Nirma University

Institute of Technology

Semester End Examination (IR), May - 2016

B. Tech. in Computer Engineering / Information Technology, Semester-VI

CE601 Design and Analysis of Algorithms

Roll / Exam No.	Supervisor's Initial with Date

Time: 3 Hours Max Marks: 100

Instructions:
1. Attempt all the questions.
2. Figures to right indicate full marks.
3. Draw neat sketches wherever necessary.

Section I

Q-1 Do as directed **[20]**

a) Consider an array A[p...r] which needs to be sorted using Quick sort. Write the "PARTITION" algorithm that puts the pivot element (last element) to its correct position in the array. State the "loop invariant" in the algorithm precisely and thereby prove its correctness. [10]

b) A sequence $<a_1, a_2, \ldots, a_n>$ of n unordered integers is given as an input to a sorting algorithm, whose average case and worst case time complexities are $O(n^2)$. Write the suitable algorithm which satisfies the above mentioned criteria. Indicate the cost of each statement of the algorithm and hence derive the expression for total running time of the algorithm (which confirms to the above stated time complexity of $O(n^2)$). [10]

Q-2 Do as directed **[18]**

a) Prove that :- $(n\log n - 10n + 50) = \Omega(n\log n)$ [6]

OR

a) Prove that :- $(\sqrt{2})^{\log n} + \log^2 n + n^4 = O(2^n)$ [6]

b) Solve the following recurrence by "Recursion tree" method :- [6]
$T(n) = 3T(n/4) + n^2$

c) Solve the following recurrence by "Change of variable" method :- [6]
$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$ ($n > 1$ and is a power of 2).

Q-3 Do as directed **[12]**

a) Let T[1...n] be a sorted array of distinct integers, some of which may be negative. Design an algorithm that can find an index i such that $1 \le i \le n$ and $T[i] = i$, provided such an index exists. Your algorithm should take a time in O(log n) in the worst case. [6]

OR

a) Rather than separating T[1..n] into two half-size arrays for the purpose of merge sorting, we might choose to separate it into three arrays of size $n/3$, $(n+1)/3$ and $(n+2)/3$, to sort each of these recursively, and then to merge the three sorted arrays. Give a more formal description of this algorithm and analyse its execution time. [6]

b) The number of additions and subtractions needed to calculate the product of two 2 X 2 matrices using Strassen's matrix [6] multiplication method seems at first to be 24. Show that this can be reduced to 15 by using auxiliary variables to avoid recalculating terms such as $m_1 + m_2 + m_4$.

Section II

Q-4 Do as directed **[20]**

a) Let $X = <x_1, x_2, \ldots, x_m>$ and $Y = <y_1, y_2, \ldots, y_n>$ be sequences, and let $Z = <z_1, z_2, \ldots, z_k>$ be any Longest Common Subsequence (LCS) of X and Y. Write an algorithm which computes the length of the LCS. Estimate the running time of the algorithm. [10]

b) How many spanning trees are possible on a graph G = (V, E) having 5 vertices? Write Kruskal's algorithm to find out the "Minimum Spanning Tree" in the weighted graph G = (V, E). Which data structure is used to implement this algorithm? Analyse the running time of the algorithm. [10]

Q-5 Do as directed **[18]**

a) Consider a set S, which contains n distinct elements. If we want to find out the i-th smallest element in S, then it can be done efficiently by using a deterministic linear time selection algorithm. Discuss the algorithm which performs this task and derive the expression which computes the total time taken by this algorithm. [6]

OR

a) Prove that :- The expected running time of the Randomized Quikcksort is O(n logn). [6]

b) Prove the following statement :- [6]
"If there exists a non-empty sub problem S_k and let a_m be an activity in S_k with the earliest finish time, then a_m is included in some maximum-size subset of mutually compatible activities of S_k."

c) Prove that :- The total number of comparisons required to determine both the minimum and the maximum of a set of n elements (n can be even or odd) is at most $3\lfloor n/2 \rfloor$. [6]

Q-6 Do as directed **[12]**

a) Ram's house contains four types of objects, whose weights are respectively 2, 3, 4 and 5 units, and whose values are 3, 5, 6 and 10. A thief enters his house with the knapsack that can carry a maximum of 8 units of weight. Apply "Backtracking" approach to fill the knapsack in a way that maximizes the value of the included objects in the knapsack. [6]

OR

a) Consider n objects and a knapsack of capacity W. For i =1, 2, …n, object i has a positive weight w_i and a positive value v_i. If the fractional selection of object i is allowed and is denoted by x_i, develop a mathematical model which maximizes the value in the knapsack. Apply "Greedy approach" to the following example so that value of the included objects in the knapsack is maximized :- [6]

Number of objects (n) = 5
Capacity of Knapsack (W) = 100

w_i	10	20	30	40	50
v_i	20	30	66	40	60

b) State whether the following statements are True or False (with proper justification) :- [6]

1) The problems 3-SAT and 2-SAT are NP-complete and in P respectively.
2) If P! =NP, then NP-complete ∩ P = Φ.
3) If we want to prove that a problem X is NP-Hard, we take a known NP-Hard problem Y and reduce Y to X.