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Inventory Models

Inventory

Inventory is a deteiles list of those items which are necessary to manufacture a product and to maintain the equipment and machinery in good working order.

Inventory is actually 'money' kept in the store room in the shap of ~~a high~~ tools, raw material, finished goods etc.

An inventory consists of usable but idle resources ~~involved~~ such as men, machines materials or money. When the resource involved is a material, the inventory is also called 'stock'.

Inventory Control

Inventory control is concerned with achieving an optimum balance between two competing objectives, are

(i) To minimize investment in inventory.

(ii) To maximize the service levels to the firm's customers and its own operating departments.

(2)

Inventory control may be defined as th scientific method of finding out how much stock should be maintained in order to meet the production demands and be able to provide right type of material at right quantities and at competitive prices.

Inventory ~~Classifi~~ Classification

Inventory may be classified as

(i) Raw inventories: They include, raw material and semifinished products supplied by anoth firm and which are raw items for the prese industry.

(ii) In-process inventories: They are semifinished ~~goods~~ goods at various stages of manufacturing cycle.

(iii) Finished inventories: They ~~in~~ are the finis goods lying in stock rooms and waiting for dispatch.

(iv) Indirect inventories: They include lubricant and other items (like spare parts, consum general items etc) needed for proper operati repair and maintenance during manufactur cycle.

Need of Inventory Control

- The necessity of inventory control is to maintain a reserve (store) of goods that will ensure manufacturing according to the production plan based on the ~~basis~~ sales requirements and the lowest possible ultimate cost.

- Losses from ~~improper inventory control include purchases in excess than what needed, the cost of slowed up production resulting from~~ improper inventory control include purchases in excess than what needed, the cost of slowed up production resulting from material not being available when wanted. Each time a machine is shut down for lacks of finished goods. Thus a factory losses money.

- To promote smooth factory operation and to prevent piling up or idle machine time proper quantity of material must be on hand when it is wanted. Proper inventory control can reduce such losses to a great extent.

Functions of Inventory Control

Following are the most important functions of Inventory Control

(a) To run the stores effectively - This includes layout, storing media (bins, shelves and open space etc), utilization of storage space, receiving and issuing procedures etc.

(b) To ensure timely availability of material and avoid built up of stock levels.

(c) Technical responsibility for the state of materials. This includes methods of storing, maintenance procedures, studies of deterioration and obsolescence.

(d) Stock Control System - physical verification (Stock-taking), records, ordering polices and ~~pro~~ procedures for the purchase of goods.

(e) Maintenance of specified raw materials, general supplies, work in process and component parts in sufficient quantities to meet the demand of production.

(f) Protecting the inventory from losses due to improper handling or storing of goods and unauthorised removal from stores.

(g) Pricing all materials supplied to the shops so as to estimate material cost.

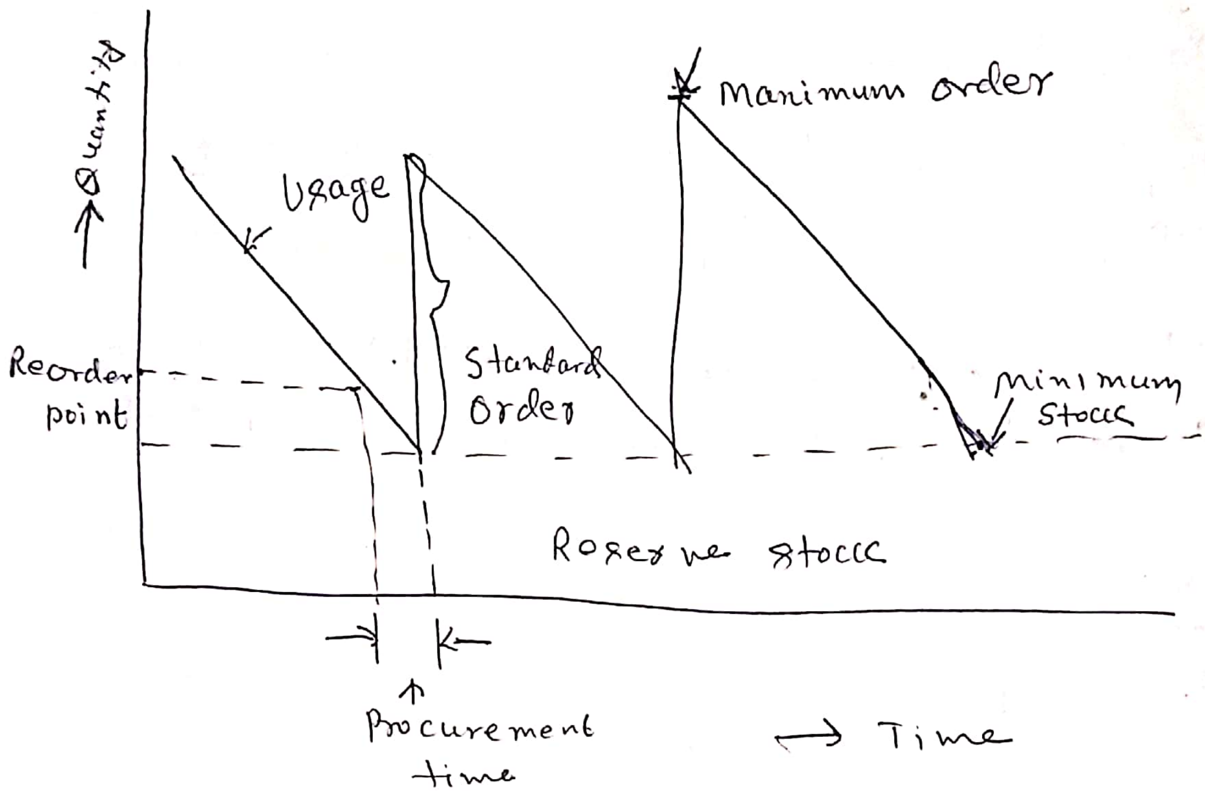
Objectives of Inventory Planning and Control

1. Timely availability of required material of acceptable quality.
2. To avoid build-up stock levels by inflow of materials much in advance of requirements or accumulation of unwanted items which though ordered earlier, but not required now.

Quantity Standards.

There are five important quantity standards used as tool to control Inventory. These are as follows.

1. The maximum Stores/Stocks/Quantity
2. The minimum Stores/Stocks/Quantity.
3. The standard Order
4. The Ordering Point
5. Lead or Procurement Time.



1. <u>The maximum Stores</u>:- This term is applied to designate the upper limit of the inventory and represents the largests quantity which in the interest of economy should generally be kept in stores.

2. <u>The Minimum Stores</u>:- The term is applied ~~to~~ to designate the lower limit of the Inventory and represents as reserve or margin of safety to be used in case of emergency. When requirements have been abnormal it is intented ~~to~~ that there must always be atleast this quantity available in stores.

(3) <u>The standard Order</u>: It is the quantity to be purchased at any time. Repeat orders for a given product are always ~~for~~ for this quantity until this is revised.

4. **The Ordering Point:** This represents the quantity required to ensure against exhaustion of the supply during the interval between the placement of an order and delivery. When the balance falls to this level, it is an indication that a new purchase order must be placed.

5. **Lead time:-** It is the time which takes the stock to reach from Re-order point to minimum stock level. It may also be defined as the time that elapses between the voicing of a need for anything and the time taken to satisfy the need.

- lead time determines the amount of material to be kept in reserve. As the lead time decreases, the reserve stock also decreases and vice versa. Therefore, the lead time analysis is very necessary, and the attempts should be made to reduce this period.

Lead time includes:

1. Time to process the enquires and to place the order.
2. Time to deliver the order to supplier.
3. Time for the supplier to fulfil the order.
4. Transportation time to reach the purchaser etc.

In setting maximum, minimum and order quantities each item should be considered separately in terms of the following factors

(a) Economic size of each purchase order.

