## Question 1

Part 1)

a) 7 is the root node

b) 1, 3, 6, 9, 12 are leaves

c) (2,10), (1,4), (8,11), (3,5) are sibiling pairs

d) {7} = 0

{2, 10} = 1

{1, 4, 8, 11} = 2

{3, 5, 9, 12} = 3

{6} = 4

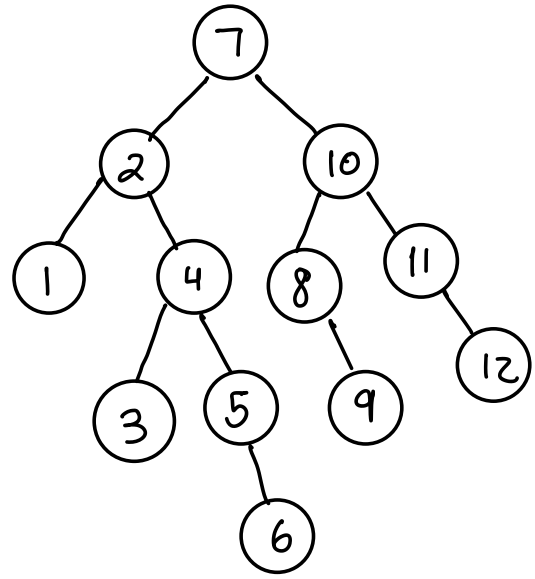
e) {7} = 4

{2, 10} = 3

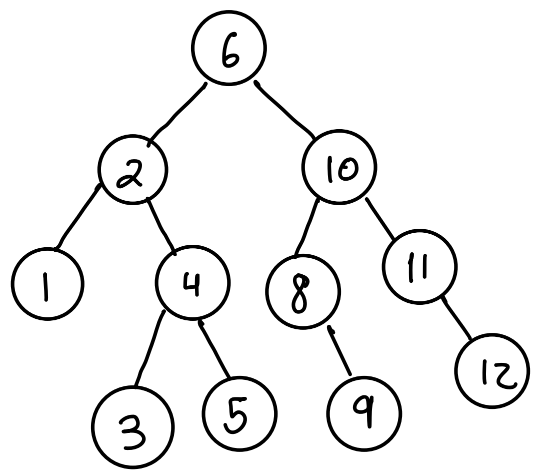
{1, 4, 8, 11} = 2

{3, 5, 9, 12} = 1

{6} = 0



Part 2)

