$\log_b a > k$

Tutorial 3
Divide and Conquer

Q.3 (1) $T(n) = 8T(n/2) + 1000n^2$

$n^3 > n^2$ $\quad O(n^3)$

(2) $T(n) = 2T(n/2) + n$

n^1 $\quad n^1$

$O(n\log n)$

(3) $T(n) = 2T(n/2) + n^2$

$O(n^2)$

(4) $T(n) = 3T(n/3) + n\log n$

Not by master $\quad$ $\left.\begin{matrix} n \\ n\log n \end{matrix}\right\}$ $\times$ polynomial larger

∴ By extended

$n\log^2 n$

(5) $T(n) = 9T(n/3) + n$

$O(n^2)$

(10)

$T(n) = T(n/2) + n(2 - \cos n)$

$\times$ Not possible

(6) $T(n) = 27T(n/3) + n^3$

$O(n^3\log n)$

(7) $T(n) = 8T(n/2) + n^3/\log n$

~~$O(n^4)$~~ $O(n^3\log(\log n))$

(8) $T(n) = 2T(n/2) + n/\log n$

$O(n\log(\log(n)))$

(9) $T(n) = 0.5T(n/2) + 1/n$

$\log_2 0.5 \quad 1/2 \; 2^{-1}$ $\quad -1 \quad -1$ $\quad O(n^{-1}\log n)$

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Q.6. $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$ By Change of Variable

$T(n) = n^{1/2}\, T(n^{1/2}) + n$ $(n = m^{2^k})$

Let $i = n^{1/2}$ Let ~~n=2^i~~ $m = 2^{2i}$

$i^2 = n$

$T(i^2) = i\,T(i) + n$ $(m^{2^k})^{1/2}$

$m^{2^{k/2}}$

$T(\cancel{2^i}) =$ $T(2^{2i}) = (2^i)\, T(2^i) + n$

$n = 2^x$

$T(2^x) = 2^{x/2} * (2^{x/2}) + n$

Q.1 (1) $T(n) - 4T(n-1) + 4T(n-2) = 0$

$x^2 - 4x + 4 = 0$ $x^2 - 2x - 2x + 4 = 0$

$(x-2)(x-2)$

$\boxed{t(n) = c_1(2)^n + c_2 \cdot n \cdot (2)^n}$

~~$8c_1 + 24c_2 - 4(4c_1$~~

$t(2) - 4t(1) + 4t(0) = 0$ $8c_1 + 24c_2 - 16c_1 - 32c_2 +$

$4c_1 + 8c_2 - 8c_1 - 8c_2 + 4c_1 = 0$ $8c_1 + 8c_2 = 0$

~~$4c_1$~~ $3c_1 + 6c_2 = 0$

$c_1 + 2c_2 = 0$ $\boxed{c_1 = -2c_2}$

$\boxed{c_1 = t_0}$

$t(2) - 4(t(1)) = -4t_0$ ~~$t(1) =$~~

$4c_1 + 8c_2 - 8c_1 - 8c_2 + 4t_0$ $t(2) = c_1(4) + 8c_2$

$-4c_1 = 4t_0$ $4c_1 + 8c_2 = 4(2c_1 + 2c_2) - 4t_0$

$\boxed{c_1 = +t_0}$ ~~$4t_0 = 8c_1 + 8c_2 - 4c_1 - 8c_2$~~

Q.1 (2) $T(n) - 5T(n-1) + 6T(n-2) = 0$ $t_2 =$ ~~$4c_1 + 9c_2$~~

$x^2 - 5x + 6 = 0$ $\boxed{c_1(2)^n + (t_0 - c_1)(3)^n}$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$ $x = 3, 2$

$t_n = c_1(2)^n + c_2(3)^n$

$\boxed{t_0 = c_1 + c_2}$ $t_1 = 2c_1 + 3c_2$

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Q.6. $T(n) = n\,T^2(n/2)$ $\quad T(1) = 1/3$