(b) Increased lock-up of capital

(c) The time required to receive the goods after requisitioning.

(d) The probable depreciation and obsolescence.

(e) The rate of demand etc.

Advantages of Inventory Control

(1) It creates buffers between input and output.

(2) It ensures against delays in deliveries

(3) It allows for possible increase in ~~output~~ output.

(4) It allows advantage of quantity discount

(5) It ensures against scarcity of materials in the market

(6) It utilizes the benefit of price ~~fluc~~ fluctuations.

Causes of Poor Inventory Control

1. Overbuying without regard to the forecast or proper estimates of demand to take advantage of favourable market.

2. Overproduction or production of goods much before the customer requires them.

3. Overstocking may also result from the desire to provide better service to the customers. Bulk production to cut down production costs also will result in large

inventories.

4. Cancellation of orders and minimum quantity stipulations by the suppliers may also give rise to large inventories.

Inventory Costs

The three costs considered in inventory control models are

1. Inventory carrying or stock holding cost.
2. Procurement costs (for bought outs) or setup costs (for made ins) and
3. shortage cost.

Inventory Carrying Costs or Stock holding costs

- They arise on account of maintaining the stocks and the interest paid on the capital tied up with the stocks. They are directly with the size of the inventory as well as the time the item is held in stock. Various components of the stock-holding cost are:

1. Cost of money or capital tied up in inventories. (15% to 20%)
2. Cost of storage space. (1 to 3%)
3. Depreciation and deterioration (0.2 to 1%)
4. Pileferage cost: It depends upon the nature of item, valuables such as gun metal bushes

and expensive tools may be more tempting, while there is hardly any possibility of heavy casting or forging being stolen. While the former must be kept under lock and key, the latter may be simply dumped in the stockyard. Pilferage cost may be taken as 1% of the stock value.

5. Obsolescence cost (approximately 5%)
6. Handling cost
7. Record keeping and administrative cost.
8. Taxes and insurance. (approx. 1 to 2%)

Procurement Costs or Setup Costs

These includes the fixed cost associated with placing of an order or setting up a machinery before starting production. They include costs of purchase, requisition, followup, receiving the goods, quality control etc.
Also called order costs or replenishment costs, they are assumed to be independent of the quantity ordered or produced but directly proportional to the number of orders placed.

At times, however, these costs may not bear any simple relationship to ~~be~~ the number of orders. More than one stock item may be ordered ~~one~~ on ~~one~~ one set of documents; the clerical staff is not divisible and without the existing staff increasing or decreasing, there may be considerable scope for changing the number ~~of~~ of orders. In such a case, the acquisition cost relationship may be quadratic or stepped instead of a straight line.

Shortage costs or Stock-out Costs

These costs are associated with either a delay in meeting demands or the inability to meet it at all. Therefore, shortage costs are usually interpreted in two ways. In case the unfilled demand can be filled at a later stage (~~back log~~ backlog case) these costs are proportional to quantity that is short as well as the delay time. They represents loss of goodwill and cost of idle equipment. In case the unfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These results in cancelled orders, lost sales, profit and even the business itself.

Inventory Control Problem

The inventory control problem consists of determination of two basic factors.

1. When to order (produce or purchase)?
2. How much to order?

When to order

This is related to the lead time. Lead time may be defined as the time interval between the placement of an order and its receipt in stocks. It may be replenishment order on an outside firm or ~~wh~~ within the works.

There should be enough stock for each item so that customers' order can be reasonably met from this stock until replenishment. This stock level, known as reorder level, has therefore, to be determined for each item. It is determined by comprisiming the cost of maintaining these stocks and the disservice to the customer if ~~has dor~~ his orders are not filled in time.

How much to order: ~~As already discussed~~

Each order has associated with it the ordering cost ~~of~~ or acquisition cost. To keep it low, the number of orders should be as few as ~~to~~ possible i.e the order size should be large. But large order size would imply high inventory-carrying cost. Thus the question of how much to order is solved by compromising between the acquisition costs and inventory carrying costs.

~~EOQ/E~~

Economic order Quantity (EOQ)/ Economic Batch Quantity (EBQ) or Economic Lot Size.

By inventory model we mean to represent the inventory state, operating conditions and costs involved through ~~mathmat~~ mathematical relationships, so as to find out Economic Order Quantity (EOQ) when source of procurement is purchase (user situation) or to find out Economic Batch Quantity (EBQ) when source of procurement is manufacturing (producer situation). Economic Order quantity or Economic Order is in fact optimum quantity ~~wt~~ with which total associated cost of inventory is minimized.

→ In user's situation supply of material is intermittent and instantaneous and demand is met from the periodically replenished stock through ~~a~~ ordered quantities. While in producer's situation the production is intermittent that causes

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supply of material at finite rate. The production rate is much higher than the demand rate. Inventory level is allowed to build up to some predetermind value and production terminated. The production facility may then be used to other work till the next run is triggered off.

Inventory models

Elementary models | Model with price breaks | model with restriction.

Elementary models → Deterministic, Probabilistic

Deterministic

model 1 - (No shortage is permitted)

1(a) Demand rate uniform, Production rate infinite

(b) Demand rate non-uniform, production rate infinite.

(c) Demand rate uniform, Production rate finite

model 2 (shortage permitted)

2(a) Demand rate uniform, production rate infinite

(b) Demand rate uniform, production rate infinite, Time interval fixed.

(c) Demand rate uniform, production rate finite

Probabilistic

3(a) Instantaneous demand setup cost zero, stock level discrete, lead time zero

3(b) Same as 3(a), except that stocks levels are continuous

4(a) continuous demand set up cost zero, stock level discrete, lead time zero

(b) Same as 4(a) except that stock levels are continuous

5(a) Continuous demand, setup cost zero Stock levels discrete, lead time significant.

(b) Same as (5)(a) except that stock levels are continuous, Demand

- Independent
  - Fixed order Quantity systems
  - Periodic review systems
    - Fixed order cycle ~~Sq~~ system
    - sS System.
- Dependent

Inventory Models (Deterministic Demand)

'Static models

(1) ~~Fundamental problem of EOQ~~

$$\begin{pmatrix} \text{Total} \\ \text{Inventory} \\ \text{cost} \end{pmatrix} = \begin{pmatrix} \text{Purchasing} \\ \text{cost} \end{pmatrix} + \begin{pmatrix} \text{Set up} \\ \text{cost} \end{pmatrix} + \begin{pmatrix} \text{Holding} \\ \text{cost} \end{pmatrix} + \begin{pmatrix} \text{Shortage} \\ \text{cost} \end{pmatrix}$$

Classic EOQ model (or fundamental EOQ model) (Wilson model)

Assumption / criteria

(I) Constant demand rate

(II) ~~Instaineous~~ Instantaneous order replenishment

(iii) No shortage cost (or No shortage is ~~per~~ permitted)

⇒ (i) Demand is known and uniform. Let Q denote the lot size in each production run (~~pur~~ or purchase)

(ii) Let D denote the total number of units produced or supplied per time period.

(iii) Shortage are not permitted, ie as soon as the level of this inventory reaches zero, the inventory is replenished. Thus the cost of shortage is assumed to be infinite.

