Numerical Simulation of Laminar Flames

by

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February 6, 2015

A thesis submitted to the
Faculty of the Graduate School of the
State University of New York at Buffalo
in partial fulfillment of the requirements for the degree of

Master of Science

Department of Mechanical and Aerospace Department

Acknowledgements

These are my acknowledgements

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This is my abstract

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2.1.1 Problem definition

The The flow of a compressible fluid is described in terms of the velocity (u^*), pressure (p^*), density (ρ^*), and temperature(T^*) fields. These fields are solutions of the compressible Navier Stokes equations that describe the dynamics of the system and that are statements of conservation of mass, momentum and energy and a state equation relating the thermodynamic variables.

The system of equation that needs to be solved reads

$$\begin{aligned} \frac{D\rho^*}{Dt^*} + \rho^* \, \nabla.u^* &= 0 && (2.1) \\ \rho^* \frac{Du^*}{Dt} &= -\nabla p^* + \nabla.\tau^* + \rho^* g^* && (2.2) \\ \rho^* C_p^* \frac{DT^*}{Dt^*} - \beta^* T^* \frac{Dp^*}{Dt^*} &= \tau^* \nabla.u^* - \nabla.q^* && (2.3) \\ & && (2.4) \end{aligned}$$

Where u^* is the velocity vector,ρ^* is the density,β^* is the thermal expansion coefficient,p^* is the pressure ,τ^* is the viscous stress term, g^* is the gravitational vector,c_p^* is the specific heat, T^* is the temperature, q^* is the heat flux vector.

for Newtonian fluid,

$$\tau^* = \mu^*(\nabla u^* + (\nabla u^*)^T)) - \frac{2}{3}\mu^* \nabla.u^* I$$

By fourier law's,

$$q^* = k^* \nabla T^*$$

μ^* is the dynamic viscocity. k^* is the thermal conductivity.

2.1.2 Low mach number asymtotic analysis

We nondimensionalize the Equations by using reference quantities denoted by the subscript ∞, e.g. farfield or stagnation conditions, and a typical length scale L^* of the considered flow. The thermodynamic reference quantities are assumed to be related by the equation of state for a perfect gas. We define the nondimensional quantities by:

$\rho = \frac{\rho^*}{\rho_\infty}$, $p = \frac{p^*}{p_\infty}$, $u = \frac{u^*}{u_\infty}$, $T = \frac{T^*}{T_\infty}$, $\mu = \frac{\mu^*}{\mu_\infty}$, $k = \frac{k^*}{k_\infty}$,$x = \frac{x^*}{L^*}$, $t = \frac{t^*}{L^*/u^*_\infty}$, $\beta^* = \frac{\beta}{\beta_{infty}}$,

$C_p^* = \frac{C_p}{C_{p_\infty}^*}$.

Using the relations above. we may write the nondimensional Navier-Stokes equations and other equations of interest as follows:

$$\frac{D\rho}{Dt} + \rho\,\nabla.u = 0 \tag{2.5}$$

$$\rho\frac{Du}{Dt} = -\frac{1}{M^2}\,\nabla p + \frac{1}{Re}\nabla.\tau + \frac{1}{Fr^2}\rho g \tag{2.6}$$

$$\rho C_p\frac{DT}{Dt} - \frac{\beta T}{\lambda}\frac{Dp}{Dt} = \frac{M^2}{Re\lambda}\,\tau\nabla.u - \frac{1}{RePr}\,\nabla.(k\nabla T) \tag{2.7}$$

$$\tag{2.8}$$

Where the following nondimensional quantities are

$$M = \frac{u_\infty}{a_\infty}$$

. a_∞ is the reference speed of sound.

$$Re = \frac{u_\infty \rho_\infty L}{\mu_\infty}$$

.

$$Pr = \frac{C_{p_\infty}\mu_\infty L}{k_\infty}$$

$$Fr = \sqrt{\frac{u_\infty^2}{g_\infty L}}$$

.

Where M is the mach number,Re is the reynolds number,Pr is the prandtl number,Fr is the froude number and λ is defined as $\frac{C_{p_\infty}T_\infty}{a_\infty^2}$.

By definition

$$\beta^* = -\frac{1}{\rho^*}\frac{\partial \rho^*}{\partial T^*}$$

. We non dimensional this equation

$$\beta^* = -\frac{1}{\rho_\infty \rho}\frac{\rho_\infty}{T_\infty}\frac{\partial \rho}{\partial T}$$

$$\beta^* = \frac{1}{T_\infty}\beta$$

Where $\beta_\infty = \frac{1}{T_\infty}$.

By definition

$$a^{*2} = \frac{\partial p^*}{\partial \rho^*}$$

. We non dimensional this equation

$$a^{*2} = \frac{p_\infty}{\rho_\infty}\frac{\partial p}{\partial \rho}$$

$$a^{*2} = \frac{p_\infty}{\rho_\infty}a^2$$

Where $a_\infty = \sqrt{\frac{p_\infty}{\rho_\infty}}$.

