^{1}$$

$$\therefore (x^2 + 2x + 3x + 6)(x-3)^3 (x+2)^2 (x-1) = 0$$

$$(x+2)(x+3)(x-3)^3 (x+2)^2 (x-1) = 0$$

$$(x-3)^3 (x+2)^3 (x+3)(x-1) = 0$$

$$\therefore t_n = c_1 (3)^n + c_2 \cdot n (3)^n + c_3 \cdot n^2 (3)^n + c_4 (-2)^n + c_5 \cdot n \cdot (-2)^n + c_6 (n^2)(-2)^n + c_7 (-3)^n + c_8 (1)^n$$

Now $t_0 = c_1 + c_4 + c_7 + c_8$

$t_1 = 3(c_1 + c_2 + c_3) - 2(c_4 + c_5 + c_6) - 3c_7 + c_8$

$t_2 = 9(c_1 + 2c_2 + 4c_3) + 4(c_4 + 2c_5 + 4c_6) + 9c_7 + c_8$

$$t(2) - 5t(1) + 6t(0) = 3n^2 - 2n + 1$$
$$= 9$$

$c_1(9 - 15 + 6) + c_2(18 - 15 + 6) + c_3(36 - 15) +$
$c_4(4 + 10 + 6) + c_5(8 + 10) + c_6(24 + 10) +$
$c_7(9 + 15 + 6) + c_8(9 - 5 + 6) = 9$

$$9c_2 + 21c_3 + 20c_4 + 18c_5 + 36c_6 + 30c_7 + c_8 = 9$$

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Q.1(b)

$$t_n = \begin{cases} 0 & n=0 \\ 2t_{n-1} + n + 2^n & \text{otherwise} \end{cases}$$

Inhomegenous eq^n.

$$t_n - 2t_{n-1} = n + 2^n$$

∴ It can be written as.

$$(x-2)(x-b_1)^{d_1+1}(x-b_2)^{d_2+1}$$

Here $b_1 = 1$ b/c $n = 1^n \cdot n$ $d_1 = 1$
$b_2 = 2$ $d_2 = 0$

$$\therefore (x-2)(x-1)^2(x-2)' = (x-2)^2(x-1)^2$$

$$\therefore t_n = c_1 r_1^n + c_2 r_2^n + c_3 r_3^n + c_4 r_4^n$$

but here multiplicity of 1 and 2 are 2

$$\therefore t_n = c_1 r_1^n + c_2 \cdot n \cdot r_2^n + c_3 \cdot r_3^n + c_4 \cdot n \cdot r_4^n$$

$$t_n = c_1(1)^n + c_2 \cdot n(1)^n + c_3 \cdot (2)^n + c_4 \cdot n \cdot (2)^n$$

$$t_n = c_1 + n \cdot c_2 + 2^n \cdot c_3 + n \cdot c_4 \cdot (2)^n$$

Now at $n=0$ $t_n = 0$

$$0 = c_1 + c_3 \qquad \boxed{c_1 = -c_3}$$

$t_1 = 2t_0 + 2 + 1$ $\qquad$ $t_2 = 2t_1 + 2 + 4$

$\boxed{t_1 = 3}$ $\qquad$ $\boxed{t_2 = 6 + 6 = 12}$

$t_3 = 2t_2 + 3 + 8$

$t_3 = 24 + 11 = 35$ $\qquad$ $\boxed{t_3 = 35}$

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$\square + \square + \square + \square + \square = \square$

Now

$$C_1 + C_2 + 2C_3 + 2C_4 = 3 \quad -(1)$$
$$C_1 + 2C_2 + 4C_3 + 8C_4 = 12 \quad -(2)$$
$$C_1 + 3C_2 + 8C_3 + 24C_4 = 35 \quad -(3)$$

solving 3 eq^n we get

$$C_2 + 2C_3 + 6C_4 = 9$$
$$C_2 + 4C_3 + 16C_4 = 23$$
$$\boxed{2C_3 + 10C_4 = 14}$$

$$-C_3 + 2C_2 + 4C_3 + 8C_4 = 12$$
$$2C_2 + 3C_3 + 8C_4 = 12$$
$$C_2 + C_3 + 2C_4 = 3$$
$$C_3 + 4C_4 = 6$$

$$C_3 + 5C_4 = 7 \qquad C_4 = 1 \qquad C_3 = 2$$
$$C_3 + 4C_4 = 6$$
$$C_1 = -2 \qquad C_2 = -1$$

$\therefore$ the total sol^n would be.

$$t_n = -2 + (n)(-1) + 2^n \cdot (2) + n \cdot (2)^n$$
$$-2 - n + 2^{n+1} + n \cdot 2^n$$

$$\boxed{t_n = 2^{n+1} + n \cdot 2^n - n - 2}$$

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Q.2. $f(n) = 100 \cdot 2^n + n^5 + n$ $\quad f(n) = O(2^n)$. $c = ?$ $n_0 = ?$

The equality for Big O would be

$$0 \le f(n) \le c\,g(n)$$

Now

$$100 \cdot 2^n + n^5 + n \le c(2^n)$$

$g(n) = 2^n$ as we need to proove $O(2^n)$

$$\therefore 100 + \frac{n^5}{2^n} + \frac{n}{2^n} \le c$$

Now as we have both denominator numeritar in terms of n we need to do hit and trial method.

