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CHAPTER-11 - Application of Derivatives

Exercise-11A

Q1.

Let side of the square be ‘a’

Then rate of change of side= $\frac{da}{dt} = 0.2 \text{ cm/s}$

Perimeter of the square will be 4a

Then rate of change of perimeter = $4\frac{da}{dt} = 4 \times 0.2$

$$\frac{dP}{dt} = 0.8 \text{ cm/s}$$

Q2.

Let the radius of the circle be r

Circumference of the circle=$2\pi r$

Also given that $\frac{dr}{dt} = 0.7\text{cm/s}$

Then rate of change of circumference $= 2\pi\frac{dr}{dt}$

$= 2 \times 3.14 \times 0.7$

So $\frac{dC}{dt} = 4.4 \text{ cm/s}$

Q3.

Let the radius of the circle be r

Area of the circle=πr^2

Also given that $\frac{dr}{dt} = 0.3 cm/s$

Then rate of change of Area $= 2\pi r\frac{dr}{dt}$

$= 2 \times 3.14 \times 10 \times 0.3$

$\frac{dA}{dt} = 18.84 \ cm^2/s$

Q4.

Let the side of the square be a

Area of the square = a^2

Also given that rate of change of side is $\frac{da}{dt} = 3\ cm/s$

Hence rate of change of Area = $2a\frac{da}{dt} = 2 \times 10 \times 3$

$\frac{dA}{dt} = 60 \ cm^2/s$

Q5.

Soap bubble will have the shape of a sphere

Let the radius of the soap bubble be r

Surface area of the soap bubble=$4\pi r^2$

$\frac{dr}{dt} = 0.2cm/s$

Then rate of change of Surface area $= 8\pi r \frac{dr}{dt}$

$= 8 \times 3.14 \times 7 \times 0.2$

$\frac{dS}{dt} = 35.2 \text{ cm}^2/s$

Q6.

Soap bubble will have the shape of a sphere

Let the radius of the soap bubble be 'r'

Volume of the soap bubble=$\frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 0.5cm/s$

Then rate of change of Volume $= 4\pi r^2 \frac{dr}{dt}$

$= 4 \times 3.14 \times 1^2 \times 0.5$

$\frac{dV}{dt} = 6.28 \text{ cm}^3/s$

Q7.

Let the radius of the balloon be' r'

Surface area of the bubble =$4\pi r^2$

Then volume of the spherical balloon will be

$$V = \frac{4}{3}\pi r^3$$

Differentiating it with respect to t we get

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$25\ cm^3/s = 4 \times \pi \times 5^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi}$$

Then rate of change of Surface area $= 8\pi r \frac{dr}{dt}$

$$= 8 \times \pi \times 5 \times \frac{1}{4\pi}$$

$$\frac{dS}{dt} = 10\ cm^2/s$$

Q8.

When we pump air in a balloon its volume changes.

Let the radius of the balloon at any arbitrary time 't' be 'r'

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$900\ cm^3/s = 4 \times \pi \times 15^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{900}{4 \times 3.14 \times 225}$$

$$\frac{dr}{dt} = 0.32\ cm/s$$

Q9.

Let the volume of the water tank be ‘V’

Then volume of the tank can be expressed as

$$V = l \times b \times h$$

$$V = 25 \times 40 \times h$$

$$\frac{dV}{dt} = 1000 \times \frac{dh}{dt}$$

$$500 = 1000 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.5 \text{ m/min}$$

Q10.

Let the radius of the circle be ‘r’

Then it is given that $\frac{dr}{dt} = 3.5 \text{ cm/s}$

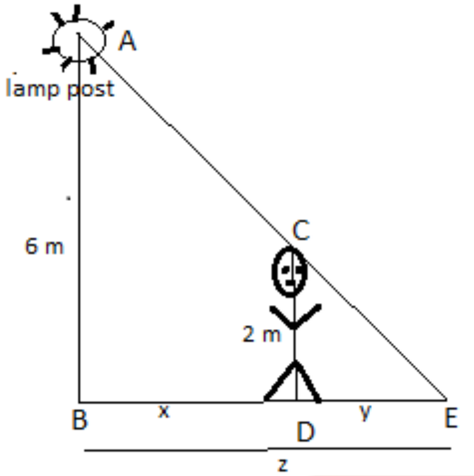
We know that area of the circle$=\pi r^2$

Then rate of change of Area $= 2\pi r \frac{dr}{dt}$

$$= 2 \times 3.14 \times 7.5 \times 3.5$$

$$= 165 \text{ cm}^2/\text{s}$$

Q11.



ABE and CDE are similar triangles.

So,

$$\frac{AB}{BE}=\frac{CD}{DE}$$

$$\frac{0.006}{x+y}=\frac{0.002}{y}$$

$$6y=2(x+y)$$

$$6\frac{dy}{dt}=2(\frac{dx}{dt}+\frac{dy}{dt})$$

$$6\frac{dy}{dt}=2(5+\frac{dy}{dt})$$

$$6\frac{dy}{dt}=10+2\frac{dy}{dt}$$

$$4\frac{dy}{dt}=10$$

$$\frac{dy}{dt}=2.5km/h$$

Q12.

Let the volume of the cone be ‘V’

Then it is given that $\frac{dV}{dt} = 1.5\ cm^3/s$

Also we know that volume of the cone can be expressed as $V = \frac{1}{3}\pi r^2 h$

So, $V = \frac{1}{3}\pi 5^2 h$

$$V = \frac{25}{3}\pi h$$

$$\frac{dV}{dt} = \frac{25}{3}\pi \frac{dh}{dt}$$

$$\frac{15}{10} = \frac{25}{3}\pi \frac{dh}{dt}$$

Q13.

It is given that $h = \frac{1}{6}r$

Also we know that volume of cone is $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi(6h)^2 h$$

$$V = 12\pi h^3$$

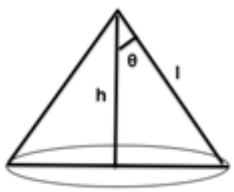
Differentiating w.r.t. t

$$\frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$18 = 36 \times 9 \times \pi \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{18\pi} cm/s$$

Q14.



Let the volume of the cone be ‘V’

It is given that $\frac{dV}{dt} = 4cm^3/s$

We know that $V = \frac{1}{3}\pi r^2 h$

$$\cos Q = \frac{h}{l} = \cos 120 = \cos(180 - 60) = -\frac{1}{2}$$

$$\sin Q = \frac{r}{l} = \sin 120 = \sin(180 - 60) = \sin 60 = \frac{\sqrt{3}}{2}$$

Also $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi\left(\frac{\sqrt{3}}{2}l\right)^2 (-\frac{1}{2}l)$$

$$V = -\frac{3}{24}\pi l^3$$

$$\frac{dV}{dt} = -\frac{9}{24}\pi l^2 \frac{dl}{dt}$$

$$4 = -\frac{3}{8}\pi 3^2 \frac{dl}{dt}$$

$$-\frac{32}{27\pi}\text{ cm/s} = \frac{dl}{dt}$$

Q15. It is given that $\frac{dV}{dt} = 15\text{ mL/s}$

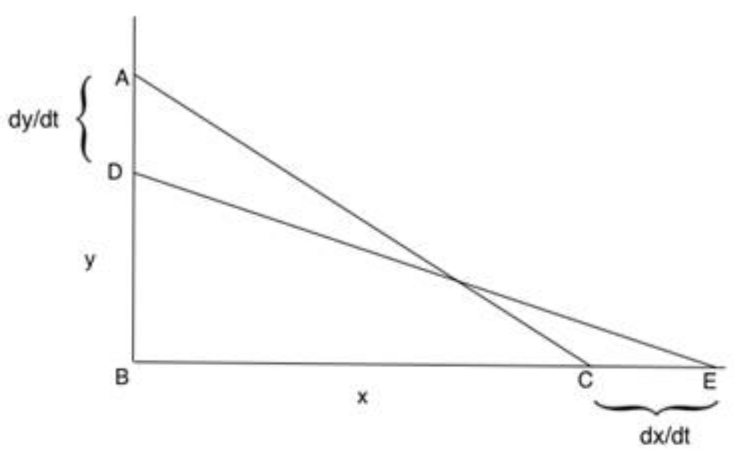
$$\frac{d}{dt}(\pi r^2 h) = 15$$

$$\frac{d}{dt}(\pi 7^2 h) = 15$$

$$49\pi\frac{dh}{dt} = 15$$

$$\frac{dh}{dt} = \frac{15}{49\pi}$$

Q16.



Let initially the ladder be at C and after applying force let it move to E

Let us assume that AB=y and BC=x

Using Pythagoras Theorem in ABC

$x^2 + y^2 = 13^2 \qquad \ldots(1)$

$5^2 + y^2 = 13^2$

$y = 12\text{cm}$

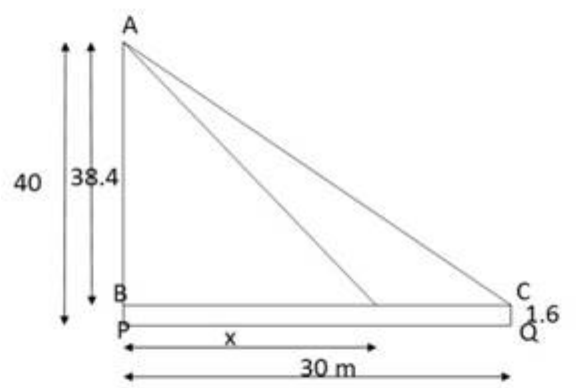
Differentiating both sides of (1) w.r.t. to t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$5.2 + 12\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{10}{12} = -\frac{5}{6}\text{cm/s}$$

Q17.



It is given in the question that $\frac{dx}{dt} = -2$ cm/s

Also in smaller right angled triangle $\tan Q = \frac{38.4}{x}$

$$Q = \tan^{-1}\frac{38.4}{x}$$

$$\frac{dQ}{dt} = \frac{1}{1+\frac{38.4^2}{x^2}}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt}=\frac{x^2}{x^2+1474.56}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt}=-\frac{1}{30^2+1474.56}.38.4.\frac{dx}{dt}$$

$$\frac{dQ}{dt}=-\frac{1}{30^2+1474.56}.38.4\times 2$$

$$\frac{dQ}{dt}=-0.032\ \text{radian/second}$$

Q18.

According to question

$$\frac{dx}{dt}=2\frac{d}{dt}(\sin x)$$

$$\frac{dx}{dt}=2\cos x\frac{dx}{dt}$$

$$\cos x=\frac{1}{2}$$

$$x=\frac{\pi}{3}$$

Q19.

It is given in the question that $\frac{dr}{dt}=10\ \text{m/s}$

We also know that $S=4\pi r^2$

Differentiating both sides by t

$$\frac{dS}{dt}=8\pi r\frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi.15.10$$

$$\frac{dS}{dt} = 1200\pi \text{ cm}^2/\text{s}$$

Q20.

It is given that $\frac{da}{dt} = 5\text{cm/s}$

We also know that Volume of a cube can be expressed as $V = a^3$

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}$$

$$\frac{dV}{dt} = 3.10^2.5$$

$$\frac{dV}{dt} = 1500 \text{ cm}^3/\text{s}$$

Q21.

It is given in the question that $\frac{da}{dt} = 2\text{cm/s}$

Also we know that area of an equilateral triangle can be expressed as $A = \frac{\sqrt{3}}{4}a^2$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}2a\frac{da}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}.10.2$$

$$\frac{dA}{dt} = 10\sqrt{3} \text{ cm}^2/\text{s}$$

Exercise-11B

Q1.

Consider $y = \sqrt{x}$.

Also it is given in question that $x = 36$ and $\Delta x = 1$.

As $y = \sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{36}}.1$

$\Rightarrow \Delta y = \frac{1}{12}$

$\therefore \Delta y = 0.08$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.08 = \sqrt{36+1} - \sqrt{36}$

$\Rightarrow 0.08 = \sqrt{37} - 6$

$\Rightarrow \sqrt{37} = 6.08$

Q2.

Consider that $y = \sqrt[3]{x}$.

Also it is given in the question that x =27 and $\Delta x = 2$.

As $y = \sqrt[3]{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{3}x^{\frac{-2}{3}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{3}27^{\frac{-2}{3}}.2$

$\Rightarrow \Delta y = \frac{2}{27}$

$\therefore \Delta y = 0.074$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.074 = \sqrt[3]{27+2} - \sqrt[3]{27}$

$\Rightarrow 0.074 = \sqrt[3]{29} - 3$

$\Rightarrow \sqrt[3]{29} = 3.074$

Q3.

Consider $y = \sqrt{x}$.

Also it is given in the question that x =25 and $\Delta x = 2$.

As $y = \sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{25}}.2$

$\Rightarrow \Delta y = \frac{1}{5}$

$\therefore \Delta y = 0.2$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.2 = \sqrt{25+2} - \sqrt{25}$

$\Rightarrow 0.2 = \sqrt{27} - 5$

$\Rightarrow \sqrt{27} = 5.2$

Q4.

Consider that $y = \sqrt{x}$.

Also it is given in the question that $x = 0.25$ and $\Delta x = -0.01$.

As $y = \sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{0.25}}.(-0.01)$

$\Rightarrow \Delta y = -0.01$

$\therefore \Delta y = -0.01$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore -0.01 = \sqrt{0.25-0.01} - \sqrt{0.25}$

$\Rightarrow -0.01 = \sqrt{0.24} - 0.5$

$\Rightarrow \sqrt{0.24} = 0.49$

Q5.

Considering $y = \sqrt{x}$.

Also it is given in the question that $x = 19$ and $\Delta x = 0.5$.

As $y = \sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{49}}.0.5$

$\Rightarrow \Delta y = \frac{0.5}{14}$

$\therefore \Delta y = 0.0357$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore\ 0.0357 = \sqrt{49+0.5} - \sqrt{49}$

$\Rightarrow 0.0357 = \sqrt{49.5} - 7$

$\Rightarrow \sqrt{49.5} = 7.0357.$

Q6.

Considering $y = \sqrt[4]{x}$.

Also it is given in the question that $x = 16$ and $\Delta x = 1$.

As $y = \sqrt[4]{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore\ \Delta y = \frac{1}{4}x^{\frac{-3}{4}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{4}16^{\frac{-3}{4}}.(-1)$

$\Rightarrow \Delta y = \frac{-1}{32}$

$\therefore\ \Delta y = -0.03125$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore\ -0.03125 = \sqrt[4]{16-1} - \sqrt[4]{16}$

$\Rightarrow -0.03125 = \sqrt[4]{15} - 2$

$\Rightarrow \sqrt[4]{15} = 1.96875$

Q7.

Considering $y = \frac{1}{x^2}$

Also it is given in the question that x =2 and $\Delta x = 0.002$.

As $y = \frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{x^3}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{-2}{x^3}.\Delta x$

$\Rightarrow \Delta y = \frac{-2}{8}.(0.002)$

$\Rightarrow \Delta y = \frac{-0.5}{1000}$

$\therefore \Delta y = -0.0005$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore -0.0005 = \frac{1}{(2.002)^2} - \frac{1}{2^2}$

$\Rightarrow -0.0005 = \frac{1}{(2.002)^2} - 0.25$

$\Rightarrow \frac{1}{(2.002)^2} = 0.2495$

Q8.

Considering $y = \log_e x$

Also it is given in the question that x =10 and $\Delta x = 0.02$.

As $y = \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{x}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{10}.(0.02)$

$\Rightarrow \Delta y = \frac{0.02}{10}$

$\therefore \Delta y = 0.002$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.002 = \log_e(10+0.02) - \log_e(10)$

$\Rightarrow 0.002 = \log_e(10.02) - 2.3026$

$\Rightarrow \log_e(10.02) = 2.3046.$

Q9.

Considering $y = \log_{10} x$

$\therefore y = \frac{\log_e x}{\log_e 10}$

$\therefore y = 0.4343 \log_e x$

Also it is given in the question that x =4 and Δx = 0.04.

As $y = 0.4343 \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{0.4343}{x}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{0.4343}{x} . \Delta x$

$\Rightarrow \Delta y = \frac{0.4343}{4} . (0.04)$

$\Rightarrow \Delta y = \frac{0.017372}{4}$

$\therefore \Delta y = 0.004343$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.004343 = \log_e(4+0.04) - \log_e(4)$

$\Rightarrow 0.004343 = \log_e(4.04) - 0.6021$

$\Rightarrow \log_e(4.04) = 0.606443.$

Q10.

Considering y = cosx

Also it is given in the question that x =60° and Δx = 1° .

As y =cos x

$\Rightarrow \frac{dy}{dx} = \sin x$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \sin x . \Delta x$

$\Rightarrow \Delta y = \sin(60)(0.01745)$

$\Rightarrow \Delta y = (0.86603)(0.01745$

$\therefore \Delta y = 0.01511$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.01511 = \cos(60+1) - \cos(60)$

$\Rightarrow 0.01511 = \cos 61° - 0.5$

$\Rightarrow \cos 61° = 0.48489$

Q11.

It is given in the question that 'x' is π /2

Value of 'π' is 22/7

22/14 is π /2

So, there will be no change.

Q12.

Let the radius of the plate be 10cm.

∴ Change in radius= $\frac{2}{100} \times 10 = 0.2$

So dr = 0.2

∴ New radius = 10+0.2 = 10.2cm

Area of circular plate = $A=\pi r^2$

∴ Change in Area = $\frac{dA}{dr}$

$\Rightarrow \frac{dA}{dr} = 2\pi r dr$

$\Rightarrow \frac{dA}{dr} = 2 \times \pi \times 10 \times 0.2$

$\Rightarrow \frac{dA}{dr} = 4\pi \text{ cm}^2$

Q13.

Time period 'T' is given with formula

∴ $T = 2\pi\sqrt{\frac{l}{g}}$

Since 2,π,g have no dimensions. So we can eliminate them.

Now $\frac{\Delta T}{T} = \frac{1}{2} \times \frac{\Delta L}{L}$

By representing in percentage error

$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$

$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$

$\Rightarrow \frac{\Delta T}{T}\% = \frac{1}{2} \times 2\%$

$\Rightarrow \frac{\Delta T}{T}\% = 1\%$

The time period becomes 1 %.

Q14.

Given: $pv^{1/4} = k$

According to question if 'V' ∴ $\frac{\Delta V}{V} \times 100 = \frac{-1}{2}$

$pv^{1/4} = k$

taking log on both sides

$\log[pv^{1/4}] = \log a$

$\log P + 1.4\log V = \log a$

Differentiating both the sides we get

$\Rightarrow \frac{1}{P}dP + \frac{1.4}{V}dV = 0$

$\Rightarrow \frac{dP}{P} + 1.4\frac{dV}{V} = 0$

Multiplying both sides by 100.

$\Rightarrow \frac{dP}{P} \times 100 + 1.4 \times \frac{dV}{V} \times 100 = 0$

$\Rightarrow \frac{dP}{P} \times 100 + 1.4\left(\frac{-1}{2}\right) = 0$

$\Rightarrow \frac{dP}{P} \times 100 = 0.7$

%error in P = 0.7%.

Q15.

Change in radius = dr = 10-9.8

$\therefore$ dr = 0.2

Volume of the sphere is given by = $V = \frac{4}{3}\pi r^3$

Change in volume = $dV = 4\pi r^2 dr$

$\therefore dV = 4\pi(10)^2 \times 0.2$

$\Rightarrow dV = 80\pi\ cm^3$

Surface area of the spherecan be represented as $A = 4\pi r^2$

Change in volume = $dA = 8\pi r dr$

$\therefore$ dA = 8π×10× 0.2

$\therefore$ dA = 16π.

Q16.

Volume of the sphere can be given by = $V = \frac{4}{3}\pi r^2$

So change in volume can be given by $dV = 4\pi r^2 dr$

It is given in the question that : Δr = 0.1

$\Rightarrow \Delta r.\frac{dV}{dr} = 4\pi r^2 \Delta r$

$\Rightarrow \Delta V = 4\pi r^2 \Delta r$

Percentage error

$\Rightarrow \frac{\Delta V}{V} = \frac{4\pi r^2}{\frac{4\pi r^3}{3}} \times 0.1$

= 0.3%

Q17.

Let ‘d’ be the diameter, ‘r’ be the radius and ‘V’ be the volume of the sphere

Volume of the sphere is given by = $V = \frac{4}{3}\pi r^2$

$\Rightarrow V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi D^3$

Let Δd be the error in d and the corresponding error in V be ΔV.

$\therefore \Delta V = \frac{dV}{dd}\Delta d = \frac{1}{2}\pi d^2 \Delta D$

$\therefore \frac{\Delta V}{V} = \frac{\frac{1}{2}\pi d^2 \Delta D}{\frac{1}{6}\pi D^3} = 3\frac{\Delta D}{D}$

Exercise-11C

Rolle’s Theorem States that if a function f is continuous on [a, b] and differentiable on (a, b) such that f(a) = f(b), then

There exists some ‘x’ in [a, b] such that f’(x) = 0

Hence the three required conditions of rolle’s theorem are

i. The function f should be continuous on [a, b]

ii. The function f should be differentiable on (a, b)

iii. Also the function f should be such that f(a) = f(b)

If the above three conditions are verified then Rolle’s theorem is applicable

Q. 1.

Given that f(x) = x^2

We know that f(x)=x^2 is a polynomial function and every polynomial function is continuous at every x ϵ R.

So f(x)=x^2 is continuous on [-1,1].

Hence condition (i) is satisfied

To check differentiability we differentiate the given function

So f'(x)=2x which exists in [-1,1].

So, f(x)=x^2 is differentiable on (-1,1).

Hence condition (ii) is satisfied

Also to check third condition f(-1)=$(-1)^2$=1 and f(1)=1^1=1

i.e. f(-1)=f(1)

Hence the three given conditions for rolle's theorem are satisfied.

So there exist at least one cϵ(-1,1) such that f'(c)=0

i.e. 2c=0

i.e. c=0

Value of c=0 ϵ (-1,1)

Thus, Rolle's theorem is satisfied

Q2.

We know $f(x)=x^2-x-12$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

So, $f(x)= x^2-x-12$ is continuous on $[-3,4]$.

To check differentiability we differentiate the given function

$f'(x)=2x-1$ which exist in $[-3,4]$.

So, $f(x)= x^2-x-12$ is differentiable on $(-3,4)$.

Hence conditions (i) and (ii) are satisfied

Also $f(-3)=(-3)^2-3-12=0$ and $f(4)=4^2-4-12=0$

i.e. $f(-3)=f(4)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (-3,4)$ such that $f'(c)=0$

i.e. $2c-1=0$

i.e. $c=\frac{1}{2}$

Value of $c=\frac{1}{2} \epsilon (-3,4)$

Rolle's theorem is satisfied.

Q3.

We know that $f(x)=x^2-5x+6$ is a polynomial function and every polynomial function is continuous for all $x \epsilon R$.

So $f(x)= x^2-5x+6$ is continuous on $[2,3]$.

To check differentiability we differentiate the given function

Here, f'(x)=2x-5 which exist in [2,3].

So, $f(x)= x^2-5x+6$ is differentiable on (2,3).

Here, $f(2)=2^2-5\times2+6=0$

And $f(3)= 3^2-5\times3+6=0$

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(2,3)$ such that f'(c)=0

i.e. 2c-5=0

i.e. $c=\frac{5}{2}$

Value of $c=\frac{5}{2}\epsilon(2,3)$

Rolle's theorem is satisfied.

Q4.

$f(x)= x^2-5x+6$ is a polynomial function and every polynomial function is continuous for all $x \epsilon R$.

$\Rightarrow f(x)= x^2-5x+6$ is continuous on [-3,6].

To check differentiability, we differentiate the given function to get f'(x)=2x-5 which exist in [-3,6].

So, $f(x)= x^2-5x+6$ is differentiable on (-3,6).

Also $f(-3)=(-3)^2-5\times(-3)+6=30$ and $f(6)= 6^2-5\times6+6=12$

i.e. $f(-3) \neq f(6)$

SO this means that last condition is not fulfilled

Hence Rolle's theorem is not applicable.

Q5.

$f(x)=x^2-4x+3$ is a polynomial function and every polynomial function is continuous for all $x \in R$.

So, $f(x)=x^2-4x+3$ is continuous on $[1,3]$.

To check differentiability we differentiate the given function to get $f'(x)=2x-4$ which exist in $[1,3]$.

So, $f(x)=x^2-4x+3$ is differentiable on $(1,3)$.

Also $f(1)=(1)^2-4(1)+3=0$ and $f(3)= (3)^2-4(3)+3=0$

i.e. $f(1)=f(3)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist atleast one $c \in (1,3)$ such that $f'(c)=0$

i.e. $2c-4=0$

i.e. $c=2$

Value of $c=2 \in (1,3)$

Rolle's theorem is satisfied.

Q6.

$f(x)=x(x-4)^2$ is a polynomial function and every polynomial function is continuous for all $x \in R$.

So, $f(x)= x(x-4)^2$ is continuous on [0,4].

To check differentiability we differentiate the given function to get $f'(x)= (x-4)^2+2x(x-4)$ which exist in [0,4].

So, $f(x)= x(x-4)^2$ is differentiable on (0,4).

Also $f(0)=0(0-4)^2=0$ and $f(4)= 4(4-4)^2=0$

i.e. $f(0)=f(4)$

Required condition of Rolle's theorem are satisfied.

Hence, there exist atleast one $c\in(0,4)$ such that $f'(c)=0$

i.e. $(c-4)^2+2c(c-4)=0$

i.e. $(c-4)(3c-4)=0$

i.e. $c=4$ or $c=3\div4$

Value of $c = \frac{3}{4} \in (0,4)$

Q7.

$f(x)=x^3- 7x^2+16x-12$ is a polynomial function and every polynomial function is continuous for all $x \in R$.

So, $f(x)= x^3- 7x^2+16x-12$ is continuous on [2,3].

To check differentiability we differentiate the given function to get $f'(x)=3x^2-14x+16$ which exist in [2,3].

Hence $f(x)= x^3- 7x^2+16x-12$ is differentiable on (2,3).

Also $f(2)= 2^3- 7(2)^2+16(2)-12=0$ and $f(3)= 3^3- 7(3)^2+16(3)-12=0$

i.e. $f(2)=f(3)$

Required conditions of Rolle's theorem are satisfied.

Rolle's theorem is applicable

So, there exist at least one $c\epsilon(2,3)$ such that $f'(c)=0$

i.e. $3c^2-14c+16=0$

i.e. $(c-2)(3c-7)=0$

i.e. $c=2$ or $c=7\div3$

Value of $c=\frac{7}{3}\epsilon(2,3)$

Q8.

$f(x)= x^3+3x^2-24x-80$ is a polynomial function and every polynomial function is continuous for all $x \epsilon R$.

So, $f(x)= x^3+3x^2-24x-80$ is continuous on [-4,5].

To check differentiability we differentiate the given function to get, $f'(x)= 3x^2+6x-24$ which exist in [-4,5].

So, $f(x)= x^3+3x^2-24x-80$ is differentiable on (-4,5).

Also f(-4)= $(-4)^3+3(-4)^2-24(-4)-80=0$ and f(5)= $(5)^3+3(5)^2-24(5)-80=0$

i.e. f(-4)=f(5)

Required conditions of Rolle's theorem are satisfied.

Rolle's theorem is applicable

Hence, there exist at least one c∈(-4,5) such that f'(c)=0

i.e. $3c^2+6c-24=0$

i.e. c=-4 or c=2

Value of c=2 ∈(-4,5)

Q9.

f(x)=(x-1)(x-2)(x-3) is a polynomial function and every polynomial function is continuous for all x ∈ R.

So, f(x)= (x-1)(x-2)(x-3) is continuous on [1,3].

To check differentiability we differentiate the given function to get,

f'(x)= (x-2)(x-3)+ (x-1)(x-3)+ (x-1)(x-2) which exist in [1,3].

So, f(x)= (x-1)(x-2)(x-3) is differentiable on (1,3).

Here, f(1)= (1-1)(1-2)(1-3) =0

And f(3)= (3-1)(3-2)(3-3) =0

i.e. f(1)=f(3)

Required conditions of Rolle's theorem are satisfied.

Rolle's theorem is applicable

Hence, there exist at least one cϵ(1,3) such that f'(c)=0

i.e. (c-2)(c-3)+ (c-1)(c-3)+ (c-1)(c-2)=0

i.e. (c-3)(2c-3)+(c-1)(c-2)=0

i.e. $(2c^2-9c+9)+(c^2-3c+2)=0$

i.e. $3c^2-12c+11=0$

i.e. $c = \frac{12 \pm \sqrt{12}}{6}$

i.e. c=2.58 or c=1.42

Value of c=1.42ϵ(1,3) and c=2.58ϵ(1,3)

Q10.

$f(x)=(x-1)(x-2)^2$ is a polynomial function and every polynomial function is continuous for all x ϵ R.

So, $f(x)= (x-1)(x-2)^2$ is continuous on [1,2].

To check differentiability we differentiate the given function to get,

$f'(x)= (x-2)^2+2(x-1)(x-2)$ which exist in [1,2].

So, $f(x)= (x-1)(x-2)^2$ is differentiable on (1,2).

Also, $f(1)= (1-1)(1-2)^2=0$ and $f(2)= (2-1)(2-2)^2=0$

i.e. f(1)=f(2)

Required conditions of Rolle's theorem are satisfied.

So Rolle's theorem is applicable.

Hence, there exist at least one $c\epsilon(1,2)$ such that $f'(c)=0$

i.e. $(c-2)^2+2(c-1)(c-2)=0$

$(3c-4)(c-2)=0$

i.e. $c=2$ or $c=4\div3$

Value of $c = \frac{4}{3} = 1.33\epsilon(1,2)$

Q11.

$f(x)=(x-2)^4(x-3)^3$ is a polynomial function and every polynomial function is continuous for all $x \epsilon R$.

