Artificial Intelligence – Assignment 2 (Theory)

Name: Dhruv Jain **Roll Number: 12CS30043**

*Answer the following questions and submit it as a **pdf file**, in moodle . This assignment should be submitted by **10th August, 2015**.*

Q1: Weighted A* can be described as best-first search with:

$f(n) = g(n) + w \cdot h(n)$ or as $f(n) = (1-w) \cdot g(n) + w \cdot h(n)$
Using one of these formulations, provide a bound on the maximum sub-optimality of a path that weighted A* can return as a function of w.

Solution:

Consider an optimal path to the goal t. If all nodes on the path were expanded by weighted A* (WA*), the solution found is optimal.

Otherwise, let n' be the deepest node on the optimal path, which is still on the OPEN list when WA* terminates. It is known from the properties of A* search that the unweighted evaluation function of n' is bounded by the optimal cost g(n') + h(n')<= C* .

So we have,

$$f(n')=g(n') + w. h(n') , f(n') <= w. (g(n') + h(n')). \text{ Consequently, } f(n')<w.C^*.$$

Let n be the arbitrary node expanded by WA*. Since it is expanded before n', f(n)<=f(n') and f(n) <=w.C*.It holds true for all nodes expanded by WA* , including the goal node t : g(t) +w.h(t) <=w.C* .Since g(t)=C and h(t)=0 therefore C<=w.C*

<u>Q2: Formulate the Rubik's cube problem as a state space search problem.</u>

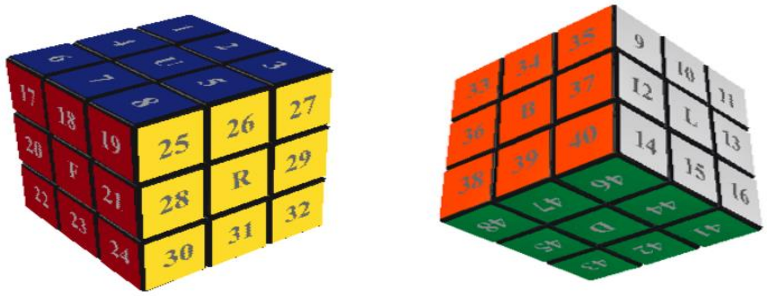
1. **Define a state representation.**

Solution:

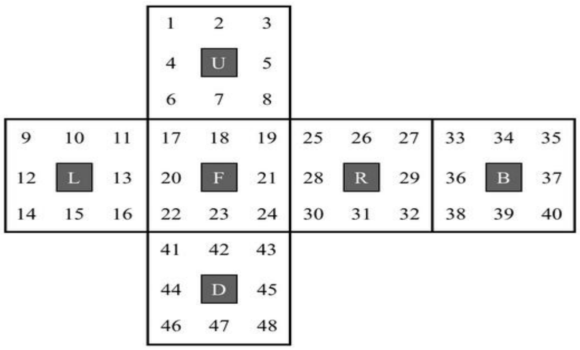
In a 3 X 3 rubix cube we have 27 possible 1*1*1 cubes, 26 are visible with one at the center.

* Six cubes are in the center of the face which don't move making them fixed reference framework, disallowing rotations of the entire cube.

* 8 are on the corners with three visible faces and 12 are on the edges with two visible



It may help to view this labeling on the flattened cube as follows.



faces.

* This is represented as an array of 20 elements, one for each cubie, the contents of which encode the position and orientation of the cubie as one of 24 different values, 8*3 for corners and the 12*2 for the edges.

* The total number of possibilities is thus 8! * 3^8 *12!*2^{12} . Since the entire problem space consists of 12 separate but isomorphic subgraphs, with no legal moves between them. Thus, the total number of states reachable from a given state is (8! * 3^8 *12!*2^{12}) /12 = 43,252,003, 274,489,856,000.

2. Give the initial and goal states in this representation

Initial state can be any of the unsolved configuration. Goal state is the one where all the faces are having the same color as that of goal state. Initial state is any state except that of goal state.

3. Define the successor function in this representation

In any state the one can move a side by 90, 180 or 270 degrees .So from one state we can easily construct all possible states achievable from that state by rotating in the above fashion which leads to a successor function.

4. Give one admissible heuristics for this problem

A heuristic can be to take the maximum of the sum of the 3D Manhattan distances of the corner cubies, and the edge cube, each divided by four. Where the Manhattan distances for a square face is calculated by the required moves from its current state to the one of goal state.