

**Akshat Shah**

**19BCE246**

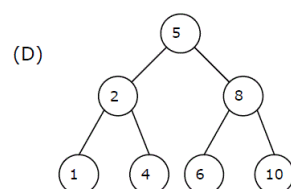
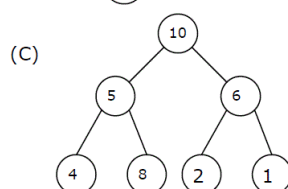
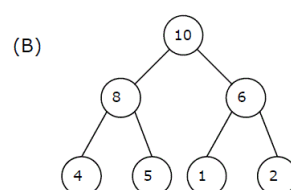
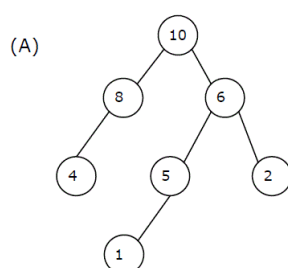
## **2CS503 Design and Analysis of Algorithms**

### **Tutorial 5: Heaps**

**September 30, 2021**

Q.1 Answer the following MCQs:

1. What is the time complexity of Build Heap operation. Build Heap is used to build a max(or min) binary heap from a given array. Build Heap is used in Heap Sort as a first step for sorting.  
A.  $O(n \log n)$   
B.  $O(n^2)$   
C.  $O(\log n)$   
**D.  $O(n)$**
2. Suppose we are sorting an array of eight integers using heapsort, and we have just finished some heapify (either maxheapify or minheapify) operations. The array now looks like this: 16 14 15 10 12 27 28 How many heapify operations have been performed on root of heap?  
A. 1  
**B. 2**  
C. 3 or 4  
D. 5 or 6
3. A max-heap is a heap where the value of each parent is greater than or equal to the values of its children. Which of the following is a max-heap?



- A. A  
**B. B**  
C. C  
D. D

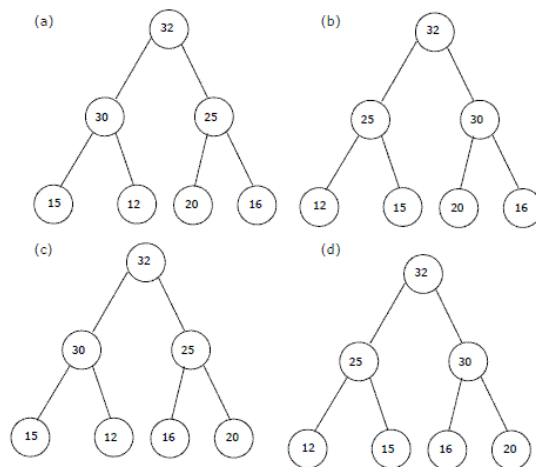
5. Consider a binary max-heap implemented using an array. Which one of the following array represents a binary max-heap?

- A. 25,12,16,13,10,8,14
- B. 25,14,13,16,10,8,12
- C. 25,14,16,13,10,8,12**
- D. 25,14,12,13,10,8,16

4. In a binary max heap containing  $n$  numbers, the smallest element can be found in time. Is an array of a sorted elements producing a min heap tree? Justify with an example.

- A.  $O(n)$**
- B.  $O(\log n)$
- C.  $O(\log \log n)$
- D.  $O(1)$

5. The elements 32, 15, 20, 30, 12, 25, 16 are inserted one by one in the given order into a Max Heap. The resultant Max Heap is.



- A. a**
- B. b
- C. c
- D. d

6. A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

- A. 10,8,7,3,2,1,5**
- B. 10,8,7,2,3,1,5
- C. 10,8,7,1,2,3,5
- D. 10,8,7,5,3,2,1

Q.2 Answer the following questions:

1. Explain the terms Binary Heap and Binomial Heap.
- A. Binary Heap is used to implement Priority Queue. Binomial Heap is an extended form of Binary Heap which is mainly used for the faster Union or Merge operations with other operations provided by Binary Heap.

#### **BINARY HEAP**

A Binomial Tree of order 0 has 1 node. A Binomial Tree of order  $k$  can be constructed by taking two binomial trees of order  $k-1$  and making one as leftmost child or other.

A Binomial Tree of order  $k$  has following properties.