So, $f(x)= (x-2)^4(x-3)^3$ is continuous on [2,3].

To check differentiability we differentiate the given function to get,

$f'(x)= 4(x-2)^3(x-3)^3+3(x-2)^4(x-3)^2$ which exist in [2,3].

So, $f(x)= (x-2)^4(x-3)^3$ is differentiable on (2,3).

Also, $f(2)= (2-2)^4(2-3)^3=0$ and $f(3)= (3-2)^4(3-3)^3=0$

i.e. $f(2)=f(3)$

Conditions of Rolle's theorem are satisfied.

Rolle's theorem is aplicable

Hence, there exist at least one $c\epsilon(2,3)$ such that $f'(c)=0$

i.e. $4(c-2)^3(c-3)^3+3(c-2)^4(c-3)^2=0$

$(c-2)^3(c-3)^2(7c-18)=0$

i.e. $c=2$ or $c=3$ or $c=18\div7$

Value of $c = \frac{18}{7} = 2.57 \epsilon (2,3)$

Q12.

$f(x) = \sqrt{1 - x^2}$ is a polynomial function and every polynomial function is continuous for all x ϵ R.

So, $f(x) = \sqrt{1 - x^2}$ is continuous on [-1,1].

To check differentiability we differentiate the given function to get $f'(x) = -\frac{x}{\sqrt{1-x^2}}$ which exist in [-1,1].

So, $f(x) = \sqrt{1 - x^2}$ is differentiable on (-1,1).

Also, $f(-1) = \sqrt{1 - (-1)^2} = 0$ and $f(1) = \sqrt{1 - 1^2} = 0$

i.e. $f(-1) = f(1)$

Conditions of Rolle's theorem are satisfied.

Rolle's theorem is applicable

Hence, there exist at least one cϵ(-1,1) such that f'(c)=0

i.e. $-\frac{c}{\sqrt{1-c^2}} = 0$

i.e. c=0

Value of c=0ϵ(-1,1)

Q13.

f(x)=cosx is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)=cosx is continuous on $[-\frac{\pi}{2},\frac{\pi}{2}]$.

To check differentiability we differentiate the given function to get,

f'(x)=-sinx which exist in $[-\frac{\pi}{2},\frac{\pi}{2}]$.

So, f(x)=cosx is differentiable on $(-\frac{\pi}{2},\frac{\pi}{2})$.

Also, $f\left(-\frac{\pi}{2}\right)=\cos\left(-\frac{\pi}{2}\right)=0$ and $f\left(\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{2}\right)=0$

i.e. $f\left(-\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}\right)$

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one $c\epsilon(-\frac{\pi}{2},\frac{\pi}{2})$ such that f'(c)=0

i.e. -sinc=0

i.e. c=0

Value of $c=0\epsilon(-\frac{\pi}{2},\frac{\pi}{2})$

Q14.

f(x)=cos2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= cos2x is continuous on [0,π].

To check differentiability we differentiate the given function to get,

f'(x)= -2sin2x which exist in [0,π].

So, f(x)=cos2x is differentiable on (0,π).

Also, f(0)=cos0=1 and f(π)=cos2π=1

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable.

Hence, there exist at least one c∈(0,π) such that f'(c)=0

i.e. -2sin2c =0

i.e. 2c=π

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2} \in (0, \pi)$

Q15.

f(x)=sin3x is a trigonometric function and we know every trigonometric function is continuous.

So, f(x)= sin3x is continuous on [0,π].

To check differentiability we differentiate the given function to get,

f'(x)= 3cos3x which exist in [0,π].

So, f(x)= sin3x is differentiable on (0,π).

Also, f(0)=sin0=0 and f(π)=sin3π=0

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one c∈(0,π) such that f'(c)=0

i.e. 3cos3c =0

i.e. $3c = \frac{\pi}{2}$

i.e. $c = \frac{\pi}{6}$

Value of $c = \frac{\pi}{6} \in (0, \pi)$

Q16.

f(x)=sin x + cos x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= sin x + cos x is continuous on $[0, \frac{\pi}{2}]$.

To check differentiability we differentiate the given function to get,

f'(x)= cos x – sin x which exist in $[0, \frac{\pi}{2}]$.

So, f(x)= sin x + cos x is differentiable on $(0, \frac{\pi}{2})$

Also , f(0)=sin0+cos0=1

And $f(\frac{\pi}{2}) = \sin\left(\frac{\pi}{2}\right) + \cos(\frac{\pi}{2}) = 1$

i.e. $f(0) = f(\frac{\pi}{2})$

Conditions of Rolle's theorem are satisfied.

Rolle's theorem is applicable.

Hence, there exist at least one $c \in (0, \frac{\pi}{2})$ such that f'(c)=0

i.e. cosc-sinc =0

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4} \in (0, \frac{\pi}{2})$

Q17.

f(x)=e^{-x} sin x is a combination of exponential and trigonometric function. Since exponential and trigonometric functions both are continuous their product will be a continuous function as well.

⇒ f(x)= e^{-x} sinx is continuous on [0,π].

To check differentiability we differentiate the given function to get,

f'(x)= e^{-x} (cosx – sinx) which exist in [0,π].

So, f(x)= e^{-x} sinx is differentiable on (0,π)

Also f(0)= e^{-0} sin0=0 and f(π)= $e^{-\pi}$sinπ =0

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one c∈(0,π) such that f'(c)=0

i.e. e^{-c} (cos c – sin c) =0

i.e. cos c-sin c = 0

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4} \epsilon (0, \pi)$

Q18.

f(x)=e^{-x} (sinx-cosx) is a combination of exponential and trigonometric function which is continuous. Since exponential and trigonometric functions both are continuous their product will be a continuous function as well.

So, f(x)= e^{-x} (sinx-cosx) is continuous on $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

To check differentiability we differentiate the given function to get,

f'(x)= e^{-x} (sinx + cosx) - e^{-x} (sinx – cosx)

= e^{-x} cosx which exist in $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

So, f(x)= e^{-x} (sinx-cosx) is differentiable on $(\frac{\pi}{4}, \frac{5\pi}{4})$

Here, $f(\frac{\pi}{4}) = e^{-\frac{\pi}{4}} (\sin\frac{\pi}{4} - \cos\frac{\pi}{4}) = 0$

And $f(\frac{5\pi}{4}) = e^{-\frac{5\pi}{4}} (\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}) = 0$

i.e. $f(\frac{\pi}{4}) = f(\frac{5\pi}{4})$

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one $c \epsilon (\frac{\pi}{4}, \frac{5\pi}{4})$ such that f'(c)=0 i.e. $e^{-c}\cos c = 0$

i.e. $\cos c = 0$

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2} \epsilon (\frac{\pi}{4}, \frac{5\pi}{4})$

Q19.

f(x) = sinx-sin2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x) = sinx-sin2x is continuous on [0,2π].

To check differentiability we differentiate the given function to get,

f'(x)= cosx-2cos2x which exist in [0,2π].

So, f(x)= sinx-sin2x is differentiable on (0,2π)

Also, f(0)= sin0-sin0 = 0 and f(2π)=sin(2π)-sin(4π) =0

i.e. f(0)=f(2π)

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one cϵ(0,2π) such that f'(c)=0

i.e. cosx-2cos2x =0

i.e. $\cos x - 4\cos^2 x + 2 = 0$

i.e. $4\cos^2 x - \cos x - 2 = 0$

i.e. $\cos x = \frac{1 \pm \sqrt{33}}{8}$

i.e. $c=32° 32'$ or $c=126°23'$

Value of $c=32°32' \epsilon (0,2\pi)$

Q20.

$f(x)=x(x+2)e^x$ is a combination of exponential and polynomial function which is continuous for all $x \epsilon R$.

$\Rightarrow f(x)= x(x+2)e^x$ is continuous on $[-2,0]$.

To check differentiability we differentiate the given function to get,

$f'(x) = (x^2+4x+2) e^x$ which exist in $[-2, 0]$.

So, $f(x) = x(x+2) e^x$ is differentiable on $(-2, 0)$.

Here, $f(-2) = (-2)(-2+2)e^{-2} =0$

And $f(0) = 0(0+2)e^0=0$

i.e. $f(-2)=f(0)$

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one $c \epsilon (-2,0)$ such that $f'(c)=0$

i.e. $(c^2+4c+2)e^c =0$

i.e. $(c+\sqrt{2})^2=0$

i.e. $c=-\sqrt{2}$

Value of $c=-\sqrt{2} \epsilon (-2,0)$

Q21.

f (x)=x(x-5)^2 is a polynomial and we know every polynomial function is continuous for all x ϵ R.

So, f(x)= x(x-5)^2 is continuous on [0,5].

To check differentiability we differentiate the given function to get,

f'(x)= (x-5)^2+ 2x(x-5) which exist in [0,5].

So, f(x)= x(x-5)^2 is differentiable on (0,5).

Also, f(0)= 0(0-5)^2=0 and f(5)= 5(5-5)^2=0

i.e. f(0)=f(5)

Conditions of Rolle's theorem are satisfied.

As a result Rolle's theorem is applicable

Hence, there exist at least one cϵ(0,5) such that f'(c)=0

i.e. (c-5)^2+ 2c(c-5)=0

i.e.(c-5)(3c-5)=0

i.e. $c = \frac{5}{3}$ or c=5

Value of $c = \frac{5}{3} \epsilon (0,5)$

Q22.

f(x)=(x-1)(2x-3) is a polynomial function and every polynomial function is continuous for all x ϵ R.

So, f(x)= (x-1)(2x-3) is continuous on [1,3].

To check differentiability we differentiate the given function to get,

f'(x)= (2x-3)+ 2(x-1) which exist in [1,3].

So, f(x) = (x-1)(2x-3) is differentiable on (1,3).

Also, f(1)= (1-1)(2(1)-3)=0 and f(5)= (3-1)(2(3)-3)=6

But clearly, $f(1)\neq f(3)$

This implies the last condition of Rolle's theorem is violated here hence Rolle's theorem is not applicable.

Q. 23.

$f(x)=x^{1/2}$ is a polynomial function and we know every polynomial function is continuous for all x ϵ R.

So, f(x)= $x^{1/2}$ is continuous on [-1,1].

To check differentiability we differentiate the given function to get, $f'(x) = \frac{1}{2x^{\frac{1}{2}}}$ which does not exist at x=0 in [-1,1].

As a result, $f(x)=x^{1/2}$ is not differentiable on (-1,1) because there exists a point in the interval where the given function is not differentiable.

Hence Rolle's theorem is not applicable.

Q24.

$f(x)=2+(x-1)^{2/3}$ is a polynomial function and every polynomial function is continuous for all $x \in R$.

$\Rightarrow f(x)= 2+(x-1)^{2/3}$ is continuous on [0,2].

To check differentiability we differentiate the given function to get, $f'(x) = \frac{2}{3(x-1)^{\frac{1}{3}}}$ which does not exist at x=1 in [0,2].

As a result $f(x)= 2+(x-1)^{2/3}$ is not differentiable on (0,2), because there exists a point in the interval where the given function is not differentiable.

So,Rolle's theorem is not applicable.

Q25.

$f(x) = \cos\frac{1}{x}$ which is not continuous at x=0

$\Rightarrow f(x) = \cos\frac{1}{x}$ is not continuous on interval [-1,1].

Condition (i) of Rolle's theorem is not satisfied.

Hence Rolle's theorem is not applicable.

Q26.

f(x)=[x] which is not continuous at x=0

$\Rightarrow$ f(x)=[x] is not continuous on [-1,1].

Condition (i) of Rolle's theorem is not satisfied.

Hence Rolle's theorem is not applicable.

Q27.

$y=x(x-4)$ is a polynomial function and every polynomial function is continuous for all $x \in R$.

So, $y= x(x-4)$ is continuous on $[0,4]$.

To check differentiability we differentiate the given function to get,

$y' = (x-4)+x$ which exist in $[0,4]$.

So, $y= x(x-4)$ is differentiable on $(0,4)$.

Also, $y(0)=0(0-4)=0$

And $y(4)= 4(4-4)=0$

i.e. $y(0)=y(4)$

Conditions of Rolle's theorem are satisfied.

Hence Rolle's theorem is applicable

Hence, there exist at least one $c \in (0,4)$ such that $y'(c)=0$

i.e. $(c-4)+c=0$

i.e. $2c-4=0$

i.e. $c=2$

Value of $c=2 \in (0,4)$

So, $y(c)=y(2)=2(2-4)=-4$

The geometric interpretation is that, the coordinate (2, -4) is the point on the given function where the tangent is parallel to x-axis.

Exercise-11D

Q1.

Since the f(x) is a polynomial function,

It will be continuous as well as differentiable in the interval [4,6].

This means

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{(36+12+3)-(16+8+3)}{6-4}$$

$$=\frac{24}{2}$$

$=12$

$\Rightarrow f'(c)=2c+2$

$\Rightarrow 2c+2=12$

$\Rightarrow c=5$

Q2.

Since the f(x) is a polynomial function,

It will be continuous as well as differentiable in the interval [0,4].

This means

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(16 + 4 - 1) - (0 + 0 - 1)}{4 - 0}$$

=5

⇒ f'(c)=2c+1

⇒ 2c+1=5

⇒ c=2

Q3.

Since the f(x) is a polynomial function,

It will be continuous as well as differentiable in the interval [1,3].

This means

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(18 - 9 + 1) - (2 - 3 + 1)}{3 - 1}$$

=5

⇒ f'(c)=4c-3

⇒ 4c-3=5

⇒ c=2

Q4.

Since the f(x) is a polynomial function, It will be continuous as well as differentiable in the interval [-1,4].

This menas

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(64 + 16 - 24) - (-1 + 1 + 6)}{4 + 1}$$

$$= \frac{50}{5}$$

=10

f' (c)=3c^2+2c-6

⇒ 3 c^2+2c-6=10

⇒ 3 c^2+2c-16=0

⇒ 3 c^2-6c+8c-16=0

⇒ 3c(c-2)+8(c-2)=0

⇒ (3c+8)(c-2)=0

$$c = 2, \frac{-8}{3}$$

Q5.

$$\mathbf{f(x) = x^3 - 18x^2 + 104x - 192}$$

Since the f(x) is a polynomial function,

It will be continuous as well as differentiable in the interval [4,6].

This means

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow f'(c)=\frac{(216-648+624-192)-(64-288+416-192)}{6-2}$$

=0

⇒ f' (c)=3c²-36c+104

=3c²-36c+10

=0

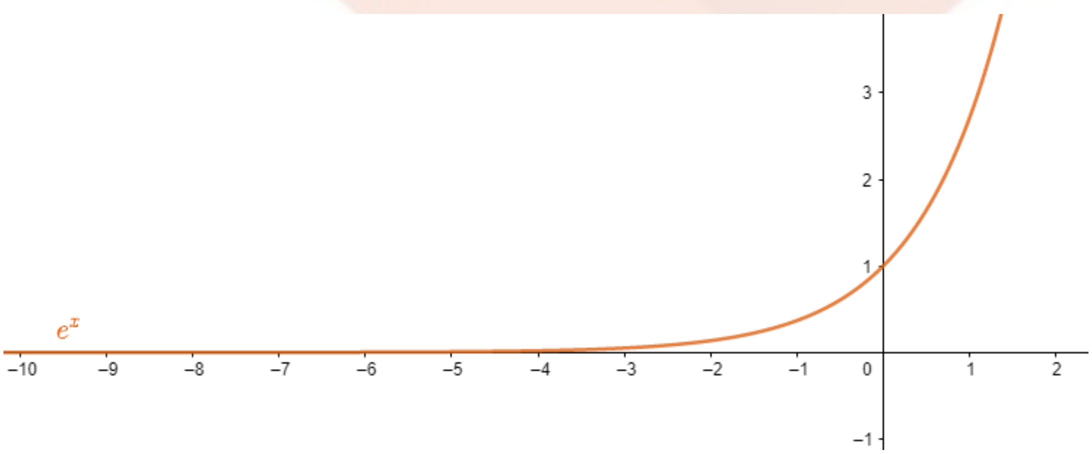
$$\Rightarrow c=\frac{36\pm\sqrt{1296-1248}}{6}$$

$$\Rightarrow c=\frac{36\pm\sqrt{48}}{6}$$

$$\Rightarrow c=6\pm\frac{2}{3}\sqrt{3}$$

Q6.

Since f(c) is continuous as well as differentiable in the interval [0,1].



$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{e-1}{1}$$

$\Rightarrow$ f' (c)=e^c

$\Rightarrow$ e^c =e-1

$$\Rightarrow \log_e e^c = \log_e(e-1)$$

$\Rightarrow$ c=$\log_e$(e-1)

Q7.

Since the f(x) is a polynomial function,

It will be continuous as well as differentiable in the interval [0,1].

This means

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{1-0}{1-0}$$

=1

$$f'(c)=\frac{2}{3}c^{\frac{1}{3}}$$

$$\Rightarrow \frac{2}{3}c^{\frac{1}{3}}=1$$

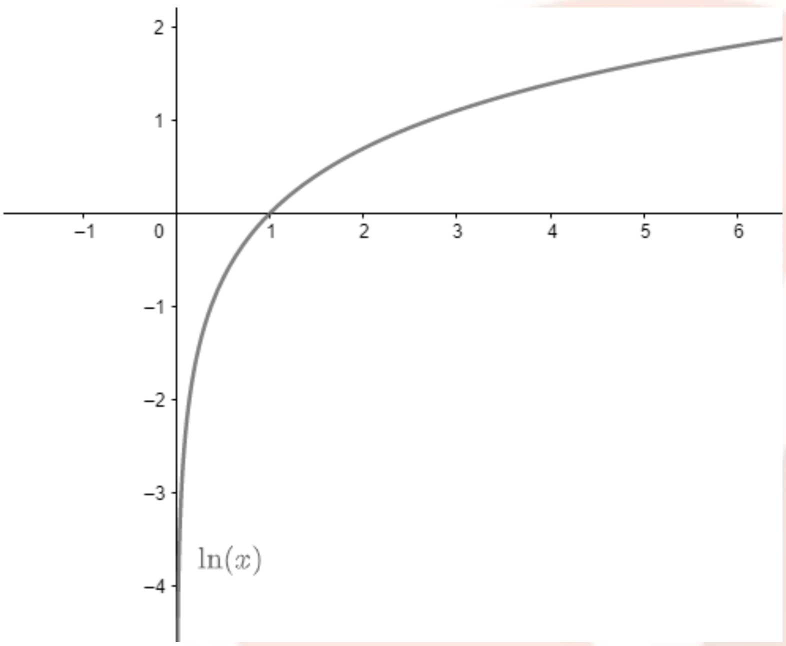
$$\Rightarrow c^{\frac{-1}{3}}=\frac{3}{2}$$

$$\Rightarrow c^{\frac{1}{3}}=\frac{2}{3}$$

$\Rightarrow\ c=\frac{8}{27}$

Q8.

Since log x is a continuous as well as differentiable function in the interval [1,e].



$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{\log e-\log 1}{e-1}$$

$$=\frac{1}{e-1}$$

$$f'(c)=\frac{1}{c}$$

$$\Rightarrow\ \frac{1}{e-1}=\frac{1}{c}$$

c=e-1

Q9.

Since $\tan^{-1}x$ is a continuous as well as differentiable function in the interval [0,1], This means,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{\tan^{-1}1-\tan^{-1}0}{1-0}$$

$$=\frac{\pi}{4}$$

$$f'(c)=\frac{1}{1+c^2}$$

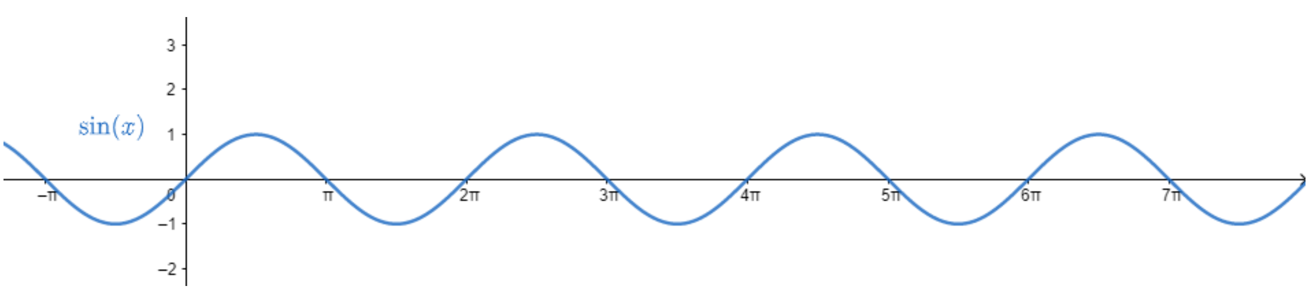
$$\Rightarrow \frac{1}{1+c^2}=\frac{\pi}{4}$$

$$\Rightarrow 1+c^2=\frac{4}{\pi}$$

$$\Rightarrow c=\sqrt{\frac{4}{\pi}-1}$$

Q10.

Since sin x is a continuous as well as differentiable function in the interval $\left[\frac{\pi}{2},\frac{5\pi}{2}\right]$, this means,



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sin\frac{5\pi}{2} - \sin\frac{\pi}{2}}{\frac{5\pi}{2} - \frac{\pi}{2}}$$

=0

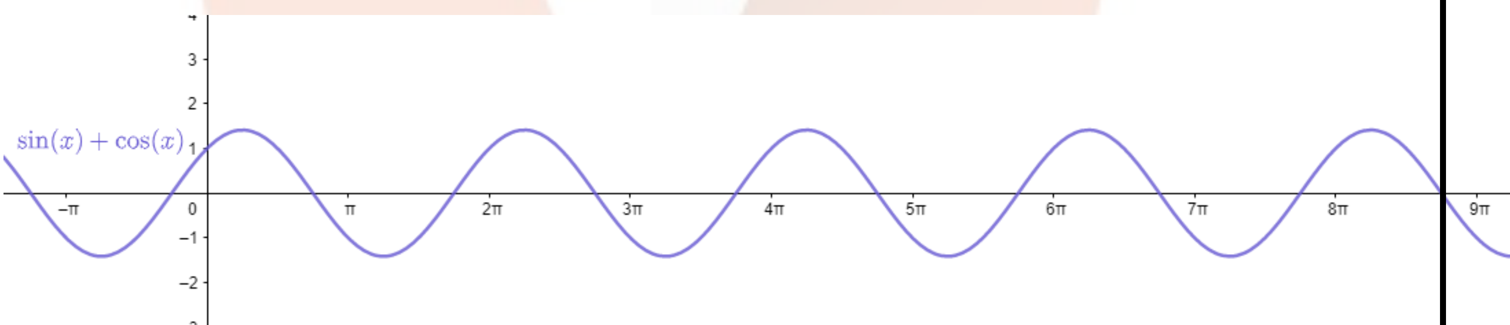
f' (c)=cos x

cos x=0

$$x = \frac{n\pi}{2}, n\epsilon\{1,2,3,4,5\}$$

Q11.

Since (sin x + cos x) is a continuous as well as differentiable function in the interval $\left[0, \frac{\pi}{2}\right]$, this means,



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sin\frac{\pi}{2} + \cos\frac{\pi}{2} - \sin 0 - \cos 0}{\frac{\pi}{2} - 0}$$

=0

f' (c)=cos x-sin x

cos x-sin x=0

$$\Rightarrow \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = 0$$

$$\Rightarrow \cos\left(x+\frac{\pi}{4}\right) = 0$$

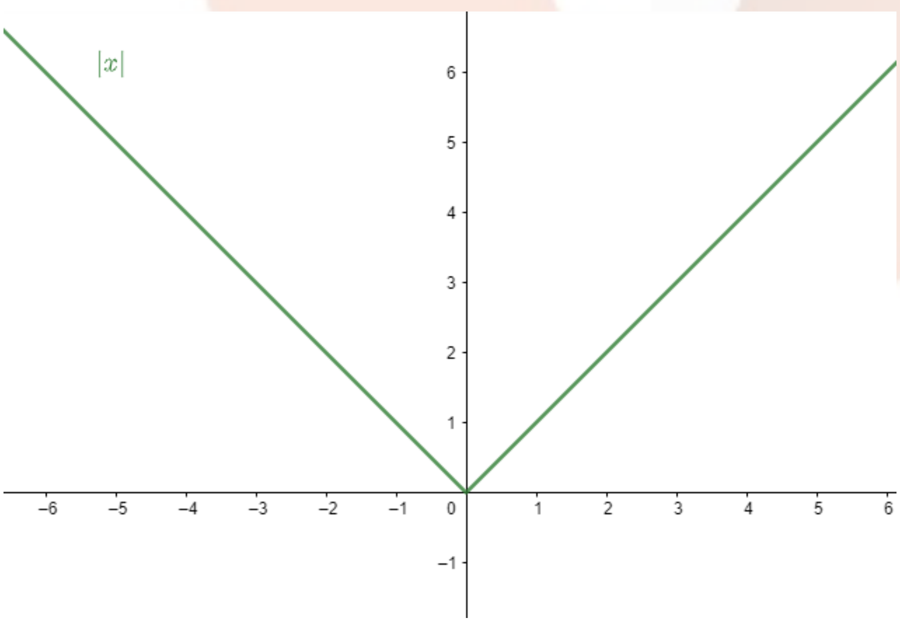
$$\Rightarrow \left(x+\frac{\pi}{4}\right) = \cos^{-1} 0$$

$$\Rightarrow \left(x+\frac{\pi}{4}\right) = 1$$

$$\Rightarrow \mathbf{x = 1 - \frac{\pi}{4}}$$

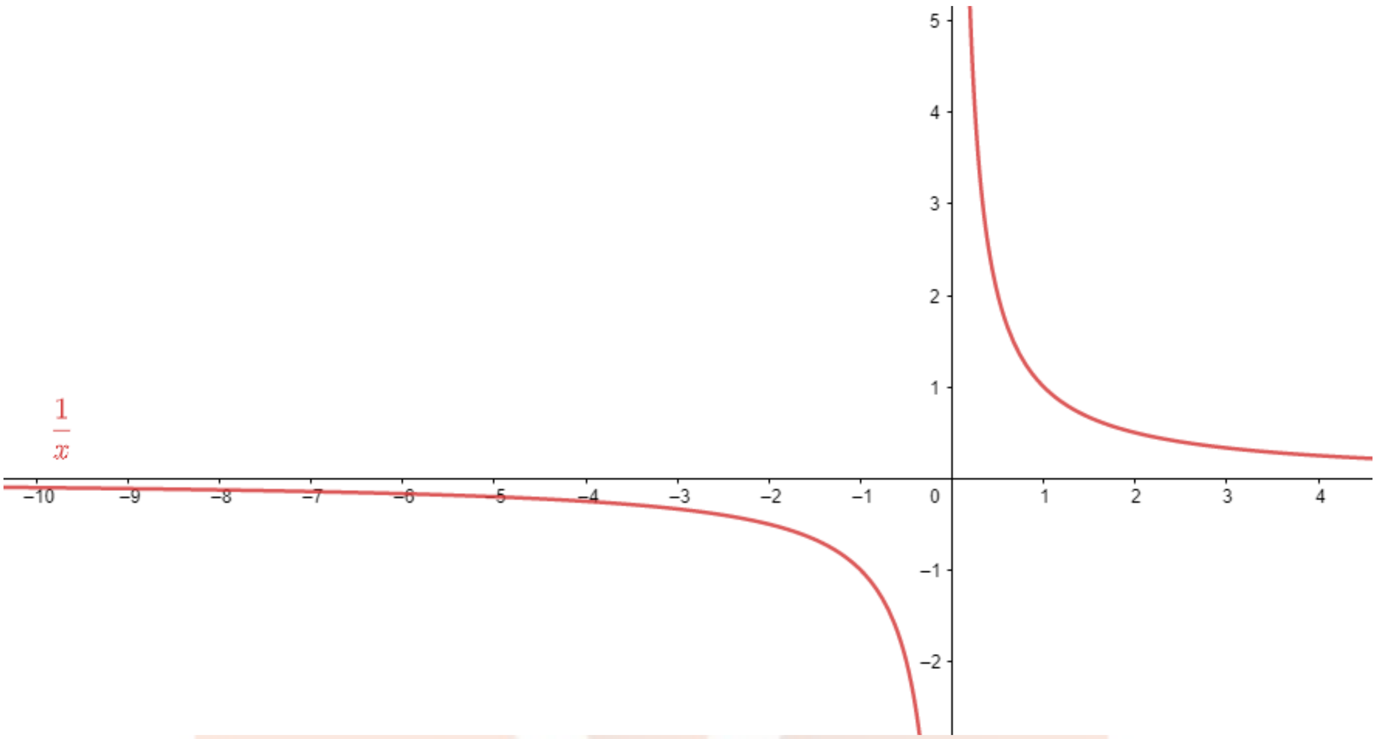
Q12.

Since f(x) is continuous in the interval [-1,1] but is non differentiable at x=0 So Lagrange's mean value theorem is not applicable to f(x)=|x|



Q13.

Since the graph is discontinuous at x=0 as shown in the graph.



So Lagrange's mean value theorem is not applicable to the above function.

Q14.

i.

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval $[0,\frac{1}{2}]$. Then,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{\frac{1}{8}-\frac{3}{4}+1-0}{\frac{1}{2}-0}$$

$=\frac{3}{4}$

f' (c)=3x^2 -6x+2

3 x^2-6x+2=3/4

12 x^2-24x+8=3

12 x^2-24x+5=0

$x=\frac{24\pm\sqrt{576-240}}{24}$

$x=1\pm\sqrt{\frac{336}{576}}$

$x=1\pm\sqrt{\frac{7}{12}}$

ii.

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [1,5].

