

**Problem B1**

(4) The two rows given can be obtained by applying the following row operations respectively:  $-2A_1 + A_3$  and  $0.5(3A_1 - A_3)$ . Denoting those two rows as  $B_1$  and  $B_2$ ,  $A_2$  can be expressed as  $2B_1 + 3B_2$ . In fact, every subsequent row  $A_n$  can be expressed as  $nB_1 + (n+1)B_2$ .

**Problem B3**

(4) There are no normal magic squares for  $n = 2$ . By the property of magic squares, given  $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$  it must be the case that  $a_1 + a_2 = a_1 + a_3$ . However, this is equivalent to  $a_2 = a_3$ , which violates the constraint that all the elements must be distinct.

$$\begin{aligned}
 1 + \dots + 3^2 &= \frac{2x_1 + 2x_2 - x_3 + 2x_1 - x_2 + 2x_3}{3} \\
 &+ \frac{-x_1 + 2x_2 + 2x_3 - 2x_1 + x_2 + 4x_3 + x_1 + x_2 + x_3 + 4x_1 + x_2 - 2x_3}{3} \\
 &+ x_1 + x_2 + x_3 \\
 45 &= 3x_1 + 3x_2 + 3x_3 \\
 15 &= x_1 + x_2 + x_3
 \end{aligned}$$

**Extra Credit** From the above, we know that the center cell must be 5 and each row and column must add up to 15. Also,  $x_1 \neq x_2$ , otherwise the cells would have repeat values. Now, without loss of generality, let  $x_2 < x_1$ . Up to rotation, we can see that the middle element of each row/column can take on the value 9. By performing case analysis on each of the four locations where 9 could be, we see that all solutions are isomorphic up to rotation and reflection.