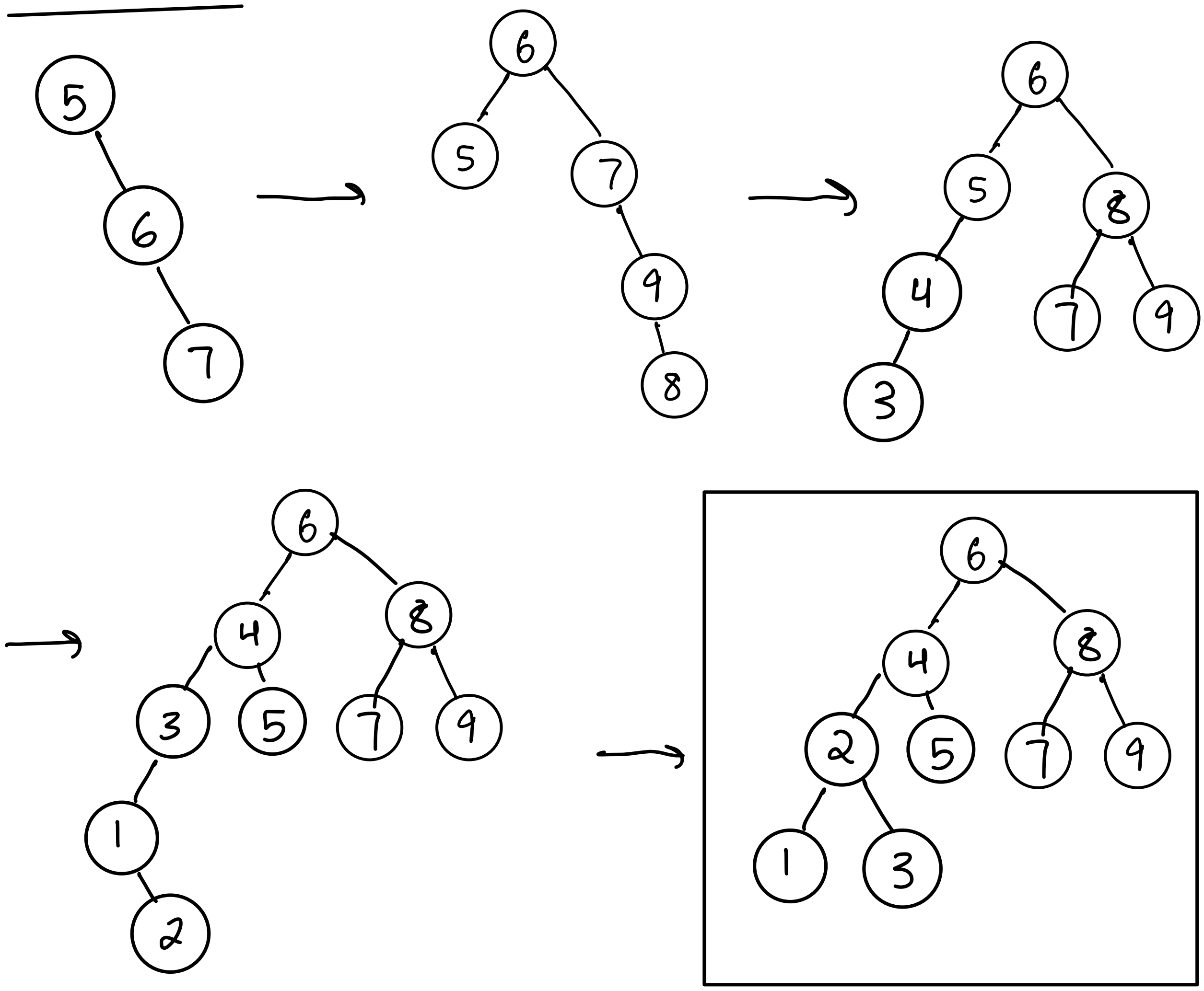


## Question 2

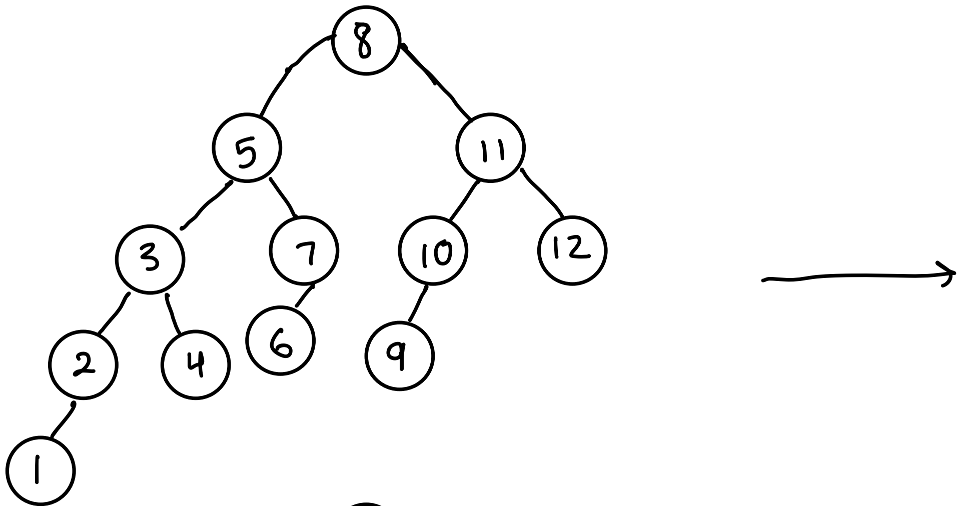
☆ code on attatched file ☆

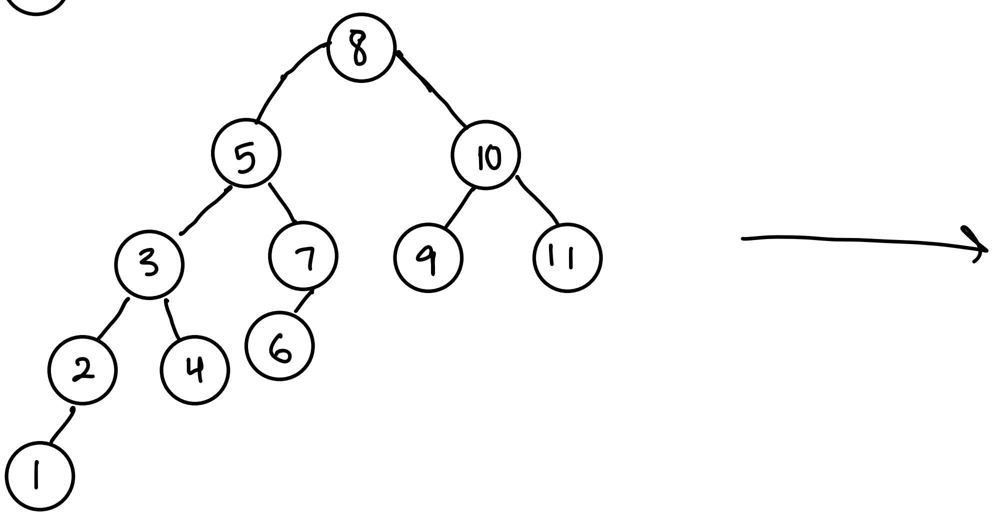
All 3 functions have time complexty of $O(n)$. It can be $O(\log(n))$ if the tree is balanced.

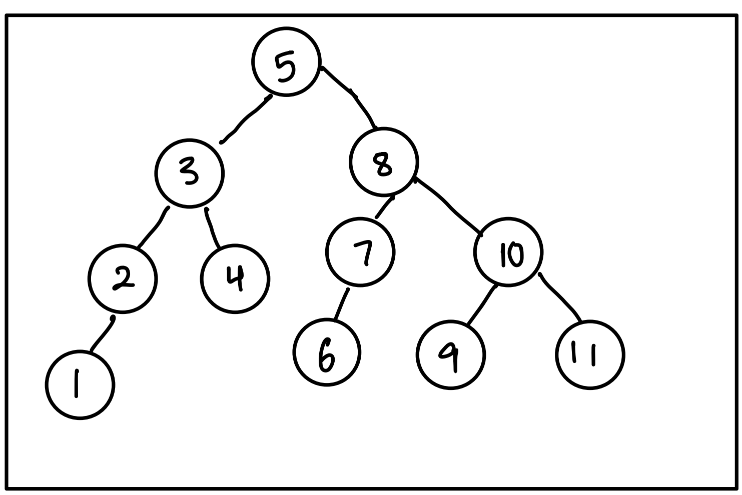
## Question 3



## Question 4



# Question 5

```
//Question 5

int AVLnodes(int height)
{
    if (height == 0)
        return 1;
    else if (height == 1)
        return 2;

    return (1 + AVLnodes(height - 1) + AVLnodes(height - 2));
}

int main()
{
    cout << AVLnodes(13) << endl;
}
```

```
(base) Dhruvs-MacBook-Pro-3:Homework dhruvmanihar$ ./a.out
986
```

986 minimum nodes

S(h) = S(h -1) + S(h - 2) + 1
S(0) = 1
S(1) = 2
S(2) = S(1) + S(0) + 1 = 4
S(3) = S(2) + S(1) + 1 = 7
S(4) = S(3) + S(2) + 1 = 12
S(5) = S(4) + S93) + 1 = 20
S(6) = S(5) + S(4) + 1 = 33
S(7) = S(6) + S(5) + 1 = 54
S(8) = S(7) + S(6) + 1 = 88
S(9) = S(8) + S(7) + 1 = 143
S(10) = S(9) + S(8) + 1 = 232
S(11) = S(10) + S(9) + 1 = 376
S(12) = S(11) + S(10) + 1 = 609
S(13) = S(12) + S(11) + 1 = 986

