

Unit-1

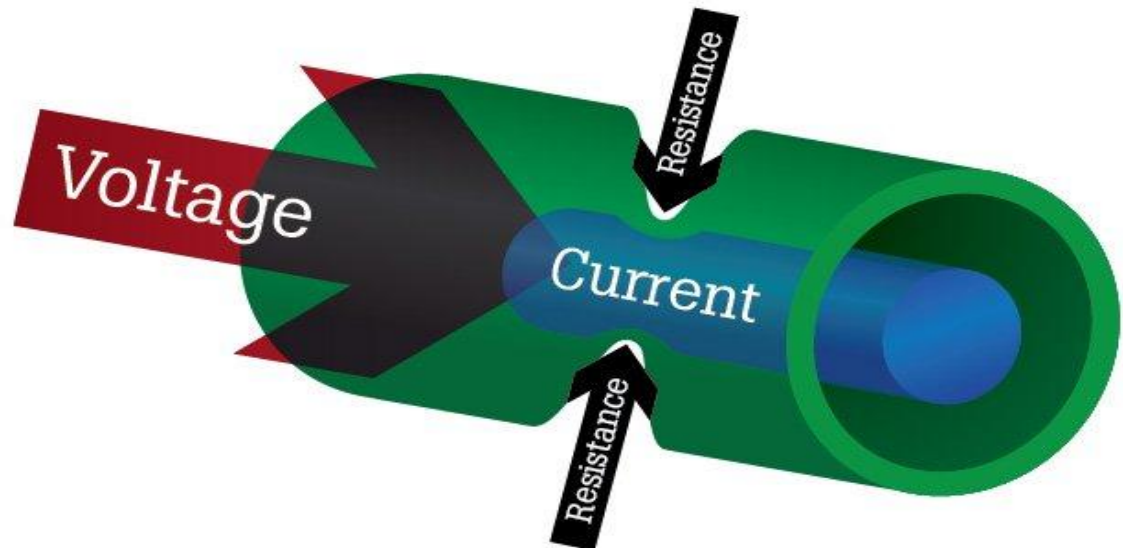
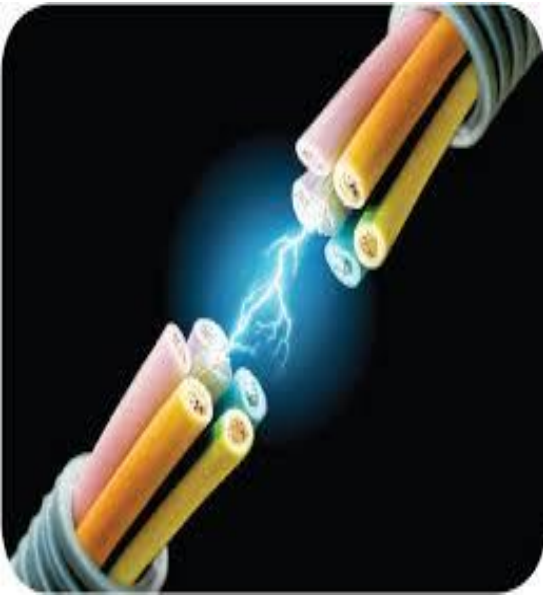
Basic Electrical Terms and Units

Prepared by:
Ankur Patel

M & V Patel Department of Electrical Engineering
CHARUSAT

ankurpatel.ee@charusat.ac.in

Mo. No. - 9978782503



Content



Current

Voltage

Resistance

Ohm's law

Resistor and its coding

Temperature co-efficient of resistance

Resistance variation with temperature

Current

- An electric current is a stream of charged particles, such as electrons or ions, moving through an electrical conductor or space.
- It is measured as the net rate of flow of electric charge past a region.
- The SI unit of electric current is the **ampere**, or amp, which is the flow of electric charge across a surface at the rate of one coulomb per second.
- Current can be measured using an ammeter.

Current

- For a steady flow of charge through a surface, the current I (in amperes) can be calculated with the following equation:

$$I = \frac{Q}{t}$$

- where Q is the electric charge transferred through the surface over a time t . If Q and t are measured in coulombs and seconds respectively, I is in amperes.
- More generally, electric current can be represented as the rate at which charge flows through a given surface as:

$$I = \frac{dQ}{dt}$$

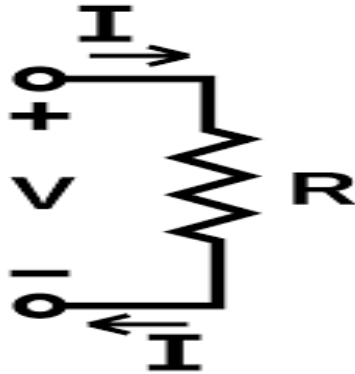
Voltage

- Voltage, electric potential difference, electric pressure or electric tension is the difference in electric potential between two points, which (in a static electric field) is defined as the work needed per unit of charge to move a test charge between the two points.
- In SI units, work per unit charge is expressed as joules per coulomb, where 1 volt = 1 joule (of work) per 1 coulomb (of charge).
- Voltage or electric potential difference is denoted symbolically by V , ΔV , or U
- A voltmeter can be used to measure the voltage (or potential difference) between two points in a system.

