

TUTORIAL-4

$$1) \quad T(n) = 3T(n/2) + n^2$$

$$a = 3, b = 2$$

$$n^{\log_b a} = n^{\log_2 3}$$

$$n^2 > n^{\log_2 3}$$

$$\therefore T(n) = \Theta(n^2)$$

$$2) \quad T(n) = 4T(n/2) + n^2$$

$$a = 4, b = 2$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$n^2 = n^2$$

$$\therefore T(n) = \Theta(n^2 \log n)$$

$$3) \quad T(n) = T(n/2) + 2^n$$

$$a = 1, b = 2$$

$$n^{\log_b a} = n^{\log_2 1} = 1$$

$$1 < 2^n$$

$$\therefore T(n) = \Theta(2^n)$$

$$4) T(n) = 2^n T(n/2) + n^2$$

Master th. not applicable as a is func of n

$$5) T(n) = 16T(n/4) + n$$

$$a=16, b=4, f(n)=n$$

$$n^{\log_b a} = n^2$$

$$n^2 > n$$

$$\therefore \Theta(n^2)$$

$$6) T(n) = 2T(n/2) + n \log n$$

$$a=2, b=2, f(n)=n \log n$$

$$n^{\log_b a} = n^{\log_2 2} = n$$

$$f(n) > n$$

$$\therefore T(n) = \Theta(n \log n)$$

$$7) T(n) = 2T(n/2) + n/\log n$$

$$a=2, b=2$$

$$n^{\log_b a} = n$$

$$n > f(n)$$

$$\therefore T(n) = \Theta(n)$$

$$8) T(n) = 2T(n/4) + n^{0.51}$$

$$a=2, b=4, f(n)=n^{0.51}$$

$$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$$

$$n^{0.51} > n^{0.5}$$

$$T(n) = \Theta(n^{0.51})$$

$$9) T(n) = 0.5 T(n/2) + 1/n$$

Master's not applicable $\because a < 1$

$$10) T(n) = 16T(n/4) + n!$$

$$a=16, b=4, f(n)=n!$$

$$n^{\log_b a} = n^2$$

$$n! > n^2$$

$$T(n) = \Theta(n!)$$

$$11) T(n) = 4T(n/2) + \log n$$

$$a=4, b=2$$

$$n^{\log_b a} = n^2$$

$$n^2 > f(n)$$

$$\therefore T(n) = \Theta(n^2)$$

$$12) T(n) = \sqrt{n} + n/2 + \log n$$

Master's not applicable as a is not constant

$$13) T(n) = 3T(n/2) + n$$

$$a=3, b=2$$

$$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$$

$$n^{1.58} > f(n)$$

$$\therefore T(n) = \Theta(n^{\log_2 3})$$

$$14) T(n) = 3T(n/3) + \sqrt{n}$$

$$a=3, b=3$$

$$n^{\log_b a} = n$$

$$n > \sqrt{n}$$

$$\therefore T(n) = \Theta(n)$$

$$15) T(n) = 4T(n/2) + cn$$

$$a=4, b=2$$

$$n^{\log_b a} = n^2$$

$$n^2 > cn$$

$$\therefore T(n) = \Theta(n^2)$$

$$16) T(n) = 3T(n/4) + n \log n$$

$$a=3, b=4$$

$$n^{\log_b a} = n^{0.79}$$

$$n^{0.79} < n \log n$$

$$\therefore T(n) = \Theta(n \log n)$$

$$17) T(n) = 3T(n/3) + n/2$$

$$n^{\log_b a} = n$$

$$n > n/2$$

$$\therefore T(n) = \Theta(n \log n)$$

$$\therefore \Theta(n) = \Theta(n/2)$$

$$a=6, b=3$$

$$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$$

$$n^{1.63} < n^2 \log n$$

$$\therefore \Theta(n^2 \log n)$$

$$19) T(n) = 4T(n/2) + n/\log n$$

$$a=4, b=2$$

$$n^{\log_b a} = n^2$$

$$n^2 > n/\log n$$

$$\therefore T(n) = \Theta(n^2)$$

$$20) T(n) = 64T(n/8) - n^2 \log n$$

Master's not applicable as $f(n)$ is not increasing

$$21) T(n) = 7T(n/3) + n^2$$

$$n^{\log_b a} = n^{1.7}$$

$$n^{1.7} < n^2$$

$$\therefore T(n) = \Theta(n^2)$$

$$22) T(n) = T(n/2) + n(2 - \cos n)$$

Master's theorem isn't applicable since condition is isolated in case 3.