$f'(c)=\frac{f(b)-f(a)}{b-a}$

$=\frac{\sqrt{25-25}-\sqrt{25-1}}{5-1}$

$=\frac{-\sqrt{24}}{4}$

$f'(c)=\frac{1}{2\sqrt{25-c^2}}(-2c)$

$\Rightarrow\ \frac{-c}{\sqrt{25-c^2}}=\frac{-\sqrt{24}}{4}$

$\Rightarrow\ 4c=\sqrt{24(25-c^2)}$

$\Rightarrow 16c^2=600-24c^2$

$\Rightarrow 40c^2=600$

$\Rightarrow c^2=15$

$\Rightarrow c=\sqrt{15}$

iii.

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [4,6].

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{\sqrt{8}-\sqrt{6}}{6-4}$$

$$=\frac{\sqrt{8}-\sqrt{6}}{2}$$

$$\mathbf{f'(c)}=\frac{1}{2\sqrt{c+2}}$$

$$\Rightarrow \frac{1}{2\sqrt{c+2}}=\frac{\sqrt{8}-\sqrt{6}}{2}$$

$$\Rightarrow \frac{1}{\sqrt{c+2}}=\frac{\sqrt{8}-\sqrt{6}}{1}$$

$$\Rightarrow \sqrt{c+2}=\frac{1}{\sqrt{8}-\sqrt{6}}\times\frac{\sqrt{8}+\sqrt{6}}{\sqrt{8}+\sqrt{6}}$$

$$\Rightarrow \sqrt{c+2}=\frac{\sqrt{8}+\sqrt{6}}{2}$$

$$\Rightarrow \mathbf{c+2}=\frac{1}{4}(8+6+2\sqrt{48})$$

$$\Rightarrow \mathbf{c}=\frac{\mathbf{3}}{\mathbf{2}}+\mathbf{2}\sqrt{\mathbf{3}}$$

$\Rightarrow$ c=4.964

Q15.

$y=x^2$

Since y is a polynomial function.

It is continuous and differentiable in [1,2]

So, there exists a c such that:

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$= \frac{4 - 1}{2 - 1}$

$= 3$

$\Rightarrow$ f' (c)=2c

$\Rightarrow$ 2c=3

$c = \frac{3}{2}$

So, the point is $\left(\frac{3}{2}, \frac{9}{4}\right)$

Q16.

$y = x^3$

Since y is a polynomial function.

It is continuous and differentiable in [1,3]

So, there exists a c such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{27 - 1}{3 - 1}$$

$$= 13$$

$\Rightarrow$ f' (c)=3c^2

$\Rightarrow$ 3c^2=13

$$\Rightarrow c = \sqrt{\frac{13}{3}}$$

$$\Rightarrow c = \frac{\sqrt{39}}{3}$$

So the point is $\left(\frac{\sqrt{39}}{3}, \frac{13\sqrt{39}}{9}\right)$

Q17.

y=x^3-3x

Since y is a polynomial function.

It is continuous and differentiable in [1,2]

So, there exists a c such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(8 - 6) - (1 - 3)}{2 - 1}$$

=4

$\Rightarrow$ f' (c)=3c^2-3

$\Rightarrow 3\ c^2 - 3 = 4$

$\Rightarrow 3c^2 = 7$

$\Rightarrow c^2 = \frac{7}{3}$

$\Rightarrow c = \pm\sqrt{\frac{7}{3}}$

So, the points are $\left(\sqrt{\frac{7}{3}}, \frac{-2}{3}\sqrt{\frac{7}{3}}\right), \left(\frac{-7}{3}\sqrt{\frac{7}{3}}, \frac{2}{3}\sqrt{\frac{7}{3}}\right)$

Q18.

$f(x) = x(1 - \log x)$

Since the function is continuous as well as differentiable

So, there exists c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow (1 - \log c) - c \times \frac{1}{c} = \frac{b(1 - \log b) - a(1 - \log a)}{b - a}$$

$$\Rightarrow \log c = \frac{b(1 - \log b) - a(1 - \log a)}{b - a}$$

$(b-a) \log c = b(1 - \log b) - a(1 - \log a)$

Exercise-11E

Q1.

Minimum value = 4

The function does not have maximum value because square of any real number will always be positive and so it will have no boundary

The minimum value exists when the expression $(5x-1)^2=0$

Therefore, minimum value=4

Q2.

Maximum value = 9

Since $(x-3)^2$ has a negative sign, the maximum value it can have is 9.

Also it has no minimum value due to negative sign.

Q3.

Maximum value = 6

Since $|x+4|$ is non-negative for all x belonging to R.

Therefore the least value it can have is 0.

Hence value of function is 6.

It has no minimum value as it the modulus function will tend to infinity and hence the function will tend to negative of infinity.

Q4.

Maximum. value = 4, Minimum value = 6

$f(x)=\sin 2x+5$

We already know that,

$-1\leq \sin\Theta\leq 1$

$-1\leq \sin 2x\leq 1$

So $-1+5 \leq \sin 2x+5 \leq 1+5$

$4 \leq \sin 2x+5 \leq 6$

So

Maximum value of $f(x)=2x+5$ is 6

And minimum value of $f(x) =2x+5$ is 4

Q5.

We already know that

$-1\leq \sin\Theta\leq 1$

$-1\leq \sin 4x\leq 1$

Adding 3 on both sides to get

$-1+3\leq \sin 4x+3\leq 1+3$

Taking modulus on both sides

$2\leq|\sin 4x+3|\leq 4$

So minimum value is 2 and maximum value is 4

Q6.

$F'(x)=4(x-3)^3=0$

$\Rightarrow X=3$

∴ local max. Vaue is 0.

Q7.

$F'(x)=2x=0$

$x=0$

So, local minimum value is 0 at $x=0$

Q8.

$F'(x)=6x^2-42x+36=0$

$\Rightarrow 6(x-1)(x-6)=0$

$\Rightarrow x=1,6$

$F''(x)=12x-42$

Hence

$F''(1)<0$,1 is the point of local max.

$F''(6)>0$, 6 is the point of local min.

$F(1)=2-21+36-20=-3$ is local maxima

$F(6)=-128$ is local minima

Q9.

$F'(x)=3x^2-12x+9=0$

$\Rightarrow 3(x-3)(x-1)=0$

$\Rightarrow x=3,1$

$F''(x)=6x-12$

$F''(3)=18-12=6>0$, 3 is the point of local minima.

$F''(1)<0$, $x=1$ is point of local maxima.

$F(3)=15$

$F(1)=19$

Q10.

$F'(x)=4x^3-124x+120=0$

$\Rightarrow 4(x^3-31x+30)=0$

For $x=1$, the given equation is 0

So, $(x-1)$ is a factor,

Then the other factor can be found as

$4(x-1)(x+6)(x-5)=0$

$\Rightarrow X=1,-6,5$

$F''(1)<0$, 1is point of maxima.

$F''(-6)$ and $f''(5)>0$, -6 and 5 are points of local minima.

$F(1)=68$

F(-6)=-1647

F(5)=-316

Q11.

$F'(x)=-3x^2+24x=0$

$\Rightarrow -3x(x-8)=0$

$\Rightarrow x=0,8$

$F''(x)=-6x+24$

$F''(0)>0$, 0 is point of local minima.

$F''(8)<0$, 8 is point of local maxima.

F(8)=251 and f(0)=-5

Q12.

$f'(x)=(x-1)2(x+2)+(x+2)^2=0$

x=0,-2

$f''(0)>0$, 0 is point of local minima.

$f''(-2)<0$, -2 is point of local maxima.

f(0)=-4

f(-2)=0

Q13.

$F'(x)=-(x-1)^3 2(x+1)-3(x-1)^2(x+1)^2=0$

$\Rightarrow x = 1, -1, -\frac{1}{5}$

Since, $f^{||}(1)$ and $f^{||}(-1) <0$, so 1 and -1 are points of local maxima

$F^{||}(-\frac{1}{5})>0$, $-\frac{1}{5}$ is a point of local minima.

$F(1)=f(-1)=0$

Also, $f\left(-\frac{1}{5}\right) = -\frac{3456}{3125}$

Q14.

$F'(x)=\frac{1}{2}-\frac{2}{x^2}=0$

$\Rightarrow x^2-4=0$

$\Rightarrow x=\pm2$

But since $x>0$, $x=2$

$F''(2) = \frac{2}{x^3}$

$= \frac{2}{8} < 0$

So, point of local minima . is 2

$F(2)=\frac{2}{2}+\frac{2}{2}=2$

Q15.

$F'(x)=6x^2-24=0$

$6(x^2-4)=0$

$6(x^2-2^2)=0$

$6(x-2)(x+2)=0$

$X=2,-2$

Evaluating value of the function at the above critical points and end points of given intervals to get to the answer

$F(2)=2(2)^3-24(2)+107=75$

$F(-2)=2(-2)^3-24(-2)+107=139$

$F(-3)=2(-3)^3-24(-3)+107=125$

$F(3)=2(3)^3-24(3)+107=89$

So x=-2 is the where there is local maxima and x=2 I where there is local minima.

Q16.

$F^{|}(x)=12x^3-24x^2+24x-48=0$

$12(x^3-2x^2+2x-4)=0$

If we put x=2 then , $x^3-2x^2+2x-4=0$

SO (x-2) is a factor

On dividing (x^3-2x^2+2x-4) by (x-2), we get,

$12(x-2)(x^2+2)=0$

X=2,4

Now, we shall evaluate the value of f at these points and the end points

$F(1)=3(1)^4-8(1)^3+12(1)^2-48(1)+1=-40$

$F(2)= 3(2)^4-8(2)^3+12(2)^2-48(2)+1=-63$

$F(4)= 3(4)^4-8(4)^3+12(4)^2-48(4)+1=257$

So local maxima is at x=4 and local minima is at x=1.

Q17.

$F^{|}(x)=\cos x-\frac{1}{2}\sin x=0$

$2\cos x=\sin x$

$\Rightarrow \frac{\pi}{6}=\frac{\pi}{3}$

$$f\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}+\frac{1}{2}\cos\frac{\pi}{2}=\frac{1}{2}$$

$$f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}+\frac{1}{2}\cos\frac{\pi}{6}=\frac{1}{2}+\frac{\sqrt{3}}{4}$$

$$f\left(\frac{\pi}{3}\right)=\sin\frac{\pi}{3}+\frac{1}{2}\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}+\frac{1}{4}$$

maximum value of the function is $\frac{3}{4}$ at $x=\frac{\pi}{6}$ and minimum value of the function is $\frac{1}{2}$ at $x=\frac{\pi}{2}$

Q18.

The given function is

$Y = x^{\frac{1}{x}}$

Taking log we obtain

$\log y = \frac{1}{x} \log x$

Differentiating above equation with respect to x we get

$\frac{1}{y}y' = -\frac{1}{x^2}\ln(x) + \frac{1}{x^2}$

$\Rightarrow y' = \frac{y}{x^2}(1 - \ln(x))$

(1-ln(x))=0

ln(x)=1

x=e

hence the maxima point is at x=e

and maximum value is $e^{\frac{1}{e}}$.

Q19.

$F(x)=x+\frac{1}{x}$

Differentiating it with respect to x and then finding critical points by equating the equation with 0

So, $F'(x)=1-\frac{1}{x^2}=0$

$X^2=1$

$x=1, x=-1$

Again differentiating it to find second derivative so that we can get to know about maxima or minima

$f''(x)=\frac{2}{x^3}$

$f''(1)=2$ and $f''(-1)=-2$

Since $f''(1)$ is positive it is the point if minima while $f''(-1)$ is negative so it is point of maxima.

minimum value-> $f(-1)=-1+\frac{1}{-1}=-2$

maximum value-> $f(1)=1+\frac{1}{1}=2$

So, maximum value is less than minimum value

Q20.

$\frac{dp}{dx}=-24-36x=0$

$\Rightarrow x=-23$

Again differentiating above differential we get

$\frac{d^2p}{dx^2}=-36$ is negative

So, maximum profit $=p\left(-\frac{2}{3}\right)$

$=49$

Q21.

Let P(x,y) be the position of the jet and the soldier be at A(3,2)

Then the distance between jet and plane can be represented as

AP=$\sqrt{(x-3)^2+(y-2)^2}$

Also it is given that $y=x^2+2$ or $y-2=x^2$

So, $AP^2=(x-3)^2 + x^4=z$ (say)

$$\frac{dz}{dx}=2(x-3)+4x^3$$

$$\frac{dz}{dx}=0$$

$\therefore$ $2x-6+4x^3=0$

Putting $x=1$

$2-6+4=0$

$\therefore$ $x-1$ is a factor

And $\frac{d^2z}{dx^2}=12x^2+2$

$\frac{dz}{dx}=0$ or $x=1$

and $\frac{d^2z}{dx^2}$(at $x=1$)>0

$\therefore$ z is minimum when $x=1, y=1+2=3$

Point is (1,3)

Q22.

f'(x)=-1+2cos x=0

$\Rightarrow \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}$

By finding the general solution, we get $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

Now, by calculating second derivative, we get $f''(\frac{\pi}{3}) < 0$ and $f''(\frac{5\pi}{3}) > 0$

So, maximum value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and minimum value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$

Exercise-11F

Q1.

Let us consider,

• x and y are the two numbers, such that x > 0 and y > 0

• Product of the numbers : x × y = 49

• Sum of the numbers : S = x + y

Now as,

x × y = 49

$y = \frac{49}{x}$ ------ (1)

Consider,

S = x + y

By substituting (1), we have

$S = x + \frac{49}{x}$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with x

$$\frac{dS}{dx} = \frac{d}{dx}\left(x + \frac{49}{x}\right)$$

$$\frac{dS}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{49}{x}\right)$$

$\frac{dS}{dx} = 1 + 49\left(\frac{-1}{x^2}\right)$ ----- (3)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dS}{dx} = 1 + 49\left(\frac{-1}{x^2}\right) = 0$$

$$= 1 - \left(\frac{49}{x^2}\right) = 0$$

$= 1 = \left(\frac{49}{x^2}\right)$

$= x^2 = 49$

$= x = \pm\sqrt{49}$

As $x > 0$, then $x = 7$

Now, for checking if the value of S is maximum or minimum at $x=7$, we will perform the second differentiation and check the value of $\frac{d^2S}{dx^2}$ at the critical value $x = 7$.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2S}{dx^2} = \frac{d}{dx}\left[1 + 49\left(\frac{-1}{x^2}\right)\right]$$

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[1] + \frac{d}{dx}\left[49\left(\frac{-1}{x^2}\right)\right]$$

$$\frac{d^2S}{dx^2} = 0 + \left[49\left(\frac{-1 \times -2}{x^3}\right)\right]$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{d^2S}{dx^2} = 49\left(\frac{2}{x^3}\right) = \frac{98}{x^3}$$

Now when $x = 7$,

$$\left[\frac{d^2S}{dx^2}\right]_{x=7} = \frac{98}{7^3} = \frac{98}{343} > 0$$

As second differential is positive, hence the critical point $x = 7$ will be the minimum point of the function S.

Therefore, the function S = sum of the two numbers is minimum at

x = 7.

From Equation (1), if x= 7

$y = \frac{49}{7} = 7$

Therefore, x = 7 and y = 7 are the two positive numbers whose

product is 49 and the sum is minimum.

Q2.

Let us consider,

- x and y are the two numbers, such that x > 0 and y > 0
- Sum of the numbers : x + y = 16
- Sum of squares of the numbers : $S = x^2 + y^2$

Now as,

x + y = 16

y = (16-x) ------ (1)

Consider,

$S = x^2 + y^2$

By substituting (1), we have

$S = x^2 + (16-y)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find

it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with x

$$\frac{dS}{dx} = \frac{d}{dx}[x^2 + (16-x)^2]$$

$$\frac{dS}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}[(16-x)^2]$$

$\frac{dS}{dx} = 2x + 2(16-x)(-1)$ ----- (3)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dS}{dx} = 2x + 2(16-x)(-1) = 0$$

$\Rightarrow 2x - 2(16 - x) = 0$

$\Rightarrow 2x - 32 + 2x = 0$

$= 4x = 32$

$\Rightarrow x = \frac{32}{4}$

$\Rightarrow x = 8$

As $x > 0$, $x = 8$

Now, for checking if the value of S is maximum or minimum at x=8, we will perform the second differentiation and check the value of $\frac{d^2S}{dx^2}$ at the critical value x = 8.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[2x + 2(16 - x)(-1)]$$

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[2x] - 2\frac{d}{dx}[16 - x]$$

$$\frac{d^2S}{dx^2} = 2 - 2[0 - 1]$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{d^2S}{dx^2} = 2 - 0 + 2 = 4$$

Now when x = 8,

$$\left[\frac{d^2S}{dx^2}\right]_{x=8} = 4 > 0$$

As second differential is positive, hence the critical point x = 8 will be the minimum point of the function S.

Therefore, the function S = sum of the squares of the two numbers is minimum at x = 8.

From Equation (1), if x= 8

$y = 16 - 8 = 8$

Therefore, $x = 8$ and $y = 8$ are the two positive numbers whose su is 16 and the sum of the squares is minimum.

Q3.

• the number 15 is divided into two numbers.

• the product of the square of one number and cube of another number is maximum.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : $x + y = 15$

• Product of square of the one number and cube of anther number :

$P = x^3 y^2$

Now as,

$x + y = 15$

$y = (15-x)$ ------ (1)

Consider,

$P = x^3y^2$

By substituting (1), we have

$P = x^3 \times (15-x)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with x

$$\frac{dP}{dx} = \frac{d}{dx}[x^3 \times (15-x)^2]$$

$$\frac{dP}{dx} = (15-x)^2 \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}[(15-x)^2]$$

$$\frac{dP}{dx} = (15-x)^2(3x^2) + x^3[2(15-x)(-1)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x,

then $\frac{d}{dx}(u \times v) = v \times \frac{d}{dx}(u) + u \times \frac{d}{dx}(v)$]

$$\frac{dP}{dx} = (15-x)^2(3x^2) + x^3[-30+2x]$$

$= 3\times[15^2 - 2\times(15)\times(x) + x^2]\, x^2 + x^3(2x-30)$

$= x^2[3\times(225 - 30x + x^2) + x(2x - 30)]$

$= x^2[675 - 90x + 3x^2 + 2x^2 - 60x]$

$= x^2[5x^2 - 120x + 675]$

$= 5x^2[x^2 - 24x + 135]$ ----- (3)

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dP}{dx} = 5x^2[x^2 - 24x + 135] = 0$$

Hence $5x^2 = 0$ (or) $x^2 - 24x + 135 = 0$

$x = 0$ (or) $x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(1)(135)}}{2 \times 1}$

$x = 0$ (or) $x = \frac{24 \pm \sqrt{576 - 540)}}{2}$

$x = 0$ (or) $x = \frac{24 \pm \sqrt{36}}{2}$

$x = 0$ (or) $x = \frac{24 \pm 6}{2}$

$x = 0$ (or) $x = \frac{24+6}{2}$ (or) $x = \frac{24-6}{2}$

$x = 0$ (or) $x = \frac{30}{2}$ (or) $x = \frac{18}{2}$

$x = 0$ (or) $x = 15$ (or) $x = 9$

Now considering the critical values of $x = 0,9,15$

Now, for checking if the value of P is maximum or minimum at $x=0,9,15$, we will perform the second differentiation and check the value of $\frac{d^2P}{dx^2}$ at the critical value $x = 0,9,15$.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2P}{dx^2} = \frac{d}{dx}[5x^2(x^2 - 24x + 135)]$$

$$\frac{d^2P}{dx^2} = (x^2 - 24x + 135)\frac{d}{dx}[5x^2] + 5x^2\frac{d}{dx}[x^2 - 24x + 135]$$

$= (x^2 – 24x + 135)(5 \times 2x) + 5x^2(2x – 24 + 0)$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x, then $\frac{d}{dx}(u \times v) = v \times \frac{d}{dx}(u) + u \times \frac{d}{dx}(v)$]

$= (x^2 – 24x + 135)(10x) + 5x^2(2x – 24)$

$= 10x^3 – 240x^2 + 1350x + 10x^3 – 120x^2$

$= 20x^3 – 360x^2 + 1350x$

$= 5x(4x^2 – 72x + 270)$

$$\frac{d^2P}{dx^2} = 5x(4x^2 - 72x + 270)$$

Now when x = 0,

$$\left[\frac{d^2P}{dx^2}\right]_{x=0} = 5 \times 0[4(0)^2 - 72(0) + 270]$$

$= 0$

So, we reject x = 0

Now when x = 15,

$$\left[\frac{d^2P}{dx^2}\right]_{x=15} = 5 \times 15[4(15)^2 - 72(15) + 270]$$

= 65 [(4 × 225) –1080+ 270]

= 65 [900– 1080+ 270]

= 65 [1170– 1080]

= 65× (90) > 0

Hence $\left[\frac{d^2P}{dx^2}\right]_{x=15} > 0$, so at x = 15, the function P is minimum

Now when x = 9,

$$\left[\frac{d^2P}{dx^2}\right]_{x=9} = 5 \times 9\,[4(9)^2 - 72\,(9) + 270]$$

= 45 [(4 × 81) – 648 + 270]

= 45 [324 – 648 + 270]

= 45 [594 – 648]

= 45 × (-54)

= -2430 < 0

As second differential is negative, hence at the critical point x = 9 will be the maximum point of the function P.

Therefore, the function P is maximum at x = 9.

From Equation (1), if x= 9

y = 15 – 9 = 6

Therefore, x = 9 and y = 6 are the two positive numbers whose sum is 15 and the product of the square of one number and cube of another number is maximum.

Q4.

the number 8 is divided into two numbers.

• the product of the square of one number and cube of another number is minimum.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : $x + y = 8$

• Product of square of the one number and cube of anther number :

$S = x^3 + y^2$

Now as,

$x + y = 8$

$y = (8-x)$ ------ (1)

Consider,

$S = x^3 + y^2$

By substituting (1), we have

$S = x^3 + (8-x)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with x

$\frac{dS}{dx} = \frac{d}{dx}[x^3 + (8-x)^2]$

$\frac{dS}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}[(8-x)^2]$

$\frac{dS}{dx} = (3x^2) + 2(8-x)(-1)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dS}{dx} = 3x^2 - 16 + 2x$

$= 3x^2 + 2x - 16$ ------ (3)

Now equating the first derivative to zero will give the critical point c.

So,

$\frac{dS}{dx} = 3x^2 + 2x - 16 = 0$

Hence $3x^2 + 2x - 16 = 0$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-16)}}{2 \times 3}$

$= \frac{-2 \pm \sqrt{4 + 192}}{6}$

$= \frac{-2 \pm \sqrt{196}}{6}$

$x = \frac{-2 \pm 14}{6}$

$x = \frac{-2+14}{6}$ (or) $x = \frac{-2-14}{6}$

$x = \frac{12}{6}$ (or) $x = \frac{-16}{6}$

x = 2 (or) x = -2.67

Now considering the critical values of x = 2,-2.67

Now, for checking if the value of P is maximum or minimum at x=2,-2.67, we will perform the second differentiation and check the value of $\frac{d^2S}{dx^2}$ at the critical value x = 2,-2.67.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[3x^2 + 2x - 16]$$

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[3x^2] + \frac{d}{dx}[2x] - \frac{d}{dx}[16]$$

= 3 (2x) + 2 (1) - 0

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

= 6x + 2

$$\frac{d^2S}{dx^2} = 6x + 2$$

Now when x = -2.67,

$$\left[\frac{d^2S}{dx^2}\right]_{x=-2.67} = 6(-2.67) + 2$$

= - 16.02 + 2 = -14.02

At x = -2.67 $\frac{d^2S}{dx^2} = -14.02 < 0$ hence, the function S will be maximum at this point.

Now consider x = 2,

$$\left[\frac{d^2S}{dx^2}\right]_{x=2} = 6\,(2) + 2$$

= 12 + 2 = 14

Hence $\left[\frac{d^2S}{dx^2}\right]_{x=2} = 14 > 0$, so at x = 2, the function S is minimum

As second differential is positive, hence at the critical point x = 2 will be the maximum point of the function S.

Therefore, the function S is maximum at x = 2.

From Equation (1), if x= 2

y = 8 – 2 = 6

Therefore, x = 2 and y = 6 are the two positive numbers whose sum is 8 and the sum of the square of one number and cube of another number is maximum.

Q5.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : $x + y = a$

• Product of square of the one number and cube of anther number :

$P = x^p y^q$

Now as,

$x + y = a$

$y = (a-x)$ ------ (1)

Consider,

$P = x^p y^q$

By substituting (1), we have

$P = x^p \times (a-x)^q$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with x

$$\frac{dP}{dx} = \frac{d}{dx}[x^p \times (a-x)^q]$$

$$\frac{dP}{dx} = (a-x)^q \frac{d}{dx}(x^p) + x^p \frac{d}{dx}[(a-x)^q]$$

$$\frac{dP}{dx} = (a-x)^q (px^{p-1}) + x^p [q(a-x)^{q-1}(-1)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x,

then $\frac{d}{dx}(u \times v) = v \times \frac{d}{dx}(u) + u \times \frac{d}{dx}(v)$]

$\frac{dP}{dx} = x^{p-1}(a-x)^{q-1}[(a-x)p - xq]$

$= x^{p-1}(a-x)^{q-1}[ap-xp-xq]$

$= x^{p-1}(a-x)^{q-1}[ap - x\ (p+q)]$ ----- (3)

Now equating the first derivative to zero will give the critical point c.

So,

$\frac{dP}{dx} = x^{p-1}(a-x)^{q-1}[ap - x\,(p+q)] = 0$

Hence $x^{p-1} = 0$ (or) $(a-x)^{q-1}$ (or) $ap - x(p+q) = 0$

$x = 0$ (or) $x = a$ (or) $x = \frac{ap}{p+q}$

Now considering the critical values of $x = 0, a$ and $x = \frac{ap}{p+q}$

Now, using the First Derivative test,

For f, a continuous function which has a critical point c, then, function has the local maximum at c, if f'(x) changes the sign from positive to negative as x increases through c, i.e. f'(x)>0 at every point close to the left of c and f'(x)<0 at every point close to the right of c.

Now when x = 0,

$$\left[\frac{dP}{dx}\right]_{x=0} = 0$$

So, we reject x = 0

Now when x = a,

$$\left[\frac{dP}{dx}\right]_{x=a} = 0$$

Hence we reject x = a

Now when $x < \frac{ap}{p+q}$,

$$\left[\frac{dP}{dx}\right]_{x<\frac{ap}{p+q}} = \left(\frac{ap}{p+q}\right)^{p-1}\left(a - \frac{ap}{p+q}\right)^{q-1}\left[ap - \frac{ap}{p+q}(p+q)\right] > 0 \text{ ---- (4)}$$

Now when $x > \frac{ap}{p+q}$,

$$\left[\frac{dP}{dx}\right]_{x>\frac{ap}{p+q}} = \left(\frac{ap}{p+q}\right)^{p-1}\left(a - \frac{ap}{p+q}\right)^{q-1}\left[ap - \frac{ap}{p+q}(p+q)\right] < 0 \text{ ---- (5)}$$

By using first derivative test, from (4) and (5), we can conclude that, the function P has local maximum at $x = \frac{ap}{p+q}$

From Equation (1), if $x = \frac{ap}{p+q}$

$$y = a - \frac{ap}{p+q} = \frac{a(p+q) - ap}{p+q} = \frac{ap}{p+q}$$

Therefore, $x = \frac{ap}{p+q}$ and $y = \frac{aq}{p+q}$ are the two positive numbers whose sum together to give the number ‘a’ and whose product of the pth power of one number and qth power of the other number is maximum.

Q6.

Rate of working of an engine R, v is the speed of the engine:

$R = 15v + \frac{6000}{v}$, where 0<v<30

For finding the maximum/ minimum of given function, we can find it by differentiating it with v and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Now, differentiating the function R with respect to v.

$$\frac{dR}{dv} = \frac{d}{dv}\left[15v + \frac{6000}{v}\right]$$

$$\frac{dR}{dv} = \frac{d}{dv}[15v] + \frac{d}{dv}\left[\frac{6000}{v}\right]$$

$$\frac{dR}{dv} = 15 + \left[\frac{6000}{v^2}\right](-1) = 15 - \frac{6000}{v^2} \text{ ----- (1)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Equating equation (1) to zero to find the critical value.

$$\frac{dR}{dv} = 15 - \frac{6000}{v^2} = 0$$

$$15 = \frac{6000}{v^2}$$

$$v^2 = \frac{6000}{15} = 400$$

$v^2 = 400$

$v = \pm\sqrt{400}$

v = 20 (or) v = -20

As given in the question 0<v<30, v = 20

Now, for checking if the value of R is maximum or minimum at v=20, we will perform the second differentiation and check the value of $\frac{d^2R}{dv^2}$ at the critical value v = 20.

Differentiating Equation (1) with respect to v again:

$$\frac{d^2R}{dv^2} = \frac{d}{dx}\left[15 - \frac{6000}{v^2}\right]$$

$$= \frac{d}{dx}[15] - \frac{d}{dx}\left[\frac{6000}{v^2}\right]$$

$$= 0 - (-2)\left[\frac{6000}{v^3}\right]$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$= 2\left[\frac{6000}{v^3}\right]$$

$\frac{d^2R}{dv^2} = \left[\frac{12000}{v^3}\right]$ ----- (2)

Now find the value of $\left(\frac{d^2R}{dv^2}\right)_{v=20}$

$$\left(\frac{d^2R}{dv^2}\right)_{v=20} = \left[\frac{12000}{(20)^3}\right] = \frac{12000}{20 \times 20 \times 20} = \frac{3}{2} > 0$$

So, at critical point v = 20. The function R is at its minimum.

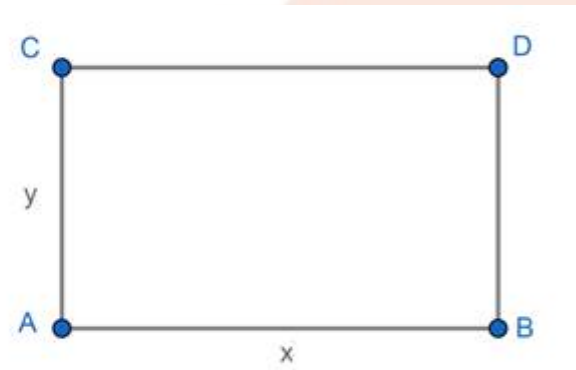
Hence, the function R is at its minimum at v = 20.