Resistance

- In electronics and electromagnetism, the electrical resistance of an object is a measure of its opposition to the flow of electric current.
- It is the property of material which opposes the flow of current in a electrical circuit.

Ohm's Law

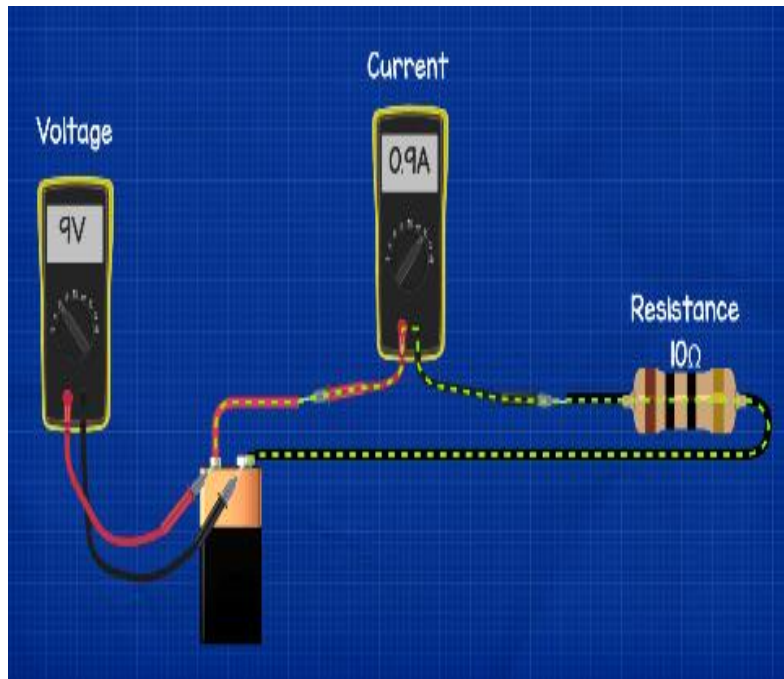


- Ohm's law states that the current through any conductor is directly proportional to voltage or potential difference between its ends, if the physical conditions of the conductor do not change.

$$I \propto V$$

or
$$I = \frac{1}{R} V$$

- Or we can state it as, $V = IR$.
- This relationship between current, voltage was discovered by German scientist Georg Simon Ohm.



Limitations of Ohm's Law

- Ohm's law is not applicable to unilateral networks.

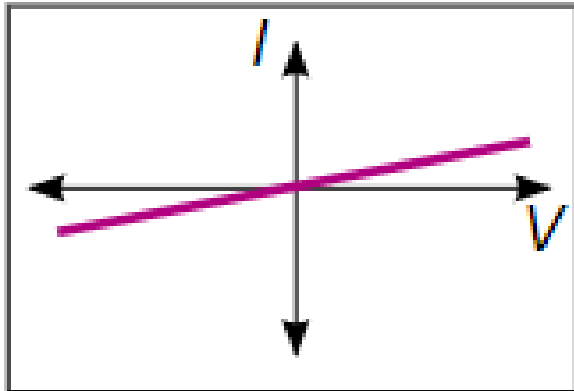
Note:- Unilateral networks allow the current to flow in one direction. Such types of network consist elements like a diode , transistor, etc.

- Ohm's law is also not applicable to non – linear elements.

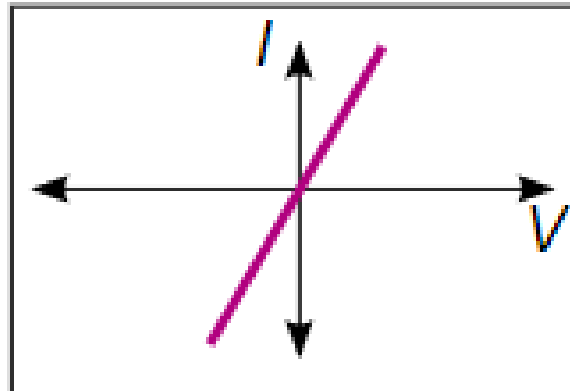
Note:- Non-linear elements are those which do not have current exactly proportional to the applied voltage that means the resistance value of those elements changes for different values of voltage and current. Examples of non – linear elements are the thyristor.