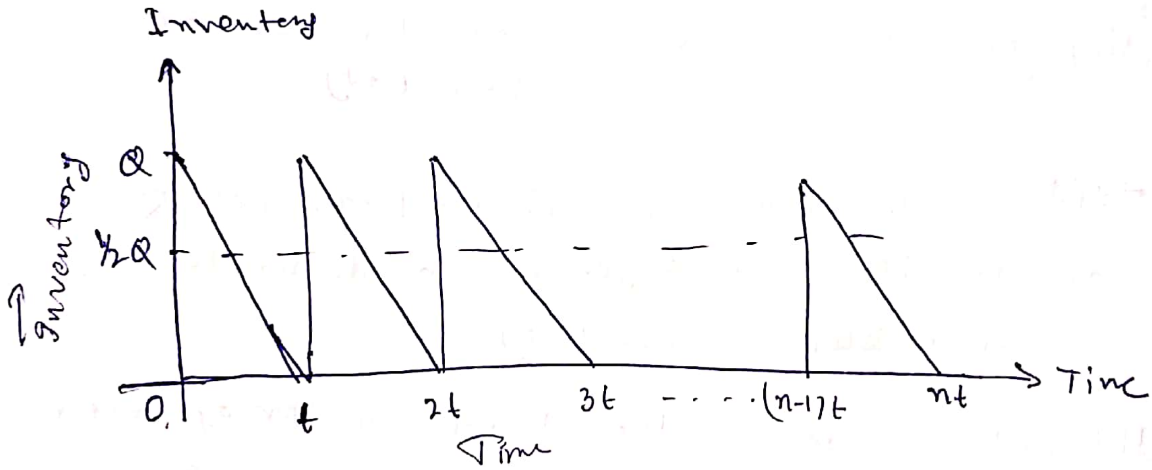
(iv) Lead time is zero

(vi) Setup cost per production run or procurement is C_s (or A)

(vi) production or supply of commodity is Instantaneous (Abundant availability)

(vii) Holding cost is C_1 per unit in inventory for a unit time i.e $C_1 = IC$, where C is the unit cost, I is called inventory carrying ~~cost~~ charges expressed as a % of the value of the average inventory.

This fundamental situation can be shown on an inventory-time diagram, with Q on the vertical axis and time on the horizontal axis.
The total time period (one year) is divided into n parts



Here it is assumed that after each time t, the Quantity Q is produced or supplied throughout the entire time period, say One year, now if n denots the total number of runs of the Quantity produced during the year, then clearly we have

$$1 = nt \quad \text{and} \quad D = nQ$$

It may be clear that the average amount of inventory at hand on any day is then $\frac{1}{2}Q$

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Total inventory over the time period 't' days is clearly the area of the first triangle i.e $\frac{1}{2}Qt$, Thus the average inventory at at any time on any given day in the 't' period is $\left(\frac{1}{2}Qt\right)\frac{1}{t} = \frac{1}{2}Q$.

- Now since each of the ~~trinac~~ triangles in above figure over a period looks like the same, i.e $\frac{1}{2}Q$ remains the average amount of inventory in each interval of length 't' during the entire period.

Annual inventory carrying cost/holding cost is therefore given by

$$f(Q) = \frac{1}{2}QC_1$$

Annual costs associated with runs of size Q are given by

$$g(Q) = nC_s = \frac{D}{Q}C_s$$

Thus total ~~cost~~ annual cost is given by

$$C_A = f(Q) + g(Q)$$
$$= \frac{1}{2}QC_1 + \frac{D}{Q}C_s$$

~~Now $\frac{dCA}{dQ}$ = 0 implies that $\frac{1}{2}C_1 - \frac{1}{2}\frac{D}{Q^2}C_s$~~

~~Put $\frac{dCA}{dQ} = 0$ then $\frac{1}{2}C_1 = \frac{1}{2}\frac{D}{Q^2}C_s$~~

$DC_s\, Q^{-2}$

$DC_s\,(-2)\, Q^{-3}$

~~$Q^2 =$~~ D

$$\frac{dC_A}{dQ} = 0, \quad \frac{d}{dQ}\left(\frac{1}{2}QC_1 + \frac{D}{Q}C_s\right) = 0$$

$$\text{or} \quad \frac{1}{2}\cancel{Q}C_1 - \frac{D}{Q^2}C_s = 0$$

$$\text{or} \quad \boxed{Q^* = \sqrt{\frac{2DC_s}{C_1}}}$$

now

$$\frac{d^2C_A}{dQ^2} = 0 + \frac{2DC_s}{Q^3} > 0 \quad (+\text{ve value})$$

hence C_A is minimum at

$$\boxed{Q^* = \sqrt{\frac{2DC_s}{C_1}}}$$

The optimum value of Q (ie Q^*) has thus obtained and is given by is known as the economic (optimum) ~~LOR~~ lot size formula due to R. H. Wilson

→ The above EOQ formula can also be expressed into terms of the economic order value terms as follows,

$$Q^0 = \sqrt{\frac{2C_sD}{IC}}$$

$$CQ^0 = \sqrt{\frac{2C_sC^2D}{I\cancel{C}}} \quad (\text{multiply both side by } C)$$

$$\boxed{V^0 = \sqrt{\frac{2C_sCD}{I}} = \sqrt{\frac{2C_sV}{I}}}$$

Where V^0 is the EOV (EOQ expressed in terms of the value) and V is the value of the total demand.

Furthermore, the other corresponding optimal quantities are given by

$$t^0 = Q^0/D$$

$$= \left(\sqrt{2DC_s/C_1} \;/ D\right)$$

$$\boxed{t^0 = \sqrt{2C_s/C_1 D}}$$

$$\boxed{C_A^0 = DC_s/Q^0 + \tfrac{1}{2} Q^0 C_1}$$

$$= \frac{DC_s}{\sqrt{\frac{2DC_s}{C_1}}} + \frac{1}{2}C_1\sqrt{\frac{2C_sD}{C_1}}$$

$$= DC_s\sqrt{\frac{C_1}{2DC_s}} + \sqrt{\frac{1}{2^{\cancel{2}}}C_1^{\cancel{2}} \times \frac{\cancel{2}C_sD}{\cancel{C_1}}}$$

$$= \sqrt{\frac{D^{\cancel{2}}C_s^{\cancel{2}}C_1}{2\cancel{D}\cancel{C_s}}} + \sqrt{\frac{C_1C_sD}{2}}$$

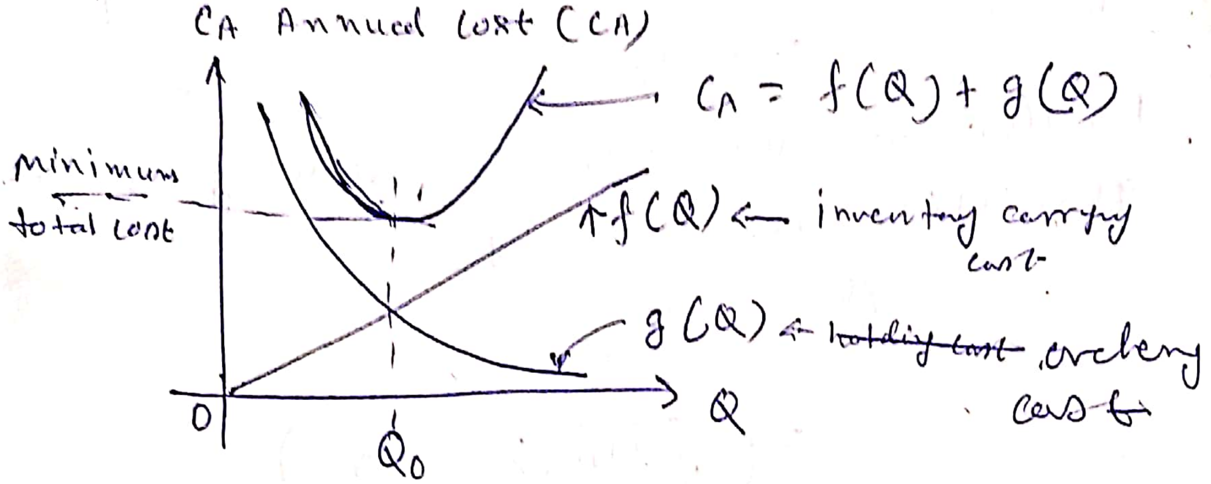
$$= 2\sqrt{\frac{C_1C_sD}{2}} = \sqrt{\frac{2^{\cancel{2}}C_1C_sD}{\cancel{2}}}$$

$$\boxed{C_A^0 = \sqrt{2C_1C_sD}}$$

C_A^0 – represents the total minimum cost

This represents the total annual cost associated with holding inventory and setting up the machine when the optimum lot size, Q^0

is produced in each run.



