Unit I

1.	**First order and First-degree Ordinary Differential Equations:**
1.1	Formation of Ordinary Differential Equation
1.2	Concept of general and particular solutions
1.3	Initial value problems
1.4	Solutions of first order and first degree differential equations: Linear, Bernoulli, Exact and non-exact differential equations

Examples

1.	Order and Degree Examples with answers (a) $3y = x^{-\frac{3}{2}}\left(\frac{dy}{dx}\right) + \frac{1000}{\frac{dy}{dx}}$ **Ans.**: Order-1; Degree-2 (b) $y = x^{-2}\left(\frac{dy}{dx}\right) + 78\left\{1 + \left(\frac{d^2y}{dx^2}\right)^2\right\}^{\frac{1}{3}}$ **Ans.**: Order-2; Degree-2 (c) $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{5}} = \left(3y + x\frac{dy}{dx}\right)^{\frac{1}{2}}$ **Ans.**: Order-2; Degree-2 (d) $x^2\left(\frac{d^2y}{dx^2}\right) = \left(y - 7x\left(\frac{dy}{dx}\right)^2\right)^{\frac{7}{3}}$ **Ans.**: Order-2; Degree-3 (e) $7\frac{d^5y}{dx^5} + cosx\frac{dy}{dx} = 0$ **Ans.**: Order-5; Degree-1 (f) $\frac{d^2y}{dx^2} = \cos(2x) + \sin(2x)$ **Ans.**: Order-2; Degree-1
2.	Obtain the differential equation of all straight lines passing through the point (a, b). **Ans.** $\mathrm{y - b = (x - a)\left(\frac{dy}{dx}\right)}$
3.	Find the differential equation that describes the family of circles passing through the origin. **Ans.** $\mathrm{(x^2 + y^2)y'' + 2\left(y'^2 + 1\right)(y - xy') = 0}$
4.	Show that $y = A\,lnx + B$is a solution of $x^2\,\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$,where A and B are constants.
5.	Show that $y = sinx$ is the solution of differential equation $\frac{dy}{dx} = y\,cotx$ with condition $y\left(\frac{\pi}{2}\right) = 1$.

6.	Solve the differential equation $sinx\frac{dy}{dx}+2y=\tan^3\left(\frac{x}{2}\right)$. **Ans.** $ytan^2\left(\frac{x}{2}\right)=\frac{\tan^5\left(\frac{x}{2}\right)}{5}+c$
7.	Solve the differential equation. $\frac{dy}{dx}+\frac{y}{x}=x^2y^2$. **Ans**. $\frac{1}{y}=-\frac{x^3}{2}+cx$
8.	Solve the differential equation $x\left[\frac{dy}{dx}+y\right]=1-y$. **Ans.** $y=\frac{1}{x}+\frac{c}{x}e^{-x}$
9.	Solve the differential equation $y\,logy\,dx+(x-logy)dy=0$. **Ans.** $x.logy=\frac{(logy)^2}{2}+c$
10.	Find the value of constant λ such that $(2x\,e^y+3\,y^2)\,\frac{dy}{dx}+\,(3\,x^2+\,\lambda\,e^y)=0$ is exact. Further, for this value of λ solve the equation. **Ans:** λ=2, $x^3+2\,e^x+\,y^3=c$
11.	Solve the differential equation $(2xy\cos(x^2)-2xy+1)\,dx+\,(\sin(x^2)-x^2+3)dy=0$. **Ans:** y $\sin(x^2)-y\,x^2+x+3y=C$
12.	Solve the differential equation $x\,(x-y)\,\frac{dy}{dx}=y\,(x+y)$. **Ans:** $x\,y^2=C$
13.	Solve the differential equation $(2xy^4e^y+2xy^3+y)\,dx+(x^2y^4e^y-x^2y^2-3x)\,dy=0$. **Ans:** $x^2\,e^y+\frac{x^2}{y}+\frac{x}{y^3}=C$
14.	Solve the differential equation $y(1+xy)\,dx+x(1-xy)\,dy=0$. **Ans:** $xy\log\frac{y}{x}=C\,xy-1$
15.	Solve the differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)\,dx+\frac{(x+x\,y^2)}{4}\,dy=0$. **Ans:** $3x^4y+\,x^4y^3+\,x^6=C$
16.	The current $i(t)$ flowing in an R-L circuit is governed by the equation $L\frac{di}{dt}+Ri=E_0sin(\omega t)$, where R is the constant resistance , L is the constant inductance and $E_0sin(\omega t)$ is the voltage at time t, E_0 and ω being constants.Find the current at any time t assuming that initially it is zero. **Ans:** $\boldsymbol{i(t)=\frac{E_0}{R^2+w^2L^2}[R\sin(\omega t)-\omega L\,cos(\omega t)]+\frac{\omega LE_0}{R^2+w^2L^2}e^{-\frac{Rt}{L}}}$