Limitations of Ohm's Law

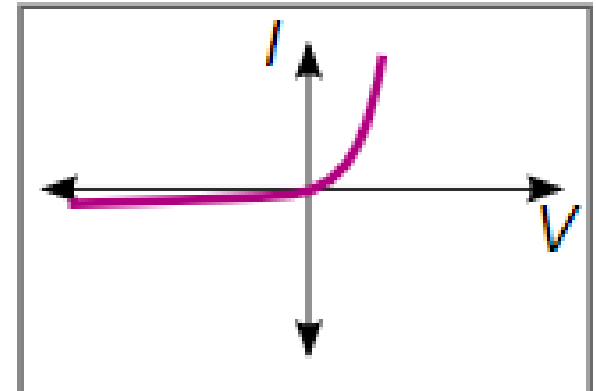
Large resistance



Small resistance

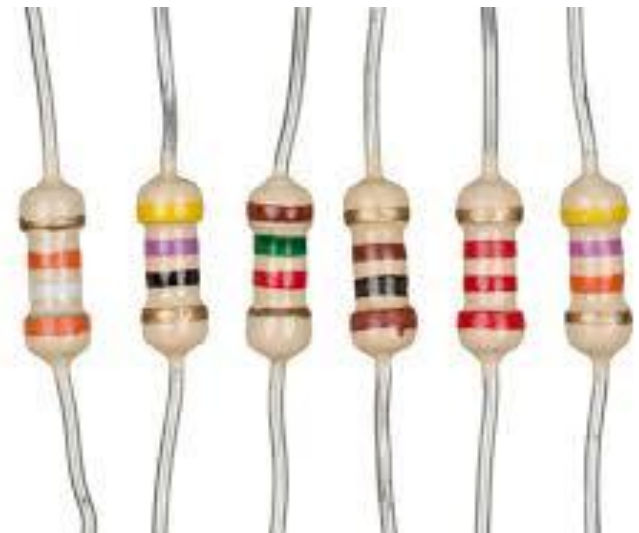
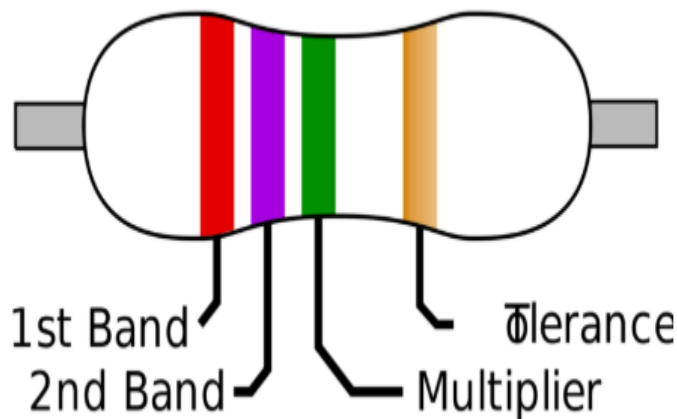


Diode



Resistors

- A resistor is an electrical component that limits or regulates the flow of electrical current in an electronic circuit.
- Resistors are one of the important blocks of electrical circuits.
- They are made up of the mixture of clay or carbon, so they are not only good conductors but good insulators.



How to read resistor color code

6-Band  = 274 Ω ± 2%, 250 ppm/K

Color
Black
Brown
Red
Orange
Yellow
Green
Blue
Violet
Grey
White
Gold
Silver

1st Digit	2nd Digit	3rd Digit
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

×

Multiplier	Tolerance	Temperature Coefficient
1 Ω		250 ppm/K
10 Ω	± 1%	100 ppm/K
100 Ω	± 2%	50 ppm/K
1k Ω		15 ppm/K
10k Ω		25 ppm/K
100k Ω	± 0.5%	20 ppm/K
1M Ω	± 0.25%	10 ppm/K
	± 0.1%	5 ppm/K
		1 ppm/K
0.1 Ω	± 5%	
0.01 Ω	± 10%	

4-Band  = 1,200 kΩ ± 5%

5-Band  = 10,000 Ω ± 1%

- Resistance of the conducting material is calculated using

$$R = \frac{\rho L}{A}$$

ρ = resistivity
 L = length
 A = cross sectional area



resistance-in-a-wire_en.html

- The resistance of a conducting material is found to
 - be directly proportional to the length L of the material.
 - be inversely proportional to the cross- section are A of the material.
 - depend on the nature of the material.
 - depend upon the temperature.
- Electrical resistivity (also called specific electrical resistance or volume resistivity) and its inverse, electrical conductivity, is a fundamental property of a material that quantifies how strongly it resists or conducts electric current.
- Resistivity is commonly represented by the Greek letter ρ (rho). The SI unit of electrical resistivity is the ohm-meter ($\Omega \cdot \text{m}$).

- The resistance of an object depends in large part on the **material** it is made of. Objects made of electrical insulators like rubber tend to have very high resistance and low conductivity, while objects made of electrical conductors like metals tend to have very low resistance and high conductivity.
- The resistance of an object also depends on the **size** and **shape** of an object because these properties are extensive rather than intensive. For example, a **wire's resistance is higher if it is long and thin, and lower if it is short and thick.**

Conductor	Insulator	Semi-Conductor
<ul style="list-style-type: none"> ➤ It is a material which has a large number of free electrons so, the current can easily flow through it. ➤ All materials with resistivity less than 10^{-8} ohm.m behave as conductor. ➤ E.g. Silver, Copper, Aluminium, Carbon and almost all metals. 	<ul style="list-style-type: none"> ➤ If the outer electrons of a material are very tightly bound to the nucleus, it becomes very difficult to remove from their orbits. Hence current cannot flow through such materials and they are known as insulators. ➤ All materials with resistivity above 10^5 ohm.m behave as insulator. ➤ E.g. Mica, Porcelain, Glass, oil, rubber, etc. 	<ul style="list-style-type: none"> ➤ It is material in which outer electrons are bound to the nucleus but the electrons can be made free by some means. For examples by adding some impurity. ➤ All material with resistivity between 10^{-8} and 10^5 ohm.m behave as semi-conductor. ➤ E.g. Germanium and Silicon.