Q7.

• Area of the rectangle is 93 cm^2.

• The perimeter of the rectangle is also fixed.

Let us consider,



• x and y be the lengths of the base and height of the rectangle.

• Area of the rectangle = A = x × y = 96 cm^2

• Perimeter of the rectangle = P = 2 (x + y)

As,

x × y = 96

$y = \frac{96}{x}$ ------ (1)

Consider the perimeter function,

P = 2 (x + y)

Now substituting (1) in P,

$P = 2\left(x + \frac{96}{x}\right)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dP}{dx} = \frac{d}{dx}\left[2\left(x + \frac{96}{x}\right)\right]$$

$$\frac{dP}{dx} = \frac{d}{dx}(2x) + 2\frac{d}{dx}\left(\frac{96}{x}\right)$$

$$\frac{dP}{dx} = 2\,(1) + 2\left(\frac{96}{x^2}\right)(-1)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right)$ ----- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right) = 0$$

$$2 = \left(\frac{192}{x^2}\right)$$

$$x^2 = \left(\frac{192}{2}\right) = 96$$

$$x = \sqrt{96}$$

$$x = \pm 4\sqrt{6}$$

As the length and breadth of a rectangle cannot be negative,

hence $x = 4\sqrt{6}$

Now to check if this critical point will determine the least perimeter, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[2 - \left(\frac{192}{x^2}\right)\right]$$

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(2) - \frac{d}{dx}\left(\frac{192}{x^2}\right)$$

$$\frac{d^2P}{dx^2} = 0 - (-2)\left(\frac{192}{x^3}\right)$$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{d^2P}{dx^2} = \left(\frac{2 \times 192}{x^3}\right)$ ------ (4)

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}}$

$$\frac{d^2P}{dx^2} = \left(\frac{2 \times 192}{(4\sqrt{6})^3}\right)$$

$$= \left(\frac{2 \times 192}{4\sqrt{6} \times 4\sqrt{6} \times 4\sqrt{6}}\right)$$

$$= \left(\frac{2 \times 192}{4\sqrt{6} \times 4\sqrt{6} \times 4\sqrt{6}}\right) = \frac{1}{\sqrt{6}}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}} = \frac{1}{\sqrt{6}} > 0$, so the function P is minimum at $x = 4\sqrt{6}$.

Now substituting $x = 4\sqrt{6}$ in equation (1):

$$y = \frac{96}{4\sqrt{6}}$$

$$y = \frac{96\sqrt{6}}{4 \times 6}$$

[By rationalizing he numerator and denominator with $\sqrt{6}$]

$\therefore y = 4\sqrt{6}$

Hence, area of the rectangle with sides of a rectangle

with $x = 4\sqrt{6}$ and $y = 4\sqrt{6}$ is 96cm^2 and has the least perimeter.

Now the perimeter of the rectangle is

$P = 2(4\sqrt{6} + 4\sqrt{6}) = 2(8\sqrt{6}) = 16\sqrt{6}$ cms

The least perimeter is $16\sqrt{6}$ cms.

Q8.

• Rectangle with given perimeter.

Let us consider,

• 'p' as the fixed perimeter of the rectangle.

• 'x' and 'y' be the sides of the given rectangle.

• Area of the rectangle, $A = x \times y$.

Now as consider the perimeter of the rectangle,

$p = 2(x + y)$

$p = 2x + 2y$

$y = \frac{p-2x}{2}$ ----- (1)

Consider the area of the rectangle,

A = x × y

Substituting (1) in the area of the rectangle,

$A = x \times \left(\frac{p-2x}{2}\right)$

$A = \frac{1}{2} \times (px - 2x^2)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{1}{2}(px - 2x^2)\right]$$

$$\frac{dA}{dx} = \frac{1}{2}\frac{d}{dx}(px) - \frac{1}{2}\frac{d}{dx}(2x^2)$$

$$\frac{dA}{dx} = \frac{1}{2}(p) - \frac{2}{2}(2x)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dA}{dx} = \frac{p}{2} - (2x)$ ----- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dA}{dx} = \frac{p}{2} - (2x) = 0$$

$$2x = \frac{p}{2}$$

$x = \frac{p}{4}$

Now to check if this critical point will determine the largest rectangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{p}{2} - (2x)\right]$$

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{p}{2}\right) - \frac{d}{dx}(2x)$$

$$\frac{d^2A}{dx^2} = 0 - 2 = -2$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{d^2A}{dx^2} = -2$ ------ (4)

Now, consider the value of $\left(\frac{d^2A}{dx^2}\right)_{x=\frac{p}{4}}$

$$\frac{d^2A}{dx^2} = -2 < 0$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=\frac{p}{4}} = -2 < 0$, so the function P is maximum at $x = \frac{p}{4}$.

Now substituting $x = \frac{p}{4}$ in equation (1):

$$y = \frac{p - 2\left(\frac{p}{4}\right)}{2}$$

$$y = \frac{p - \frac{p}{2}}{2} = \frac{p}{4}$$

$\therefore y = \frac{p}{4}$

As $= y = \frac{p}{4}$ the sides of the taken rectangle are equal, we can clearly say that a largest rectangle which has a given perimeter is a square.

Q9.

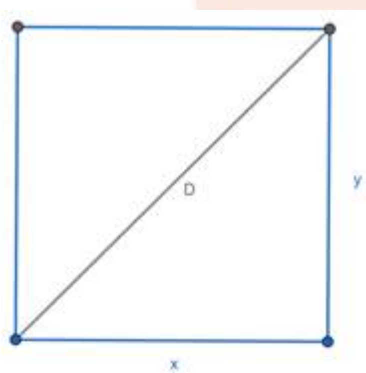
• Rectangle with given perimeter.

Let us consider,

• 'p' as the fixed perimeter of the rectangle.

• 'x' and 'y' be the sides of the given rectangle.

• Diagonal of the rectangle, $D = \sqrt{x^2 + y^2}$. (using the hypotenuse formula)



Now as consider the perimeter of the rectangle,

p = 2(x +y)

p = 2x + 2y

$y = \frac{p-2x}{2}$ ----- (1)

Consider the diagonal of the rectangle,

$D = \sqrt{x^2 + y^2}$

Substituting (1) in the diagonal of the rectangle,

$$D = \sqrt{x^2 + \left(\frac{p-2x}{2}\right)^2}$$

[squaring both sides]

$Z = D^2 = x^2 + \left(\frac{p-2x}{2}\right)^2$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dZ}{dx} = \frac{d}{dx}\left[x^2 + \left(\frac{p-2x}{2}\right)^2\right]$$

$$\frac{dZ}{dx} = \frac{d}{dx}(x^2) + \frac{1}{4}\frac{d}{dx}[(p-2x)^2]$$

$$\frac{dZ}{dx} = 2x + \frac{1}{4}[2(p-2x)(-2)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

= 2x - p + 2x

$\frac{dZ}{dx} = 4x - p$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dZ}{dx} = 4x - p = 0$

$4x - p = 0$

$4x = p$

$x = \frac{p}{4}$

Now to check if this critical point will determine the minimum diagonal, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$\frac{d^2Z}{dx^2} = \frac{d}{dx}[4x - p]$

$\frac{d^2Z}{dx^2} = \frac{d}{dx}(4x) - \frac{d}{dx}(p)$

$= 4 + 0$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{d^2Z}{dx^2} = 4$ ------ (4)

Now, consider the value of $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{p}{4}}$

$\frac{d^2Z}{dx^2} = 4 > 0$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{p}{4}} = 4 > 0$, so the function Z is minimum at $x = \frac{p}{4}$.

Now substituting $x = \frac{p}{4}$ in equation (1):

$$y = \frac{p - 2\left(\frac{p}{4}\right)}{2}$$

$$y = \frac{p - \frac{p}{2}}{2} = \frac{p}{4}$$

$$\therefore\ y = \frac{p}{4}$$

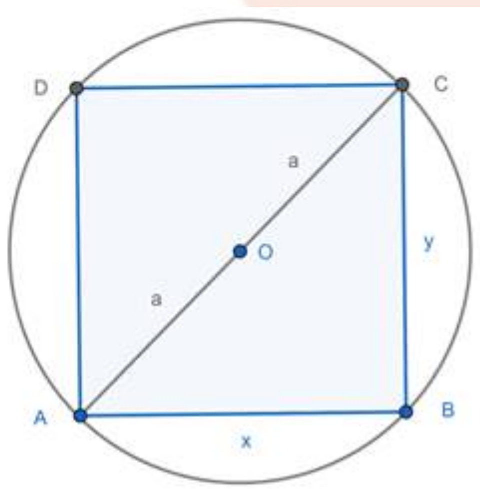
As $x = y = \frac{p}{4}$ the sides of the taken rectangle are equal, we can clearly say that a rectangle with minimum diagonal which has a given perimeter is a square.

Q10.

Rectangle is of maximum perimeter.

The rectangle is inscribed inside a circle.

The radius of the circle is ‘a’.



Let us consider,

• ‘x’ and ‘y’ be the length and breadth of the given rectangle.

• Diagonal $AC^2 = AB^2 + BC^2$ is given by $4a^2 = x^2+y^2$ (as AC = 2a)

• Perimeter of the rectangle, P = 2(x+y)

Consider the diagonal,

$4a^2 = x^2 + y^2$

$y^2 = 4a^2 - x^2$

$y = \sqrt{4a^2 - x^2}$ ---- (1)

Now, perimeter of the rectangle, P

P = 2x + 2y

Substituting (1) in the perimeter of the rectangle.

$P = 2x + 2\sqrt{4a^2 - x^2}$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to x:

$$\frac{dP}{dx} = \frac{d}{dx}\left[2x + 2\sqrt{4a^2 - x^2}\right]$$

$$\frac{dP}{dx} = \frac{d}{dx}(2x) + 2\frac{d}{dx}[\sqrt{4a^2 - x^2}]$$

$$\frac{dP}{dx} = 2 + 2[\frac{1}{2}(4a^2 - x^2)^{-\frac{1}{2}}(-2x)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dP}{dx} = 2 - \frac{2x}{\sqrt{4a^2 - x^2}}$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dP}{dx} = 2 - \frac{2x}{\sqrt{4a^2 - x^2}} = 0$$

$$2 = \frac{2x}{\sqrt{4a^2 - x^2}}$$

$$\sqrt{4a^2 - x^2} = x$$

[squaring on both sides]

$4a^2 - x^2 = x^2$

$2x^2 = 4a^2$

$x^2 = 2a^2$

$$x = \pm a\sqrt{2}$$

$$x = a\sqrt{2}$$

[as x cannot be negative]

<u>Now to check if this critical point will determine the maximum diagonal, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[2 - \frac{2x}{\sqrt{4a^2 - x^2}}\right]$$

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(2) - \frac{d}{dx}\left(\frac{2x}{\sqrt{4a^2 - x^2}}\right)$$

$$\frac{d^2P}{dx^2} = 0 - \left[\frac{\sqrt{4a^2 - x^2}\,\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(\sqrt{4a^2 - x^2})}{(\sqrt{4a^2 - x^2})^2}\right]$$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x, then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$]

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{4a^2 - x^2}\,(2) - (2x)\frac{1}{2}(4a^2 - x^2)^{-\frac{1}{2}}(-2x)}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{4a^2 - x^2}\,(2) + (2x^2)(4a^2 - x^2)^{-\frac{1}{2}}}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{2\sqrt{4a^2 - x^2} + \frac{2x^2}{\sqrt{4a^2 - x^2}}}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{2\,(4a^2 - x^2) + 2x^2}{(4a^2 - x^2)^{\frac{3}{2}}}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{8a^2}{(4a^2 - x^2)^{\frac{3}{2}}}\right] \text{ ----- } (4)$$

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}}$

$$\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\left[\frac{8a^2}{(4a^2 - (a\sqrt{2})^2)^{\frac{3}{2}}}\right]$$

$$\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\left[\frac{8a^2}{(4a^2 - 2a^2)^{\frac{3}{2}}}\right] = -\frac{8a^2}{(2a^2)^{\frac{3}{2}}} = -\frac{8a^2}{2\sqrt{2}\,a^3} = -\frac{2\sqrt{2}}{a}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\frac{2\sqrt{2}}{a} < 0$, so the function P is maximum at $x = a\sqrt{2}$.

Now substituting $x = a\sqrt{2}$ in equation (1):

$y = \sqrt{4a^2 - (a\sqrt{2})^2}$

$y = \sqrt{4a^2 - 2a^2} = \sqrt{2a^2}$

$\therefore\ y = a\sqrt{2}$

As $x = y = a\sqrt{2}$ the sides of the taken rectangle are equal, we can clearly say that a rectangle with maximum perimeter which is inscribed inside a circle of radius 'a' is a square.

Q11.

• Sum of perimeter of square and circle.

Let us consider,

• 'x' be the side of the square.

• 'r' be the radius of the circle.

• Let 'p' be the sum of perimeters of square and circle.

p = 4x + 2πr

Consider the sum of the perimeters of square and circle.

p = 4x + 2πr

4x = p – 2πr

$x = \frac{p-2\pi r}{4}$ ---- (1)

Sum of the area of the circle and square is

$A = x^2 + \pi r^2$

Substituting (1) in the sum of the areas,

$$A = \left(\frac{p - 2\pi r}{4}\right)^2 + \pi r^2$$

$$A = \frac{1}{16}[p^2 + 4\pi^2 r^2 - 4\pi pr] + \pi r^2 \text{ ------ (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to r:

$$\frac{dA}{dr} = \frac{d}{dr}\left[\frac{1}{16}[p^2 + 4\pi^2 r^2 - 4\pi pr] + \pi r^2\right]$$

$$\frac{dA}{dr} = \frac{1}{16}\frac{d}{dr}(p^2 + 4\pi^2 r^2 - 4\pi pr) + \frac{d}{dr}[\pi r^2]$$

$$\frac{dA}{dr} = \frac{1}{16}(0 + 8\pi^2 r - 4\pi p) + 2\pi r$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$]

$$\frac{dA}{dr} = \frac{\pi^2 r}{2} - \frac{\pi p}{4} + 2\pi r \text{ ------ (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dA}{dr} = \frac{\pi^2 r}{2} - \frac{\pi p}{4} + 2\pi r = 0$$

$$\left(\frac{\pi^2}{2} + 2\pi\right)r - \frac{\pi p}{4} = 0$$

$$r = \frac{\frac{\pi p}{4}}{\frac{\pi^2}{2} + 2\pi} = \frac{2\pi p}{4(\pi^2 + 4\pi)} = \frac{\pi p}{2(\pi^2 + 4\pi)}$$

$$r = \frac{\pi p}{2\pi(\pi + 4)} = \frac{p}{2(\pi + 4)}$$

$$r = \frac{p}{2(\pi + 4)}$$

<u>Now to check if this critical point will determine the least of the sum of the areas of square and circle, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2A}{dr^2} = \frac{d}{dx}\left[\frac{\pi^2 r}{2} - \frac{\pi p}{4} + 2\pi r\right]$$

$$\frac{d^2A}{dr^2} = \frac{d}{dr}\left(\frac{\pi^2 r}{2}\right) - \frac{d}{dr}\left(\frac{\pi p}{4}\right) + \frac{d}{dr}(2\pi r)$$

$$\frac{d^2A}{dr^2} = \frac{\pi^2}{2} - 0 + 2\pi$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{d^2A}{dr^2} = \frac{\pi^2}{2} + 2\pi$ ----- (4)

Now, consider the value of $\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}}$

$$\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}} = \frac{\pi^2}{2} + 2\pi$$

As $\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}} = \frac{\pi^2}{2} + 2\pi > 0$, so the function A is minimum at $r = \frac{p}{2(\pi+4)}$.

Now substituting $r = \frac{p}{2(\pi + 4)}$ in equation (1):

$$x = \frac{p - 2\pi\left(\frac{p}{2(\pi + 4)}\right)}{4}$$

$$x = \frac{p(\pi + 4) - \pi p}{4 \times (\pi + 4)} = \frac{\pi p + 4p - \pi p}{4\pi + 16} = \frac{4p}{4(\pi + 4)}$$

$$x = \frac{p}{\pi + 4}$$

As the side of the square,

$$x = \frac{p}{\pi + 4}$$

$$x = 2\left[\frac{p}{2(\pi + 4)}\right] = 2r$$

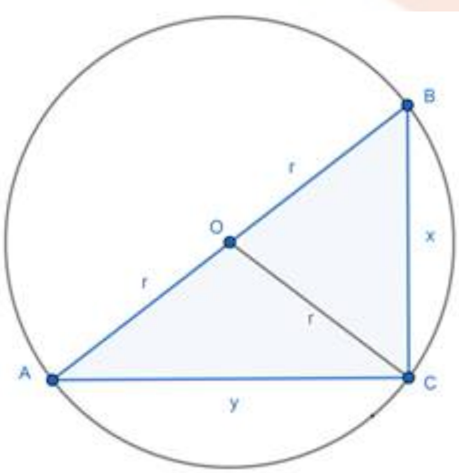
[as $r = \frac{p}{2(\pi+4)}$]

Therefore, side of the square, x = 2r = diameter of the circle.

Q12.

• A right angle triangle is inscribed inside the circle.

• The radius of the circle is given.



Let us consider,

• 'r' is the radius of the circle.

• 'x' and 'y' be the base and height of the right angle triangle.

• The hypotenuse of the $\Delta ABC = AB^2 = AC^2 + BC^2$

$AB = 2r$, $AC = y$ and $BC = x$

Hence,

$4r^2 = x^2 + y^2$

$y^2 = 4r^2 - x^2$

$y = \sqrt{4r^2 - x^2}$ --- (1)

Now, Area of the ΔABC is

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A = \frac{1}{2} \times x \times y$$

Now substituting (1) in the area of the triangle,

$$A = \frac{1}{2} x (\sqrt{4r^2 - x^2})$$

[Squaring both sides]

$Z = A^2 = \frac{1}{4} x^2 (4r^2 - x^2)$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is

<u>because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to x:

$$\frac{dZ}{dx} = \frac{d}{dx}\left[\frac{1}{4}x^2(4r^2 - x^2)\right]$$

$$\frac{dZ}{dx} = \frac{1}{4}\left[(4r^2 - x^2)\frac{d}{dx}(x^2) + x^2\frac{d}{dx}(4r^2 - x^2)\right]$$

$$\frac{dZ}{dx} = \frac{1}{4}[(4r^2 - x^2) \times (2x) + x^2(0 - 2x)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x,

then $\frac{d}{dx}(u.v) = v\frac{du}{dx} + u\frac{dv}{dx}$]

$$\frac{dZ}{dx} = \frac{1}{4}[8r^2x - 2x^3 - 2x^3]$$

$$\frac{dZ}{dx} = \frac{1}{4}[8r^2x - 4x^3] = \frac{4x}{4}[2r^2 - x^2]$$

$\frac{dZ}{dx} = 2r^2x - x^3$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{dx} = 2r^2x - x^3 = 0$$

$$2r^2x = x^3$$

$$x^2 = 2r^2$$

$$x = \pm\sqrt{2r^2}$$

$$x = r\sqrt{2}$$

[as the base of the triangle cannot be negative.]

Now to check if this critical point will determine the maximum area of the triangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[2r^2x - x^3]$$

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}(2r^2x) - \frac{d}{dx}(x^3)$$

$$\frac{d^2Z}{dx^2} = 2r^2 - 3x^2 \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now, consider the value of $\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}}$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}} = 2r^2 - 3(r\sqrt{2})^2 = 2\,r^2 - 6r^2 = -4r^2$$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}} = -4r^2 < 0$, so the function A is maximum at $x = r\sqrt{2}$.

Now substituting $x = r\sqrt{2}$ in equation (1):

$$y = \sqrt{4r^2 - (r\sqrt{2})^2}$$

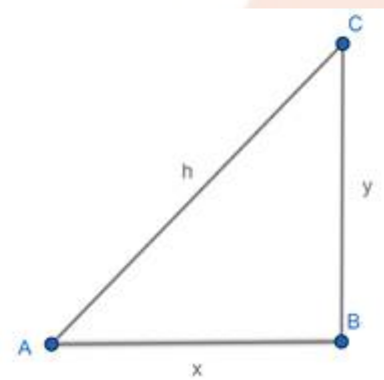
$$y = \sqrt{4r^2 - 2r^2} = \sqrt{2r^2} = r\sqrt{2}$$

As $x = y = r\sqrt{2}$, the base and height of the triangle are equal, which means that two sides of a right angled triangle are equal, Hence the given triangle, which is inscribed in a circle, is an isosceles triangle with sides AC and BC equal.

Q13.

• A right angle triangle.

• Hypotenuse of the given triangle is given



Let us consider,

• 'h' is the hypotenuse of the given triangle.

• 'x' and 'y' be the base and height of the right angle triangle.

• The hypotenuse of the $\Delta ABC = AC^2 = AB^2 + BC^2$

$AC = h$, $AB = x$ and $BC = y$

Hence,

$h^2 = x^2 + y^2$

$y^2 = h^2 - x^2$

$y = \sqrt{h^2 - x^2}$ --- (1)

Now, perimeter of the ΔABC is

P = h + x + y

Now substituting (1) in the area of the triangle,

$P = h + x + \sqrt{h^2 - x^2}$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dP}{dx} = \frac{d}{dx}\left[h + x + \sqrt{h^2 - x^2}\right]$$

$$\frac{dP}{dx} = \left[\frac{d}{dx}(h) + \frac{d}{dx}(x) + \frac{d}{dx}(\sqrt{h^2 - x^2})\right]$$

$$\frac{dP}{dx} = 0 + 1 + \frac{1}{2}\left(\frac{-2x}{\sqrt{h^2 - x^2}}\right)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dP}{dx} = 1 - \frac{x}{\sqrt{h^2 - x^2}}$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dP}{dx} = 1 - \frac{x}{\sqrt{h^2 - x^2}} = 0$$

$$\frac{x}{\sqrt{h^2 - x^2}} = 1$$

$$x = \sqrt{h^2 - x^2}$$

[squaring on both sides]

$x^2 = h^2 - x^2$

$x^2 = \frac{h^2}{2}$

$x = \pm\sqrt{\frac{h^2}{2}}$

$x = \frac{h}{\sqrt{2}}$

[as the base of the triangle cannot be negative.]

<u>Now to check if this critical point will determine the maximum perimeter of the triangle, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[1 - \frac{x}{\sqrt{h^2 - x^2}}\right]$$

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(1) - \frac{d}{dx}\left(\frac{x}{\sqrt{h^2 - x^2}}\right)$$

$$\frac{d^2P}{dx^2} = 0 - \left[\frac{\sqrt{h^2 - x^2}\,\frac{d}{dx}(x) - x\,\frac{d}{dx}\left(\sqrt{h^2 - x^2}\right)}{\left(\sqrt{h^2 - x^2}\right)^2}\right]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ if u and v are two functions of x,

then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$]

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{h^2 - x^2}\,(1) - x\left(\frac{-2x}{2\sqrt{h^2 - x^2}}\right)}{h^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{(\sqrt{h^2-x^2})^2+x^2}{h^2-x^2\sqrt{h^2-x^2}}\right] = -\left[\frac{h^2}{(h^2-x^2)\sqrt{h^2-x^2}}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{h^2}{(h^2-x^2)^{\frac{3}{2}}}\right]$$

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=\frac{h}{\sqrt{2}}}$

$$\left(\frac{d^2P}{dx^2}\right)_{x=\frac{h}{\sqrt{2}}} = -\left[\frac{h^2}{(h^2-\left(\frac{h}{\sqrt{2}}\right)^2)^{\frac{3}{2}}}\right] = -\left[\frac{h^2}{(\frac{h^2}{2})^{\frac{3}{2}}}\right] = -\left[\frac{h^2}{(\frac{h^2}{2})^{\frac{3}{2}}}\right] = -\frac{2^{\frac{3}{2}}}{h}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=\frac{h}{\sqrt{2}}} = -\frac{2^{\frac{3}{2}}}{h} < 0$, so the function A is maximum at $x = \frac{h}{\sqrt{2}}$.

Now substituting $x = \frac{h}{\sqrt{2}}$ in equation (1):

$$y = \sqrt{h^2 - \left(\frac{h}{\sqrt{2}}\right)^2}$$

$$y = \sqrt{\frac{h^2}{2}} = \frac{h}{\sqrt{2}}$$

As $x = y = \frac{h}{\sqrt{2}}$, the base and height of the triangle are equal, which means that two sides of a right angled triangle are equal,

Hence the given triangle is an isosceles triangle with sides AB and BC equal.

Q14.

Let us consider,

• ‘x’ and ‘y’ be the other two sides of the triangle.

Now, perimeter of the ΔABC is

8 = 3 + x + y

y = 8-3-x = 5-x

y = 5-x --- (1)

Consider the Heron’s area of the triangle,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where $s = \frac{a+b+c}{2}$

As perimeter = a + b+ c = 8

$$s = \frac{8}{2} = 4$$

Now Area of the triangle is given by

$$A = \sqrt{8\,(8-3)(8-x)(8-y)}$$

Now substituting (1) in the area of the triangle,

$$A = \sqrt{4(4-3)(4-x)(4-(5-x))}$$

$$A = \sqrt{4(4-x)(x-1)}$$

$$A = \sqrt{4(\,4x-4-x^2+x)} = \sqrt{4(\,5x-x^2-4)}$$

$$A = \sqrt{4(\,5x-x^2-4)}$$

[squaring on both sides]

$Z = A^2 = 4(5x - x^2 - 4)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to x:

$$\frac{dZ}{dx} = \frac{d}{dx}[4(5x - x^2 - 4)]$$

$$\frac{dZ}{dx} = 4\frac{d}{dx}(5x) - 4\frac{d}{dx}(x^2) - 4\frac{d}{dx}(4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dZ}{dx} = 4(5) - 4(2x) - 0$$

$\frac{dZ}{dx} = 20 - 8x$------ (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{dx} = 20 - 8x = 0$$

20 – 8x = 0

8x = 20

$$x = \frac{5}{2}$$

<u>Now to check if this critical point will determine the maximum area of the triangle, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with x:

$\frac{d^2Z}{dx^2} = \frac{d}{dx}[20 - 8x]$

$\frac{d^2Z}{dx^2} = -8$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{5}{2}} = -8 < 0$, so the function A is maximum at $x = \frac{5}{2}$.

Now substituting $x = \frac{5}{2}$ in equation (1):

y = 5 – 2.5

y = 2.5

As x = y = 2.5, two sides of the triangle are equal,

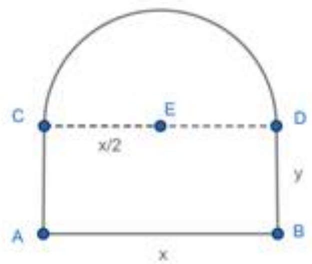
Hence the given triangle is an isosceles triangle with two sides equal to 2.5 cm and the third side equal to 3cm.

Q15.

• Window is in the form of a rectangle which has a semicircle mounted on it.

• Total Perimeter of the window is 10 metres.

• The total area of the window is maximum.



Let us consider,

• The breadth and height of the rectangle be ‘x’ and ‘y’.

• The radius of the semicircle will be half of the base of the rectangle.

Given Perimeter of the window is 10 meters:

$$10 = (x + 2y) + \frac{1}{2}\left[2\pi\left(\frac{x}{2}\right)\right]$$

[as the perimeter of the window will be equal to one side (x) less to the perimeter of rectangle and the perimeter of the semicircle.]

$$10 = (x + 2y) + \left(\frac{\pi x}{2}\right)$$

From here,

$$2y = 10 - x - \left(\frac{\pi x}{2}\right) = \frac{20 - 2x - \pi x}{2}$$

$$y = \frac{20-2x-\pi x}{4} \text{ ----- (1)}$$

Now consider the area of the window,

Area of the window = area of the semicircle + area of the rectangle

$$A = \frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right] + xy$$

Substituting (1) in the area equation:

$$A = \frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right] + x\left(\frac{20 - 2x - \pi x}{4}\right)$$

$$A = \frac{1}{8}[\pi x^2] + \left(\frac{20x - 2x^2 - \pi x^2}{4}\right)$$

$$A = \frac{\pi x^2 - 2\pi x^2 + 40x - 4x^2}{8}$$

$$A = \frac{1}{8}[x^2(\pi - 2\pi - 4) + 40x] \text{ ------ (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{1}{8}[x^2(\pi - 2\pi - 4) + 40x]\right]$$

$$\frac{dA}{dx} = \frac{1}{8}\frac{d}{dx}(x^2(\pi - 2\pi - 4)) + \frac{1}{8}\frac{d}{dx}(40x)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dA}{dx} = \frac{1}{8}[2x(-\pi - 4)] + \frac{1}{8}(40)$$

$$\frac{dA}{dx} = \frac{1}{4}[x(-\pi - 4)] + 5 \text{ ------ (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dA}{dx} = \frac{1}{4}[x(-\pi - 4)] + 5 = 0$$

$$\frac{1}{4}[x(-\pi - 4)] + 5 = 0$$

$\frac{1}{4}[x(4+\pi)] = 5$

x (4 +π) = 20

$x = \frac{20}{(4+\pi)}$

Now to check if this critical point will determine the maximum area of the window, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}[x(-\pi-4)] + 5\right]$

$\frac{d^2A}{dx^2} = \frac{d}{dx}[x(-\pi-4)] + \frac{d}{dx}(5)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{d^2A}{dx^2} = (-\pi-4)(1) + 0 = -(\pi+4)$ ------ (4)

As $\left(\frac{d^2A}{dx^2}\right)_{x=\frac{20}{(4+\pi)}} = -(\pi+4) < 0$, so the function A is maximum at $x = \frac{20}{(4+\pi)}$.