Problem of EOQ with finite rate of replenishment

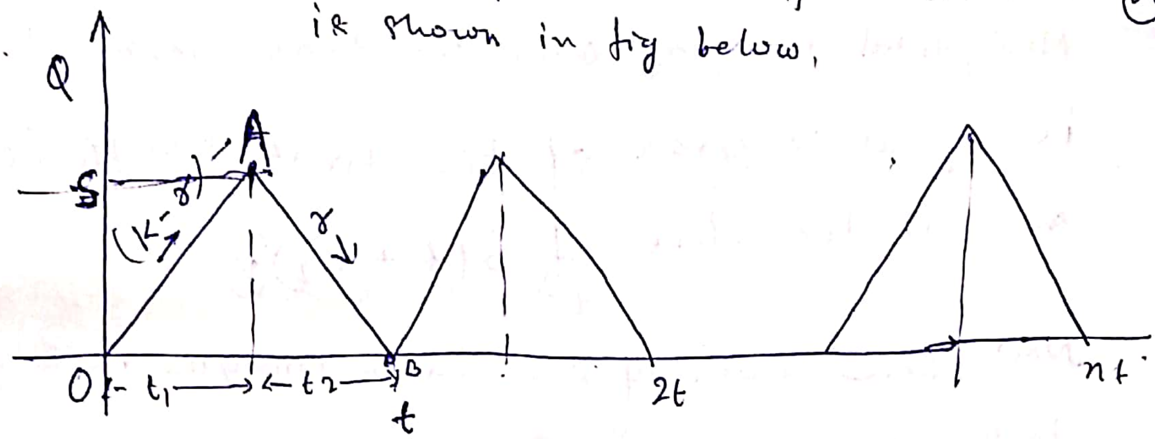
In this problem all the assumptions are same as the previous model except that the rate of replenishment of inventory is ~~infinite~~ finite, say 'K' units of quantity per unit time and consumption rate 'r' unit ~~units~~ per unit time.

Assume that each production run of length 't' consists of two part t_1 and t_2, such that

(i) the inventory is building up at a constant of $(K-r)$ units per unit of time during 't_1'

(ii) There is no replenishment (or production) during time t_2 and the inventory is decreasing at the rate ~~per~~ 'r' per unit of time during 't_2'.

The graphical representation of the situation is shown in fig below.



~~The graphical~~

At the end of period t_1, let the level of inventory be S; and the end of the period t_2 let it be zero then, clearly we have

$$\underline{S = t_1(K - r)} \quad \text{and} \quad \underline{S = t_2 r}$$

If Q be the lot size the we must have

$$\underline{S = Q - r t_1}$$

because Q is the total quantity produced during time t_1. and $r t_1$ is the quantity utilized during t_1.

~~Thus $S = (Q - S)(K - r)/r$~~

$$S = Q - r t_1$$

or $S = K t_1 - r t_1$ ($\because$ $Q = K t_1$
$= (K - r) t_1$ $\qquad t_1 = Q/K$

$$\underline{S = (K - r) Q/K}$$

$Q = K t_1$
$\Rightarrow t_1 = Q/K$
$S = K t_1 - r t_1$
$= t_1(K - r)$
or
$S = (K - r) Q/K$

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Now, total inventory over the time period t.

is equal to area of the first triangle (OAB)

and is therefore $\frac{1}{2} S(t_1 + t_2)$

Now since each of the ~~triang~~ triangle in a year looks the same, $\frac{1}{2} S$, remains to be the average amount of inventory in each interval of length $t_1 + t_2$, during the entire year.

→ Thus the annual inventory holding cost is given by $\frac{1}{2} S c_1$ which is equal to $\left[\frac{1}{2}(K-r)Q/K\right] \times c_1$

Also, the setup cost for one unit of time is $c_s / (t_1 + t_2)$

[One unit means one year]

Since $t_1 + t_2 = \frac{S}{K-r} + \frac{S}{r} = \frac{SK}{r(K-r)}$

Now $\frac{K}{r(\cancel{K-r})} \times \frac{(\cancel{K-r})}{\cancel{K}} \times Q$

$t_1 + t_2 = \frac{Q}{r}$

The total cost for one unit of time is given by

$$\boxed{C = \frac{1}{2}\left(\frac{K-r}{K}\right) Q c_1 + \frac{r}{Q} c_s}$$

$$\frac{dC}{dQ} = \frac{1}{2}\left(\frac{K-r}{K}\right) c_1 - \frac{r}{Q^2} c_s$$

$$\Rightarrow \boxed{Q^0 = \sqrt{\frac{2C_s}{C_1}} \cdot \sqrt{\frac{rK}{K-r}}}$$

$$\frac{\partial^2 C}{\partial Q^2} = \cancel{1}\, 0 + \frac{2 r C_s}{Q^3}$$ is positive for all values of Q, the minimum (optimum) value of Q is given by.

$$Q^0 = \sqrt{\frac{2C_s}{C_1}} \sqrt{\frac{rK}{K-r}}$$

and consequently the other optimum values are given by

$$\boxed{t^0 = \frac{Q^0}{r} = \sqrt{\frac{2C_s}{rC_1}} \cdot \sqrt{\frac{K}{K-r}}}$$

and $$C^0 = \frac{1}{2}\left(\frac{K-r}{K}\right) Q^0 C_1 + \frac{r}{Q^0} C_s$$

$$\boxed{C^0 = \sqrt{2C_1 C_s r} \sqrt{\left(\frac{K-r}{K}\right)}}$$ ~~is the minimum~~

is the minimum cost equation for the problem

Note If $K \to r$, then $C^0 \to 0$, this shows that there will be no holding cost and no setup cost.

→ If $K \to \infty$, ie when the production rate become infinite, the above problem reduces to the one considered in first model.

(28) - The inventory holding cost ~~cost~~ per unit of time is reduced from the cost ~~above~~ discussed in previous model in the ratio $(1 - r/k) : 1$ for minimum cost, although the set up cost remains the same.

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Problems of EOQ with Shortage

In a business, if shortage occurs then these can be classified into the following two categories.

(a) As soon as the desired units of a certain commodity arrive in ~~in~~ inventory, the back orders are satisfied.

(b) Shortage are lost sales.

In the first case demand of the customer is met in the begining of new production run, where as in the second case the customers moves ~~t~~ to other firm to fulfill his requirements.

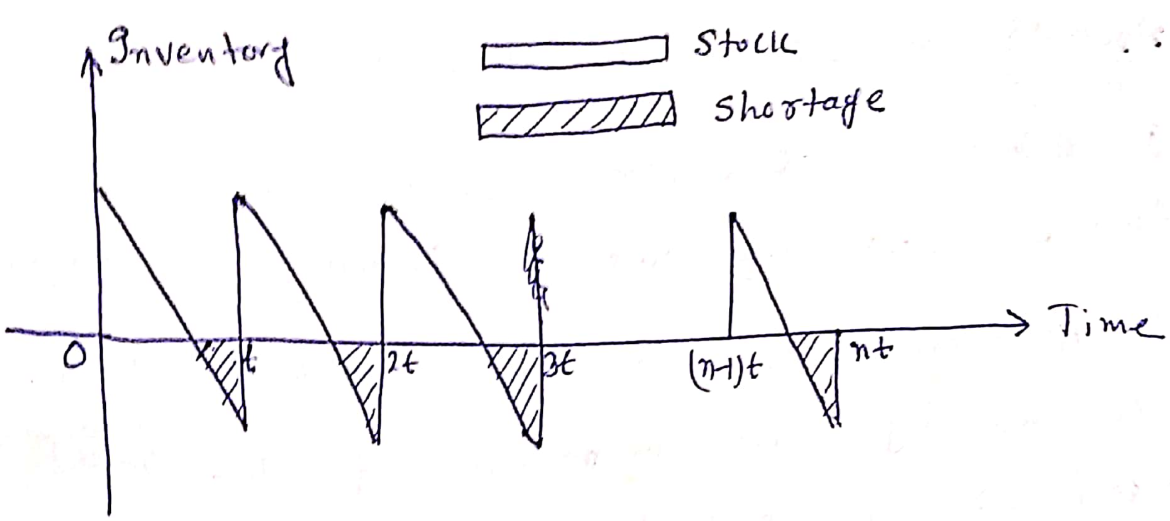
→ This section deals with those problems of shortages ~~where~~ where back orders are entertained.

