CV Homework 3

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1.1 Planar Homography

The plane Π constrains the value of P to be 2 dimensional. If you set the dimension normal to the plane to be z, then the point P can be described only using x and y.

The existing projection matrix M moves from the world to the image coordinates.

The existing M $\begin{bmatrix} m11 & m12 & m13 & m14 \\ m21 & m22 & m23 & m24 \\ m31 & m32 & m33 & m34 \end{bmatrix}$ can be rewritten as a 3x3 matrix when you drop out the third dimension.

$M_1 * \begin{matrix} x \\ y \\ z \\ 1 \end{matrix}$ can be rewritten as $M_{1\Pi} * \begin{matrix} x \\ y \\ 1 \end{matrix}$ where $M_{1\Pi}$ is 3x3. The p and q equations can be rewritten as

$p = M_{1\Pi} P_{\Pi}$

$q = M_{2\Pi} P_{\Pi}$

$P = M_{2\Pi}^{-1} q$

$p = M_{1\Pi} M_{2\Pi}^{-1} q$ You can then say that $H = M_{1\Pi} M_{2\Pi}^{-1}$

1.2 Homography by Rotation

$p = Hq$

$p = H(K_2[R\ 0]P) = K_1[I\ 0]P$

$K_2[R\ 0]P$ can be rewritten as $K_2 R\hat{P}$ because the fourth column is a zero vector. This means that the scale factor drops off the P value, leaving only a 3x1 xyz vector. This also leads to $K_1[I0]P$ being set to $K_1[I]\hat{P}$ With the new 3x3 values for the projection and 3x1 values for the point $\hat{P}$ you can rewrite

$p = K_1 I \hat{P}$ as

$p = K_1 I[K_2 R]^{-1} q$
$p = K_1 R^{-1} K_2^{-1} q$
$p = K_1 R^T K_2^{-1} q$
with $H = K_1 R^T K_2^{-1}$

1.3 Sufficiency

The planar homography is not sufficient because it assumes either 0 translation of the camera or that the points are on a plane Π. Since the world is three dimensional and not planar, all the points in the world P will not be on the same plane. An example would be points that can be occluded from view at different vantage points by the 3D structure of an object.

1.4 Deriving H

1.4.a

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$x_i = H_{11}u_i + H_{12}v_i + H_{13}$
$y_i = H_{21}u_i + H_{22}v_i + H_{23}$
$1 = H_{31}u_i + H_{32}v_i + H_{33}$
Dividing the first and second equations by the first you get
$x_i = \frac{H_{11}u_i+H_{12}v_i+H_{13}}{H_{31}u_i+H_{32}v_i+H_{33}}$
$y_i = \frac{H_{21}u_i+H_{22}v_i+H_{23}}{H_{31}u_i+H_{32}v_i+H_{33}}$
$x_i(H_{31}u_i + H_{32}v_i + H_{33}) = H_{11}u_i + H_{12}v_i + H_{13}$
$y_i(H_{31}u_i + H_{32}v_i + H_{33}) = H_{21}u_i + H_{22}v_i + H_{23}$
$x_i(H_{31}u_i + H_{32}v_i + H_{33}) - (H_{11}u_i + H_{12}v_i + H_{13}) = 0$
$y_i(H_{31}u_i + H_{32}v_i + H_{33}) - (H_{21}u_i + H_{22}v_i + H_{23}) = 0$ These are the 2 equations that make up the A matrix. This is why there are 2N rows in A.
$a_x = \begin{bmatrix} -u_i & -v_i & -1 & 0 & 0 & 0 & x_iu_i & x_iv_i & x_i \end{bmatrix}$
$a_y = \begin{bmatrix} 0 & 0 & 0 & -u_i & -v_i & -1 & y_iu_i & y_iv_i & y_i \end{bmatrix}$

$$A = \begin{bmatrix} a_{1x} \\ a_{1y} \\ . \\ . \\ . \\ a_{Nx} \\ a_{Ny} \end{bmatrix}$$

1.4.b

There are 9 elements in h.

$$h = \begin{bmatrix} h11 \\ h12 \\ h13 \\ h21 \\ h22 \\ h23 \\ h31 \\ h32 \\ h33 \end{bmatrix}$$

1.4.c

A is 2Nx9

1.4.d

There are 8 degrees of freedom in H because of the scale factor. Each correspondance gives us 4 variables (x,y,u,v) and 2 equations.

1.4.e

$A = USV^T$ from SVD.
blah blah h = last column of V.