Now substituting $x = \frac{20}{(4+\pi)}$ in equation (1):

$y = \frac{20 - \left(\frac{20}{(4+\pi)}\right)(\pi+2)}{4}$

$y = \frac{20(4+\pi) - (20)(\pi+2)}{4(4+\pi)} = \frac{20[4+\pi-\pi-2]}{4(4+\pi)} = \frac{20 \times 2}{4(4+\pi)}$

$y = \frac{5 \times 2}{(4+\pi)} = \frac{10}{(4+\pi)}$

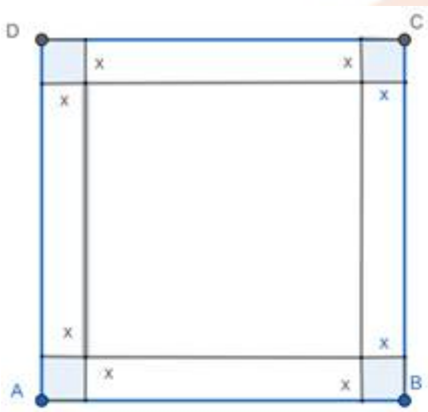
Hence the given window with maximum area has breadth,

$x = \frac{20}{(4+\pi)}$ and height, $y = \frac{10}{(4+\pi)}$

Q16.

• Side of the square piece is 12 cms.

• the volume of the formed box is maximum.



Let us consider,

• 'x' be the length and breadth of the piece cut from each vertex of the piece.

• Side of the box now will be (12-2x)

• The height of the new formed box will also be 'x'.

Let the volume of the newly formed box is :

$V = (12-2x)^2 \times (x)$

$V = (144 + 4x^2 - 48x)\ x$

$V = 4x^3 - 48x^2 + 144x$ ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (1) with respect to x:

$$\frac{dV}{dx} = \frac{d}{dx}[4x^3 - 48x^2 + 144x]$$

$$\frac{dV}{dx} = 12x^2 - 96x + 144 \text{ -------- } (2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$$

$x^2 - 8x + 12 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2}$$

$$x == \frac{8 \pm 4}{2}$$

x = 6 or x = 2

x = 2

[as x = 6 is not a possibility, because 12-2x = 12-12= 0]

Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2V}{dx^2} = \frac{d}{dx}[12x^2 - 96x + 144]$$

$\frac{d^2V}{dx^2} = 24x - 96$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dx^2}\right)_{x=2} = 24(2) - 96 = 48 - 96 = -48$$

As $\left(\frac{d^2V}{dx^2}\right)_{x=2} = -48 < 0$, so the function A is maximum at x = 2

Now substituting x = 2 in 12 – 2x, the side of the considered box:

Side = 12-2x = 12 - 2(2) = 12-4= 8cms

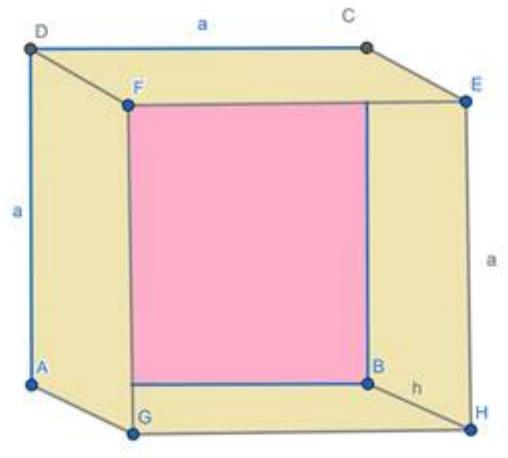
Therefore side of wanted box is 8cms and height of the box is 2cms.

Now, the volume of the box is

$V = (8)^2 \times 2 = 64 \times 2 = 128cm^3$

Hence maximum volume of the box formed by cutting the given 12cms sheet is $128cm^3$ with 8cms side and 2cms height.

Q17.



Let us consider,

• The side of the square base of the box be 'a' units. (pink coloured in the figure)

• The breadth of the 4 sides of the box will also be 'a'units (skin coloured part).

• The depth of the box or the length of the sides be 'h' units (skin coloured part).

Now, the area of the box =

(area of the base) + 4 (area of each side of the box)

So as area of the box is given c^2,

$c^2 = a^2 + 4ah$

$h = \frac{c^2 - a^2}{4a}$ ---- (1)

Let the volume of the newly formed box is :

$V = (a)^2 \times (h)$

[substituting (1) in the volume formula]

$$V = a^2 \times \left(\frac{c^2 - a^2}{4a}\right)$$

$$V = \left(\frac{ac^2 - a^3}{4}\right) \text{------ (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with a and then equating it to zero. This is because if the function f(a) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to a:

$$\frac{dV}{da} = \frac{d}{da}\left[\left(\frac{ac^2 - a^3}{4}\right)\right]$$

$$\frac{dV}{da} = \frac{c^2}{4} - \frac{3a^2}{4} \text{------- (3)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dV}{da} = \frac{c^2}{4} - \frac{3a^2}{4} = 0$$

$c^2 - 3a^2 = 0$

$$a^2 = \frac{c^2}{3}$$

$$a = \pm\sqrt{\frac{c^2}{3}}$$

$$a = \frac{c}{\sqrt{3}}$$

[as 'a' cannot be negative]

Now to check if this critical point will determine the maximum Volume of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2V}{da^2} = \frac{d}{dx}\left[\frac{c^2}{4} - \frac{3a^2}{4}\right]$$

$$\frac{d^2V}{da^2} = 0 - \frac{3\times2\times a}{4} = -\frac{3a}{2} \text{ ----- } (4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{da^2}\right)_{a=\frac{c}{\sqrt{3}}} = -\frac{3\left(\frac{c}{\sqrt{3}}\right)}{2} = -\frac{c\sqrt{3}}{2}$$

As $\left(\frac{d^2V}{da^2}\right)_{a=\frac{c}{\sqrt{3}}} = -48 - \frac{c\sqrt{3}}{2} < 0$, so the function V is maximum at $a = \frac{c}{\sqrt{3}}$

Now substituting a in equation (1)

$$h = \frac{c^2 - \left(\frac{c}{\sqrt{3}}\right)^2}{4\left(\frac{c}{\sqrt{3}}\right)} = \frac{\frac{2c^2}{3}}{\frac{4c}{\sqrt{3}}} = \frac{c\sqrt{3}}{6} = \frac{c}{2\sqrt{3}}$$

$$\therefore h = \frac{c}{2\sqrt{3}}$$

Therefore side of wanted box has a base side, $a = \frac{c}{\sqrt{3}}$ is and height of the box, $h = \frac{c}{2\sqrt{3}}$.

Now, the volume of the box is

$$V = \left(\frac{c}{\sqrt{3}}\right)^2 \times \left(\frac{c}{2\sqrt{3}}\right)$$

$$V = \frac{c^2}{3} \times \left(\frac{c}{2\sqrt{3}}\right) = \frac{c^3}{6\sqrt{3}}$$

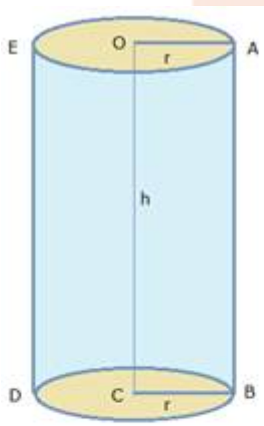
$$\therefore V = \frac{c^3}{6\sqrt{3}}$$

Q18.

• The can is cylindrical with a circular base

• The volume of the cylinder is 1 litre = 1000 cm^2.

• The surface area of the box is minimum as we need to find the minimum dimensions.



Let us consider,

• The radius base and top of the cylinder be ‘r’ units. (skin coloured in the figure)

• The height of the cylinder be ‘h’units.

• As the Volume of cylinder is given, V = $1000cm^3$

The Volume of the cylinder= $\pi r^2 h$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$ ---- (1)

The Surface area cylinder is = area of the circular base + area of the circular top + area of the cylinder

$S = \pi r^2 + \pi r^2 + 2\pi rh$

$S = 2\pi r^2 + 2\pi rh$

[substituting (1) in the volume formula]

$S = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$S = 2\left[\pi r^2 + \left(\frac{1000}{r}\right)\right]$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. <u>This is because if the function f(r) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to r:

$$\frac{dS}{dr} = \frac{d}{dr}\left[2\left[\pi r^2 + \left(\frac{1000}{r}\right)\right]\right]$$

$$\frac{dS}{dr} = 2\,(2\pi r) + \left(\frac{1000}{r^2}\right)(-1)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dS}{dr} = 2\,(2\pi r) - 2\left(\frac{1000}{r^2}\right)$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dS}{dr} = 2\,(2\pi r) - 2\left(\frac{1000}{r^2}\right) = 0$$

$$2\,(2\pi r) - 2\left(\frac{1000}{r^2}\right) = 0$$

$$2\pi r = \frac{1000}{r^2}$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

<u>Now to check if this critical point will determine the minimum surface area of the box, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2\,(2\pi r) - 2\left(\frac{1000}{r^2}\right)\right]$$

$$\frac{d^2S}{dr^2} = 4\pi - \frac{2 \times 1000 \times (-2)}{r^3} = 4\pi + \frac{4000}{r^3} \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{500}{\pi}}} = 4\pi + \frac{4000}{\left(\sqrt[3]{\frac{500}{\pi}}\right)^3} = 4\pi + \frac{4000 \times \pi}{500} = 4\pi + 8\pi = 12\pi$$

As $\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{500}{\pi}}} = 12\pi > 0$, so the function S is minimum at $r = \sqrt[3]{\frac{500}{\pi}}$

Now substituting r in equation (1)

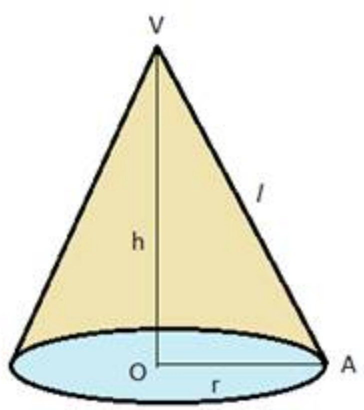
$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2} = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$$

$$\therefore\ h = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$$

Therefore the radius of base of the cylinder, $r = \sqrt[3]{\frac{500}{\pi}}$ and height of the cylinder, $h = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$ where the surface area of the cylinder is minimum.

Q19.

- The volume of the cone.
- The cone is right circular cone.
- The cone has least curved surface.



Let us consider,

- The radius of the circular base be ‘r’ cms.

• The height of the cone be ‘h’ cms.

• The slope of the cone be ‘l’ cms.

Given the Volume of the cone = $\pi r^2 l$

$$V = \frac{\pi r^2 h}{3}$$

$h = \frac{3v}{\pi r^2}$ ----- (1)

The Surface area cylinder is = πrl

$S = \pi rl$

$$S = \pi r\left(\sqrt{h^2 + r^2}\right)$$

[substituting (1) in the Surface area formula]

$$S = \pi r\left[\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}\right]$$

[squaring on both sides]

$$Z = S^2 = \pi^2 r^2\left(\frac{9V^2}{\pi^2 r^4} + r^2\right)$$

$Z = \pi^2\left(\frac{9V^2}{\pi^2 r^2} + r^4\right)$----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function Z has a maximum/minimum at a point c then Z’(c) = 0.

Differentiating the equation (2) with respect to r:

$$\frac{dZ}{dr} = \frac{d}{dr}\left[\pi^2\left(\frac{9V^2}{\pi^2 r^2} + r^4\right)\right]$$

$$\frac{dZ}{dr} = \pi^2\left(\frac{9V^2}{\pi^2}\right)\frac{d}{dr}\left(\frac{1}{r^2}\right) + \pi^2\frac{d}{dr}(r^4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dZ}{dr} = \left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3)$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{dr} = \left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3) = 0$$

$$\pi^2(4r^3) = \frac{18V^2}{r^3}$$

$2\pi^2 r^6 = 9V^2$ ---- (4)

<u>Now to check if this critical point will determine the minimum surface area of the cone, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2Z}{dr^2} = \frac{d}{dr}\left[\left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3)\right]$$

$$\frac{d^2Z}{dr^2} = \frac{-18V^2(-3)}{r^4} + \pi^2(4 \times 3r^2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{d^2Z}{dr^2} = \frac{54V^2}{r^4} + \pi^2(12r^2)$$

Now let us find the value of

$$\left(\frac{d^2Z}{dr^2}\right) = \frac{54V^2}{r^4} + \pi^2(12r^2) > 0$$

As $\left(\frac{d^2Z}{dr^2}\right) > 0$, so the function $Z = S^2$ is minimum

Now consider, the equation (4),

$$9V^2 = 2\pi^2 r^6$$

Now substitute the volume of the cone formula in the above equation.

$$9\left(\frac{\pi r^2 h}{3}\right)^2 = 2\pi^2 r^6$$

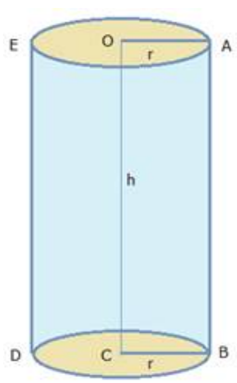
$$\pi^2 r^4 h^2 = 2\pi^2 r^6$$

$$2r^2 = h^2$$

$$h = r\sqrt{2}$$

Hence, the relation between h and r of the cone is proved when S is the minimum.

Q20.



Let us consider,

• The radius base and top of the cylinder be ‘r’ units. (skin coloured in the figure)

• The height of the cylinder be ‘h’units.

• As the Volume of cylinder is given, $V = 100cm^3$

The Volume of the cylinder= $\pi r^2 h$

$100 = \pi r^2 h$

$h = \frac{100}{\pi r^2}$ ---- (1)

The Surface area cylinder is = area of the circular base + area of the circular top + area of the cylinder

$S = \pi r^2 + \pi r^2 + 2\pi rh$

$S = 2\pi r^2 + 2\pi rh$

[substituting (1) in the volume formula]

$S = 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$

$S = 2\left[\pi r^2 + \left(\frac{100}{r}\right)\right]$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. <u>This is because if the function f(r) has a maximum/minimum at a point c then f’(c) = 0.</u>

Differentiating the equation (2) with respect to r:

$\frac{dS}{dr} = \frac{d}{dr}\left[2\left[\pi r^2 + \left(\frac{100}{r}\right)\right]\right]$

$\frac{dS}{dr} = 2(2\pi r) + \left(\frac{100}{r^2}\right)(-1)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dS}{dr} = 2(2\pi r) - 2\left(\frac{100}{r^2}\right)$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dS}{dr} = 2(2\pi r) - 2\left(\frac{100}{r^2}\right) = 0$

$2(2\pi r) - 2\left(\frac{100}{r^2}\right) = 0$

$2\pi r = \frac{100}{r^2}$ ---- (4)

Now to check if this critical point will determine the minimum surface area of the box, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with r:

$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2(2\pi r) - 2\left(\frac{100}{r^2}\right)\right]$

$\frac{d^2S}{dr^2} = 4\pi - \frac{2\times100\times(-2)}{r^3} = 4\pi + \frac{400}{r^3}$ ----- (5)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{50}{\pi}}} = 4\pi + \frac{400}{\left(\sqrt[3]{\frac{50}{\pi}}\right)^3} = 4\pi + \frac{400 \times \pi}{50} = 4\pi + 8\pi = 12\pi$$

As $\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{50}{\pi}}} = 12\pi > 0$, so the function S is minimum at $r = \sqrt[3]{\frac{50}{\pi}}$

As S is minimum from equation (4)

$$2\pi r = \frac{100}{r^2}$$

$$2\pi r = \frac{V}{r^2}$$

$V = 2\pi r^3$

Now in equation (1) we have,

$$h = \frac{V}{\pi r^2} = \frac{2\pi r^3}{\pi r^2}$$

h = 2r = diameter

Therefore when the total surface area of a cone is minimum, then height of the cone is equal to twice the radius or equal to its diameter.

Q21.

Let r be the radius of the base and h the height of a cylinder.

The surface area is given by,

$S = 2\pi r^2 + 2\pi rh$

$h = \frac{S - 2\pi r^2}{2\pi r}$ ....(1)

Let V be the volume of the cylinder.

Therefore, $V = \pi r^2 h$

$V = \pi r^2 \left(\frac{S-2\pi r^2}{2\pi r}\right)$…….Using equation 1

$$V = \frac{Sr - 2\pi r^3}{2}$$

Differentiating both sides w.r.t r, we get,

$\frac{dV}{dr} = \frac{S}{2} - 3\pi r^2$……….(2)

For maximum or minimum, we have,

$$\frac{dV}{dr} = 0$$

$\Rightarrow \frac{S}{2} - 3\pi r^2 = 0$

$\Rightarrow S = 6\pi r^2$

$2\pi r^2 + 2\pi rh = 6\pi r^2$

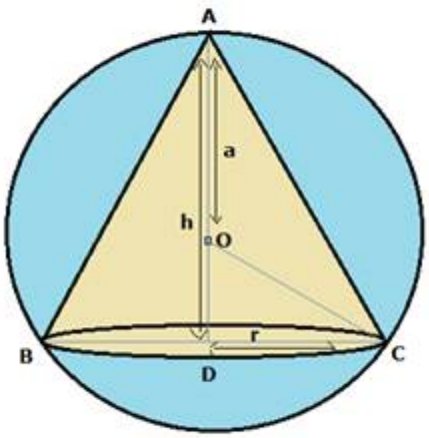
$h = 2r$

Differentiating equation 2, with respect to r to check for maxima and minima, we get,

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$

Hence, V is maximum when h = 2r or h = diameter

Q22.



Let us consider,

• The radius of the sphere be ‘a’ units.

• Volume of the inscribed cone be ‘V’.

• Height of the inscribed cone be ‘h’.

• Radius of the base of the cone is ‘r’.

Given volume of the inscribed cone is,

$V = \frac{\pi r^2 h}{3}$

Consider OD = (AD-OA) =(h-a)

Now let $OC^2 = OD^2 + DC^2$, here OC = a, OD = (h-a), DC = r,

So $a^2 = (h-a)^2 + r^2$

$r^2 = a^2 - (h^2 + a^2 - 2ah)$

$r^2 = h(2a - h)$ ----- (1)

Let us consider the volume of the cone:

$$V = \frac{1}{3}(\pi r^2 h)$$

Now substituting (1) in the volume formula,

$$V = \frac{1}{3}(\pi h(2a - h)h)$$

$V = \frac{1}{3}(2\pi h^2 a - \pi h^3)$ ---- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with h and then equating it to zero. <u>This is because if the function V(r) has a maximum/minimum at a point c then V'(c) = 0.</u>

Differentiating the equation (2) with respect to h:

$$\frac{dV}{dh} = \frac{d}{dh}\left[\frac{1}{3}(2\pi h^2 a - \pi h^3)\right]$$

$$\frac{dV}{dh} = \frac{1}{3}(2\pi a)(2h) - \frac{1}{3}(\pi)(3h^2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dV}{dh} = \frac{1}{3}[4\pi ah - 3\pi h^2]$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dV}{dh} = \frac{1}{3}[4\pi ah - 3\pi h^2] = 0$$

$4\pi ah - 3\pi h^2 = 0$

$h(4\pi a - 3\pi h) = 0$

$h = 0$ (or) $h = \frac{4\pi a}{3\pi} = \frac{4a}{3}$

$h = \frac{4a}{3}$

[as h cannot be zero]

Now to check if this critical point will determine the maximum volume of the inscribed cone, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with h:

$$\frac{d^2V}{dh^2} = \frac{d}{dh}\left[\frac{1}{3}[4\pi ah - 3\pi h^2]\right]$$

$$\frac{d^2V}{dh^2} = \frac{1}{3}[4\pi a - (3\pi)(2h)] = \frac{\pi}{3}[4a - 6h] \text{ ----- } (4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dh^2}\right)_{h=\frac{4a}{3}} = \frac{\pi}{3}\left[4a - 6\left(\frac{4a}{3}\right)\right] = \frac{4a\pi}{3}[1-2] = -\frac{4a\pi}{3}$$

As $\left(\frac{d^2V}{dh^2}\right)_{h=\frac{4a}{3}} = -\frac{4a\pi}{3} < 0$, so the function V is maximum at $h = \frac{4a}{3}$

Substituting h in equation (1)

$$r^2 = \left(\frac{4a}{3}\right)\left(2a - \frac{4a}{3}\right)$$

$$r^2 = \left(\frac{4a}{3}\right)\left(2a - \frac{4a}{3}\right)$$

$$r^2 = \frac{8a^2}{9}$$

As V is maximum, substituting h and r in the volume formula:

$V = \frac{1}{3}\pi\left(\frac{8a^2}{9}\right)\left(\frac{4a}{3}\right)$

$V = \frac{8}{27}\left(\frac{4}{3}\pi a^3\right)$

$V = \frac{8}{27}$(volume of the sphere)

Therefore when the volume of a inscribed cone is maximum, then it is equal to $\frac{8}{27}$ times of the volume of the sphere in which it is inscribed.

Q23.

The pth power of a number exceeds by a fraction to be the greatest.

Let us consider,

- 'x' be the required fraction.
- The greatest number will be $y = x - x^p$ ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function y(x)has a maximum/minimum at a point c then y'(c) = 0.</u>

Differentiating the equation (1) with respect to x:

$\frac{dy}{dx} = \frac{d}{dx}(x - x^p)$

$\frac{dy}{dx} = 1 - px^{p-1}$ ---- (2)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dy}{dx} = 1 - px^{p-1} = 0$$

$1 = px^{p-1}$

$$x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$$

Now to check if this critical point will determine the if the number is the greatest, we need to check with second differential which needs to be negative.

Consider differentiating the equation (2) with x:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[1 - px^{p-1}]$$

$$\frac{d^2y}{dx^2} = -p(p-1)x^{p-2} \text{ ----- (3)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2y}{dx^2}\right)_{x=\left(\frac{1}{p}\right)^{\frac{1}{p-1}}} = -p(p-1)\left(\left(\frac{1}{p}\right)^{\frac{1}{p-1}}\right)^{p-2}$$

As $\left(\frac{d^2y}{dx^2}\right)_{x=\left(\frac{1}{p}\right)^{\frac{1}{p-1}}} = -p(p-1)\left(\left(\frac{1}{p}\right)^{\frac{1}{p-1}}\right)^{p-2} < 0$, so the number y is greatest at $x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$

Hence, the y is the greatest number and exceeds by a

fraction $x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$

Q24.

Let us consider,

• The co-ordinates of the point be P(x,y)

• As the point P is on the curve, then $y^2 = 4x$

$x = \frac{y^2}{4}$

• The distance between the points is given by,

$D^2 = (x-2)^2 + (y+8)^2$

$D^2 = x^2 - 4x + 4 + y^2 + 64 + 16y$

Substituting x in the distance equation

$$D^2 = \left(\frac{y^2}{4}\right)^2 - 4\left(\frac{y^2}{4}\right) + y^2 + 16y + 68$$

$Z = D^2 = \frac{y^4}{16} + 16y + 68$ ---- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with y and then equating it to zero. This is because if the function Z(x) has a maximum/minimum at a point c then Z'(c) = 0.

Differentiating the equation (2) with respect to y:

$$\frac{dZ}{dy} = \frac{d}{dy}\left(\frac{y^4}{16} + 16y + 68\right)$$

$\frac{dZ}{dy} = \frac{4y^3}{16} + 16 = \frac{y^3}{4} + 16$ ---- (2)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dZ}{dy} = \frac{y^3}{4} + 16 = 0$$

$y^3 + 64 = 0$

$(y + 4)(y^2 – 4y + 16) = 0$

$(y+4) = 0$ (or) $y^2 – 4y + 16 = 0$

$y = -4$

(as the roots of the $y^2 – 4y + 16$ are imaginary)

<u>Now to check if this critical point will determine the distance is mimimum, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (2) with y:

$$\frac{d^2Z}{dy^2} = \frac{d}{dy}\left[\frac{y^3}{4} + 16\right]$$

$\frac{d^2Z}{dy^2} = \frac{3y^2}{4}$ ----- (3)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2Z}{dy^2}\right)_{y=-4} = \frac{3}{4}(-4)^2 = 12$$

As $\left(\frac{d^2Z}{dy^2}\right)_{y=-4} = 12 > 0$, so the Distance D^2 is minimum at y = -4

Now substituting y in x, we have

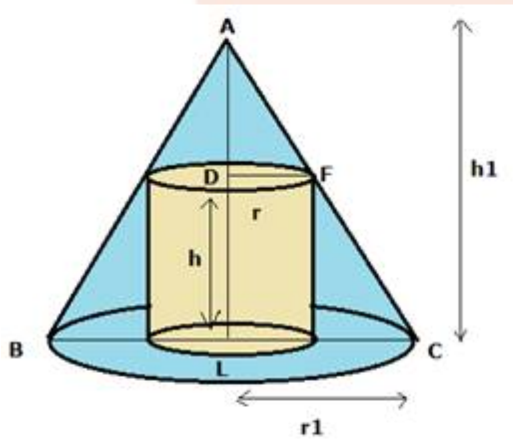
$$x = \frac{(-4)^2}{4} = 4$$

So, the point P on the curve $y^2 = 4x$ is (4,-4) which is at nearest from the (2,-8)

Q25.

• A right circular cylinder is inscribed inside a cone.

• The curved surface area is maximum.



Let us consider,

• 'r_1' be the radius of the cone.

• 'h_1' be the height of the cone.

• 'r' be the radius of the inscribed cylinder.

• 'h' be the height of the inscribed cylinder.

DF = r, and AD = AL – DL = $h_1 - h$

Now, here ΔADF and ΔALC are similar,

Then

$$\frac{AD}{AL} = \frac{DF}{LC} \Rightarrow \frac{h_1 - h}{h_1} = \frac{r}{r_1}$$

$$h_1 - h = \frac{rh_1}{r_1}$$

$$h = h_1 - \frac{rh_1}{r_1} = h_1\left(1 - \frac{r}{r_1}\right)$$

$$h = h_1\left(1 - \frac{r}{r_1}\right) \text{ ----- (1)}$$

Now let us consider the curved surface area of the cylinder,

S = 2πrh

Substituting h in the formula,

$$S = 2\pi r\left[h_1\left(1 - \frac{r}{r_1}\right)\right]$$

$$S = 2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1} \text{ ---- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function S(r) has a maximum/minimum at a point c then S'(c) = 0.

Differentiating the equation (2) with respect to r:

$$\frac{dS}{dr} = \frac{d}{dr}\left[2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1}\right]$$

$$\frac{dS}{dr} = 2\pi h_1 - \frac{2\pi h_1(2r)}{r_1}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1}$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} = 0$$

$$\frac{4\pi h_1 r}{r_1} = 2\pi h_1$$

$$r = \frac{2\pi h_1 r_1}{4\pi h_1}$$

$$r = \frac{r_1}{2}$$

<u>Now to check if this critical point will determine the maximum volume of the inscribed cylinder, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2\pi h_1 - \frac{4\pi h_1 r}{r_1}\right]$$

$\frac{d^2S}{dr^2} = 0 - \frac{4\pi h_1}{r_1} = -\frac{4\pi h_1}{r_1}$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\frac{d^2S}{dr^2}_{r=\frac{r_1}{2}} = -\frac{4\pi h_1}{r_1}$$

As $\frac{d^2S}{dr^2}_{r=\frac{r_1}{2}} = -\frac{4\pi h_1}{r_1} < 0$, so the function S is maximum at $r = \frac{r_1}{2}$

Substituting r in equation (1)

$$h = h_1\left(1 - \frac{\frac{r_1}{2}}{r_1}\right)$$

$h = h_1\left(1 - \frac{1}{2}\right) = \frac{h_1}{2}$ --- (5)

As S is maximum, from (5) we can clearly say that $h_1 = 2h$ and

$r_1 = 2r$

this means the radius of the cone is twice the radius of the cylinder or equal to diameter of the cylinder.

Q26.

- Closed cuboid has square base.
- The volume of the cuboid is given.
- Surface area is minimum.

Let us consider,

- The side of the square base be 'x'.
- The height of the cuboid be 'h'.
- The given volume, $V = x^2h$

$h = \frac{V}{x^2}$ ----- (1)

Consider the surface area of the cuboid, Surface Area =

2(Area of the square base) + 4(areas of the rectangular sides)

$S = 2x^2 + 4xh$

Now substitute (1) in the Surface Area formula

$S = 2x^2 + 4x\left(\frac{V}{x^2}\right)$

$S = 2x^2 + \left(\frac{4V}{x}\right)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function S(x) has a maximum/minimum at a point c then S'(c) = 0.

Differentiating the equation (2) with respect to x:

$\frac{dS}{dx} = \frac{d}{dx}\left[2x^2 + \left(\frac{4V}{x}\right)\right]$

$\frac{dS}{dx} = 2(2x) + 4V\left(\frac{-1}{x^2}\right)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dS}{dx} = 4x - \frac{4V}{x^2}$ ------- (3)

To find the critical point, we need to equate equation (3) to ero.