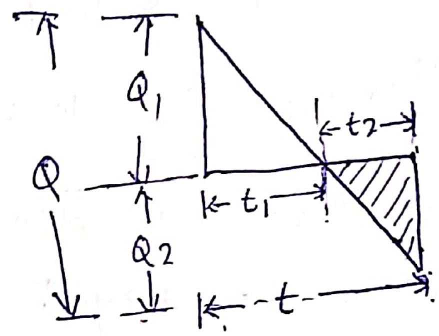
~~Problem~~ Production is Instantaneous: ~~If The~~ This problem is same as we have discussed earlier, except that here shortages are permitted

→ Let C_2 be the shortage cost per unit of time per unit of quantity.

→ The inventory situation can also be illustrated graphically as shown in ~~fig~~

(30)





Here the total time period is one year and is divided into n equal parts, say interval t. Further, this time interval t is divided into two parts t_1 and t_2, s.t. $t = t_1 + t_2$

During the interval t_1, the items are drawn from the inventory as needed and during t_2, orders for the item are being accumulated but not filled. Then at the end of the interval t an amount Q is produced (or delivered). The amount Q has been divided into Q_1 and Q_2 such that $Q = Q_1 + Q_2$ ✓

Where Q_1 denotes the amount which goes into inventory, and Q_2 denotes the amount which is immediatly taken to satisfy past orders or unfilled demand.

The problem now is concerned with the areas of triangles above the time axis (representing item in inventory) and below the same axis (representing items in shortages).

Now Total inventory over ~~time~~ the time period $t_1 = \frac{1}{2} Q_1 t_1$

Average inventory at any time $= (\frac{1}{2} Q_1 t_1)/t$ ✓

Annual inventory holding cost $= C_1 (\frac{1}{2} Q_1 t_1)/t$ ✓

Simillarly

Total amount shortage over time period t

$= \frac{1}{2} Q_2 t_2$ ✓

Annual shortage costs $= C_2 (\frac{1}{2} Q_2 t_2)/t$ ✓

Annual costs associated with runs of size $Q = n C_s = \frac{D}{Q} C_s$

[Since $\frac{D}{Q}$ runs produced in each year]

∴ Total annual cost is given by

$$C_A = \left[C_1 (\tfrac{1}{2} Q_1 t_1) + C_2 (\tfrac{1}{2} Q_2 t_2)\right]/t + \frac{D}{Q} C_s$$

Now using the relationship for simillar triangles, we have $\frac{t_1}{t} = \frac{Q_1}{Q}$ and $\frac{t_2}{t} = \frac{Q_2}{Q}$

ie $t_1 = \frac{Q_1}{Q} t$ and $t_2 = \frac{Q_2}{Q} t$

Substituting these values ~~we get~~ we get

$$\boxed{C_A = \frac{1}{2} C_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} C_2 \left[\frac{(Q - Q_1)^2}{Q} \right] + C_s \left[D/Q \right]}$$

since $Q_2 = Q - Q_1$

For determining the optimum values of Q_1 and Q so as to optimise C_A we have

$$\frac{\partial C_A}{\partial Q_1} = 0 \Rightarrow Q_1 = C_2 Q / (C_1 + C_2),$$

$$\frac{\partial C_A}{\partial Q} = 0 \Rightarrow Q = \sqrt{\frac{2 C_s D + C_1 Q_1^2}{C_2} + Q_1^2}$$

and $\frac{\partial^2 C_A}{\partial Q_1^2} > 0$, $\frac{\partial^2 C_A}{\partial Q^2} > 0$ for these values of Q_1 and Q

Thus the optimum Quantities are given by (on simplification)

$$\boxed{Q^0 = \sqrt{\frac{2 C_s D}{C_1}} \sqrt{\frac{(C_1 + C_2)}{C_2}}}$$

$$\boxed{Q_1^0 = \left(\frac{C_2}{C_1 + C_2} \right) Q^0 = \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{\frac{2 C_s D}{C_1}}}$$

The minimum annual cost equation therefore is

$$C_A^0 = \tfrac{1}{2} C_1 (Q_1^0)^2 / Q^0 + \tfrac{1}{2} C_2 (Q^0 - Q_1^0)^2 / Q^0 + C_s (D/Q^0)$$

or $$\boxed{C_A^0 = \sqrt{2C_1 C_s D}\sqrt{\frac{C_2}{C_1+C_2}}}$$

Hence we conclude

"The optimal policy is ~~so~~ to supply Q^0 units in each run of length 't' and should plan to supply Q_1^0 units on schedule in each run, where

$$Q^0 = \sqrt{\frac{2C_s(C_1+C_2)D}{C_1C_2}} \text{ and } Q_1^0 = \left(\frac{C_2}{C_1+C_2}\right)Q^0$$

(1) Remarks If $C_1 > 0$ and $C_2 = \infty$, shortage are prohibited, ~~In this case $Q_1^0 = Q^0$~~

$Q_1^0 = Q^0 = \sqrt{2C_sD/C_1}$ and each batch Q^0 is used entirely for inventory.

2. If $C_1 = \infty$ and $C_2 > 0$, inventory are prohibited. In this case $Q_1^0 = 0$, $Q^0 = \sqrt{2C_sD/C_2}$ and each batch is used only to fill back orders.

3. If shortage costs are negligible, then $C_1 > 0$ and $C_2 \to 0$, In this case $Q_1^0 \to 0$ and $Q^0 \to \infty$.

(same as above)

④ If inventory costs are negligible, then $C_1 \to 0$ and $C_2 > 0$. In this case $Q^0 = \infty$ and $Q_1^0 \to \infty$ i.e $Q_1^0 \to Q^0$. Thus ~~as~~ inventory costs become very small, increasingly large batches should be produced and used entirely as inventory for future demands.

⑤ When inventories and shortages are equally costly, i.e when $C_1 = C_2$, $\frac{C_2}{C_1+C_2} = 1/2$

This in this case

$$Q^0 = \sqrt{2}\sqrt{\frac{2C_S D}{C_1}} = (1.414)\sqrt{\frac{2C_S D}{C_1}}$$

Replenishment of Inventory with finite rate

In this problem all the assumptions are same as in the previous model except that rate of replenishment of inventory is finite, say K units of ~~quality~~ quantity per time