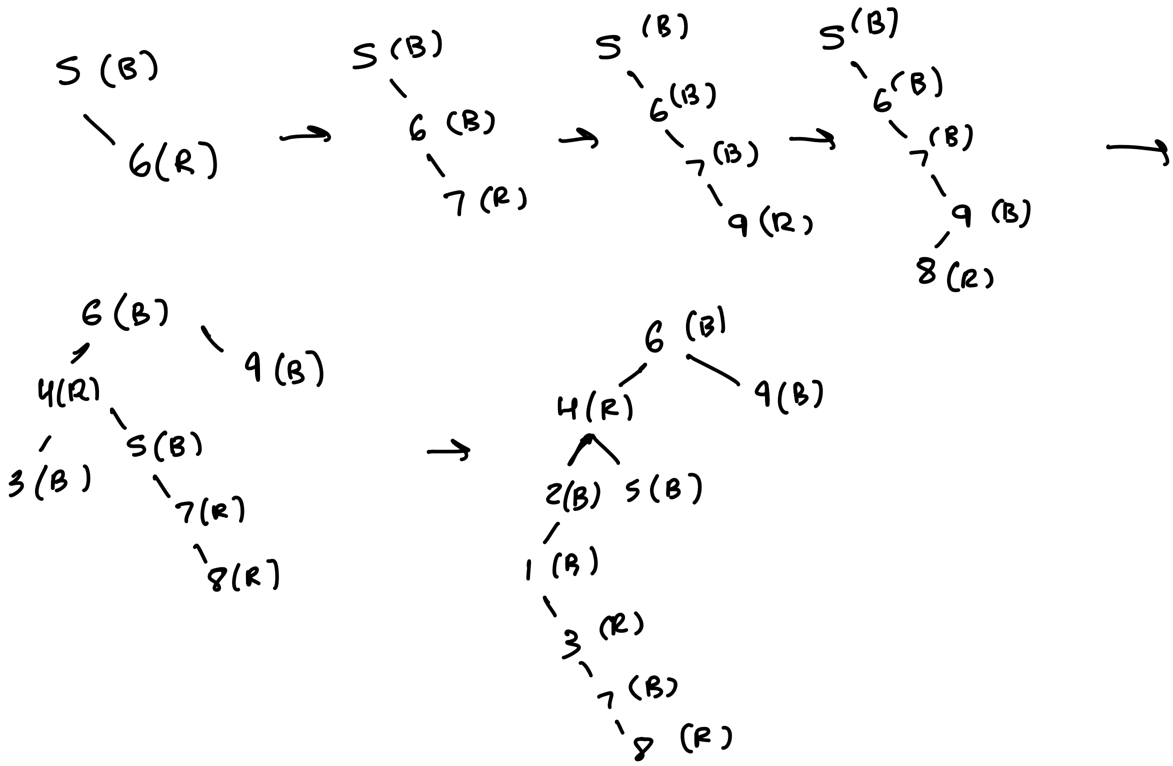
# Question 6

Every AVL tree can be colored as a red-black tree, because both data structures satisfy the same set of balance conditions on their nodes. Specifically, both AVL trees and red-black trees require that the heights of the left and right subtrees of every node differ by at most 1.

We can simply color all nodes of the AVL tree black, since this will satisfy the red-black tree balance conditions. This is because a black node can have at most one red child, and if a black node has a red child, then its other child must also be black. Since every node in an AVL tree satisfies the balance conditions, it follows that coloring all nodes black will satisfy the red-black tree balance conditions as well.

However, not all red-black trees are AVL trees. In particular, red-black trees can have unbalanced heights between black nodes, as long as the difference is no more than a factor of two. This means that some red-black trees may violate the AVL tree height balance condition, which requires that the heights of the left and right subtrees differ by at most one. Therefore, while every AVL tree can be colored as a red-black tree, not every red-black tree is an AVL tree.

# Question 7


5 (B)
6 (R)
5 (B)
6 (B)
7 (R)
5 (B)
6 (B)
7 (B)
9 (R)
5 (B)
6 (B)
7 (B)
9 (B)
8 (R)
6 (B)
4 (R)
9 (B)
3 (B)
5 (B)
7 (R)
8 (R)
6 (B)
4 (R)
9 (B)
2 (B)
5 (B)
1 (B)
3 (R)
7 (B)
8 (R)