(Q1)

$$E(\omega) = \frac{1}{2} \sum_{n=1}^{N} r_n (t_n - \omega^T \phi(x_n))^2$$

find the optimal ω

$$E(\omega) = \frac{1}{2} \sum_{n=1}^{N} \left(\sqrt{r_n}\, t_n - \sqrt{r_n}\, \omega^T \phi(x_n) \right)^2$$

let $\sqrt{r_i}\, t_i = t_i'$ $\forall\, i \in 1$ to N

$\sqrt{r_i}\, \phi(x_i) = \phi'(x_i)$ $\forall\, i \in 1$ to N

we get

$$E(\omega) = \frac{1}{2} \sum_{n=1}^{N} \left(t_n' - \omega^T \phi'(x_n) \right)^2$$

so we get (as we know)

$$\omega_{ML} = (\Psi^T \Psi)^{-1} \Psi^T t'$$

where $t' = \begin{pmatrix} t_1' \\ t_2' \\ \vdots \\ t_N' \end{pmatrix} = \begin{pmatrix} \sqrt{r_1}\, t_1 \\ \sqrt{r_2}\, t_2 \\ \vdots \\ \sqrt{r_N}\, t_N \end{pmatrix}$

and $\Psi = \begin{pmatrix} \sqrt{r_1}\, \phi_0(x_1) & \sqrt{r_1}\, \phi_1(x_1) & \cdots & \sqrt{r_1}\, \phi_{M-1}(x_1) \\ \sqrt{r_2}\, \phi_0(x_2) & \sqrt{r_2}\, \phi_1(x_2) & \cdots & \sqrt{r_2}\, \phi_{M-1}(x_2) \\ \vdots & & & \\ \sqrt{r_N}\, \phi_0(x_N) & \cdots & & \sqrt{r_N}\, \phi_{M-1}(x_N) \end{pmatrix}$