2.1.3 Asymptotic analysis

We do asymptotic analysis with respect to Mach number. We use the following equation in terms of Mach number

$$\xi(x,t,M) = \xi^0(x,t) + M\xi^1(x,t) + M^2\xi^2(x,t) + O(M^3)$$

Where ξ can be u, p, ρ. We use non dimensional form of navier stokes equation do the asymptotic equation. We show the example of asymptotic analysis for mass conservation.

$$\frac{D}{Dt}(\rho^0 + M\rho^1 + M^2\rho^2 + O(M^3)) \; + \; (\rho^0 + M\rho^1 + M^2\rho^2 + O(M^3)) \; \nabla.(u^0 + Mu^1 + M^2u^2 + O(M^3)) = 0$$

Collecting equal order terms together we get,

$$\left(\frac{D\rho^0}{Dt} \; + \; \rho^0 \; \nabla.u^0\right) + M\left(\frac{\partial\rho^1}{\partial t} + + \rho^1 \; \nabla.u^0 + + \rho^0 \; \nabla.u^1\right) + O(M^2) = 0$$

. Therefore zeroth order terms can be written.

$$\frac{D\rho^0}{Dt} \; + \; \rho^0 \; \nabla.u^0 \quad = \quad 0 \tag{2.9}$$

The asymptotic relation for momentum are obtained are as follows. The zero, first and second order momentum equation are written as follows respectively

$$M^{-2}\nabla p^{(0)} \quad = \quad 0 \tag{2.10}$$
$$M^{-1}\nabla p^{(1)} \quad = \quad 0 \tag{2.11}$$
$$\rho^0\frac{Du^0}{Dt} \quad = \quad -\nabla p^2 \; + \; \nabla.\tau^0 + \rho^0 g \tag{2.12}$$

The asymptotic relation for momentum are obtained are as follows.

$$\rho^0 C_p^0 \frac{DT^0}{Dt} \; - \; \frac{\beta^0 T^0}{\lambda}\frac{dp^0}{dt} \quad = \quad -\frac{1}{RePr}\; \nabla.(k^0 \nabla T^0) \tag{2.13}$$

We have seen from momentum asymptotic analysis that $p^0 = p^0(t)$ and therefore

$$\frac{Dp^0}{Dt} = \frac{dp^0}{dt}$$

.

The Equation of state is given by the following equation

$$\frac{D\rho^*}{Dt^*} \quad = \quad -\beta^* T^* \frac{D\rho^*}{Dt^*} + \alpha^* \rho^* \; \frac{Dp^*}{Dt^*} \tag{2.14}$$

Where $\beta^* = -\frac{1}{\rho^*}\frac{\partial\rho^*}{\partial T^*}$ and $\alpha^* = \frac{1}{\rho^*}\frac{\partial\rho^*}{\partial p^*}$. We nondimensionalise these quantities. As before $\beta_\infty = \frac{1}{T_\infty}$ and $\alpha = \frac{1}{p_\infty}\alpha$. Therefore $\alpha_i nfty = \frac{1}{p_\infty}$

We non dimensionalise the equation of state by using previous relationships and get the following non dimensional equation

$$\frac{D\rho}{Dt} \quad = \quad -\beta T \frac{D\rho}{Dt} + \alpha\rho \; \frac{Dp}{Dt} \tag{2.15}$$

Thus the nondimensional equation of state is same as that of the dimensional form. Now same asymptotic expansion of the state equation is done in order to get zeroth order state equation which is given below.

$$\frac{D\rho^0}{Dt} = -\beta^0 T^0 \frac{D\rho^0}{Dt} + \alpha^0 \rho^0 \frac{Dp^0}{Dt} \tag{2.16}$$

From mass conservation equation and since $p^0 = p^0(t)$, we get the following equation

$$-\nabla.u^0 = -\beta^0 \frac{D\rho^0}{Dt} + \alpha^0 \frac{dp^0}{dt} \tag{2.17}$$

From energy conservation equation

$$-\rho^0 C_p^0 \nabla.u^0 = \frac{1}{RePr}\nabla.q^0 + \left(\frac{C_p^0\lambda - 1}{\lambda}\right) \frac{dp^0}{dt} \tag{2.18}$$

The final equation obtained by applying various thermodynamic properties are following

$$-\nabla.u + \frac{1}{T}\frac{DT}{Dt} = \frac{1}{p^0}\frac{dp^0}{dt} \tag{2.19}$$

$$\rho\frac{Du}{Dt} = -\frac{1}{M^2}\nabla p + \frac{1}{Re}\nabla.\tau + \frac{1}{Fr^2}\rho g \tag{2.20}$$

$$\rho C_p\frac{DT}{Dt} - \frac{1}{\gamma - 1}\frac{dp^0}{dt} = -\frac{1}{RePr}\nabla.(k\nabla T) \tag{2.21}$$

$$\tag{2.22}$$