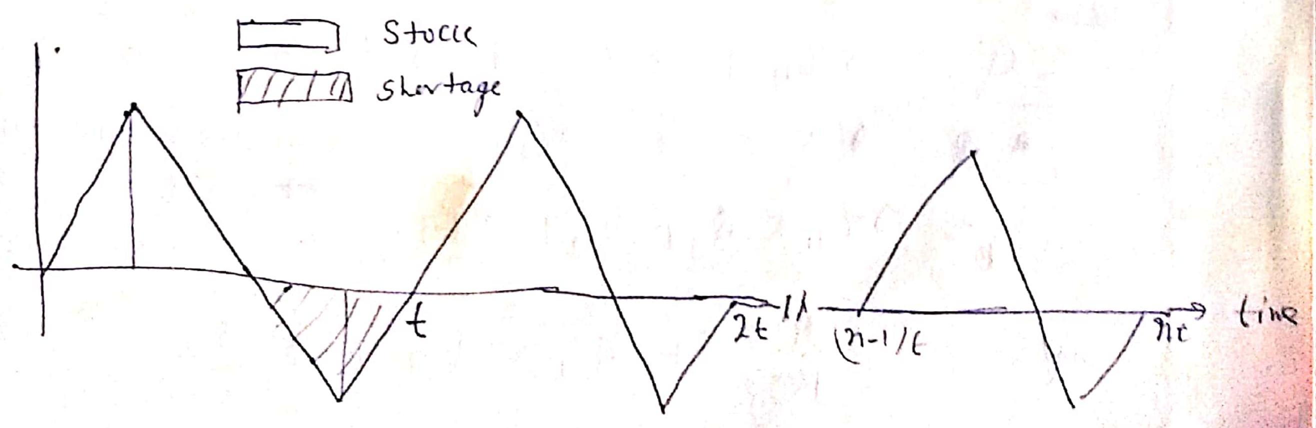
Assume that each production run of length 't' Consists of two parts t_1 and t_2 which are further sub divided into two parts say t_{11} and t_{12}, ~~or~~ t_{21} and t_{22} where;

(i) Inventory is building up at a constant rate of $(K-r)$ units per unit of time during t_{11}.

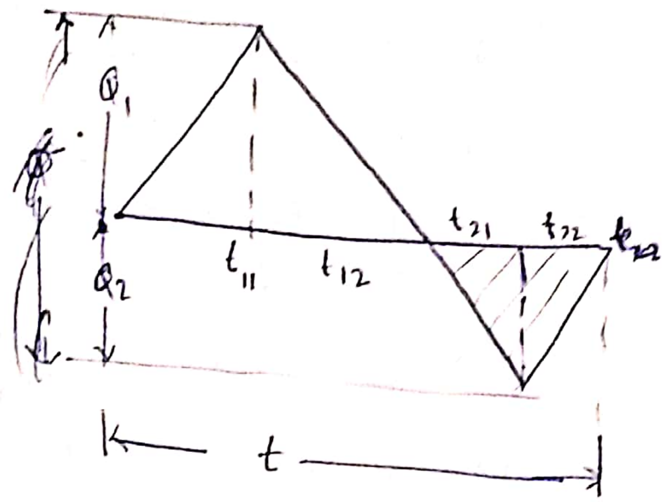
(ii) No replenishment during time t_{12} and inventory is decreasing at the rate r per unit of time.

(iii) Shortage is building up at a constant rate of r per unit of time during time t_{21}

The graphical representation of time situation is as follows



$Q - r t_{11}$



From the above figure, we can see that at the end of t_{11} the level of inventory is Q_1 and at the end of period t_{12} inventory becomes nil. Now shortages start and ~~supp~~ suppose that the shortages build up of quantity Q_2 up to time t_{21} and let then these shortages be filled up during time t_{22}.

Then obviously

$$Q_1 = t_{11}(K - r) \qquad Q_1 = t_{12}\, r$$

$$Q_2 = t_{21}\, r \qquad \text{and } Q_2 = t_{22}(K - r)$$

Now Q is the lot size then

$\cancel{Q_1 = Q - Q_2 = r t_{11} \quad K t_{11} -}$

Then

$$Q = r t_{11} + r t_{12} + r t_{21} + r t_{22}$$

$[\because Q = r t \text{ or } t = Q/r]$ (Total consumption).

$$Q = r t_{11} + Q_1 + Q_2 + r t_{22} \qquad = r t$$

where $t = t_{11} + t_{12} + t_{21} + t_{22}$

$$Q = r \times \frac{Q_1}{K - r} + Q_1 + Q_2 + r\,\frac{Q_2}{K - r}$$

or $$\boxed{Q = Q_1\left[\frac{r}{K-r}+1\right] + Q_2\left[1+\frac{r}{K-r}\right]}$$

$$= Q_1\left[\frac{\cancel{r}+K-\cancel{r}}{K-r}\right] + Q_2\left[\frac{K-\cancel{r}+\cancel{r}}{K-r}\right]$$

$$= Q_1 \cdot \frac{K}{K-r} + \frac{Q_2 K}{K-r}$$

or $$\frac{K-r}{K} Q = Q_1 + Q_2$$

or $$\boxed{Q_1 = \left(\frac{K-r}{K}\right) Q - Q_2}$$

Now total inventory over the time period t

$$= \frac{1}{2} Q_1 (t_{11} + t_{12})$$

$$= \frac{1}{2} Q t_1$$

Total amount of shortage over the time period t

$$= \frac{1}{2} Q_2 (t_{21} + t_{22})$$

$$= \frac{1}{2} Q_2 t_2$$

Setup cost for one unit of time

$$= \frac{C_s}{(t_1 + t_2)} \; (= t)$$

$$= \frac{C_s}{t}$$

$Q = rt$
$t = Q/r$

$$= \frac{C_s}{Q/r} = \frac{rC_s}{Q}$$

Total cost for One unit of time

C = Inventory carrying cost + Shortage cost + Set up cost

$$= c_1\left(\tfrac{1}{2}Q_1 t_1\right)/t + c_2\left(\tfrac{1}{2}Q_2 t_2\right)/t + C_s/t$$

$$\Big[\ t_1 = t_{11} + t_{12} = \frac{Q_1}{K-r} + \frac{Q_1}{r}$$

$$= Q_1\left[\frac{r + K - r}{(K-r)(r)}\right] = \frac{K Q_1}{r(K-r)}$$

$$t_2 = t_{21} + t_{22} = \frac{Q_2}{r} + \frac{Q_2}{K-r}$$

$$= Q_2\left[\frac{1}{r} + \frac{1}{K-r}\right]$$

$$= Q_2\left[\frac{K - r + r}{r(K-r)}\right] = \frac{KQ_2}{r(K-r)}$$

$$t = \frac{Q}{r}\ \Big]$$

$$\therefore C = \left[\frac{1}{2}c_1 Q_1 \times \frac{KQ_1}{r(K-r)} \times \frac{r}{Q}\right] + \left[\frac{1}{2}c_2 Q_2 \frac{KQ_2}{r(K-r)} \times \frac{r}{Q}\right] + \left[C_s \frac{r}{Q}\right]$$

$$= \cancel{\frac{1}{2Q}c_1}\ \cancel{\frac{1}{2Q}}\Big[c_1$$

$$= \frac{1}{2Q}\frac{K}{K-r}\left[c_1 Q_1^2 + c_2 Q_2^2\right] + \frac{r}{Q}C_s$$

$$\boxed{C = \frac{1}{2Q}\frac{K}{K-r}\left[c_1\left(\frac{K-r}{K}Q - Q_2\right)^2 + c_2 Q_2^2\right] + \frac{r}{Q}C_s}$$

For determining the optimum values of Q, Q_1 and Q_2 so to minimize C we have

$$\frac{\partial C}{\partial Q_2} = 0 \Rightarrow C_1\left[2Q_2 - \cancel{2}\frac{K-r}{K}Q\right] + 2C_2Q_2 = 0$$

$$\frac{\partial C}{\partial Q_2} = 0 = \frac{1}{2Q}\frac{K}{K-r}\left[C_1 \times 2\left(\frac{K-r}{K}Q - Q_2\right)\times(-1) + 2C_2Q_2\right] + 0 = 0$$

or $$2C_1\left[\frac{(K-r)}{K}.Q - Q_2\right] = 2C_2Q_2$$

or $$C_2Q_2 + C_1Q_2 = C_1\frac{(K-r)}{K}Q$$

or $$Q_2(C_1+C_2) = C_1\frac{(K-r)}{K}Q$$

or $$\boxed{Q_2 = \left(\frac{C_1}{C_1+C_2}\right)\times\left(\frac{K-r}{K}\right)\times Q}$$

$$\frac{\partial C}{\partial Q} = 0 \Rightarrow -\frac{rC_s}{Q^2} + C_1\left\{\left[\tfrac{1}{2}(K-r)/K\right] - \frac{Q_2^2K}{2Q^2(K-r)}\right\} - \frac{C_2Q_2^2.K}{2Q^2(K-r)} = 0$$

or $$-\frac{rC_s}{Q^2} + \frac{C_1(K-r)}{2K} - \frac{C_2^2}{2(C_1+C_2)}\cdot\frac{K-r}{K} = 0$$

or $$Q = \sqrt{\frac{2C_s(C_1+C_2)}{C_1C_2}}\sqrt{\frac{Kr}{K-r}}$$

Since $\frac{\partial^2 C}{\partial Q^2} > 0$ and $\frac{\partial^2 C}{\partial Q_2^2} > 0$ for all values of Q and Q_2 the optimum value of