Temperature co-efficient of resistance

- Resistance of almost all the materials changes with the change in the temperature.

R_0 = Resistance of the material at 0 degree Celsius

R_t = Resistance of the material at t degree Celsius

- Then the change in resistance $\Delta R = R_t - R_0$ is found to be

- I. directly proportional to its initial resistance, and
- II. directly proportional to the change in temperature.

- Thus,

$$\Delta R \propto R_0 t \quad \Rightarrow \quad (R_t - R_0) \propto R_0 t \quad \Rightarrow \quad (R_t - R_0) = \alpha R_0 t$$

α = constant known as the temperature co-efficient of resistance.

Continue....

- The variation of resistance with change in temperature of any material is governed by this property.

$$\alpha = \frac{(R_t - R_0)}{R_0 t} \quad (1)$$

- Temperature co-efficient of resistance α is defined as the change in resistance per unit rise in temperature per ohm original resistance.
- Usually temperature co-efficient is taken at a particular reference temperature which is normally taken as 0 degree Celsius.
- It is denoted by α_0
- Rewriting the equation (1), $R_t = R_0 * [1 + \alpha_0 * t]$

Resistance at different temperatures

- If at a standard temperature of 0 degree Celsius a material has a resistance of R_0 ohms,
 - at t_1 degree Celsius resistance of R_1 ohms and
 - at t_2 degree Celsius resistance of R_2 ohms, then

$$R_1 = R_0 * [1 + \alpha_0 * t_1] \text{ and}$$
$$R_2 = R_0 * [1 + \alpha_0 * t_2]$$

$$\frac{R_2}{R_1} = \frac{[1 + \alpha_0 * t_2]}{[1 + \alpha_0 * t_1]}$$

Continue....

- If temperature co-efficient α_0 and resistance R_0 are not given then the relation between the known resistance R_{t_1} and t_1 degree Celsius and the unknown resistance R_{t_2} and t_2 degree Celsius can be found as follows:

R_{t_1} = resistance at t_1 degree Celsius

R_{t_2} = resistance at t_2 degree Celsius

- Suppose,

α_{t_1} = temperature co-efficient of resistance at t_1 degree Celsius

α_{t_1} = Slope of the graph / resistance at t_1 degree Celsius

Slope of the graph = $\alpha_{t_1} * R_{t_1}$

Continue....

Slope of the graph=AB/BC

$$= \frac{R_{t2} - R_{t1}}{t_2 - t_1}$$

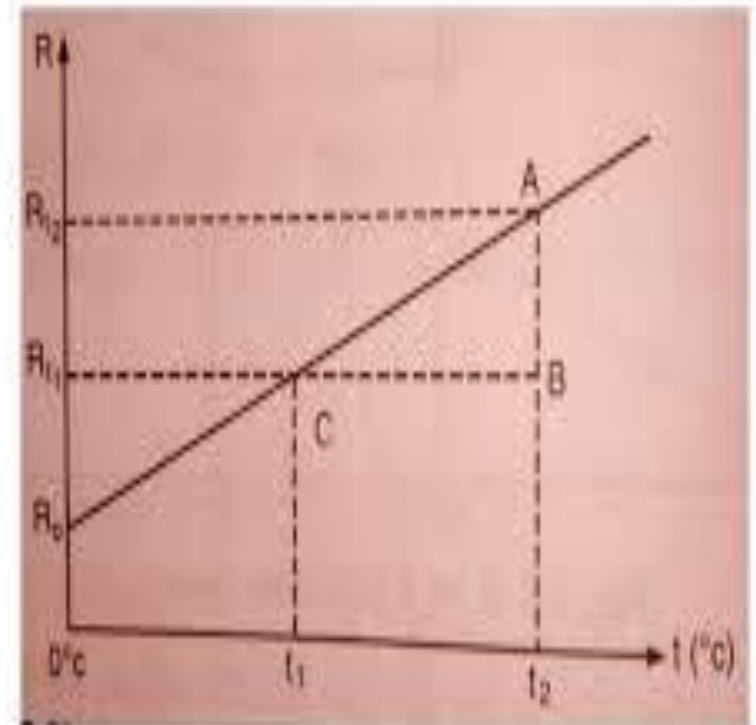
$$\alpha_{t1} * R_{t1} = \frac{R_{t2} - R_{t1}}{t_2 - t_1}$$

$$R_{t2} - R_{t1} = \alpha_{t1} * R_{t1} * (t_2 - t_1)$$

$$R_{t2} = \alpha_{t1} * R_{t1} * (t_2 - t_1) + R_{t1}$$

$$= R_{t1} [1 + \alpha_{t1} * (t_2 - t_1)]$$

Resistance at Different temperature



Temperature Co-efficient at Various Temperatures

- Consider a conductor having resistances R_0 and R_1 at temperatures 0°C and $t_1^\circ\text{C}$ respectively. Let α_0 and α_1 be the temperature co-efficients of resistance of the conductor at 0°C and $t_1^\circ\text{C}$ respectively. It is desired to establish the relationship between α_1 and α_0 .
- As proved:

$$\alpha_0 = \frac{\text{Slope of graph}}{R_0}$$

Similarly,

$$\alpha_1 = \frac{\text{Slope of graph}}{R_1}$$

$$\therefore \text{Slope of graph} = \alpha_0 R_0 \quad \therefore \text{Slope of graph} = \alpha_1 R_1$$