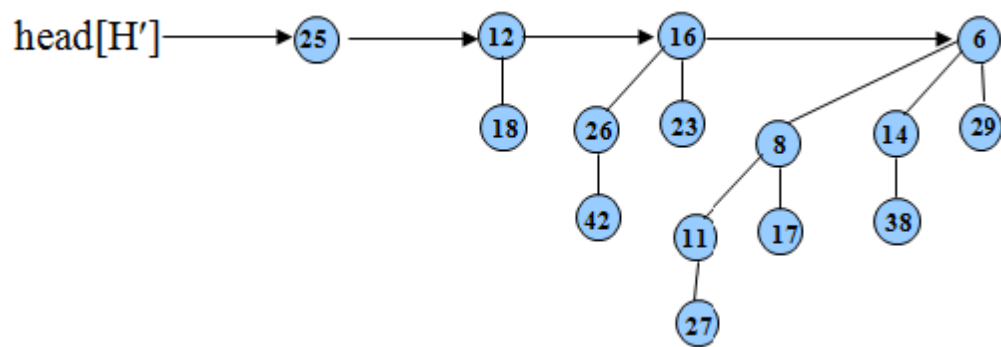
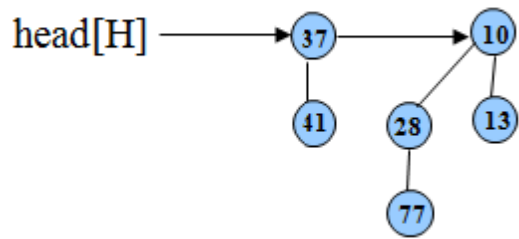
- It has exactly  $2^k$  nodes.
- It has depth as  $k$ .
- There are exactly  $\binom{k}{i}$  nodes at depth  $i$  for  $i = 0, 1, \dots, k$ .
- The root has degree  $k$  and children of root are themselves Binomial Trees with order  $k-1, k-2, \dots, 0$  from left to right.

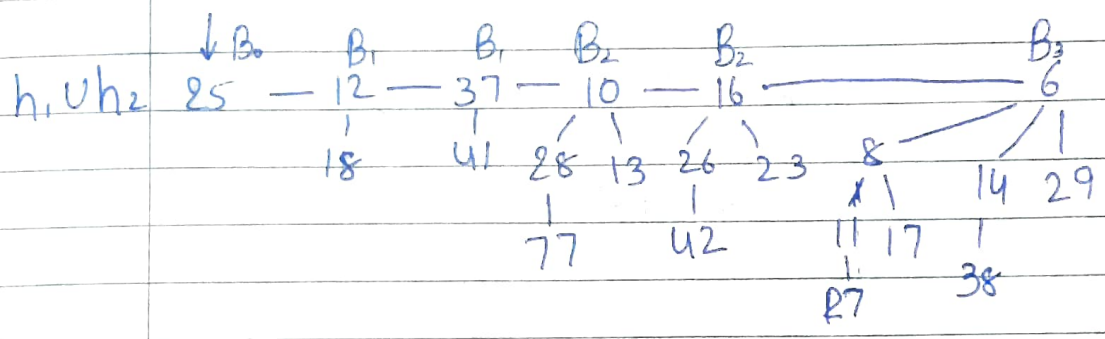
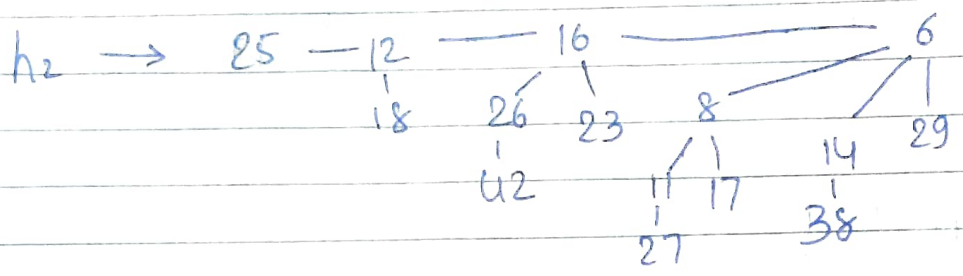
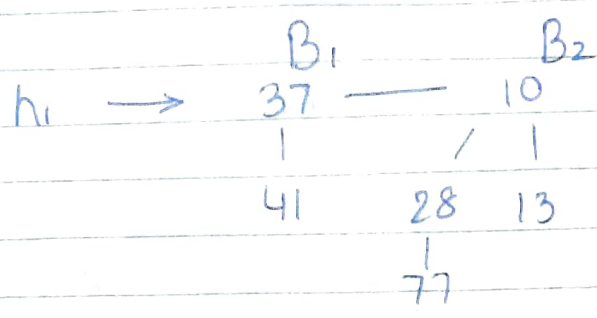
#### **BINOMIAL HEAP**

A Binomial Heap is a set of Binomial Trees where each Binomial Tree follows Min Heap property. And there can be at most one Binomial Tree of any degree.

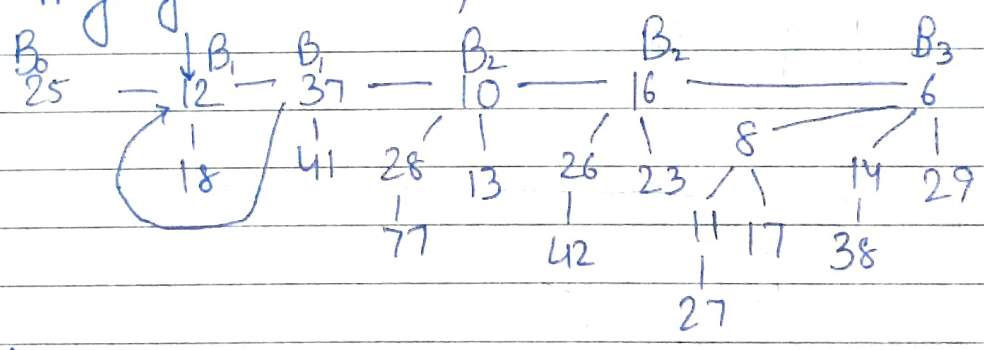
A Binomial Heap with 12 nodes. It is a collection of 2 Binomial Trees of orders 2 and 3 from left to right.

