Unit IV

4	**Matrix Algebra II**
4.1	Revision of matrices and determinant.
4.2	Eigenvalues and Eigenvectors of matrices
4.3	Eigenvalues and Eigenvector of special matrices
4.4	Cayley-Hamilton's Theorem and its applications.
4.5	Crout's method of LU decomposition

Examples

1.	The sum of the Eigen values of 3×3 matrix is 6 and the product of the Eigen value is also 6. If one of the Eigen values is one, find the other two Eigen values. **Ans.** $2, 3$
2.	If the product of two Eigen values of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16, find the third Eigen value. **Ans.** 2
3.	Find the Eigen values and Eigen vectors of the matrix $$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}.$$ **Ans.** $\lambda = 1,\ X = k\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$; $\lambda = 2, X = k\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$; $\lambda = 3\ , X = k\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ where $k \in \mathbf{R} - \{0\}$
4.	Find the Eigen values and Eigen vectors of the following matrix: $$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$ **Ans.** $\lambda = 1,\ X = k_1\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + k_2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ where $k_1\ , k_2 \in \mathbf{R} - \{0\}$
6.	Find the Eigen values and Eigen vectors of the following matrix: $$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

	Ans. $\lambda = 2,\ X = k\begin{bmatrix}-1\\1\\0\end{bmatrix}$; $\lambda = 3, X = k\begin{bmatrix}1\\0\\0\end{bmatrix}$; $\lambda = 5\ , X = k\begin{bmatrix}3\\2\\1\end{bmatrix}$ where $k \in \mathbf{R} - \{0\}$.
7.	Find the Eigen values and corresponding Eigen vectors of the matrix $$\begin{bmatrix}2 & 1 & 1\\2 & 3 & 2\\3 & 3 & 4\end{bmatrix}.$$ **Ans.** $\lambda = 1{,}1,\ X = k_1\begin{bmatrix}-1\\1\\0\end{bmatrix} + k_2\begin{bmatrix}-1\\0\\1\end{bmatrix}$; $\lambda = 7, X = k_3\begin{bmatrix}1/3\\2/3\\1\end{bmatrix}$ where $k_1\ , k_2, k_3 \in \mathbf{R} - \{0\}$
8.	Find the characteristic roots of the roots of the matrix $A = \begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}$ and verify Cayley Hamilton theorem for this matrix . Find A^{-1} and also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.
9.	Using Cayley Hamilton theorem for evaluate $2A^4 - 5A^3 - 7A + 6I$ for $A = \begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$. **Ans.** $\begin{bmatrix}36 & 32\\32 & 52\end{bmatrix}$
10.	Find the value of $A^5 - 5A^4 + 6A^3 - A^2 + 3I_3$ for a matrix $A = \begin{bmatrix}1 & 1 & 2\\1 & 2 & 1\\1 & 1 & 2\end{bmatrix}$. **Ans.** $\begin{bmatrix}14 & 15 & 15\\15 & 24 & 20\\15 & 20 & 24\end{bmatrix}$
11.	Factorize the matrix $A = \begin{bmatrix}2 & 3 & 1\\1 & 2 & 3\\3 & 1 & 2\end{bmatrix}$ into the LU form. **Ans.** $L = \begin{bmatrix}1 & 0 & 0\\1/2 & 1 & 0\\3/2 & -7 & 1\end{bmatrix}$; $U = \begin{bmatrix}2 & 3 & 1\\0 & 1/2 & 5/2\\0 & 0 & -17\end{bmatrix}$
12.	Factorize the matrix $A = \begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$ into the LU form. **Ans.** $L = \begin{bmatrix}1 & 0 & 0\\-1 & 1 & 0\\1/2 & -1 & 1\end{bmatrix}$; $U = \begin{bmatrix}-2 & 2 & -3\\0 & 3 & -9\\0 & 0 & -15/2\end{bmatrix}$
13.	Solve the following system of linear equations using LU decomposition method. $x + y + z = 1,\ 4x + 3y - z = 6,\ 3x + 5y + 3z = 4.$ **Ans.** $L = \begin{bmatrix}1 & 0 & 0\\4 & -1 & 0\\3 & 2 & -10\end{bmatrix}$; $U = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 5\\0 & 0 & 1\end{bmatrix}$ and $x = 1, y = \frac{1}{2}, z = -\frac{1}{2}$.

14.	Consider the vibration system of a car stereo modeled by the stiffness matrix $S = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}.$ **(a)** Find the eigenvalues of the matrix S. **(b)** Determine the corresponding eigenvectors.