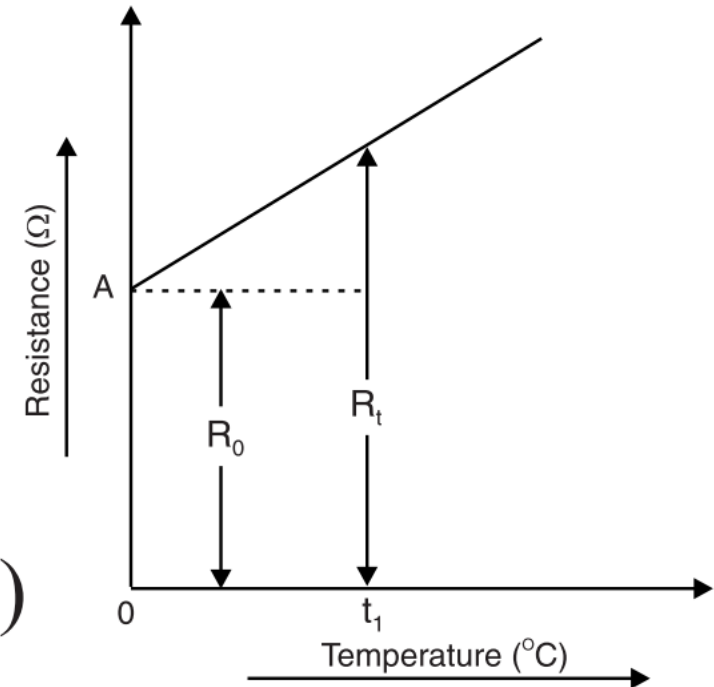
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- Since the slope of temperature/resistance graph is constant,

$$\alpha_0 R_0 = \alpha_1 R_1$$

$$\alpha_1 = \frac{\alpha_0 R_0}{R_1} = \frac{\alpha_0 R_0}{R_0 (1 + \alpha_0 t_1)}$$

$$[\because R_1 = R_0 (1 + \alpha_0 t_1)]$$



$$\therefore \alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \quad \dots(i)$$

Similarly,

$$\alpha_2 = \frac{\alpha_0}{1 + \alpha_0 t_2}$$

...(ii)

Continue....

- Subtracting the reciprocal of eq. (i) from the reciprocal of eq. (ii),

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{1 + \alpha_0 t_2}{\alpha_0} - \frac{1 + \alpha_0 t_1}{\alpha_0} = t_2 - t_1$$

$$\therefore \alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$

...(iii)

- Eq. (i) gives the relation between α_1 and α_0 while Eq. (iii) gives the relation between α_2 and α_1 .

Effect of temperature on Resistance

Metals	Alloys	Insulators, Semi-Conductors & Electrolytes
<ul style="list-style-type: none"> ➤ Resistance of pure metals increases with the rise in temperature. ➤ The increase in resistance is large and regular. ➤ Metals have a positive temperature co-efficient of resistance. ➤ E.g. Copper, Aluminium 	<ul style="list-style-type: none"> ➤ Resistance of alloys increases with the rise in temperature. ➤ But the increase in resistance is irregular and small. ➤ Alloys have a very low value of positive temperature co-efficient of resistance. ➤ E.g. Nichrome 	<ul style="list-style-type: none"> ➤ Resistance of Insulators, Semi-Conductors & Electrolytes decreases with the rise in temperature. ➤ As temperature is increased, many free electrons are created. ➤ So there is a drop in the value of the resistance. ➤ Hence they have a negative temperature co-efficient of resistance. ➤ E.g. Rubber, Oil, Plastic

1. A wire has a resistance of 2 ohms. It has been stretched to the length 3 times that of original, what will be the resistance of wire?

Solution:- We know that

$$R = \frac{\rho l}{a}$$

$$= \frac{\rho \times l}{a} \times \frac{l}{l} \quad (\because \text{Multiply and divide by 'l'})$$

$$= \frac{\rho l^2}{a l}$$

$$= \frac{\rho l^2}{V} \quad [\because V = a l]$$

By stretching the wire, volume remains unchanged

$$\therefore R \propto l^2$$

In other words

$$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1} \right)^2$$

$$\text{Now } R_1 = 2 \, \Omega \quad l_2 = 3 \, l_1$$

$$\therefore R_2 = 2 \left(\frac{3 l_1}{l_1} \right)^2$$

$$= 2 (3)^2$$

$$\boxed{= 18 \, \Omega}$$

2. Calculate the resistance of 100 m length of a wire having a uniform cross section area of 0.1 mm^2 if the wire is made of Manganin having a resistivity of $50 \times 10^{-8} \text{ ohm-m}$. If the wire is drawn out to three times its original length, find out new resistance.