Q, Q_2 so as to minimize C are given by

$$\boxed{Q^0 = \sqrt{\frac{2C_s(C_1+C_2)}{C_1C_2}}\sqrt{\frac{Kr}{K-r}}}$$

and $$\boxed{Q_2^0 = \sqrt{\frac{2C_sC_1}{(C_1+C_2)C_2}}\sqrt{\frac{K}{r(K-r)}}}$$

and $$\boxed{Q_1^0 = \frac{K-r}{K}Q^0 - Q_2^0 = \sqrt{\frac{2C_2C_s}{C_1(C_1+C_2)}}\sqrt{\frac{r(K-r)}{K}}}$$

Hence the minimum cost equation is

$$\boxed{C^0 = \sqrt{\frac{2C_1C_2C_s}{C_1+C_2}}\sqrt{\frac{r(K-r)}{K}}}$$

Remarks

→ If $K=\infty$, the problem is in an complete agreement with the instantaneous supply and Shortage is ~~at~~ allowed

→ If $C_2=\infty$ the problem reduced to ~~to~~ that ~~of~~ of finite rate of replacement and shortage is not allowed.

→ If $K \to \infty$, and $C_2=\infty$, the problem will become an Instantaneous supply and shortage is not allowed.

Inventory control Techniques - Uncertain demand.

When the demand cannot be completely pre-determined, it is said to be uncertain, and ~~it~~ can fluctuate on either way.

In many practical situations, it is observed that neither the consumption rate of material is constant throughout the year nor is the lead time. To face these uncertainties in consumption rate and lead time, an extra stock is termed as "Buffer stock" (or safety stock or Reserved stock).

Determining the Buffer stock and Re-order Level.

For determining the buffer stock, we approximate the estimated maximum lead time and normal lead time for ~~a po~~ a particular item.

The buffer stock then would be equal to the difference of the maximum and normal lead times multiplied by the consumption rate of that item during the lead time.

Thus if B denotes the buffer stock, and $(L_m - L_n)$ is the difference of the maximum ~~and~~ (L_m) and normal (L_n) lead times, r is the consumption rate during the lead time; then

$$B = \gamma(L_m - L_n)$$

Now if we do not maintain a buffer stock, then the total requirement for inventory during normal lead time will be γL_n, where L_n denotes the ~~lead~~ normal lead time.

This assumption implies that as soon as the stock reaches a level γL_n we place an order for 'Q' quantity. This point is generally called the Re-order level (ROL). This policy of ROL results in shortages about half the time. To avoid this we add a buffer stock and place an order when stock reaches $B + \gamma L_n$

ie $ROL = B + \gamma L_n$. where $B = (L_m - L_n)\gamma$

Determining Optimum Buffer Stock

When the Buffer stock maintained is very low, the inventory holding cost would be low but shortages will occur very frequently and the cost of shortages would be very high. As against this if the buffer stock maintained is rather large, shortages would be rather rare, resulting into low shortages costs but the inventory costs would be high. Hence it becomes necessary to strike a balance between the cost of shortages and cost of inventory holding to arrive at an optimum buffer ~~stoc~~ stock.

Inventory Control with price Breaks

In the real world, it is not always true that the unit cost of an item is independent of the quantity procured. Often discounts are ~~are~~ offered for the purchase of large quantities. These discounts take the form of price breaks.

Conditions

(i) Demand is known and is uniform

(ii) Shortages are not permitted

(iii) production ~~for~~ supply or purchase is instantaneous.

→ Let Q be the lot size in each production run, D be the total number of units produced or supplied over the ~~to~~ entire time period, and C_s be the set-up cost per production run,

$$1 = nt, \quad D = nQ.$$

When n is the total number of runs during the entire time period, and t is time ~~for~~ period unit (or interval between two runs)

Let the cost of manufacturing (or purchasing) ~~the~~ be K_1 per unit, and holding cost P (in %) per unit. The the average number of month inventories (or t' inventories in t time)

$$\frac{1}{2} Q t = \frac{1}{2} \frac{Q^2}{D} \qquad \left(\begin{array}{l} 1 = nt \\ D = nQ \\ n = D/Q \therefore 1 = \frac{D}{Q} \times t \\ t = Q/D \end{array} \right)$$

(or average inventory per unit time $\frac{1}{2} Qt / t = 1/2 Q$)

Now the annual (periodical) costs associated with runs of size Q are given by

$$f(Q) = nQK_1 + nC_s \quad \left(\begin{array}{l}\text{purchase +} \\ \text{setup or ordering cost}\end{array}\right)$$

$$= \frac{D}{Q}(QK_1 + C_s) \qquad (n = D/Q)$$

$$f(Q) = DK_1 + \frac{D}{Q} C_s$$

Annual inventory holding costs associated with purchasing costs ~~are~~ are given by

$$h(Q) = \frac{1}{2} Q \times P K_1 \quad \left(\begin{array}{l}\text{\% of inventory} \\ \text{carrying cost per unit price}\end{array}\right)$$

Thus the total expected cost is given by

$$C_A(Q) = f(Q) + h(Q)$$

$$= DK_1 + \frac{D}{Q} C_s + \frac{1}{2} Q K_1 P.$$

$$\frac{d}{dQ} C_A(Q) = 0 \Rightarrow Q = \sqrt{\frac{2C_s D}{K_1 P}}$$

Also $\frac{d^2}{dQ^2} C_A(Q) \geq 0$

Thus the optimum value of Q has been obtained and is given by $Q^0 = \sqrt{\frac{2C_s D}{K_1 P}}$

$$C_A^0 = DK_1 + \frac{D}{Q^0} C_s + \frac{1}{2} Q^0 K_1 P$$

$$= \sqrt{2C_s D K_1 P} + DK_1$$

We now proceed to consider the generalization when purchase cost is subject to price breaks.

Purchase Inventory Problem with One price Break

When there is only one price break (one quantity discount), the situation may be illustrated as follows

Range of Quantity	Purchase cost per unit
$0 \leq Q_1 < b$	K_{11}
$b \leq Q_2$	K_{12}

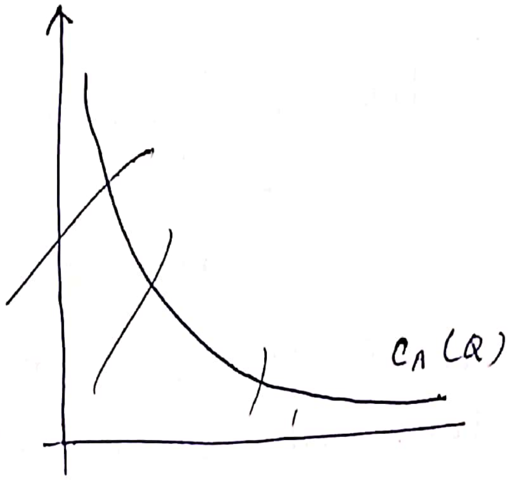
where b is that quantity at and beyond which the quantity discount applies and $K_{12} < K_{11}$

Now total expected cost $C_A(Q_1)$ for any purchase quantity Q_1 in the range $0 \leq Q_1 < b$ will be

$$C_A(Q_1) = \frac{D}{Q_1} C_s + \frac{1}{2} Q_1 K_{11} P + DK_{11}$$

Similarly, for any purchase quantity Q_2, in range $Q_2 \geq b$, the total expected cost $C_A(Q_2)$ will be given by

$$C_A(Q_2) = \frac{D}{Q_2} C_s + \frac{1}{2} Q_2 K_{12} P + DK_{12}$$



In order to obtain the optimum purchase quantity. we proceed as follows

Step-1. Compute Q_2^0 and compare with the quantity b,

Step 2 If $Q_2^0 \geq b$, the optimum lot size size will be Q_2^0 (ie ~~considering~~ considering K_{12} as purchasing cost)

Step 3 If $Q_2^0 < b$, the quantity discount will not be applicable to purchase ~~qua~~ quantity Q_2^0.