At $n=1$ $\quad f(n) = 100 + \frac{1}{2} + \frac{1}{2} = 101$

At $n=2$ $\quad f(n) = 100 + \frac{32}{4} + \frac{2}{4} = 108.5$

At $n=3$ $\quad f(n) = \cancel{100+}\ 130.75$

At $n=4$ $\quad f(n) = 164.25$

~~Here we need to find n where $n^5 < 2^n$ so at approx $n = 23$ the values get started decreasing~~

At $n=5$ $\quad f(n) = 197.81$

At $n=6$ $\quad f(n) = 121.59$

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so at $n=5$ the $f(n)$ is Max which is 197.81 $\therefore$ we can say $\boxed{c \simeq 198}$

and can assume $n_0 = 6$

$\therefore$ At $n \geq n_0$ the condition $f(n) \leq cg(n)$ holds for $\boxed{c = 198 \quad n_0 = 6}$

Q. 3

```
int seqSearch(int A[], int &x, int n)
{
  int i                                        S1
for (int i=0 ; i<n && a[i] != x ; i++)         S2
 {
    if (i==n)                                  S3
        return -1;                             S4
 }
return i;                                      S5
}
```

For S1 :-

cost $= C_0$ time $= 1$

For S2 :-

cost $= C_1$ time $= n+1$

For S3 :- time $= n$ cost $= C_2$

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For S4:-

cost = C_4 time = 1

For S5:-

cost = C_4 time = 1

∴ Total time consumed is

$C_0(1) + C_1(n+1) + C_2(n) + C_4(1) + C_4(1)$

$= C_1 n + C_1 + C_2 n + C_0 + C_3 + C_4$

$n(C_1 + C_2) + C_0 + C_1 + C_3 + C_4$

$C_1 + C_2$ = some constant a

$C_0 + C_1 + C_3 + C_4$ = some constant b

∴ $n(a) + b$

∴ For the worst case the time complexity would be $O(n)$.

For the best case if we subsitute $n=1$

then $T(1) = a+b$

∴ For the best case it would be $O(K) = O(1)$

For the average case it would be again $O(n)$ as $O(\frac{n}{2}) = O(n)$

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Q.4.

```
int fun (int n)
{
    int i                          -> S1
    for (i=1; i<=n; i++)           -> S2
        printf(" Hello World");    -> S3
}
```

For S_1 cost $= C_0$ time $= 1$;
For S_2 cost $= C_1$ time $= n+1$;
For S_3 cost $= C_2$ time $= n$;

$\therefore \quad T(n) = C_0(1) + C_1(n+1) + C_2(n)$

$\therefore \quad T(n) = C_0 + C_1 + n(C_1 + C_2)$

$T(n) = an + b$

$\therefore$ $T(n)$ time complexity is $O(n)$ as the highest polynomial degree is ~~of~~ n

$\therefore$ $O(n)$ is the time complexity of above fragment.

Q.5. (a) $T(n) = 4T(n/2) + n^2 \times \sqrt{n}$

$T(n) = 4T(n/2) + n^{2+\frac{1}{2}}$

$4T(n/2) + n^{5/2}$

$\therefore$ here $a = 4$ $b = 2$ $k = 5/2$

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So $\log_b a = 2$ $2 < 5/2$

$\therefore$ condition 3 Matches
So the ans would be $\boxed{O(n^{5/2})}$

(B) $T(n) = 3T(n/3) + n\log n$

Here we can clearly $\log_b a = 1$ $a=3$ $b=3$
and also $K=1$

so $f(n) = n\log n$ $n^{\log_b a} = n$

$\frac{f(n)}{n} = \log n$ which is not polynomially larger

but though using Extended Master theorem we can solve thes.

$p = 1$ $\therefore$ sol^n is $\boxed{O(n\log^2 n)}$

(C) $T(n) = 2T(n/2) + n^2$

$a=2$ $b=2$ $K=2$
$\log_b a = 1$ $\frac{n^2}{n} = n$

so polynomially larges.

Thus we can write it as $\boxed{O(n^2)}$