Solution:-

$$l = 100 \text{ m}$$

$$a = 0.1 \text{ mm}^2 = 0.1 \times 10^{-6} \text{ m}^2$$

$$\rho = 50 \times 10^{-8} \text{ } \Omega\text{m}$$

$$(i) \quad R = \frac{\rho l}{a}$$

$$= \frac{50 \times 10^{-8} \times 100}{0.1 \times 10^{-6}}$$

$$\boxed{= 500 \text{ } \Omega}$$

$$\begin{aligned}
 \text{(ii)} \quad R &= \frac{\rho l}{a} \\
 &= R = \frac{\rho l}{a} \times \frac{l}{l} \quad (\text{multiply by } l \text{ to the numerator and denominator}) \\
 &= \frac{\rho l^2}{V} \quad (\because V = a \times l)
 \end{aligned}$$

As wire is drawn its length l and cross section a may vary but the volume remains constant.

$$\therefore R \propto l^2$$

As length is increased 3 times

$$l_2 = 3 l_1$$

$$\therefore \frac{R_2}{R_1} = \left(\frac{l_2}{l_1} \right)^2 = \left(\frac{3}{1} \right)^2$$

$$\therefore \boxed{R_2 = 9 R_1}$$

3. A resistance wire 10 m. long and cross section area 10 mm² at 0 degree Celsius passes, a current of 10 A, when connected to a d.c. supply of 200 Volts. Calculate:

I. Resistivity of the material

II. Current which will flow through the wire when the temp. rises to 50 degree Celsius. Given $\alpha_0 = 0.0003$ per degree Celsius.

Solution:-

$$l = 10 \text{ m}, \quad a = 10 \text{ mm}^2 = 10 \times 10^{-6} \text{ m}^2$$

$$I_0 = 10 \text{ A}, \quad V = 200 \text{ V}, \quad \alpha_0 = 0.0003$$

$$\rho = ? \quad I_{50} = ?$$

$$\text{At } 0^\circ\text{C}, \quad I_0 = \frac{V}{R_0} \quad \therefore 10 = \frac{200}{R_0}$$

$$\therefore R_0 = 20 \, \Omega$$

$$R_0 = \frac{\rho_0 l}{a} \quad \therefore 20 = \frac{\rho_0 \times 10}{10 \times 10^{-6}}$$

$$\therefore \boxed{\rho_0 = 20 \times 10^{-6} \, \Omega \text{ m}}$$

Now resistance will increase with the increase in temperature

$$R_t = R_0 (1 + \alpha_0 t_1)$$

$$R_{50} = 20 (1 + 0.0003 \times 50)$$

$$= 20.3 \, \Omega$$

$$I_{50} = \frac{V}{R_{50}} = \frac{200}{20.3} = 9.85 \text{ A}$$

$$\therefore \boxed{I_{50} = 9.85 \text{ A}}$$

4. A copper coil has a resistance of 12.2 ohm at 28 degree Celsius and 14.4 ohms at 44 degree Celsius, find:-

- I. Temperature co-efficient of resistance at 0 degree Celsius
- II. Resistance of coil at 0 degree Celsius
- III. Temperature co-efficient of resistance at 60 degree Celsius
- IV. Resistance of coil at 75 degree Celsius

Solution:-

$$R_1 = 12.2 \, \Omega$$

$$t_1 = 28^\circ\text{C}$$

$$R_2 = 14.4 \, \Omega$$

$$t_2 = 44^\circ\text{C}$$

$$(i) \quad \frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1}$$

$$\frac{14.4}{12.2} = \frac{1 + \alpha_0 \times 44}{1 + \alpha_0 \times 28}$$

$$1.18 + 33 \alpha_0 = 1 + 44 \alpha_0$$

$$11 \alpha_0 = 0.18$$

$$\alpha_0 = \boxed{0.016 \, ^\circ\text{C}^{-1}}$$

$$(ii) \quad R_t = R_0 (1 + \alpha_0 t_1)$$

$$R_0 = \frac{12.2}{1 + (0.016 \times 28)}$$

$$= \frac{12.2}{1.448}$$

$$R_0 = \boxed{8.425 \, \Omega}$$

$$(iii) \quad \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$$

$$= \frac{0.016}{1 + (0.016 \times 60)}$$

$$\alpha_t = \boxed{0.0082 \, ^\circ\text{C}^{-1}}$$

$$(iv) \quad R_t = R_0 (1 + \alpha_0 t)$$

$$= 8.425 (1 + 0.016 \times 75)$$

$$R_t = \boxed{18.54 \, \Omega}$$

5. A 100 W, 200 V bulb is connected in series with a 100 W, 250 V, bulb across 250 V supply. Calculate (i) Circuit current (ii) Voltage across each lamp assume bulb resistance to remain unchanged.

