LID DRIVEN CAVITY : USING PROJECTION METHOD (FDM)

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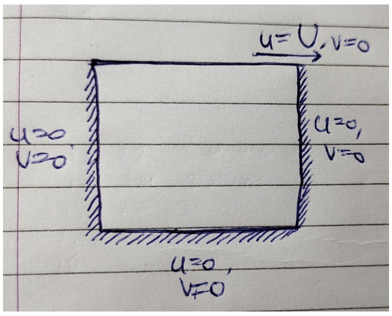


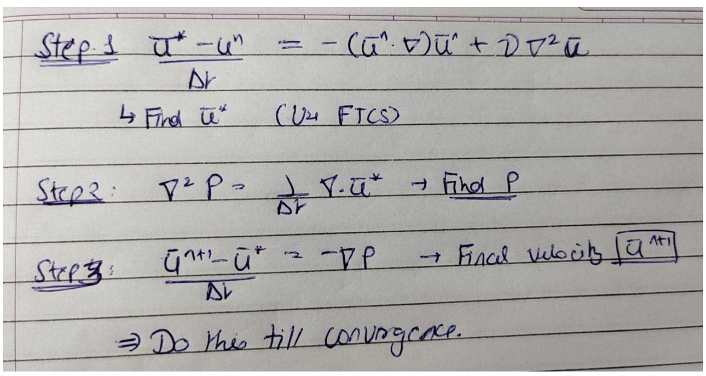
Lid driven cavity is a classic benchmark in Computational Fluid Dynamics. It aims to visualise and verify vortex formation inside a cavity whose top is moving at a fixed speed and all other sides are fixed. In our case, we are using Finite Differences Schemes involving Projection Method and Vorticity-Stream Function method to numerically simulate flows of Re=100,400 on a 2d grid of 129 x 129 grid points. We are then verifying our result by calculating RMSE metric of velocity values corresponding to those given in the famous Ghia et. al. paper.

Projection Method:

Projection method is a numerical technique which allows us to decouple pressure and velocity fields. It has two steps:

Predictor Step: Compute an intermediate velocity field (without considering pressure) using the momentum equation.

Corrector Step: Solve a Poisson equation for pressure, then correct the velocity field to enforce the incompressibility condition ($\nabla \cdot u = 0$).



RESULTS:

1. Here are the benchmark values of horizontal velocity at x = 0.5 given by Ghia in his paper.

TABLE I

Results for *u*-velocity along Vertical Line through Geometric Center of Cavity

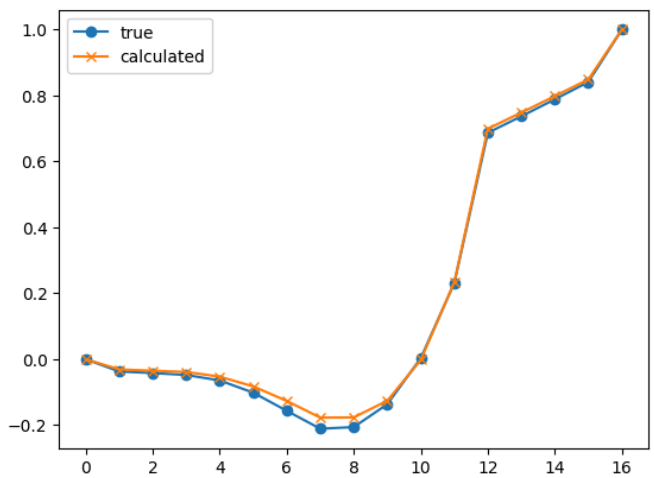
129-grid pt. no.	*y*	Re 100	400	1000	3200	5000	7500	10,000
129	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
126	0.9766	0.84123	0.75837	0.65928	0.53236	0.48223	0.47244	0.47221
125	0.9688	0.78871	0.68439	0.57492	0.48296	0.46120	0.47048	0.47783
124	0.9609	0.73722	0.61756	0.51117	0.46547	0.45992	0.47323	0.48070
123	0.9531	0.68717	0.55892	0.46604	0.46101	0.46036	0.47167	0.47804
110	0.8516	0.23151	0.29093	0.33304	0.34682	0.33556	0.34228	0.34635
95	0.7344	0.00332	0.16256	0.18719	0.19791	0.20087	0.20591	0.20673
80	0.6172	−0.13641	0.02135	0.05702	0.07156	0.08183	0.08342	0.08344
65	0.5000	−0.20581	−0.11477	−0.06080	−0.04272	−0.03039	−0.03800	0.03111
59	0.4531	−0.21090	−0.17119	−0.10648	−0.86636	−0.07404	−0.07503	−0.07540
37	0.2813	−0.15662	−0.32726	−0.27805	−0.24427	−0.22855	−0.23176	−0.23186
23	0.1719	−0.10150	−0.24299	−0.38289	−0.34323	−0.33050	−0.32393	−0.32709
14	0.1016	−0.06434	−0.14612	−0.29730	−0.41933	−0.40435	−0.38324	−0.38000
10	0.0703	−0.04775	−0.10338	−0.22220	−0.37827	−0.43643	−0.43025	−0.41657
9	0.0625	−0.04192	−0.09266	−0.20196	−0.35344	−0.42901	−0.43590	−0.42537
8	0.0547	−0.03717	−0.08186	−0.18109	−0.32407	−0.41165	−0.43154	−0.42735
1	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