2. Given the two heaps H and H' find the union of these two heaps.

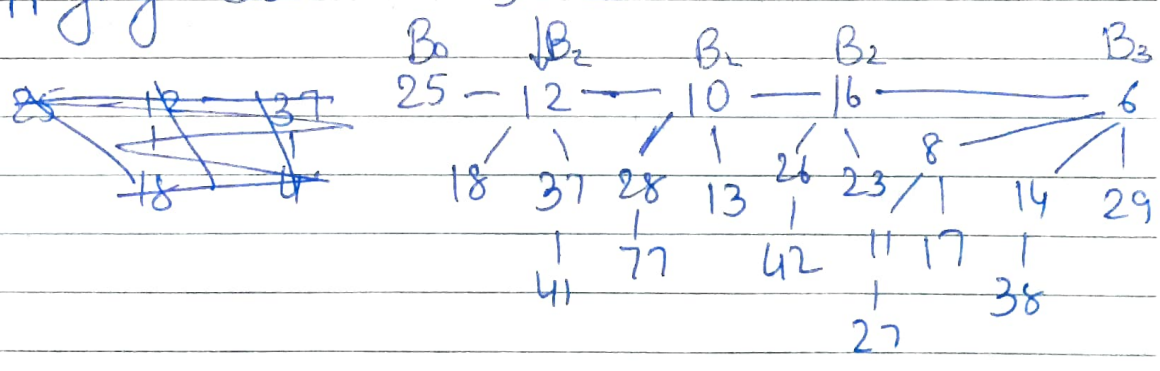




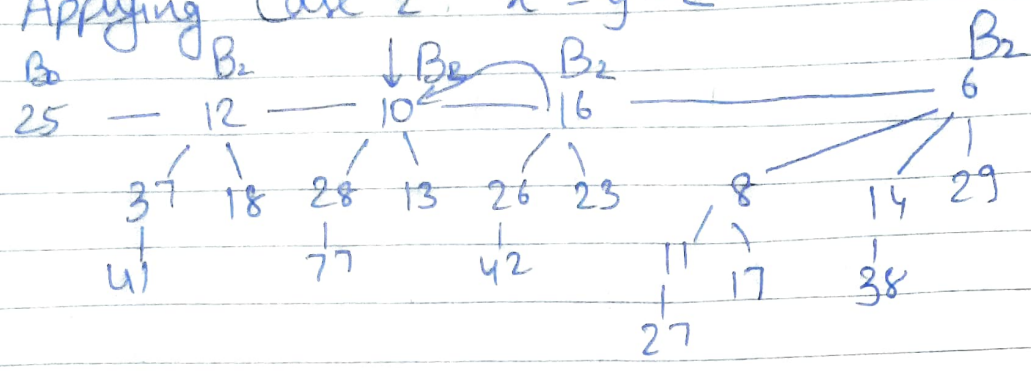
Applying Case 1:  $x \neq y$



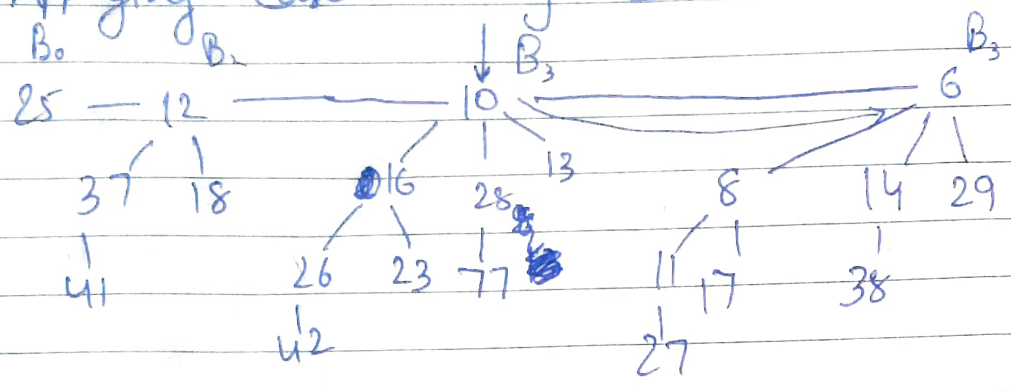
Applying Case 3:  $x = y \neq z$



Applying Case 2:  $x = y = z$



Applying Case 3:  $x = y \neq z$



Applying Case 3:  $x = y \neq z$