Solution:- Resistance of bulb 1,

$$\begin{aligned} R_1 &= \frac{V_1^2}{P_1} \\ &= \frac{(200)^2}{100} \\ &= 400 \, \Omega \end{aligned}$$

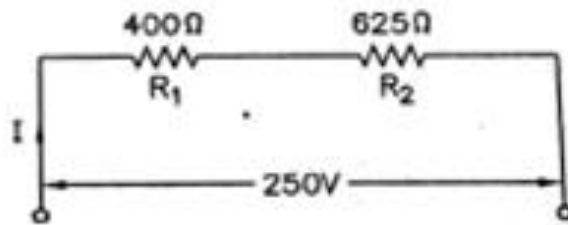


FIG. 1.18

$$\text{Total resistance} = 400 + 625 = 1025 \, \Omega$$

$$\text{Current } I = \frac{V}{R} = \frac{250}{1025} = \boxed{0.244 \, \text{A}}$$

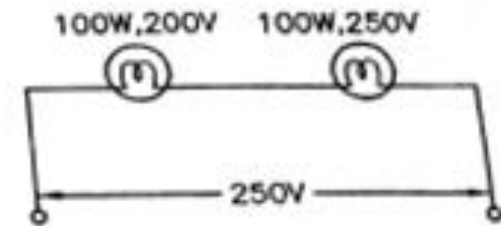


FIG. 1.17

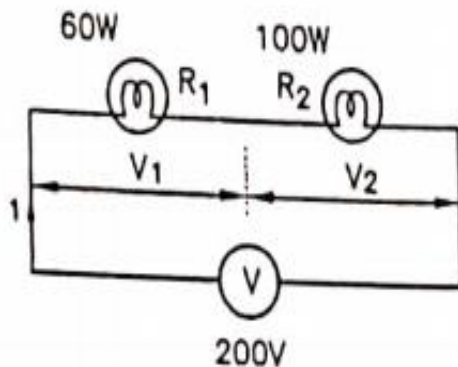
Similarly resistance of bulb 2,

$$\begin{aligned} R_2 &= \frac{V_2^2}{P_2} \\ &= \frac{(250)^2}{100} \\ &= 625 \, \Omega \end{aligned}$$

- Voltage across 200 V bulb:- $IR_1 = 0.244 \times 400 = 97.6 \text{ V}$
- Voltage across 250 V bulb;- $IR_2 = 0.244 \times 625 = 152.5 \text{ V}$

6. Two bulbs rated 250 V, 60 W and 100 w respectively are connected in series across 200 V. Find voltage across each bulb.

Solution:-



Resistance of bulb-1

$$\begin{aligned}
 R_1 &= \frac{V^2}{P} \\
 &= \frac{(250)^2}{60} \\
 &= 1041.66 \, \Omega
 \end{aligned}$$

Resistance of bulb-2

$$\begin{aligned}
 R_2 &= \frac{V^2}{P} \\
 &= \frac{(250)^2}{100} \\
 &= 625 \, \Omega
 \end{aligned}$$

The bulbs are in series, so the current is same. Hence the voltage across each bulb is proportional to its resistance

$$\therefore \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

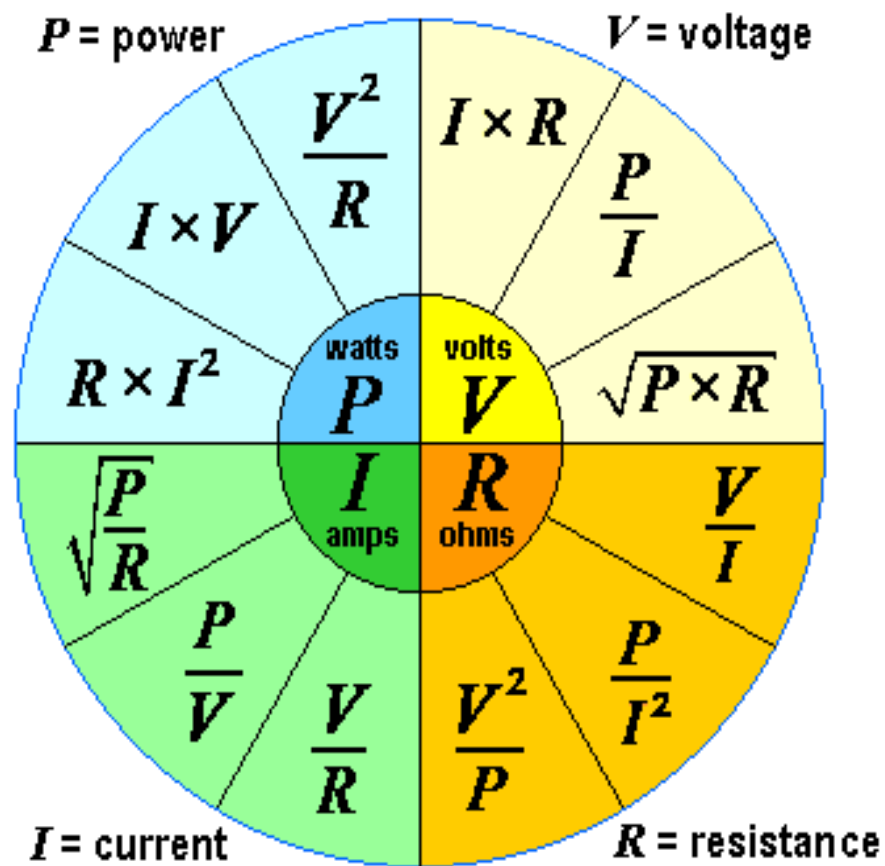
$$\therefore \frac{V_1}{V_1 + V_2} = \frac{R_1}{R_1 + R_2}$$

$$\therefore V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V \quad \left[\because V = V_1 + V_2 \right]$$

$$= \left(\frac{1041.66}{1041.66 + 625} \right) \times 200$$

$$= 125 \text{ V}$$

$$\begin{aligned} \therefore V_2 = V - V_1 &= 200 - 125 \\ &= 75 \text{ V} \end{aligned}$$



Temperature coefficient of resistance

- The change in resistance of a material with change in temperature can be expressed by means of temperature coefficient of resistance.