For our purpose, we will use only the Re = 100 and Re = 400 values.

Number of main iterations = 10,000

Number of poisson iterations per main iteration = 300

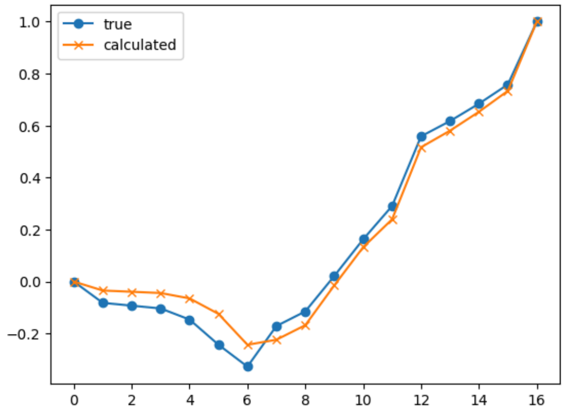
1. **For Re = 100:**



Using a python program, we compared and plotted the true and calculated values of horizontal velocities at x =0 and various vertical grid points corresponding to the paper.

RMSE = 0.0099

2. For Re = 400



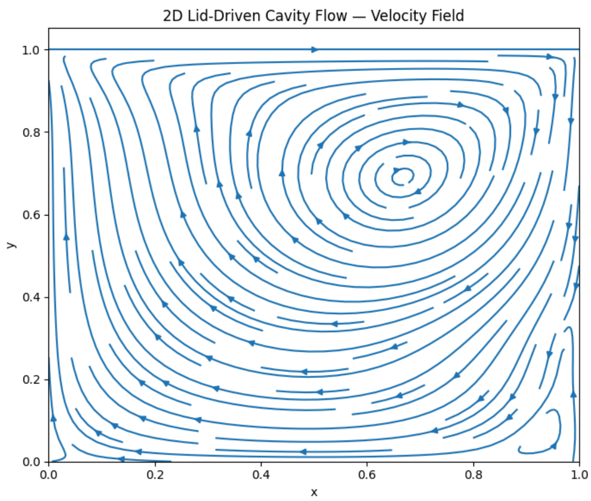
For Re = 400, there is clearly some mismatch between the calculated and the true values.