$\frac{dS}{dx} = 4x - \frac{4V}{x^2} = 0$

$4x = \frac{4V}{x^2}$

$x^3 = V$

Now to check if this critical point will determine the minimum surface area, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$$\frac{d^2S}{dx^2} = \frac{d}{dx}\left[4x - \frac{4V}{x^2}\right]$$

$$\frac{d^2S}{dx^2} = 4 + \frac{8V}{x^3} \text{ ----- } (4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\frac{d^2S}{dx^2}_{x=V^{\frac{1}{3}}} = 4 + \frac{8V}{V} = 12$$

As $\frac{d^2S}{dx^2}_{x=V^{\frac{1}{3}}} = 12 > 0$, so the function S is minimum at $x = \sqrt[3]{V}$

Substituting x in equation (1)

$$h = \frac{V}{x^2} = \frac{x^3}{x^2} = x$$

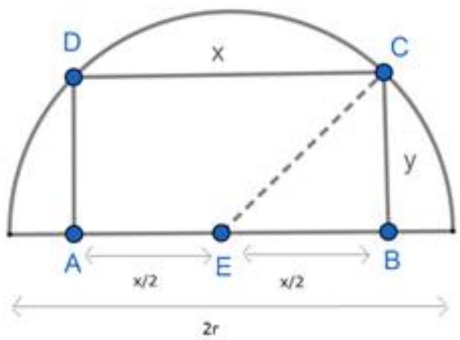
h = x

As S is minimum and h = x, this means that the cuboid is a cube.

Q27.

• Radius of the semicircle is ‘r’.

• Area of the rectangle is maximum.



Let us consider

• The base of the rectangle be 'x' and the height be 'y'.

Consider the ΔCEB,

$CE^2 = EB^2 + BC^2$

As CE = r, $EB = \frac{x}{2}$ and CB = y

$r^2 = \left(\frac{x}{2}\right)^2 + y^2$

$y^2 = r^2 - \left(\frac{x}{2}\right)^2$ ---- (1)

Now the area of the rectangle is

$A = x \times y$

Squaring on both sides

$A^2 = x^2 y^2$

Substituting (1) in the above Area equation

$A^2 = x^2\left[r^2 - \left(\frac{x}{2}\right)^2\right]$

$Z = A^2 = x^2r^2 - x^2\frac{x^2}{4} = x^2r^2 - \frac{x^4}{4}$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function Z(x) has a maximum/minimum at a point c then Z'(c) = 0.

Differentiating the equation (2) with respect to x:

$\frac{dZ}{dx} = \frac{d}{dx}\left[x^2r^2 - \frac{x^4}{4}\right]$

$\frac{dZ}{dx} = r^2\,(2x) - \frac{4x^3}{4}$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dZ}{dx} = 2xr^2 - x^3$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dZ}{dx} = 2xr^2 - x^3 = 0$

$x(2r^2 - x^2) = 0$

$x = 0$ (or) $x^2 = 2r^2$

$x = 0$ (or) $x = r\sqrt{2}$

$x = r\sqrt{2}$

[as x cannot be zero]

Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[2xr^2 - x^3]$$

$$\frac{d^2Z}{dx^2} = 2r^2 - 3x^2 \text{ ----- } (4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\frac{d^2Z}{dx^2}_{x=r\sqrt{2}} = 2r^2 - 3(r\sqrt{2})^2 = 2r^2 - 6r^2 = -4r^2$$

As $\frac{d^2Z}{dx^2}_{x=r\sqrt{2}} = -4r^2 < 0$, so the function Z is maximum at $x = r\sqrt{2}$

Substituting x in equation (1)

$$y^2 = r^2 - \left(\frac{r\sqrt{2}}{2}\right)^2 = r^2 - \frac{r^2}{2} = \frac{r^2}{2}$$

$$y = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$$

As the area of the rectangle is maximum, and $x = r\sqrt{2}$ and $y = \frac{r\sqrt{2}}{2}$

So area of the rectangle is

$$A = r\sqrt{2} \times \frac{r\sqrt{2}}{2}$$

$A = r^2$

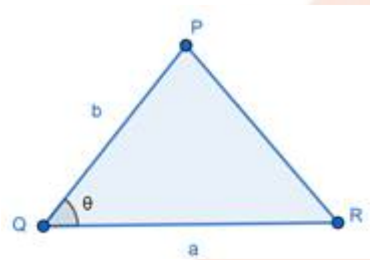
Hence the maximum area of the rectangle inscribed inside a semicircle is r^2 square units.

Q28.

• The length two sides of a triangle are ‘a’ and ‘b’

• Angle between the sides ‘a’ and ‘b’ is θ.

• The area of the triangle is maximum.



Let us consider,

The area of the ΔPQR is given be

$A = \frac{1}{2}\, ab \sin\theta$ ---- (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with θ and then equating it to zero. This is because if the function A (θ) has a maximum/minimum at a point c then A’(c) = 0.

Differentiating the equation (1) with respect to θ:

$$\frac{dA}{d\theta} = \frac{d}{d\theta}\left[\frac{1}{2}\, ab \sin\theta\right]$$

$\frac{dA}{d\theta} = \frac{1}{2}\, ab \cos\theta$ ---- (2)

[Since $\frac{d}{dx}(\sin\theta) = \cos\theta$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dA}{d\theta} = \frac{1}{2}ab\cos\theta = 0$$

Cos θ = 0

$$\theta = \frac{\pi}{2}$$

<u>Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (2) with θ :

$$\frac{d^2A}{d\theta^2} = \frac{d}{d\theta}\left[\frac{1}{2}ab\cos\theta\right]$$

$\frac{d^2A}{d\theta^2} = -\frac{1}{2}ab\sin\theta$ ----- (2)

[Since $\frac{d}{dx}(\cos\theta) = -\sin\theta$]

Now let us find the value of

$$\frac{d^2A}{d\theta^2}_{\theta=\frac{\pi}{2}} = -\frac{1}{2}ab\sin\left(\frac{\pi}{2}\right) = -\frac{1}{2}ab$$

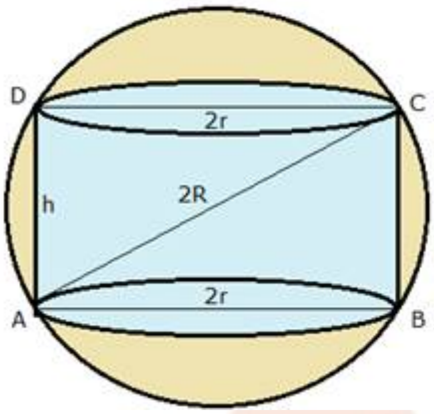
As $\frac{d^2A}{d\theta^2}_{\theta=\frac{\pi}{2}} = -\frac{1}{2}ab < 0$, so the function A is maximum at $\theta = \frac{\pi}{2}$

As the area of the triangle is maximum when $\theta = \frac{\pi}{2}$

Q29.

• Radius of the sphere is $5\sqrt{3}$.

• Volume of cylinder is maximum.



Let us consider,

• The radius of the sphere be ‘R’ units.

• Volume of the inscribed cylinder be ‘V’.

• Height of the inscribed cylinder be ‘h’.

• Radius of the cylinder is ‘r’.

Now let $AC^2 = AB^2 + BC^2$, here $AC = 2R$, $AB = 2r$, $BC = h$,

So $4R^2 = 4r^2 + h^2$

$r^2 = \frac{1}{4}[4R^2 - h^2]$ ----- (1)

Let us consider, the volume of the cylinder:

$V = \pi r^2 h$

Now substituting (1) in the volume formula,

$V = \pi h\left(\frac{1}{4}[4R^2 - h^2]\right)$

$V = \frac{\pi}{4}(4R^2h - h^3)$ ---- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with h and then equating it to zero. This is because if the function V(h) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (2) with respect to h:

$$\frac{dV}{dh} = \frac{d}{dh}\left[\frac{\pi}{4}(4R^2h - h^3)\right]$$

$$\frac{dV}{dh} = \frac{4R^2\pi}{4} - \frac{\pi}{4}(3h^2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dV}{dh} = R^2\pi - \frac{3h^2\pi}{4}$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dV}{dh} = R^2\pi - \frac{3h^2\pi}{4} = 0$$

$3h^2\pi = 4R^2\pi$

$$h^2 = \frac{4}{3}R^2 = \frac{4}{3}(5\sqrt{3})^2 = \frac{4}{3}(25 \times 3) = 100$$

h = 10

[as h cannot be negative]

Now to check if this critical point will determine the maximum volume of the inscribed cone, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with h:

$$\frac{d^2V}{dh^2} = \frac{d}{dh}\left[R^2\pi - \frac{3h^2\pi}{4}\right]$$

$\frac{d^2V}{dh^2} = 0 - \frac{3\,(2h)\pi}{3} = -2h\pi$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dh^2}\right)_{h=10} = -2h\pi = -2(10)\pi = -20\pi$$

As $\left(\frac{d^2V}{dh^2}\right)_{h=10} = -20\pi < 0$, so the function V is maximum at h=10

Substituting h in equation (1)

$$r^2 = \frac{1}{4}\left[4(5\sqrt{3})^2 - (10)^2\right]$$

$$r^2 = \frac{1}{4}\left[4\,(25 \times 3) - 100\right]$$

$$r^2 = \frac{300 - 100}{4} = \frac{200}{4} = 50$$

As V is maximum, substituting h and r in the volume formula:

V = π (50) (10)

V = 500π cm^3

Therefore when the volume of a inscribed cylinder is maximum and is equal 500π cm^3

Q30.

• Capacity of the square tank is 250 cubic metres.

• Cost of the land per square meter Rs.50.

• Cost of digging the whole tank is Rs. $(400 \times h^2)$.

• Where h is the depth of the tank.

Let us consider,

• Side of the tank is x metres.

• Cost of the digging is; $C = 50x^2 + 400h^2$ ---- (1)

• Volume of the tank is; $V = x^2h$; $250 = x^2h$

$h = \frac{250}{x^2}$ ----- (2)

Substituting (2) in (1),

$$C = 50x^2 + 400\left(\frac{250}{x^2}\right)^2$$

$$C = 50x^2 + \frac{400 \times 62500}{x^4} \text{ ----- (3)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function C(x) has a maximum/minimum at a point c then C'(c) = 0.</u>

Differentiating the equation (3) with respect to x:

$$\frac{dC}{dx} = \frac{d}{dx}\left[50x^2 + \frac{400 \times 62500}{x^4}\right]$$

$$\frac{dC}{dx} = 50(2x) + \frac{25000000(-4)}{x^5}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dC}{dx} = 100x - \frac{10^8}{x^5}$ ------- (4)

To find the critical point, we need to equate equation (4) to zero.

$$\frac{dC}{dx} = 100x - \frac{10^8}{x^5} = 0$$

$x^6 = 10^6$

$x = 10$

Now to check if this critical point will determine the minimum volume of the tank, we need to check with second differential which needs to be positive.

Consider differentiating the equation (4) with x:

$$\frac{d^2C}{dx^2} = \frac{d}{dx}\left[100x - \frac{10^8}{x^5}\right]$$

$\frac{d^2C}{dx^2} = 100 - \frac{10^8(-5)}{x^6} = 100 + \frac{10^8(5)}{x^6}$ ----- (5)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2C}{dx^2}\right)_{x=10} = 100 + \frac{10^8(5)}{(10)^6} = 100 + 500 = 600$$

As $\left(\frac{d^2C}{dx^2}\right)_{x=10} = 600 > 0$, so the function C is minimum at x=10

Substituting x in equation (2)

$$h = \frac{250}{(10)^2} = \frac{250}{100} = \frac{5}{2}$$

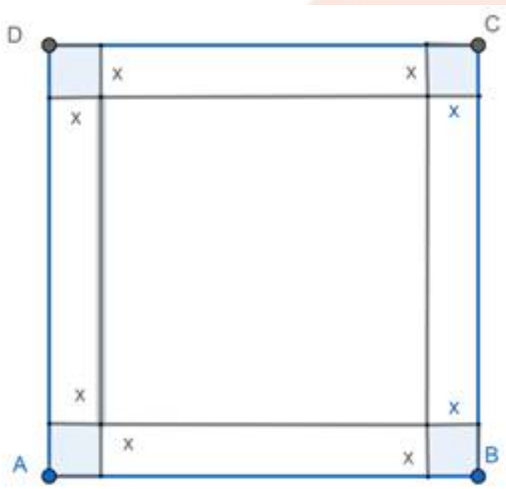
h = 2.5 m

Therefore when the cost for the digging is minimum, when x = 10m and h = 2.5m

Q31.

• Side of the square piece is 18 cms.

• the volume of the formed box is maximum.



• ‘x’ be the length and breadth of the piece cut from each vertex of the piece.

• Side of the box now will be (18-2x)

• The height of the new formed box will also be ‘x’.

Let the volume of the newly formed box is :

$V = (18-2x)^2 \times (x)$

$V = (324+ 4x^2 – 72x)\ x$

$V = 4x^3 -72x^2 +324x$ ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function V(x) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (1) with respect to x:

$$\frac{dV}{dx} = \frac{d}{dx}[4x^3 - 72x^2 + 324x]$$

$$\frac{dV}{dx} = 12x^2 - 144x + 324 \text{ -------- } (2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$$

$x^2 - 12x + 27 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(27)}}{2(1)} = \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2}$$

$$x == \frac{12 \pm 6}{2}$$

x = 9 or x =3

x= 2

[as x = 9 is not a possibility, because 18-2x = 18-18= 0]

Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2V}{dx^2} = \frac{d}{dx}[12x^2 - 144x + 324]$$

$\frac{d^2V}{dx^2} = 24x - 144$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dx^2}\right)_{x=3} = 24(3) - 144 = 72 - 144 = -72$$

As $\left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$, so the function V is maximum at x = 3cm

Now substituting x = 3 in 18 – 2x, the side of the considered box:

Side = 18-2x = 18 - 2(3) = 18-6= 12cm

Therefore side of wanted box is 12cms and height of the box is 3cms.

Now, the volume of the box is

$V = (12)^2 \times 3 = 144 \times 3 = 432cm^3$

Hence maximum volume of the box formed by cutting the given 18cms sheet is $432cm^3$ with 12cms side and 3cms height.

Q32.

• The tank is square base open tank.

• The cost of the construction to be least.

Let us consider,

• Side of the tank is x metres.

• Height of the tank be 'h' metres.

• Volume of the tank is; $V = x^2h$

• Surface Area of the tank is $S = x^2 + 4xh$

• Let Rs.P is the price per square.

Volume of the tank,

$h = \frac{V}{x^2}$ ---- (1)

Cost of the construction be:

$C = (x^2 + 4xh)P$ ---- (2)

Substituting (1) in (2),

$C = \left[x^2 + 4x\frac{V}{x^2}\right]P$

$C = [x^2 + \frac{4V}{x}]P$ ----- (3)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function C(x) has a maximum/minimum at a point c then C'(c) = 0.</u>

Differentiating the equation (3) with respect to x:

$\frac{dC}{dx} = \frac{d}{dx}\left[x^2 + \frac{4V}{x}\right]P$

$\frac{dC}{dx} = \left[(2x) + \frac{4V(-1)}{x^2}\right]P$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dC}{dx} = \left[2x - \frac{4V}{x^2}\right]P$ ------- (4)

To find the critical point, we need to equate equation (4) to zero.

$\frac{dC}{dx} = \left[2x - \frac{4V}{x^2}\right]P = 0$

$x^3 = 2V$

Now to check if this critical point will determine the minimum volume of the tank, we need to check with second differential which needs to be positive.

Consider differentiating the equation (4) with x:

$\frac{d^2C}{dx^2} = P\frac{d}{dx}\left[2x - \frac{4V}{x^2}\right]$

$\frac{d^2C}{dx^2} = \left[2 - \frac{4V(-2)}{x^3}\right]P = \left[2 + \frac{8V}{x^3}\right]P$ ----- (5)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$\left(\frac{d^2C}{dx^2}\right)_{x=(2V)^{\frac{1}{3}}} = \left[2 + \frac{8V}{2V}\right]P = [2+4]P = 6P$

As $\left(\frac{d^2C}{dx^2}\right)_{x=(2V)^{\frac{1}{3}}} = 6P > 0$, so the function C is minimum at $x = \sqrt[3]{2V}$

Substituting x in equation (2)

$$h = \frac{V}{(2V)^{\frac{2}{3}}} = \frac{V\sqrt[3]{(2V)}}{2V} = \frac{1}{2}\sqrt[3]{2V}$$

$$h = \frac{1}{2}\sqrt[3]{2V}$$

Therefore when the cost for the digging is minimum,

when $x = \sqrt[3]{2V}$ and $h = \frac{1}{2}\sqrt[3]{2V}$

Q33.

• Length of the wire is 36 cm.

• The wire is cut into 2 pieces.

• One piece is made to a square.

• Another piece made into a equilateral triangle.

Let us consider,

• The perimeter of the square is x.

• The perimeter of the equilateral triangle is (36-x).

• Side of the square is $\frac{x}{4}$

• Side of the triangle is $\frac{(36-x)}{3}$

Let the Sum of the Area of the square and triangle is

$$A = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{36-x}{3}\right)^2$$

$$A = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{4}\left(12-\frac{x}{3}\right)^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144+\frac{x^2}{9}-8x\right)$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144+\frac{x^2}{9}-8x\right) \text{ --- (1)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function A(x) has a maximum/minimum at a point c then A'(c) = 0.</u>

Differentiating the equation (1) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144+\frac{x^2}{9}-8x\right)\right]$$

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(0+\frac{2x}{9}-8\right)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9}-8\right) \text{ ----- (2)}$$

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9} - 8\right) = 0$$

$$\frac{2x}{16} = \frac{\sqrt{3}}{4}\left(8 - \frac{2x}{9}\right)$$

$$\frac{2x}{16} = 2\sqrt{3} - \frac{\sqrt{3}x}{18}$$

$$\frac{2x}{16} + \frac{\sqrt{3}x}{18} = 2\sqrt{3}$$

$$x\left(\frac{2\,(9) + \sqrt{3}(8)}{144}\right) = 2\sqrt{3}$$

$$x\left(\frac{18 + 8\sqrt{3}}{144}\right) = 2\sqrt{3}$$

$$x = 2\sqrt{3}\left(\frac{144}{18 + 8\sqrt{3}}\right) = \frac{144\sqrt{3}}{(9 + 4\sqrt{3})}$$

<u>Now to check if this critical point will determine the minimum area, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9} - 8\right)\right]$$

$$\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{\sqrt{3}}{4}\left(\frac{2}{9}\right) = \frac{9+4\sqrt{3}}{72}$$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2A}{dx^2}\right)_{x=\frac{144\sqrt{3}}{(9+4\sqrt{3})}} = \frac{9+4\sqrt{3}}{72}$$

As $\left(\frac{d^2A}{dx^2}\right)_{x=\frac{144\sqrt{3}}{(9+4\sqrt{3})}} = \frac{9+4\sqrt{3}}{72} > 0$, so the function A is minimum at

$$x = \frac{144\sqrt{3}}{(9+4\sqrt{3})}$$

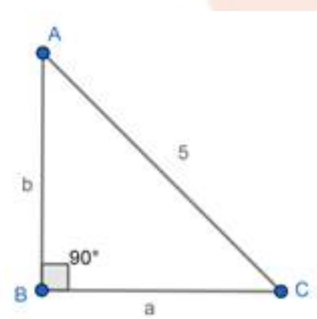
Now, the length of each piece

is $x = \frac{144\sqrt{3}}{(9+4\sqrt{3})}$ cm and $36 - x = 36 - \frac{144\sqrt{3}}{(9+4\sqrt{3})} = \frac{324}{(9+4\sqrt{3})}$ cm

Q34.

• The triangle is right angled triangle.

• Hypotenuse is 5cm.



• The base of the triangle is ‘a’.

• The adjacent side is ‘b’.

Now $AC^2 = AB^2 + BC^2$

As AC = 5, AB = b and BC = a

$25 = a^2 + b^2$

$b^2 = 25 - a^2$ ---- (1)

Now, the area of the triangle is

$$A = \frac{1}{2} ab$$

Squaring on both sides

$$A^2 = \frac{1}{4} a^2 b^2$$

Substituting (1) in the area formula

$Z = A^2 = \frac{1}{4} a^2(25 - a^2)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with a and then equating it to zero. <u>This is because if the function Z (x) has a maximum/minimum at a point c then Z'(c) = 0.</u>

Differentiating the equation (2) with respect to a:

$$\frac{dZ}{da} = \frac{d}{da}\left[\frac{1}{4} a^2(25 - a^2)\right]$$

$$\frac{dZ}{da} = \frac{1}{4}\left[25\,(2a) - 4a^3\right]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dZ}{da} = \frac{25a}{2} - a^3$ ----- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{da} = \frac{25a}{2} - a^3 = 0$$

$$a\left(\frac{25}{2} - a^2\right) = 0$$

a=0 (or) $a = \frac{5}{\sqrt{2}}$

$a = \frac{5}{\sqrt{2}}$

[as a cannot be zero]

Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with a:

$$\frac{d^2Z}{da^2} = \frac{d}{da}\left[\frac{25a}{2} - a^3\right]$$

$\frac{d^2Z}{da^2} = \frac{25}{2} - 3a^2$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2Z}{da^2}\right)_{a=\frac{5}{\sqrt{2}}} = \frac{25}{2} - 3\left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25}{2} - \frac{(3)25}{2} = -25$$

As $\left(\frac{d^2Z}{da^2}\right)_{a=\frac{5}{\sqrt{2}}} = -25 < 0$, so the function A is maximum at $a = \frac{5}{\sqrt{2}}$

Substituting value of A in (1)

$$b^2 = 25 - \frac{25}{2} = \frac{25}{2}$$

$$b = \frac{5}{\sqrt{2}}$$

Now the maximum area is

$$A = \frac{1}{2}\left(\frac{5}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}}\right) = \frac{25}{4}$$

$$\therefore A = \frac{25}{4}\ \text{cm}^2$$

Exercise-11G

Q1.

Domain of the function is R

Finding derivative f'(x)=5

Which is greater than 0

Mean strictly increasing in its domain i.e R

Q2.

Domain of the function is R

Finding derivative f'(x)=-2

Which is less than 0

Means strictly decreasing in its domain i.e R

Q3.

Domain of the function is R

Finding derivative i.e f'(x)=a

As given in question it is given that a>0

Derivative>0

Means strictly increasing in its domain i.e R

Q4.

Domain of the function is R

finding derivative i.e $f'(x)=2e^x$

As we know e^x is strictly increasing its domain

$f'(x)>0$

hence f(x) is strictly increasing in its domain

Q5.

Domain of function is **R.**

$f'(x)=2x$

for $x>0$ $f'(x)>0$ i.e. increasing

for $x<0$ $f'(x)<0$ i.e. decreasing

hence it is neither increasing nor decreasing in R

Q6.

For $x>0$

Modulus will open with + sign

$f(x)=+x$

$\Rightarrow$ f'(x)=+1 which is <0

for x<0

Modulus will open with -ve sign

f(x)=-x => f'(x)=-1 which is >0

hence f(x) is increasing in x>0 and decreasing in x<0

Q7. f(x)=ln(x)

$f'(x) = \frac{1}{x}$

for x<0

f'(x)=-ve →increasing

for x>0

f'(x)=+ve →decreasing

f(x) in increasing when x>0 i.e x∈(0,∞)

Q8.

Consider f(x)=log$_a$ x

domain of f(x) is x>0

$f'(x) = \frac{1}{x}\ln(a)$

⇒ for a>1, ln(a)>0,

hence f'(x) >0 which means strictly increasing.

⇒ for 0<a<1, ln(a)<0,

hence f'(x)<0 which means strictly decreasing.

Q9.

Consider $f(x)=3^x$

The domain of f(x) is R.

$f'(x)=3^x\ln(3)$

3^x is always greater than 0 and ln(3) is also + ve.

Overall f'(x) is >0 means strictly increasing in its domain i.e. R.

Q10.

Consider $f(x)=x^3-15x^2+75x-50$

Domain of the function is R.

$f'(x)=3x^2-30x+75$

$=3(x^2-10+25)$

$=3(x-5)(x-5)$

$=3(x-5)^2$

$f'(x)=0$ for $x=5$

for $x<5$

f'(x)>5

and

for x>5

f'(x)>5

we can see throughout R the derivative is +ve but at x=5 it is 0 so it is increasing.

Q11.

$$f(x) = \left(x - \frac{1}{x}\right)$$

domain of function is R-{0}

$$f'(x) = 1 + \frac{1}{x^2}$$

f'(x) ∀ x ∈ R is greater than 0.

Q12.

$$f(x) = \frac{1}{x} + 5$$

domain of function is R-{0}

$$f'(x) = -\frac{1}{x^2}$$

for all x, f'(x)<0

Hence function is decreasing.

Q13.

Consider $f(x) = \frac{1}{(1+x^2)}$,

$$f'(x) = -\frac{2x}{(1+x^2)^2}$$

for $x \geq 0$,

$f'(x)$ is -ve.

hence function is decreasing for $x \leq 0$

Q14.

$f(x)=x^3+x^{-3}$

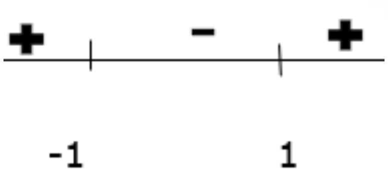
$f'(x)=3x^2-3x^{-4}$

$=3(x^2-1/x^4)$

$$= 3\left(\frac{x^3-1}{x^2} . \frac{x^3+1}{x^2}\right)$$

$$= \frac{3(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{x^4}$$

Root of $f'(x)$=1and-1



Here we can clearly see that $f'(x)$ is decreasing in [-1,1]

So, $f(x)$ is decreasing in interval [-1,1]

Q15.

Consider $f(x) = \frac{x}{\sin x}$,

$$f'(x) = \frac{\sin x - x.\cos x}{\sin^2 x}$$

$$f'(x) = \frac{\cos x(\tan x - x)}{\sin^2 x}$$

in $\left]0, \frac{\pi}{2}\right[$ cos>0,

tan x-x>0,

$\sin^2 x>0$

hence f’(x)>0,

so, function is increasing in the given interval.

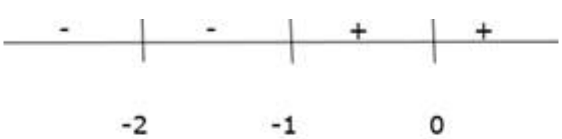
Q16.

Consider $f(x) = \log(1+x) - \frac{2x}{(x+2)}$,

$$f'(x) = \frac{1}{1+x} - \frac{4}{(x+2)^2}$$

$$= \frac{(x+2)^2 - 4(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{x^2}{(x+1)(x+2)^2}$$



Clearly we can see that f’(x)>o for x>-1.

Hence function is increasing for all x>-1

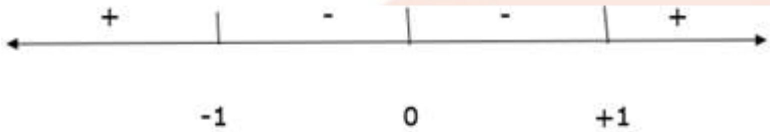
Q17.

Consider $f(x) = \left(x + \frac{1}{x}\right)$

$f'(x) = 1 - \frac{1}{x^2}$

$f'(x) = \frac{x^2 - 1}{x^2}$

$= \frac{x - 1 . x + 1}{x^2}$



We can see f'(x) <0 in [-1,1]

i.e. f(x) is decreasing in this interval.

We can see f'(x) >0 in (-∞, -1) ∪(1, ∞)

i.e. f(x) is increasing in this interval.

Q18.

Consider $f(x) = \frac{(x-2)}{(x+1)}$,

$f'(x) = \frac{3}{(x+1)^2}$

f'(x) at x=-1 is not defined

and for all x ∈ R- {-1}

f'(x)>0

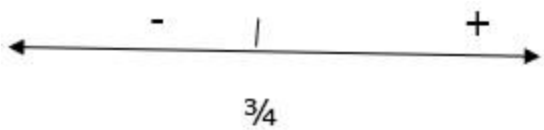
hence f(x) is increasing.

Q19.

$f(x)=(2x^2-3x)$

$f'(x)=4x-3$

$f'(x)=0$ at $x=3/4$



Clearly we can see that function is increasing for $x\in[3/4, \infty)$ and is decreasing for $x\in(-\infty,3/4)$

Q20.

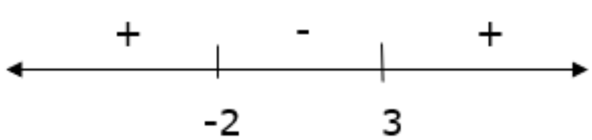
$f(x)=2x^3-3x^2-36x+7$

$f'(x)=6x^2-6x-36$

$f'(x)=6(x^2-x-6)$

$f'(x)=6(x-3)(x+2)$

$f'(x)$ is 0 at $x=3$ and $x=-2$



$F'(x)>0$ for $x \in (-\infty, -2] \cup [3, \infty)$

hence in this interval function is increasing.

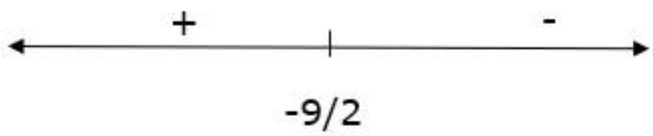
$F'(x)<0$ for $x \in (-2 ,3)$

hence in this interval function is decreasing.

Q21.

$f(x)=6-9x-x^2$

$f'(x)=-(2x+9)$



We can see that f(x) is increasing for $x \in \left(-\infty, -\frac{9}{2}\right]$ and decreasing in $x \in \left(-\frac{9}{2}, \infty\right)$

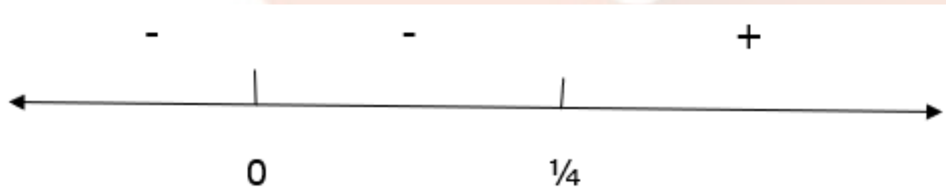
Q22.