Let $n = 2^i$

$T(2^i) = 2^i\, T^2(2^{i-1})$

$\log T(2^i) = \log_2(2^i) + \log_2(T^2(2^{i-1}))$

Let $\log(T(2^i)) = u_i$

$\therefore\ u_i = i + 2u_{i-1}$

$u_i - 2u_{i-1} = i$

$\therefore$ ~~Fi~~ $u_i = (x-2)(x-1)^2$

$u_i = c_1(2)^i + c_2(1)^i + c_3 n(1)^i$

$\therefore$ $\log(T(2^i))$

$i = c_1(2)^i + c_2 + c_2 \cdot i - 2(c_1(2)^{i-1} + c_2 + c_2(i-1))$

$i = \cancel{c_1(2)^i} + \cancel{c_2} + c_2 \cdot i - \cancel{c_1 2^i} - \cancel{c_2} - c_2(i-1)$

$c_2 \cdot i - c_2(i-1)$

$g(k) = \frac{T(2^k)}{2^k}$

Q.4 $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

Let $n = 2^k$

$\therefore T(2^k) = 2^{k/2} \cdot T(2^{k/2}) + 2^k$

$\frac{T(2^k)}{2^k} \quad \frac{T(2^{k/2})}{2^{k/2}} + 1$

Let $g(k) = \frac{T(2^k)}{2^k}$

$g(k) = g(k/2) + 1$ $\qquad O(2^{\log_2 n} \log_2 2^k)$

$a = 1 \quad b = 2$

$\log_2 1 = 0 \quad (n = 0) \quad \therefore$ $\qquad O(n \cdot k$

$g(k) = O(\log k)$

$\frac{T(2^k)}{2^k} = \log k$

$T(2^k) = 2^k(\log k)$

$k = \log_2 n$ $\qquad = O(2^k \log n)$

Q.5 $T(n) = \begin{cases} 0 & n=0 \\ 3T(n/2)+n & \text{otherwise} \end{cases}$

We tabulate the value of recurrence first few power of 2.

n	1	2	4	8	16	32
$T(n)$	1	5	19	65	211	665

→ Powers of 2 b/c $n/2$ power of 2

We need to analyze pattern followed

For e.g. $T(4) = 3T(2)+4$

$T(4) = 3(3.1+2)+4$

$= 3^2.2^0 + 3^1 2^1 + 3^0 2^2$

$\sum 3^{k} 2^{k-i}$ $\boxed{3^{k-i} 2^{i}}$

$i=0$ to k

$\boxed{k = \text{powers of } 2}$ till n

$\{\log_2(n)$

$$\sum_{i=0}^{k} 3^{k-i} 2^{i} = 3^{k} \sum_{i=0}^{k} (2/3)^{i}$$

$$3^{k} \times \frac{\left(1-\left(\frac{2}{3}\right)^{k+1}\right)}{1-\frac{2}{3}} = \boxed{3^{k+1} - 2^{k+1}}$$

$\frac{1-r^n}{1-r}$

$k = \log_2 n$ $\quad 3^{\log_2 n+1} - 2^{\log_2 n+1}$

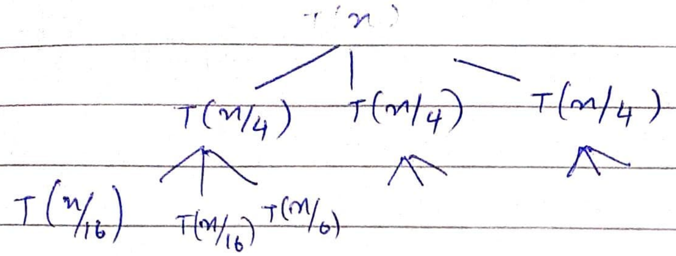
$3(n^{\log_2 3}) - 2(n)$

$\therefore \boxed{O(n^{\log_2 3})}$

Q.5 ~~$T(n) = 3T(n/2) + n$~~ (n is a power of 2)

Q.2.