Let, R_0 = resistance of the conductor at 0°C

R_t = resistance of the conductor at $t^\circ\text{C}$.

It has been found that in the normal range of temperatures, the increase in resistance, $\Delta R = R_t - R_0$ depends

- directly on its initial resistance,
- directly on rise in temperature,
- on the nature of the material of the conductor.

Combining (i) and (ii), we get

$$R_t - R_0 \propto R_0 (t - 0)$$

$$R_t - R_0 \propto R_0 t$$

$$\text{or } R_t - R_0 = \alpha R_0 t$$

$$\therefore \boxed{R_t = R_0 [1 + \alpha_0 t]}$$

Where α is constant and is known as the temperature coefficient of resistance of the material of conductor.

Rearranging the equation.

$$\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\Delta R}{R_0 \times t}$$

... .. 1.11

1.16.1 Definition of Temperature Coefficient of Resistance :

- It is defined as the change in resistance (ΔR) per ohm original resistance per degree change in temperature.
- It is denoted by α

Mathematically,

$$\alpha = \frac{\text{change in resistance}}{\text{original value of resistance} \times \text{change in temperature}} = \frac{\Delta R}{R_0 \times t}$$

Unit :

- From equation (1.11), we have

$$\alpha = \frac{\Delta R}{R_0 \times t} = \frac{\text{ohm}}{\text{ohm} \times ^\circ\text{C}} = \frac{1}{^\circ\text{C}} = ^\circ\text{C}^{-1}$$

- Thus the unit of temperature coefficient of resistance is $1/^\circ\text{C}$ or $^\circ\text{C}^{-1}$ or ohms/ohm/ $^\circ\text{C}$

Computation of α at different temperature:

- Suppose a conductor of resistance R_0 at 0°C is heated to a temperature $t^\circ\text{C}$. Its resistance at $t^\circ\text{C}$ is R_t . Hence

$$\begin{aligned} R_t &= R_0 [1 + \alpha_0 (t - 0)] \\ &= R_0 [1 + \alpha_0 t] \end{aligned} \quad \dots \dots \dots (1)$$

- where, α_0 = temp. coeff. of resistance at 0°C . Now, suppose we have a conductor of resistance R_t at $t^\circ\text{C}$. Let this conductor be cooled down to 0°C . So the resistance at 0°C i.e. R_0 is given by

$$\begin{aligned} R_0 &= R_t [1 + \alpha_t (0 - t)] \\ &= R_t [1 - \alpha_t \times t] \end{aligned} \quad \dots \dots \dots (2)$$

- where, α_t = temperature coefficient of resistance at $t^\circ\text{C}$.
- From equation (2)

$$\begin{aligned} R_0 &= R_t - R_t \times \alpha_t \times t \\ \therefore \alpha_t &= \frac{R_t - R_0}{R_t \times t} \end{aligned} \quad \dots \dots \dots (3)$$

- Substituting the value of R_t from equation (1), we have

$$\begin{aligned} \alpha_t &= \frac{R_0 [1 + \alpha_0 t] - R_0}{R_0 [1 + \alpha_0 t] \times t} \\ &= \frac{R_0 [1 + \alpha_0 t - 1]}{R_0 \times t \times [1 + \alpha_0 t]} \\ &= \frac{\alpha_0 t}{t (1 + \alpha_0 t)} \\ &= \frac{\alpha_0}{1 + \alpha_0 t} \end{aligned}$$

$$\therefore \boxed{\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}}$$

(ii) Relation between α_{t_1} and α_{t_2} :

- Let α_{t_1} and α_{t_2} are temperature coefficient of resistances at t_1 °C and t_2 °C respectively.

Then
$$\alpha_{t_1} = \frac{\alpha_0}{1 + \alpha_0 t_1} \quad \dots \dots \dots (5)$$

$$\alpha_{t_2} = \frac{\alpha_0}{1 + \alpha_0 t_2} \quad \dots \dots \dots (6)$$

From equation (5) from equation (6)

$$\frac{1}{\alpha_{t_2}} - \frac{1}{\alpha_{t_1}} = \frac{1 + \alpha_0 t_2}{\alpha_0} - \frac{1 + \alpha_0 t_1}{\alpha_0}$$

$$= \frac{1 + \alpha_0 t_2 - 1 - \alpha_0 t_1}{\alpha_0}$$

$$= \frac{\alpha_0 (t_2 - t_1)}{\alpha_0}$$

$$= (t_2 - t_1)$$

$$\therefore \frac{1}{\alpha_{t_2}} = \frac{1}{\alpha_{t_1}} + (t_2 - t_1)$$

$$\therefore \boxed{\alpha_{t_2} = \frac{1}{\frac{1}{\alpha_{t_1}} + (t_2 - t_1)}}$$

... .. (7)