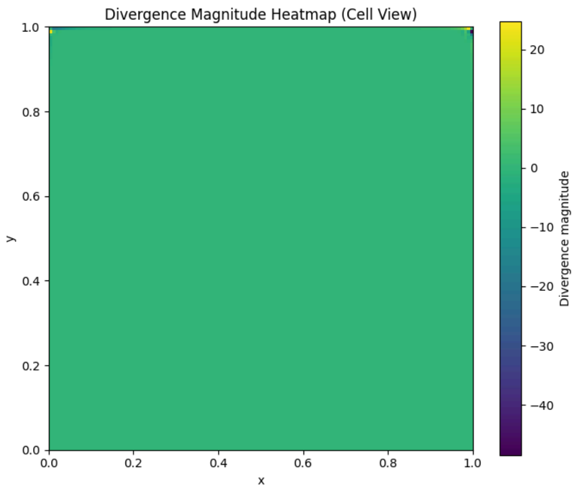
RMSE = 0.0356

- This mismatch of values could be attributed to the use of central differencing scheme in the convective portion of the navier stokes equation. Using upwind schemes would be better for higher Re values.

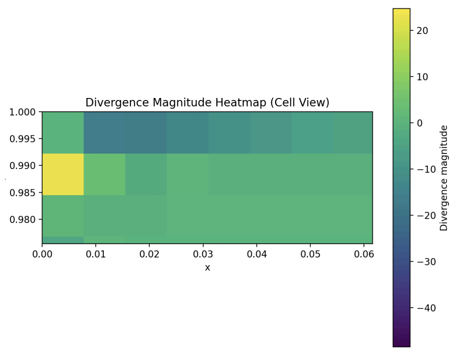
Some Colourful Fluid Dynamics:

The following figures are for Re = 400



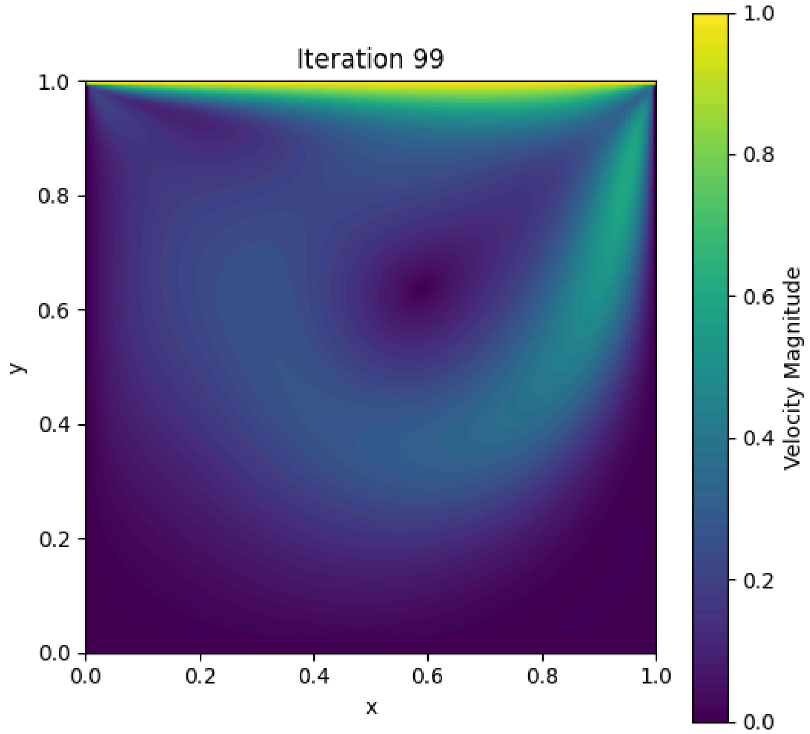


Zooming into the upper left corner of the divergence plot:



- This seems to be a major issue with the Pressure Correction Method. A fluid flow that is incompressible must have its divergence zero at all the points. But that is not the case here.
- We were able to significantly reduce the divergence by applying a **one-sided difference scheme** at the boundaries for pressure.

Here's the final output for Re = 400



for video, kindly check the presentation

** Also, as both of us have tried to solve this problem using two different methods, I request you to kindly read Varun's report on Vorticity Stream Function Method.