(1) $T(n) = 3T(n/4) + n^2$



n^2

$3 \times \left(\frac{n}{4}\right)^2$

$9 \times \left(\frac{n}{16}\right)^2$

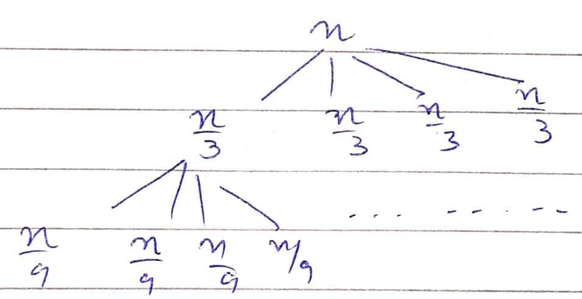
$\therefore \; n^2 + 3 \times \left(\frac{n}{4}\right)^2 + 9 \times \left(\frac{n}{16}\right)^2 + 27 \times \left(\frac{n}{64}\right)^2 + \ldots \infty$

$\therefore \; n^2\left(1 + \frac{3}{16} + \frac{9}{16^2} + \frac{27}{16^3} + \ldots \infty\right)$

$\therefore \; n^2\left(\frac{1}{1-\frac{3}{16}}\right) \quad n^2\left(\frac{16}{13}\right) \approx \boxed{O(n^2)}$

$\frac{9}{16^2}$ $\frac{3}{16}$ $\frac{27}{16^3}$ $\frac{9}{64}$

(2) $T(n) = 4T(n/3) + n$



n

$4\left(\frac{n}{3}\right)$

$16\left(\frac{n}{9}\right)$

$\therefore \; n + \left(\frac{4}{3}\right)^1 n + \left(\frac{4}{3}\right)^2 n + \left(\frac{4}{3}\right)^3 n$

$\therefore \; n\left(1 + \frac{4}{3} + \left(\frac{4}{3}\right)^2 + \ldots\right)$

$n\left(\frac{\left(\frac{4}{3}\right)^k - 1}{\frac{4}{3} - 1}\right)$

~~$n\left(\frac{1}{1-\frac{4}{3}}\right)$~~

$K = \log_3 4$

$3n\left(\left(\frac{4}{3}\right)^k - 1\right)$

$\therefore$ overal $\left(\frac{4}{3}\right)^k \times n$

$\left(\frac{4}{3}\right)^{\log_3 n} \times n = \frac{4^{\log_3 n}}{\cancel{n}} \times \cancel{n}$ $\boxed{n^{\log_3 4}}$ $\boxed{\Theta(n^{\log_3 4})}$

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(3) $T(n) = 4T(n/2) + n$

$$n$$

$$\frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2}$$

$$\frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \ldots\ldots$$

n

$4\left(\frac{n}{2}\right)$

$16\left(\frac{n}{4}\right)$

$= n + 2\times n + 4\times n + 8\times n + \ldots$

$= n\,(1+2+4+8+\ldots$

$n\,(1+2^{1}+2^{2}+2^{3}+\ldots)$

$n\,\frac{(2^{k}-1)}{1}$ $\qquad k = \log_2 n$

$n\,(2^{\log_2 n}-1)$

$n\,(n-1) = n^2$ $\qquad \boxed{\theta(n^2)}$

(4) $T(n) = T(9n/10) + T(n/10) + n$

$$n$$

$$\frac{9n}{10} \qquad n/10$$

$$\frac{9}{10}\left(\frac{9}{10}n\right) \quad \frac{9}{10}\left(\frac{n}{10}\right) \quad \frac{1}{10}\left(\frac{9n}{10}\right) \quad \frac{1}{10}\left(\frac{n}{10}\right)$$

$n + n(1) + n\left(\frac{81+9+9+1}{100}\right) + \ldots$

$n + n(1) + n\left(\frac{100}{100}\right) + \ldots$

n k times.

k^{th} shortest value will be $\log_{10} n$

k^{th} longest value will be $\log_{\frac{10}{9}} n$

$\therefore$ taking longest one.

$n(\log_{\frac{10}{9}} n)$

~~$n(\log_{10}$~~ Here the base doesn't matter

$\therefore$ $\boxed{\theta(n) = n \log n}$