Consider $f(x) = \left(x^4 - \frac{x^3}{3}\right)$

$f'(x)=4x^3-x^2$

$=x^2(4x-1)$

$F'(x)=0$ for $x=0$ and $x=1/4$



Function f(x) is decreasing for $x \in (-\infty, 1/4]$ and increasing in $x \in (1/4, \infty)$

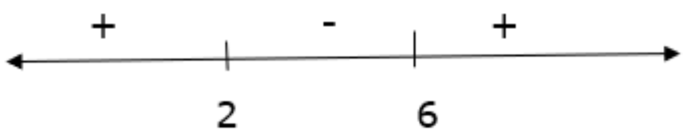
Q23.

$f(x)=x^3-12x^2+36x+17$

f’(x)=$3x^2$-24x+36

f’(x)=3(x^2-8x+12)

=3(x-6)(x-2)



Function f(x) is decreasing for x ∈ [2,6] and increasing in x ∈ (-∞,2) ∪ (6, ∞)

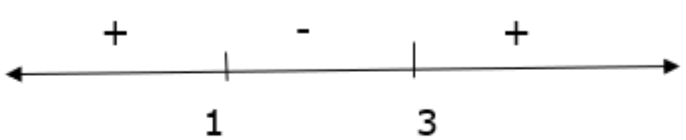
Q24.

f(x)=x^3-6x^2+9x+10

f’(x)=3x^2-12x+9

f’(x)=3(x^2-4x+3)

=3(x-3)(x-1)



Function f(x) is decreasing for x ∈ [1,3] and increasing in x ∈ (-∞,1) ∪ (3, ∞)

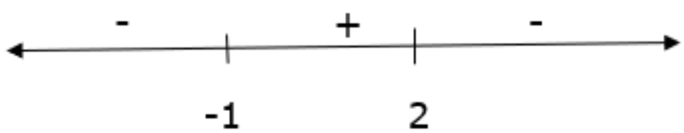
Q25.

f(x)=-2x^3+3x^2+12x+6

$f'(x)=-6x^2+6x+12$

$f'(x)=-6(x^2-x-2)$

$=-6(x-2)(x+1)$



Function f(x) is increasing for $x \in [-1,2]$ and decreasing in $x \in (-\infty,-1) \cup (2, \infty)$

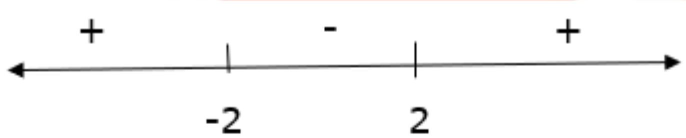
Q26.

$f(x)=2x^3-24x+5$

$f'(x)=6x^2-24$

$f'(x)=6(x^2-4)$

$6(x-2)(x+2)$



Function f(x) is decreasing for $x \in [-2,2]$ and increasing in $x \in (-\infty,-2) \cup (2, \infty)$

Q27.

$f(x)=(x-1)(x-2)^2$=x2-4x+4 * x-1= x3-4x2+4x-x2+4x-4

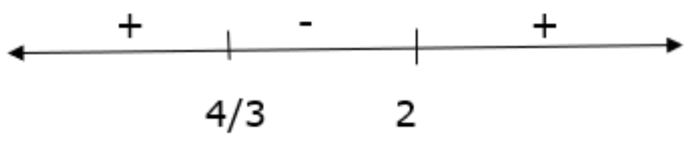
$f(x)=x^3-5x^2+8x-4$

$f'(x)=3x^2-10x+8$

$f'(x)=3x^2-6x-4x+8$

$=3x(x-2)-4(x-2)$

$=(3x-4)(x-2)$



Function f(x) is decreasing for $x \in [4/3,2]$ and increasing n $x \in (-\infty,4/3) \cup (2, \infty)$

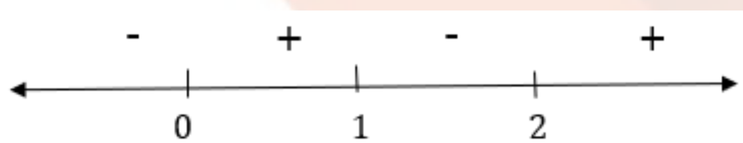
Q28.

$f(x)=x^4-4x^3+4x^2+15$

$f'(x)=4x^3-12x^2+8x$

$= 4x(x^2-3x+2)$

$=4x(x-1)(x-2)$



Function f(x) is decreasing for $x \in (-\infty,0] \cup [1, 2]$ and increasing in $x \in (0,1) \cup (2, \infty)$

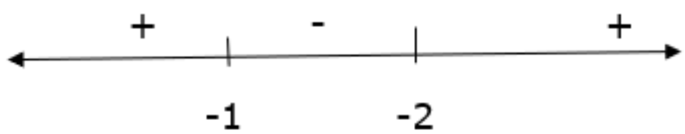
Q29.

$f(x)=2x^3+9x^2+12x+15$

$f'(x)=6x^2+18x+12$

$f'(x)=6(x^2+3x+2)$

$=6(x+2)(x+1)$



Function f(x) is decreasing for x ∈ [-1,-2] and increasing in x ∈ (-∞,-1) ∪ (-2, ∞)

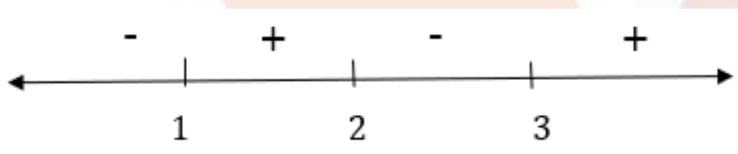
Q30.

$f(x)=x^4-8x^3+22x^2-24x+21$

$f'(x)=4x^3-24x^2+44x-24$

$= 4(x^3 -6x^2+11x-6)$

$=4(x-3)(x-1)(x-2)$



Function f(x) is decreasing for x ∈ (-∞,1] ∪ [2, 3] and increasing in x ∈ (1,2) ∪ (3, ∞)

Q31.

$f(x)=3x^4-4x^3-12x^2+5$

$f'(x)=12x^3-12x^2-24x$

$=12x(x^2 - x - 2)$

$=12(x)(x+1)(x-2)$

Function f(x) is decreasing for $x \in (-\infty,-1] \cup [0, 2]$ and increasing in $x \in (-1,0) \cup (2, \infty)$

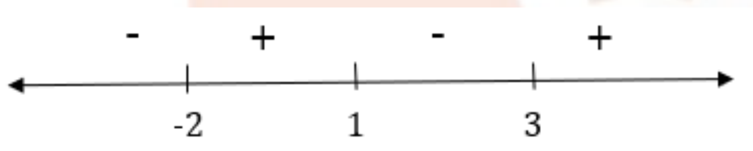
Q32.

$f'(x) = \frac{12x^3}{10} - \frac{12x^2}{5} - 6x + \frac{36}{5}$

$f'(x)=(12x^3-24x^2-60x+72)/10$

$=1.2(x^3-2x^2-5x+6)$

$=1.2(x-1)(x-3)(x+2)$



Function f(x) is decreasing for $x \in (-\infty,-2] \cup [1, 3]$ and increasing in $x \in (-2,1) \cup (3, \infty)$

Exercise-11H

Q1.

i. $\frac{dy}{dx} = 3x^2 - 1$

$\frac{dy}{dx}$ at $(x = 2) = 11$

ii. $\frac{dy}{dx} = 4x + 3\cos x$

$\frac{dy}{dx}$ at $(x = 0) = 3$

iii. $\frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$

$\frac{dy}{dx}$ at $\left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$

Q2.

$m : \frac{dy}{dx} = 3x^2 - 2$

m at (1, 6) = 1

Tangent : y – b = m(x – a)

y – 6 = 1(x – 1)

x – y + 5 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

y – 6 = -1(x – 1)

x + y – 7 = 0

Q3.

$m : 2y\frac{dy}{dx} = 4a$

m at $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = m$

Tangent : $y - b = m(x - a)$

$y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$

$m^2x - my + a = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$

$m^2x + m^3y - 2am^2 - a = 0$

Q4.

$m : \frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$m \text{ at } (a\cos\theta, b\sin\theta) = \frac{-b\cos\theta}{a\sin\theta}$

Tangent : $y - b = m(x - a)$

$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$

$bx\cos\theta + ay\sin\theta = ab$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$ax \sec\theta - by \operatorname{cosec}\theta = a^2 - b^2$

Q5.

$m : \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$

$m \text{ at } (a\sec\theta, b\tan\theta) = \dfrac{b\sec\theta}{a\tan\theta}$

Tangent : y – b = m(x – a)

$y - b\tan\theta = \dfrac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$

$bx\sec\theta - ay\tan\theta = ab$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - b\sin\theta = \dfrac{-a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$by\,\text{cosec}\theta + ax\sec\theta = (a^2 + b^2)$

Q6.

$m : \frac{dy}{dx} = 3x^2$

m at (1, 1) = 3

Tangent : y – b = m(x – a)

y – 1 = 3(x – 1)

y = 3x – 2

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 1 = \dfrac{-1}{3}(x - 1)$

x + 3y = 4

Q7.

$m: 2y\frac{dy}{dx} = 4a$

m at $(at^2, 2at) = 1/t$

Tangent : $y - b = m(x - a)$

$y - 2at = \frac{1}{t}(x - at^2)$

$x - ty + at^2 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 2at = -t(x - at^2)$

$tx + y = at^3 + 2at$

Q8.

$m: \frac{dy}{dx} = 2\cot x(-\mathrm{cosec}^2 x) + 2\,\mathrm{cosec}^2 x$

m at $(x = \pi/4) = 2(-2) + 2(2) = 0$

Tangent : $y - b = m(x - a)$

$y - 1 = 0(x - \pi/4)$

$y = 1$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 1 = \frac{-1}{0}\left(x - \frac{\pi}{4}\right)$

$x = \pi/4$

Q9.

$m : 32x + 18y\frac{dy}{dx} = 0$

$m \text{ at } (2, y_1) = \frac{-32}{9y_1}$

$16(2)^2 + 9(y_1)^2 = 144$

$y_1 = \frac{4\sqrt{5}}{3}$

Tangent : $y - b = m(x - a)$

$y - \frac{4\sqrt{5}}{3} = \frac{-32}{9\frac{4\sqrt{5}}{3}}(x - 2)$

$8x + 3\sqrt{5}y - 36 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - \frac{4\sqrt{5}}{3} = \frac{9\frac{4\sqrt{5}}{3}}{32}(x - 2)$

$9\sqrt{5x} - 24y + 14\sqrt{5} = 0$

Q10.

$m : \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

m at $(x = 1) = 2$

y at $(x = 1) = (1)^4 - 6(1)^3 + 13(1)^2 - 10(1) + 5 = 3$

Tangent : $y - b = m(x - a)$

$y - 3 = 2(x - 1)$

$2x - y + 1 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 3 = \frac{-1}{2}(x - 1)$

$x + 2y - 7 = 0$

Q11.

m : $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

m at $\left(\frac{a^2}{4}, \frac{a^2}{4}\right) = -1$

$y - b = m(x - a)$

$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$

$2(x + y) = a^2$

Q12.

m at $(x_1, y_1) = \frac{b^2 x_1}{a^2 y_1}$

At (x_1, y_1) : $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow b^2x_1^2 - a^2y_1^2 = a^2b^2$

$y - b = m(x - a)$

$y - y_1 = \frac{b^2 x_1}{a^2 y_1}(x - x_1)$

$a^2y_1y - a^2y_1^2 = b^2x_1x - b^2x_1^2$

$b^2x_1x - a^2y_1y = a^2b^2$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Q13.

$m : \frac{dy}{dx} = 4\sec^3x(\tan x \sec x) - 4\tan^3x(\sec^2x)$

$m \text{ at } \left(x = \frac{\pi}{3}\right) = 4(2)^3(\sqrt{3} \times 2) - 4(\sqrt{3})^3(2)^2 = 16\sqrt{3}$

At $x = \pi/3$, $y = 7$

$y - b = m(x - a)$

$y - 7 = 16\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$3y - 48\sqrt{3}x + 16\sqrt{3}\pi - 21 = 0$

Q14.

$m : \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2x)$

$\frac{dy}{dx} \text{ at } \left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$

At $x = \pi/2$, $y = 4$

$y - b = \frac{-1}{m}(x - a)$

$y - 4 = \frac{1}{12}\left(x - \frac{\pi}{2}\right)$

$24y - 2x + \pi - 96 = 0$

Q15.

m : $\frac{dy}{dx} = 6x^2$

m at (x = 2) = 24

m at (x = -2) = 24

We know that if the slope of curve at two different point is equal then straight lines are parallel at that points.

Q16.

We know that if two straight lines are parallel then their slope are equal. So, slope of required tangent is also equal to 4.

m : $\frac{dy}{dx} = \frac{-2x}{3} = 4$

x = -6 and y = -11

y – b = m(x – a)

y – (-11) = 4(x – (-6))

4x – y + 13 = 0

Q17.

If the tangent is parallel to y-axis it means that it's slope is not defined or 1/0.

m : $2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-(2x-2)}{(2y-4)} = \frac{1}{0}$$

$2y - 4 = 0 \Rightarrow y = 2$

$x^2 + (2)^2 - 2x - 4(2) + 1 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3$ and $x = -1$

So, the requied points are (-1, 2) and (3, 2).

Q18.

If the tangent is parallel to x-axis it means that it's slope is 0

$$m: \ 2x + 2y\frac{dy}{dx} - 2 = 0$$

$2x + 2y(0) - 2 = 0$

$x = 1$

$(1)^2 + y^2 - 2(1) - 3 = 0$

$\Rightarrow y^2 = 4 \Rightarrow y = 2$ and $y = -2$

So, the requied points are (1, 2) and (1, -2).

Q19.

We know that if the slope of two tangent of a curve are satisfies a relation $m_1m_2 = -1$, then tangents are at right angles

$$m: \ \frac{dy}{dx} = 2x - 5$$

m_1 at (2, 0) = -1

m_2 at (3, 0) = 1

$m_1m_2 = (-1)(1) = -1$

So, we can say that tangent at (2, 0) and (3, 0) are at right angles.

Q20.

If tangent is pass through origin it means that equation of tangent is

$y = mx$

Let us suppose that tangent is made at point (x_1, y_1)

$y_1 = x_1^2 + 3x_1 + 4$ …(1)

$m : \frac{dy}{dx} = 2x + 3$

m at $(x_1, y_1) = 2x_1 + 3$

Equation of tangent : $y_1 = (2x_1 + 3)x_1$ …(2)

On compairing eq(1) and eq(2)

$x_1^2 + 3x_1 + 4 = (2x_1 + 3)x_1$

$x_1^2 - 4 = 0 \Rightarrow x_1 = 2$ and -2

At $x_1 = 2, y_1 = 14$

At $x_1 = -2, y_1 = 2$

So, required points are (2, 14) and (-2, 2)

Q21.

Slope of y = x – 11 is equal to 1

$$m : \frac{dy}{dx} = 3x^2 - 11$$

$3x^2 - 11 = 1 \Rightarrow x = 2$ and -2

At x = 2

From the equation of curve, $y = (2)^3 - 11(2) + 5 = -9$

From the equation of tangent, $y = 2 - 11 = -9$

At x = -2

From the equation of curve, $y = (-2)^3 - 11(-2) + 5 = 19$

From the equation of tangent, $y = -2 - 11 = -13$

So, the final answer is (2, -9) because at x = -2, y is come different from the equation of curve and tangent which is not possible.

Q22.

If tangent is parallel to the line x + 3y = 4 then it's slope is -1/3.

$$m : 4x + 6y\frac{dy}{dx} = 0$$

$$m = \frac{-2x}{3y} = \frac{-2x}{3\sqrt{\frac{14-2x^2}{3}}} = \frac{-1}{3}$$

$$2x = \sqrt{\frac{14-2x^2}{3}}$$

$4x^2 = \frac{14 - 2x^2}{3}$

x = 1 and -1

At x = 1, y = 2 and y = -2 (not possible)

At x = -1, y = -2 and y = 2 (not possible)

y – b = m(x – a)

At (1, 2)

$y - 2 = \frac{-1}{3}(x - 1)$

3y + x = 7

At (-1, -2)

$y - (-2) = \frac{-1}{3}(x - (-1))$

y + x = -7

Q23.

If tangent is perpendicular to the line x – 2y + 1 = 0 then it's -1/m is -2.

$m : 2x + 2\frac{dy}{dx} = 0$

m = -x = 1/2

x = -1/2

At x = -1/2, y = 31/8

$$y - b = \frac{-1}{m}(x - a)$$

At (-1/2, 31/8)

$$y - \frac{31}{8} = \frac{-1}{\frac{1}{2}}\left(x - \left(-\frac{1}{2}\right)\right)$$

16x + 8y – 23 = 0

Q24.

We know that if tangent is parallel to x-axis then it's slope is equal to 0.

$$m : \frac{dy}{dx} = 4x - 6$$

4x – 6 = 0 ⇒ x = 3/2

At x = 3/2 , y = -17/2

So, the required points are $\left(\frac{3}{2}, \frac{-17}{2}\right)$.

Q25.

If the tangent is parallel to chord joining the points (3, 0) and (4, 1) then slope of tangent is equal to slope of chord.

$$m = \frac{1 - 0}{4 - 3} = 1$$

$$m : \frac{dy}{dx} = 2(x - 3)$$

2(x – 3) = 1 ⇒ x = 7/2

At x = 7/2, y = 1/4

So, the required points are $\left(\frac{7}{2},\frac{1}{4}\right)$.

Q26.

If curves cut at right angle if $8k^2 = 1$ then vice versa also true. So, we have to prove that $8k^2 = 1$ if curve cut at right angles.

If curve cut at right angle then the slope of tangent at their intersecting point satisfies the relation $m_1m_2 = -1$

We have to find intersecting point of two curves.

$x = y^2$ and $xy = k$ then $y = k^{\frac{1}{3}}$ and $x = k^{\frac{2}{3}}$

$$m_1 : \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$m_1 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{1}{2k^{\frac{1}{3}}}$$

$$m_2 : \frac{dy}{dx} = \frac{-k}{x^2}$$

$$m_2 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{-k}{k^{\frac{4}{3}}} = -\frac{1}{k^{\frac{1}{3}}}$$

$$m_1m_2 = -1$$

$$\left(\frac{1}{2k^{\frac{1}{3}}}\right)\left(-\frac{1}{k^{\frac{1}{3}}}\right) = -1$$

$$k^{\frac{2}{3}} = \frac{1}{2} \Rightarrow k^2 = \frac{1}{8} \Rightarrow 8k^2 = 1$$

Q27.

If the two curve touch each other then the tangent at their intersecting point formed a angle of 0.

We have to find the intersecting point of these two curves.

$xy = a^2$ and $x^2 + y^2 = 2a^2$

$\Rightarrow x^2 + \left(\frac{a^2}{x}\right)^2 = 2a^2$

$\Rightarrow x^4 - 2a^2x^2 + a^4 = 0$

$\Rightarrow (x^2 - a^2) = 0$

$\Rightarrow x = +a$ and $-a$

At $x = a$, $y = a$

At $x = -a$, $y = -a$

$m_1 : \frac{dy}{dx} = \frac{-a^2}{x^2}$

m_1 at $(a, a) = -1$

m_1 at $(-a, -a) = -1$

$m_2 : \ 2x + 2y\frac{dy}{dx} = 0$

m_2 at $(a, a) = -1$

m_2 at $(-a, -a) = -1$

At (a, a)

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

At (-a, -a)

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

So, we can say that two curves touch each other because the angle between two tangent at their intersecting point is equal to 0.

Q28.

If the two curve cut orthogonally then angle between their tangent at intersecting point is equal to 90º.

We have to find their intersecting point.

$x^3 - 3xy^2 + 2 = 0$ …(1) and $3x^2y - y^3 - 2 = 0$ …(2)

On adding eq (1) and eq (2)

$x^3 - 3xy^2 + 2 + 3x^2y - y^3 - 2 = 0$

$x^3 - y^3 - 3xy^2 + 3x^2y = 0$

$(x - y)^3 = 0 \Rightarrow x = y$

Put x = y in eq (1)

$y^3 - 3y^3 + 2 = 0 \Rightarrow y = 1$

At y = 1, x = 1

$m_1 : 3x^2 - 3\left(x \times 2y\frac{dy}{dx} + y^2\right) = 0$

m_1 at (1, 1) = 0

$m_2: 3\left(x^2\frac{dy}{dx}+2xy\right)-3y^2\frac{dy}{dx}=0$

m_2 at (1, 1) = -2/0

At (1, 1)

$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$

$\tan\theta=\frac{m_2\left(1-\frac{m_1}{m_2}\right)}{m_2\left(\frac{1}{m_2}+m_1\right)}$

$\tan\theta=\frac{(1-0)}{(0+0)}=\text{not defined}\Rightarrow\theta=\frac{\pi}{2}$

So, we can say that two curve cut each other orthogonally because angle between two tangent at their intersecting point is equal to 90º.

Q29.

$m:\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$

$m \text{ at } \left(\theta=\frac{\pi}{4}\right)=\frac{-1}{1+\sqrt{2}}=1-\sqrt{2}$

At $\theta=\frac{\pi}{4}, x=\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)$ and $y=\left(1+\frac{1}{\sqrt{2}}\right)$

y – b = m(x – a)

$y-\left(1+\frac{1}{\sqrt{2}}\right)=(1-\sqrt{2})\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$

$y=(1-\sqrt{2})x+\frac{(\sqrt{2}-1)\pi}{4}+2$

Q30.

$m: \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$

$m \text{ at } \left(t = \frac{\pi}{4}\right) = \frac{2\sqrt{2}}{3}$

At $t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}$ and $y = 0$

$y - b = m(x - a)$

$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$

$4x - 3\sqrt{2}y - 2\sqrt{2} = 0$

Objective Exercise

Q1.

Given that $y=2^x$

Taking log both sides, we get

$\log_e y = x\log_e 2$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e 2 \text{ or } \frac{dy}{dx} = \log_e 2 \times y$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

Q2.

Given that $y = \log_{10} x$

Using the property that $\log_a b = \frac{\log_e b}{\log_e a}$, we get

$$y = \frac{\log_e x}{\log_e 10}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x\log_e 10}$$

Q3.

Given that $y = e^{\frac{1}{x}}$

Taking log both sides, we get

$$\log_e y = \frac{1}{x} \text{ (Since } \log_a b^c = c\log_a b)$$

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2} \text{ or } \frac{dy}{dx} = -\frac{1}{x^2} \times y$$

Hence $\frac{dy}{dx} = -\frac{1}{x^2} \times e^{\frac{1}{x}}$

Q4.

Let $y=f(x)=x^x$

Taking log both sides, we get

$\log_e y = x \times \log_e x$-(1) (Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1$$

$$\Rightarrow \frac{dy}{dx} = y \times (1 + \log_e x)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x(1 + \log_e x)$$

Q5.

Let $y=f(x)=x^{\sin x}$

Taking log both sides, we get

$\log_e y = \sin x \times \log_e x$ -(1) (Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log_e x \times \cos x$$

$$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{\sin x}{x} + \log_e x \cos x\right)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x\left(\frac{\sin x + x\log_e x \cos x}{x}\right)$$

Q6.

Let $y = f(x) = x^{\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sqrt{x} \times \log_e x$ -(1)

(Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \times \frac{1}{x} + \log_e x \times \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = f'(x)$$

$$= x^{\sqrt{x}}\left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$$

Q7.

Given that $y = e^{\sin\sqrt{x}}$

Taking log both sides, we get

$$\log_e y = \sin\sqrt{x}$$

$$(\text{Since } \log_a b^c = c \log_a b)$$

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

Or

$$\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}} \times y$$

Hence $\frac{dy}{dx} = \frac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$

Q8.

Given that $y=(\tan x)^{\cot x}$

Taking log both sides, we get

$\log_e y=\cot x\times\log_e\tan x$ (Since $\log_a b^c=c\log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx}=\cot x\times\frac{1}{\tan x}\times\sec^2 x-\log_e\tan x\times\operatorname{cosec}^2 x=\operatorname{cosec}^2 x(1-\log_e\tan x)$$

Hence, $\frac{dy}{dx}=\operatorname{cosec}^2 x(1-\log_e\tan x\times y=\operatorname{cosec}^2 x(1-\log_e\tan x)(\tan x)^{\cot x}$

Q9.

Given that $y=(\sin x)^{\log_e x}$

Taking log both sides, we get

$\log_e y=\log_e x\times\log_e\sin x$ (Since $\log_a b^c=c\log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx}=\log_e x\times\frac{1}{\sin x}\times\cos x+\log_e\sin x\times\frac{1}{x}$$

$$=\frac{x\cot x\log_e x+\log_e\sin x}{x}$$

Hence, $\frac{dy}{dx}=\frac{x\cot x\log_e x+\log_e\sin x}{x}\times y$

$$=\frac{x\cot x\log_e x+\log_e\sin x}{x}(\sin x)^{\log_e x}$$

Q10.

Given that $y=\sin(x^x)$

Let $x^x=u$, then $y=\sin u$

Differentiating with respect to x, we get

$\frac{dy}{dx}=\cos u\times\frac{du}{dx}=\cos(x^x)\frac{du}{dx}$ –(1)

Also, $u=x^x$

Taking log both sides, we get

$\log_e u=x\times\log_e x$

$(\text{Since } \log_a b^c=c\log_a b)$

Differentiating with respect to x, we get

$$\frac{1}{u}\frac{du}{dx}=x\times\frac{1}{x}+\log_e x\times 1$$

$$\Rightarrow\frac{du}{dx}=u\times(1+\log_e x)$$

$\Rightarrow\frac{du}{dx}=x^x(1+\log_e x)$ -(2)

From (1) and (2), we get

$$\frac{dy}{dx}=\cos(x^x)x^x(1+\log_e x)$$

Q11.

Given that $y=\sqrt{x\sin x}$

Squaring both sides, we get

$y^2 = x\sin x$

Differentiating with respect to x, we get

$2y\frac{dy}{dx} = x\cos x + \sin x$ or $\frac{dy}{dx} = \frac{x\cos x+\sin x}{2y}$

Hence, $\frac{dy}{dx} = \frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

Q12.

Given that $xy=e^{x+y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we get

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Or

$$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$$

Hence, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

Q13.

Given that x+y=sin(x+y)

Differentiating with respect to x, we get

$1+\frac{dy}{dx}=\cos(x+y)\left(1+\frac{dy}{dx}\right)$ or $(\cos(x+y)-1)\left(1+\frac{dy}{dx}\right)=0$

Hence, cos(x+y)=1 or $\frac{dy}{dx}=-1$

If cos(x+y)=1 then, x+y=2nπ, n∈ℤ

Hence x+y=sin(2nπ)=0 or y=-x

Differentiating with respect to x, we get

$$\frac{dy}{dx}=-1$$

Hence, $\frac{dy}{dx}=-1$

Q14.

Given that $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

Or

$$\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$$

Q15.

Given that $x^y=y^x$

Taking log both sides, we get

$y\log_e x = x\log_e y$

$(\text{Since } \log_a b^c = c\log_a b)$

Differentiating with respect to x, we get

$$\frac{y}{x}+\log_e x\frac{dy}{dx}=\frac{x}{y}\frac{dy}{dx}+\log_e y$$

$$\Rightarrow \frac{x-y\log_e x}{y}\frac{dy}{dx}=\frac{y-x\log_e y}{x}$$

Hence $\frac{dy}{dx}=\frac{y(y-x\log_e y)}{x(x-y\log_e x)}$

Q16.

Given that $x^p y^q=(x+y)^{p+q}$

Taking log both sides, we get

$\log_e x^p y^q = (p+q)\log_e(x+y)$

$(\text{Since } \log_a b^c = c\log_a b)$

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x^p + \log_e y^q = (p+q)\log_e(x+y)$

$p\log_e x + q\log_e y = (p+q)\log_e(x+y)$

Differentiating with respect to x, we get

$$\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{xq-yp}{y(x+y)}\right)=\frac{xq-yp}{x(x+y)}$$

Hence, $\frac{dy}{dx}=\frac{y}{x}$

Q17.

Given that $y = x^2 \sin\frac{1}{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = x^2 \cos\frac{1}{x} \times -\frac{1}{x^2} + 2x\sin\frac{1}{x} = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

Q18.

$y=\cos^2 x^3=(\cos(x^3))^2$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 2\cos(x^3) \times -\sin(x^3) \times 3x^2$$

Using 2sinAcosA=sin2A

$$\frac{dy}{dx} = -3x^2 \sin(2x^3)$$

Q19.

Given that $y = \log_e\left(x + \sqrt{x^2 + a^2}\right)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{1}{2\sqrt{x^2 + a^2}} \times 2x\right)$$

Hence, $\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \times \frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

Q20.

Given that $y = \log_e \frac{1+\sqrt{x}}{1-\sqrt{x}}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1-\sqrt{x}}{1+\sqrt{x}} \times \frac{(1-\sqrt{x}) \times \frac{1}{2\sqrt{x}} - (1+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(1-\sqrt{x})^2} = \frac{1}{(1-x)\sqrt{x}}$$

Q21.

Given that $y = \log_e \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{\left(\sqrt{1+x^2}-x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x+1\right)-\left(\sqrt{1+x^2}+x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x-1\right)}{\left(\sqrt{1+x^2}-x\right)^2}$$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

Q22.

Given that $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Using, $\cos^2\theta + \sin^2\theta = 1$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$y = \sqrt{\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

Dividing by $\sin\frac{x}{2}$ in numerator and denominator, we get

$$y = \frac{\cot\frac{x}{2} + 1}{\cot\frac{x}{2} - 1} = \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\left(\text{Using } \cot\left(\frac{\pi}{4} - A\right) = \frac{\cot A + 1}{\cot A - 1}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\text{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \times -\frac{1}{2}$$

Hence, $\frac{dy}{dx} = \frac{1}{2}\text{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

Q23.

Given that $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$

Multiplying by cos x in numerator and denominator, we get

$$y = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$= \tan\left(\frac{x}{2}\right)$$

Differentiating with respect to x, we get

$y = \sec^2\frac{x}{2} \times \frac{1}{2}$

$= \frac{1}{2}\sec^2\frac{x}{2}$

Q24.