~~For~~ Here in order to obtain optimum purchase quantity, ~~need~~ need to compare total expected cost for $Q = Q_1^0$ (ie unit cost as K_{11}) with $Q = b$ (ie unit cost as K_{12})

Whenever the total cost is less that will be the optimum quantity ~~(a~~

ie min of $C_A(Q_1^0)$ and $C_A(b)$.

It is well known that every organization consumes several items of stores. As all the items are not of equal importance, a high degree of control on inventories of each item is neither practical nor worthwhile. Therefore, it becomes necessary to classify the items in groups depending upon their importance. Such a classification is know as the principle of selective control as applied to inventories.

A-B-C Analysis (Always Better Control)

This selective control technique approach is based on the annual usage value of various items. Some items like bolts and nuts, though equally important, cost less than some items like engines. So we can have safely stock bolts etc but not the engines because of latter's heavy cost. There is ~~thus~~ thus less control required over stocking of bolts, nuts etc. but greater emphasis should be given on control over stocking of big items like engines.

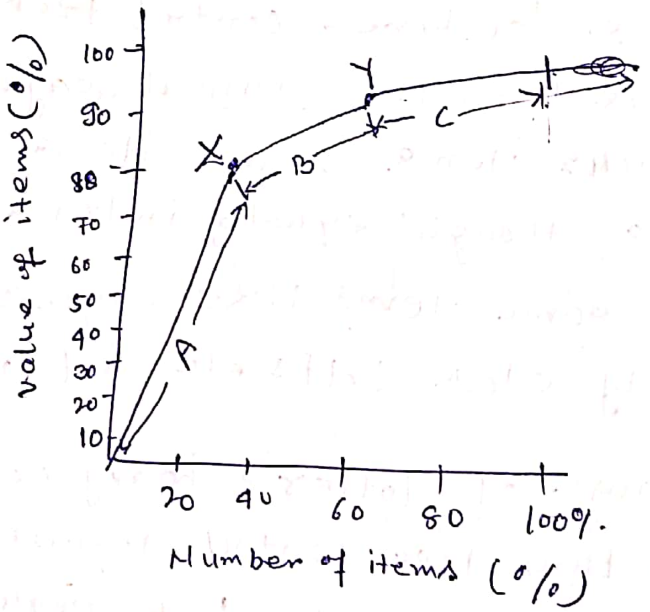
ABC analysis separates inventory items into three classes, say A, B and C in

(49) descending order of usages value.

Based on ABC analysis an average pattern of percentages of items and percentages of their annual usage value may be worked out as follows

Class	Percentage of items (%) (Approx)	Percentage of Annual Usage (%) (Approx)
A	10	80
B	20	15
C	70	5

These percentage are shown in the following figure.



The two points X and Y where the curve changes its shape provide the three segments A, B and C. From points X and Y and the tabulated data it is ~~the~~ seen that 'A' items which form only 10% of the total number of items form highest value among all the items, while C which is 70% forms only 5% of the total value.

ABC analysis helps to concentrate efforts in area which need it most. It gives the most effective and rewarding control with the least amount of controlling.

In case of 'A' items careful attention is paid for inventory control, estimates of requirements, purchase scheduling, safety stocks etc, while 'B' items are paid less attention and 'C' are paid least attention.

- High value 'A' items ~~are like~~ have lower safety stocks because of the cost of production is so high. The low value items carry much higher safety stocks. Sometimes even visual control is provided over low value 'C' items.
- In With 'ABC' control, it is also possible to reduce investment in inventories.

ABC Analysis Procedure

Step-1: Determine the number of units sold or used in the past 12 months period.

Step-2: Determine the unit-cost standard for each item

Step 3: Compute the annual consumption value (in rupees) of each consumed item by multiplying annual consumption (of units) with the unit price.

Step 4: Arrange these items in descending order of the usage value computed above.

Step 5: Obtain the cumulative total of the number of items and the usage value.

Step 6: Convert the cumulated totals of no. of items and usage values into percentages of the grand totals.

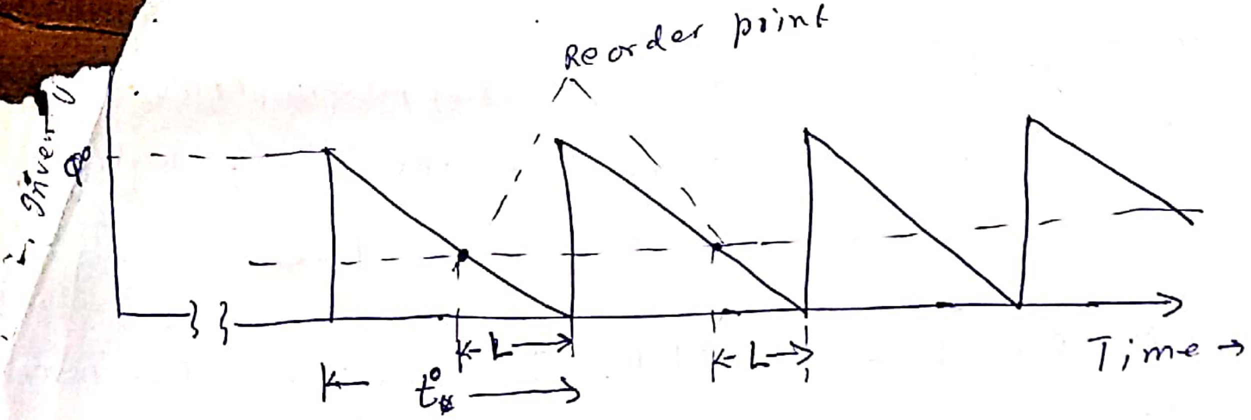
Step 7 Plot the two percentages on the graph paper.

Step 8: Identify the cut-off points X and Y where the curve sharply changes its shape. This provides three segment A, B and C. From points X and Y and the tabulated data the usages value classification points A, B and C may fined and generalised over the entire population of the stock items.

It is usual in most industries ~~to fin~~ to find about 10% of the items accounting for as much as 80% of the capital in inventories.

Scanned with CamScanner

time



Q^0 = Economic order quantity

L = Lead time

D = Demand in per unit time (It can be per day, per month, per year, the unit of lead time should also be same like day, month or year)

t^0 = Optimum ordering cycle length.

...rder point (ROL) = LD units when $L \leq t^0$

... $L > t^0$,

Then we have to take L_e (i.e. effective lead time) where L_e is the effective Lead time

$$L_e = L - nt^0$$

where 'n' is the largest integer not exceeding $\frac{L}{t^0}$. This result is justified because of after n cycles of t_0 each the the inventory situation acts as if the interval between placing an order and re

another is Le. Thus the reorder point occ

at LeD units and the inventory policy,

Can be restated as ~~order the~~

order the quantity Q^0 whenever the inventory level drops to LeD units.

(Ad100)