Given that $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$y = \sqrt{\tan\left(\frac{\pi}{4} + x\right)}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4} + x\right)}} \times \sec^2\left(\frac{\pi}{4} + x\right) \times 1$

Hence, $\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+x\right)}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}$

Q25.

Given that $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and Using $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, we get

$y = \tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or $y = \tan^{-1}\tan\frac{x}{2}$

$y = \frac{x}{2}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Q26.

Given that $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Dividing numerator and denominator with cosx, we get

$$y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

Using $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right) = \frac{\pi}{4}+x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 1$$

Q27.

Given that $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$$\Rightarrow y = \tan^{-1}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$=\frac{\pi}{4}-\frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=-\frac{1}{2}$$

Q28.

Given that $y=\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using $1-\cos x=2\sin^2\frac{x}{2}$ and $1+\cos x=2\cos^2\frac{x}{2}$, we get

$$y=\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}=\tan^{-1}\tan\left(\frac{x}{2}\right)=\frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{2}$$

Q29.

Given that $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Dividing by bcosx in numerator and denominator, we get

$$y = \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

Let $\frac{a}{b} = \tan\alpha \Rightarrow \alpha = \tan^{-1}\frac{a}{b}$

Then $y = \tan^{-1}\left(\frac{\tan\alpha - \tan x}{1 + \tan\alpha\tan x}\right)$

Using $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$, we get

$$y = \tan^{-1}\tan(\alpha - x) = \alpha - x = \tan^{-1}\frac{a}{b} - x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -1$$

Q30.

Given that $y=\sin^{-1}(3x-4x^3)$

Let $x=\sin\theta$

$\Rightarrow \theta=\sin^{-1}x$

Then, $y= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

Using $\sin3\theta=3\sin\theta - 4\sin^3\theta$, we get

$y=\sin^{-1}(\sin3\theta)=3\theta=3\sin^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

Q31.

Given that $y=\cos^{-1}(4x^3-3x)$

Let $x=\cos\theta$

$\Rightarrow \theta=\cos^{-1}x$

Then, $y=\cos^{-1}(4\cos^3\theta-3\cos\theta)$

Using $\cos3\theta=4\cos^3\theta-3\cos\theta$, we get

$y=\cos^{-1}(\cos3\theta)=3=3\cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$$

Q32.

Given that $y = \tan^{-1}\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{x} = \tan B$, then $A = \tan^{-1}\sqrt{a}$ and $B = \tan^{-1}\sqrt{x}$

Hence, $y = \tan^{-1}\frac{\tan A+\tan B}{1-\tan A\tan B}$

Using $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$, we get

$y=\tan^{-1}\tan(A+B)=A+B$

$= \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$$

Q33.

Given that $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y = \frac{x^2-1}{x^2+1}$ or $\sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1$$

$$= \frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$$y = \tan^{-1}(-\tan 2\theta)$$

Using $-\tan x = \tan(-x)$, we get

$$y = \tan^{-1}(\tan(-2\theta))$$

$= -2\theta$

$=-2\tan^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{-2}{1+x^2}$$

Q34.

Given that $y=\tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x^2=\tan\theta$

$\Rightarrow \theta=\tan^{-1}x^2$

Hence, $y=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we get

$$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}(x^2)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+x^4}\times 2x=\frac{2x}{1+x^4}$$

Q35.

Given that $y=\tan^{-1}(-\sqrt{x})$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+(-\sqrt{x})^2}\times\frac{-1}{2\sqrt{x}}=\frac{-1}{2\sqrt{x}(1+x)}$$

Q36.

Given that y=cos^{-1}x^3

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^3)^2}} \times 3x^2 = \frac{-3x^2}{\sqrt{1-x^6}}$$

Q37.

Given that y=tan^{-1}(sec x + tan x)

Hence, $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we get

$$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{2}$$

Q38.

Given that $y=\cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$ and using $\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$

Hence, $y=\frac{\pi}{2}-\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we get

$$y=\frac{\pi}{2}-\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+x^2}$$

Q39.

Given that $y=\sqrt{\frac{1+x}{1-x}}$

Let $x=-\cos\theta \Rightarrow \theta=\cos^{-1}(-x)$.

Using $1-\cos\theta=2\sin^2\frac{\theta}{2}$ and $1+\cos\theta=2\cos^2\frac{\theta}{2}$, we get

$$y=\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}=\tan\left(\frac{\theta}{2}\right)$$

Differentiating with respect to x, we get

$\frac{dy}{dx}=\sec^2\left(\frac{\theta}{2}\right)\times\frac{1}{2}\frac{d\theta}{dx}$ –(1)

Since, x=-cosθ ⇒ $2\cos^2\frac{\theta}{2}=1+\cos\theta=1-x$ or $\sec^2\left(\frac{\theta}{2}\right)=\frac{2}{1-x}$ -(2)

Also, since θ=cos^{-1}(-x), therefore $\frac{d\theta}{dx}=\frac{1}{\sqrt{1-x^2}}$ –(3)e

Substituting (2) and (3) in (1), we get

$$\frac{dy}{dx}=\frac{2}{1-x}\times\frac{1}{2}\times\frac{1}{\sqrt{1-x^2}}=\frac{1}{(1-x)\sqrt{1-x^2}}=\frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

Q40.

Given that $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$$\Rightarrow \sec y=\frac{x^2+1}{x^2-1}$$

Since tan^{2}x=sec^{2}x-1, therefore

$$\tan^2 y=\left(\frac{x^2+1}{x^2-1}\right)^2-1=\frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y=-\frac{2x}{1-x^2}$ or $y=\tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let x=tanθ ⇒ θ=tan^{-1}x

Hence, $y=\tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$$y = \tan^{-1}(-\tan 2\theta)$$

Using -tan x=tan(-x), we get

$$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Q41.

$$\Rightarrow\ y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$$

$$\Rightarrow\ \sec y\ = \frac{1}{2x^2-1}$$

$\Rightarrow$ cos y=$2x^2$ -1

$\Rightarrow$ y=$\cos^{-1}$ ($2x^2$ -1)

Put x = cos θ

$\Rightarrow$ y = $\cos^{-1}$(2 $\cos^2$ θ – 1)

$\Rightarrow$ y = $\cos^{-1}$(cos 2 θ)

$\Rightarrow$ y = 2θ

But θ = $\cos^{-1}$x.

$$\Rightarrow\ \frac{dy}{dx} = \frac{d(\cos^{-1}x)}{dx}$$

$$\Rightarrow\ \frac{dy}{dx} = 2.\frac{d(\cos^{-1}x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2.\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

Q42.

Put x = tan θ

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow y = \frac{\theta}{2}$$

$\theta = \tan^{-1}x$

$$\Rightarrow y = \frac{\tan^{-1}x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Q43.

Put x = cos2θ

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2}+\frac{\sqrt{1-\cos 2\theta}}{2}\right)$$

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2}+\frac{\sqrt{2.\sin^2\theta}}{2}\right)$$

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}}+\frac{\sin 2\theta}{\sqrt{2}}\right)$$

$$\Rightarrow\ y=\sin^{-1}(\sin\left(\frac{\pi}{4}+2\theta\right)$$

$$\Rightarrow\ y=\frac{\pi}{4}+2\theta.$$

$$\Rightarrow\ \frac{dy}{d\theta}=2$$

Put $\theta=\frac{\cos^{-1}x}{2}$

$$\Rightarrow\ \frac{d\theta}{dx}=\frac{-1}{4\sqrt{1-x^2}}$$

$$\therefore\ \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$$

Q44.

$x = at^2$

$$\therefore\ \frac{dx}{dt}=2at$$

$$\therefore\ \frac{dt}{dx}=\frac{1}{2at}$$

Y = 2at

$\therefore \frac{dy}{dt} = 2a$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 2a \times \frac{1}{2at}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{t}$

Q45.

x = a sec θ

$\therefore \frac{dx}{d\theta} = a\sec\theta.\tan\theta$

$\therefore \frac{d\theta}{dx} = \frac{1}{a\sec\theta.\tan\theta}$

y = b tan θ

$\therefore \frac{dy}{d\theta} = b.\sec^2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = b.\sec^2\theta \times \frac{1}{a\sec\theta.\tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b.\frac{1}{\cos\theta}}{a.\frac{\sin\theta}{\cos\theta}}$

$$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\csc\theta$$

Q46.

$x = a.\cos^2\theta$

$$\therefore \frac{dx}{d\theta} = -2\,a\cos\theta\,.\sin\theta$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{-1}{2\,a.\cos\theta\,.\sin\theta}$$

$y = b.\sin^2\theta$

$$\therefore \frac{dy}{d\theta} = 2b\sin\theta\,.\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2b\sin\theta.\cos\theta \times \frac{-1}{2\,a\cos\theta\,.\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}$$

Q47.

$x = a(\cos\theta + \theta\sin\theta)$

$$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$$

$y = a(\sin\theta - \theta\cos\theta)$

$$\therefore \frac{dy}{d\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$$

$$\Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + \theta a\sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta$$

Q48.

Given:

$\Rightarrow y = x^{x^{x^{\dots\infty}}}$

We can write it as

$\Rightarrow y = x^y$

Taking log of both sides we get

$\log y = y \log x$

Differentiating

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y.\frac{1}{x}$$

$$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1-\log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-\log x)}$$

Q49.

Given:

$$\Rightarrow y = \sqrt{x + \sqrt{x + \sqrt{x +}}} \ldots \infty$$

We can write it as

$$\Rightarrow y = \sqrt{x + y}$$

Squaring we get

$\Rightarrow y^2 = x + y$

Differentiating

$$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(2y-1)}$$

Q50.

Given:

$$\Rightarrow y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x +}}} \ldots \infty$$

We can write it as

$$\Rightarrow y = \sqrt{\sin x + y}$$

Squaring we get

$\Rightarrow y^2 = \sin x + y$

Differentiating

$$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(2y-1)}$$

Q51.

We can write it as

$\Rightarrow y = e^{x+y}$

$\log y = (x + y) \log e$

Differentiating

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = 1\left(\frac{y}{1-y}\right)$$

Q52.

Since f(x) is continuous on 0.

$$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{3x} = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin 5x}{3x}\times\frac{5x}{5x}=f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin 5x}{5x}\times\frac{5x}{3x}=f(0)$$

$$\Rightarrow f(0)=\frac{5}{3}$$

$$\Rightarrow k=\frac{5}{3}$$

Q53.

Left hand limit =

$$\Rightarrow \lim_{x\to 0^-} f(x)$$

$$\Rightarrow \lim_{h\to 0} f(0-h)$$

$$\Rightarrow \lim_{h\to 0} h.\sin\left(\frac{-1}{h}\right)$$

$$\Rightarrow \lim_{h\to 0} -h.\frac{\sin\left(\frac{-1}{h}\right)}{-\frac{1}{h}}\times\frac{-1}{h} = 1$$

Right hand limit =

$$\Rightarrow \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{h\to 0} f(0+h)$$

$$\Rightarrow \lim_{h\to 0} h.\sin\left(\frac{1}{h}\right)$$

$$\Rightarrow \lim_{h\to 0} h.\frac{\sin\left(\frac{1}{h}\right)}{\frac{1}{h}}\times\frac{1}{h}$$

$= 1$

As L.H.L = R.H.L

F(x) is continuous.

Q54.

$\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x\to 0}\frac{3x+4\tan x}{x}$$

$$\Rightarrow f(x) = \lim_{x\to 0}\frac{3x}{x}+\frac{4\tan x}{x}$$

$$\Rightarrow f(x) = 3+4\lim_{x\to 0}\frac{\tan x}{x}$$

$\Rightarrow$ f(x) = 3+4

$\therefore$ K = 7.

Q55.

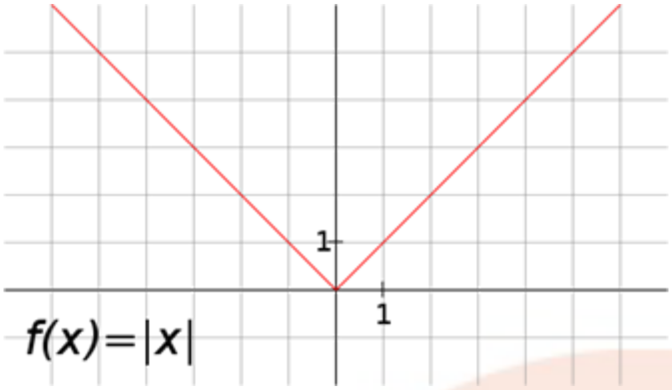
$f(x) = x^{3/2}$

$$\Rightarrow f'(x) = \frac{3}{2\sqrt{x}}$$

As $x\rightarrow 0$, $f'(x)\rightarrow\infty$

$\therefore$ f'(x) does not exist

Q56.



in the graph we can see f(x) is Continuous on 0.

But it has sharp curve on x = 0 which implies it is not differentiable.

Q57.

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuity at x =2.

For continuity at x=2,

L.H.L = $\lim_{x\to 2^-}(1+x) = 3$

R.H.L = $\lim_{x\to 2^+}(5-x) = 3$

f(2) = 1+2 = 3

∴ f(x) is continuous at x = 2

Now for differentiability.

$$\Rightarrow f'(2^-) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$$

$\Rightarrow f'(2^-) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{2-h-2}$

$\Rightarrow f'(2^-) = \lim_{h\to 0} \frac{1+2-h-3}{2-h-2} = \lim_{h\to 0} \frac{-h}{-h} = 1.$

$\Rightarrow f'(2^+) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$\Rightarrow f'(2^+) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{2+h-2}$

$\Rightarrow f'(2^-) = \lim_{h\to 0} \frac{5-(2-h)-3}{2+h-2}$

$= \lim_{h\to 0} \frac{h}{-h}$

=-1

As, f'(2^-) is not equal to f(2^+)

∴ f(x) is not differentiable.

Q58.

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x =2.

L.H.L = $\lim_{x\to 2^-}(kx+5)$

$\Rightarrow \lim_{h\to 0}(k(2-h)+5)$

⇒ k(2-0)+5 = 2k+5

R.H.L $= \lim_{x\to 2^+}(x+1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

⇒ 2+0+1

=3

As f(x) is continuous

∴ 2k+5 = 3

K = -1.

Q59.

Given:

$\Rightarrow f(x) = \frac{1-\cos 4x}{8x^2}$ is continuous at x = 0.

⇒ 1-cos4x = $2\sin^2 2x$

$\Rightarrow f(x) = \lim_{x \to 0} \frac{2\sin^2 2x}{8x^2}$

$\Rightarrow f(x) = \lim_{x \to 0} \frac{2\sin^2 2x}{2 \times 4x^2}$

$\Rightarrow f(x) = \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^2$

⇒ f(x) = 1

∴ Kk= 1

Q60.

F(x) is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{\sin^2 ax}{x^2}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{\sin^2 ax}{x^2} \times \frac{a^2}{a^2}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$$

$\Rightarrow f(x) = a^2$

$\therefore k = a^2$

Q61.

Given: f(x) is continuous at x= π/2.

$$\therefore \text{L.H.L} = \lim_{x \to \frac{\pi}{2}} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

Putting $x = \frac{\pi}{2} - h$;

As $x \to \frac{\pi}{2}^-$ then $h \to 0.$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k. \lim_{h \to 0} \frac{\sin h}{h}$$

$\therefore$ L.H.L = k

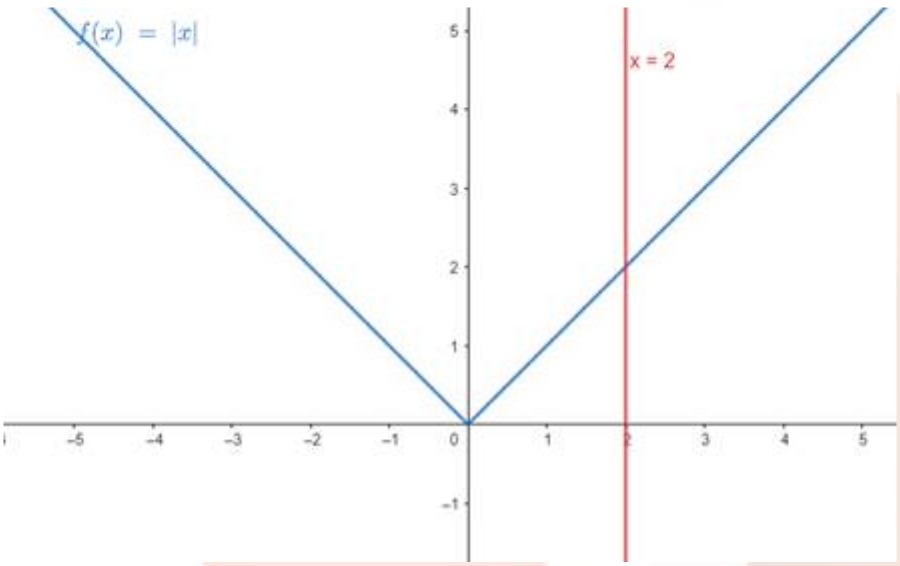
As it is continuous which implies right hand limit equals left hand limit equals the value at that point.

$\therefore$ k=3.

Q62.

Given:

Let us see that graph of the modulus function.



We can see that f(x) = |x| is neither continuous and nor differentiable at x = 2. Hence, D is the correct answer.

Q63.

$\Rightarrow f(x) = \frac{x^2-2x-3}{x+1}$ is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x\to -1} \frac{(x+1)(x-3)}{x+1}$$

$$\Rightarrow f(x) = \lim_{x\to -1} x-3$$

$\Rightarrow$ f(x) = -4

$\therefore$ K = 1.

Q64.

Given:

$f(x) = x^3+6x^2+15x-12.$

$f'(x) = 3x^2+12x+15$

$f'(x) = 3x^2+12x+12+3$

$f'(x) = 3(x^2+4x+4)+3$

$f'(x) = 3(x+2)^2+3$

As square is a positive number

$\therefore$ f'(x) will be always positive for every real number

Hence $f'(x) > 0$ for all $x \in R$

$\therefore$ f(x) is strictly increasing.

Q65.

$f(x) = -x^3+3x^2-3x+4.$

$f'(x) = -3x^2+6x-3$

$f'(x) = -3(x^2-2x+1)$

$f'(x) = -3(x-1)^2$

As f'(x) has –ve sign before 3

$\Rightarrow$ f'(x) is decreasing over R.

Q66.

Given:

$f(x) = 3x+\cos 3x$

$f'(x) = 3-3\sin 3x$

$f'(x) = 3(1-\sin 3x)$

$\sin 3x$ varies from$[-1,1]$

when $\sin 3x$ is 1 $f'(x) = 0$ and $\sin 3x$ is -1 $f'(x) = 6$

As the function is increasing in 0 to 6.

$\therefore$ The function is increasing on R.

Q67.

Given:

$f(x) = x^3+6x^2+9x+3.$

$f'(x) = 3x^2+12x+9 = 0$

$f'(x) = 3(x^2+4x+3) =0$

$f'(x) = 3(x+1)(x+3) = 0$

$x = -1$ or $x =-3$

for $x>-1$ $f(x)$ is increasing

for $x<-3$ $f(x)$ is increasing

But for $-1<x<-3$ it is decreasing.

Q68.

Given:

$f(x) = x^3-27x+8.$

$f'(x) = 3x^2-27x = 0$

$f'(x) = 3(x^2-9) =0$

$f'(x) = 3(x-3)(x+3) = 0$

$x = 3$ or $x =-3$

for $x>3$ $f(x)$ is increasing

for $x<-3$ $f(x)$ is increasing

$\therefore$ for $|x|>3$ $f(x)$ is increasing.

Q69.

Given: $f(x)$ is $\sin x$

$\therefore f'(x) = \cos x$

$\Rightarrow f'(x) =\cos x$

$= 0$

$\Rightarrow$ for $x \epsilon \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f'(x)$ is increasing

$\therefore$ $f(x)$ is increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Q70.

$\Rightarrow f(x) = \frac{2x}{\log x}$

$\Rightarrow f'(x) = \frac{2.\log x - 2}{\log^2 x}$

Put f'(x) = 0

We get

$\Rightarrow \frac{2.\log x - 2}{\log^2 x} = 0$

$\Rightarrow 2.\log x = 2$

$\log x = 1$

$\Rightarrow x = e$

We only have one critical point

So, we can directly say x>e f(x) would be increasing

$\therefore$ f(x) will be increasing in (e, ∞)

Q71.

Given:

$f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x$

Multiply and divide by $\sqrt{2}$.

$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)$

$\Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}.\cos x+\cos\frac{\pi}{4}.\sin x\right)$

$\Rightarrow \sqrt{2}(\sin\left(\frac{\pi}{4}+x\right))$

$\Rightarrow f'(x)=\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$

For f(x) to be decreasing .f'(x)<0

$\Rightarrow f'(x)=\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)<0$

$\Rightarrow \pi<x+\frac{\pi}{4}<2\pi$

($\because$ $\sin\theta<0$ for $\pi<\theta<2\pi$)

$\Rightarrow \pi-\frac{\pi}{4}<x<2\pi-\frac{\pi}{4}$

$\Rightarrow \frac{3\pi}{4}<x<\frac{7\pi}{4}$

$\therefore$ f(x) decreases in the interval.

$\Rightarrow \left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$

Q72.

$\Rightarrow f(x)=\frac{x}{\sin x}$

$\Rightarrow f'(x)=\frac{\sin x-x\cos x}{\sin x}$

Now see

In (0,1) sin x is increasing and cos x is decreasing sin x – x cos x will be increasing

∴ f(x) is increasing in (0,1)

Q73.

Given: $f(x) = x^x$.

⇒ $f'(x)=(\log x+1)\, x^x$

⇒ keeping f'(x) = 0

We get

⇒ $x = 0 \text{ or } x = \frac{1}{e}$

Now

When x>1/e the function is increasing

x<0 function is increasing.

But in the interval (0,1/e) the function is decreasing.

Q74.

Given $f(x) = x^2.e^{-x}$

⇒ $f'(x) = 2x.\, e^{-x} – x^2\, e^{-x}$

⇒ Put f'(x) = 0

⇒ $- (x^2– 2x)e^{-x} = 0$

⇒ x = 0 or x =2.

Now as there is a -ve sign before f'(x)

When $x>2$ the function is decreasing

$x<0$ function is decreasing

But in the interval $(0,2)$ the function is increasing.

Q75.

$f(x) = \sin x - kx$

$f'(x) = \cos x - k$

$\therefore$ f decreases, if $f'(x) \leq 0$

$\Rightarrow \cos x - k \leq 0$

$\Rightarrow \cos x \leq k$

So, for decreasing $k \geq 1$.

Q76.

Given:

$\Rightarrow f(x) = (x+1)^3.(x-3)^3$

$\Rightarrow f'(x) = 3(x+1)^2(x-3)^3 + 3(x-3)^3\ (x+1)^3$

Put $f'(x) = 0$

$\Rightarrow 3(x+1)^2(x-3)^3 = -3(x-3)^2(x+1)^3$

$\Rightarrow x-3 = -(x+1)$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

When x>1 the function is increasing.

x<1 function is decreasing.

So, f(x) is increasing in $(1, \infty)$.

Q77.

$\Rightarrow f(x)=[x(x-3)]^2$

$\Rightarrow f'(x)=2[x(x-3)]=0$

$\Rightarrow x = 3 \text{ and } x = \frac{3}{2}$

When x> 3/2 the function is increasing

X<3 function is increasing.

$\Rightarrow \left(0, \frac{3}{2}\right) \cup (3, \infty)$ Function is increasing.

Q78.

Given $f(x) = kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

$\Rightarrow f'(x) = 3(kx^2 - 6x + 3) > 0$

$\Rightarrow kx^2 - 6x + 3 > 0$

For quadratic equation to be greater than 0. a>0 and D<0.

$\Rightarrow k>0 \text{ and } (-6)^2 - 4(k)(3) < 0$

$\Rightarrow 36 - 12k < 0$

$\Rightarrow$ 12k>36

$\Rightarrow$ k>3

$\therefore$ k>3.

Q79.

$$\Rightarrow f(x) = \frac{x}{x^2+1}$$

$$\Rightarrow f'(x) = \frac{x^2-2x^2+1}{x^2+1}$$

$$\Rightarrow f'(x) = -\frac{x^2-1}{x^2+1}$$

$\Rightarrow$ For critical points f'(x) = 0

When f'(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f'(x) is multiplied by –ve sign we get

For x>1 function is decreasing

or x<-1 function is decreasing

But between -1 to 1 function is increasing.

$\therefore$ Function is increasing in(-1,1).

Q 80.

f(x) =x^2+kx+1

For increasing

$f'(x) = 2x+k$

$k \geq -2x$

thus,

$k \geq -2.$

Least value of -2.

Q81.

$f(x) = |x|$

Now to check the maxima and minima at $x = 0$.

It can be easily seen through the option.

See $|x|$ is x for $x>0$ and $-x$ for $x<0$

That is no matter if you put a number greater than zero or number less than zero you will get positive answer.

$\therefore$ for $x = 0$ we will get minima.

Q82.

Given: $f(x) = x^x$.

$\Rightarrow f'(x)=(\log x+1)\, x^x$

$\Rightarrow$ keeping $f'(x) = 0$

We get

$\Rightarrow\ x = 0 \text{ or } x = \frac{1}{e}$

$\Rightarrow\ f''(x) = x^x(1+\log x)\left[1+\log x+\frac{1}{x(1+\log x)}\right]$

When x is greater than zero,

We get a maximum value as the function will be negative.

Therefore,

$F(x) = x^x$

$F(e) = \left(\frac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

Hence, C is the correct answer.

Q83.

$f(x) = \frac{\log x}{x}$

$\therefore\ f'(x) = \frac{\log x - x.\frac{1}{x}}{x^2}$

$\Rightarrow$ f'(x) = log x – 1

$\Rightarrow$ Put f'(x) = 0

We get x =e

F''(x) = 1/x

Put x =e in f''(X)

1/e is point of maxima

$\therefore$ The max value is 1/e.

Q84.

We can go through options for this question

Option a is wrong because 0 is not included in $(-\pi,0)$

At $x = -\pi/4$ value of $f(x)$ is $-\sqrt{2} = -1.41$

At $x = -\pi/3$ value of $f(x)$ is -2.

At $x = -\pi/2$ value of $f(x)$ is -1.

$\therefore$ $f(x)$ has max value at $x = -\pi/2$.

Which is -1.

Q85.

Given: $x>0$ and $xy = 1$

We need to find the minimum value of $(x + y)$.

$$\Rightarrow y = \frac{1}{x}$$

$$\Rightarrow f(x) = x + \frac{1}{x}$$

$$\Rightarrow f(x) = \frac{x^2+1}{x}$$

$$\Rightarrow f'(x) = \frac{x.2x - (x^2+1).1}{x^2}$$

$$\Rightarrow f'(x) = \frac{x^2-1}{x^2}$$

$$\Rightarrow f''(x) = \frac{x^2(2x) - (x^2-1).2x}{x^4}$$

$\Rightarrow f''(x) = \frac{2x}{x^4}$

$\Rightarrow f''(x) = \frac{2}{x^3}$

For maximum or minimum value f'(x) = 0.

$\therefore \frac{x^2 - 1}{x^2} = 0$

∴ x = 1 or x =-1

f''(x) at x = 1.

∴ f''(x) = 2.

F''(x)>0 it is decreasing and has minimum value at x = 1

At x = -1

f''(x) = -2

f''(x)<0 it is increasing and has maximum value at x = -1.

∴ Substituting x = 1 in f(x) we get

f(x) = 2.

∴ The minimum value of given function is 2.

Q86.

$\Rightarrow f(x) = x^2 + \frac{250}{x}$

$\Rightarrow f'(x) = 2x - \frac{250}{x^2} = 0$

$\Rightarrow 2x^3 = 250$

$\Rightarrow x^3 = 125$

$\Rightarrow x = 5$

Substituting x = 5 in f(x) we get

$f(x) = 25+50$

$f(x) = 75.$

Q87.

Given:

$f(x) = 3x^4-8x^3-48x+25.$

$F'(x) = 12x^3-24x^2-48 = 0$

$F'(x) = 12(x^3-2x^2-4) = 0$

Differentiating again, we get,

$F''(x) = 3x^2 - 4x = 0$

$x(3x - 4) = 0$

$x = 0$ or $x = 4/3$

Putting the value in equation, we get,

$f(x) = -39$

Hence, C is the correct answer.

Q88.

$f(x) = (x-2)(x-3)^2$

$f(x) = (x-2)(x^2-6x+9)$

$f(x) = x^3-8x^2+21x-18.$

$f'(x) = 3x^2-16x+21$

$f''(x) = 6x-16$

For maximum or minimum value $f'(x) = 0$.

$\therefore 3x^2-9x-7x+21 = 0$

$\Rightarrow 3x(x-3)-7(x-3)=0$

$\Rightarrow x = 3$ or $x = 7/3$.

$f''(x)$ at $x = 3$.

$\therefore f''(x) = 2$

$f''(x)>0$ it is decreasing and has minimum value at $x = 3$

At $x = 7/3$

$F''(x) = -2$

$F''(x)<0$ it is increasing and has maximum value at $x = 7/3$.

Substituting $x = 7/3$ in $f(x)$ we get

$\Rightarrow \left(\frac{7}{3}-2\right)\left(\frac{7}{3}-3\right)^2$

$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$

$\Rightarrow \frac{4}{27}$

Q89.

f(x) = $e^x + e^{-x}$

$\Rightarrow f(x) = e^x + \frac{1}{e^x}$

$\Rightarrow f(x) = \frac{e^{2x}+1}{e^x}$

f(x) is always increasing at x = 0it has the least value

$\Rightarrow f(x) = \frac{1+1}{1} = 2$

∴ The least value is 2.