CV Homework 3

Eric Feuvrier Danziger

1.1 Planar Homography

The plane Π constrains the value of P to be 2 dimensional. If you set the dimension normal to the plane to be z, then the point P can be described only using x and y.

The existing projection matrix M moves from the world to the image coordinates.

The existing M $\begin{bmatrix} m11 & m12 & m13 & m14 \\ m21 & m22 & m23 & m24 \\ m31 & m32 & m33 & m34 \end{bmatrix}$ can be rewritten as a 3x3 matrix when you drop out the third dimension.

$M_1 * \begin{matrix} x \\ y \\ z \\ 1 \end{matrix}$ can be rewritten as $M_{1\Pi} * \begin{matrix} x \\ y \\ 1 \end{matrix}$ where $M_{1\Pi}$ is 3x3. The p and q equations can be rewritten as

$p = M_{1\Pi} P_{\Pi}$

$q = M_{2\Pi} P_{\Pi}$

$P = M_{2\Pi}^{-1} q$

$p = M_{1\Pi} M_{2\Pi}^{-1} q$ You can then say that $H = M_{1\Pi} M_{2\Pi}^{-1}$

1.2 Homography by Rotation

$p = Hq$

$p = H(K_2[R\ 0]P) = K_1[I\ 0]P$

$K_2[R\ 0]P$ can be rewritten as $K_2 R\hat{P}$ because the fourth column is a zero vector. This means that the scale factor drops off the P value, leaving only a 3x1 xyz vector. This also leads to $K_1[I0]P$ being set to $K_1[I]\hat{P}$ With the new 3x3 values for the projection and 3x1 values for the point $\hat{P}$ you can rewrite

$p = K_1 I\hat{P}$ as

$p = K_1 I [K_2 R]^{-1} q$
$p = K_1 R^{-1} K_2^{-1} q$
$p = K_1 R^T K_2^{-1} q$
with $H = K_1 R^T K_2^{-1}$

1.3 Sufficiency

The planar homography is not sufficient because it assumes either 0 translation of the camera or that the points are on a plane Π. Since the world is three dimensional and not planar, all the points in the world P will not be on the same plane. An example would be points that can be occluded from view at different vantage points by the 3D structure of an object.

1.4 Deriving H

1.4.a

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$x_i = H_{11}u_i + H_{12}v_i + H_{13}$
$y_i = H_{21}u_i + H_{22}v_i + H_{23}$
$1 = H_{31}u_i + H_{32}v_i + H_{33}$
Dividing the first and second equations by the first you get
$x_i = \frac{H_{11}u_i + H_{12}v_i + H_{13}}{H_{31}u_i + H_{32}v_i + H_{33}}$
$y_i = \frac{H_{21}u_i + H_{22}v_i + H_{23}}{H_{31}u_i + H_{32}v_i + H_{33}}$
$x_i(H_{31}u_i + H_{32}v_i + H_{33}) = H_{11}u_i + H_{12}v_i + H_{13}$
$y_i(H_{31}u_i + H_{32}v_i + H_{33}) = H_{21}u_i + H_{22}v_i + H_{23}$
$x_i(H_{31}u_i + H_{32}v_i + H_{33}) - (H_{11}u_i + H_{12}v_i + H_{13}) = 0$
$y_i(H_{31}u_i + H_{32}v_i + H_{33}) - (H_{21}u_i + H_{22}v_i + H_{23}) = 0$ These are the 2 equations that make up the A matrix. This is why there are 2N rows in A.
$a_x = \begin{bmatrix} -u_i & -v_i & -1 & 0 & 0 & 0 & x_iu_i & x_iv_i & x_i \end{bmatrix}$
$a_y = \begin{bmatrix} 0 & 0 & 0 & -u_i & -v_i & -1 & y_iu_i & y_iv_i & y_i \end{bmatrix}$

$$A = \begin{bmatrix} a_{1x} \\ a_{1y} \\ . \\ . \\ . \\ a_{Nx} \\ a_{Ny} \end{bmatrix}$$

1.4.b

There are 9 elements in h.

$$h = \begin{bmatrix} h11 \\ h12 \\ h13 \\ h21 \\ h22 \\ h23 \\ h31 \\ h32 \\ h33 \end{bmatrix}$$

1.4.c

A is 2Nx9

1.4.d

There are 8 degrees of freedom in H because of the scale factor. Each correspondance gives us 4 variables (x,y,u,v) and 2 equations.

1.4.e

The sum squared error of $Ah = 0$ can be written
$f(h) = \frac{1}{2}(Ah - 0)^T(Ah - 0)$
$f(h) = \frac{1}{2}(Ah)^T(Ah)$
$f(h) = \frac{1}{2}h^T A^T Ah$
taking the derivative of f with respect to h and setting it to zero
$\frac{d}{dh}f = 0 = \frac{1}{2}(A^T A + (A^T A)^T)h$
$= A^T Ah$
Looking at the eigen-decomposition of $A^T A$, we see that h should equal the eigenvector of $A^T A$ that is zero or closest to zero. This is the smallest column of V from SVD.
(From *Homography Estimation* by David Kreigman)

3.1.b

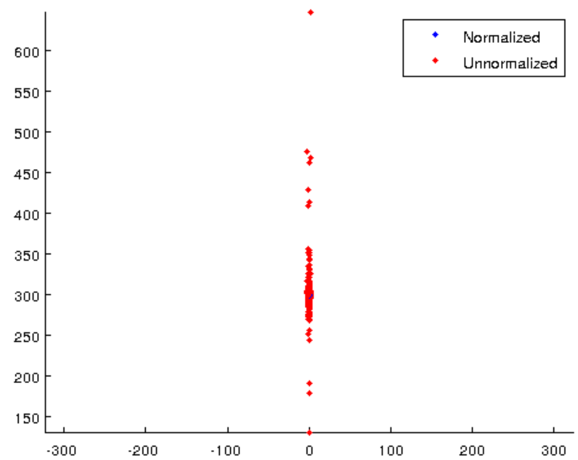

Figure 1: Normalized vs Unnormalized

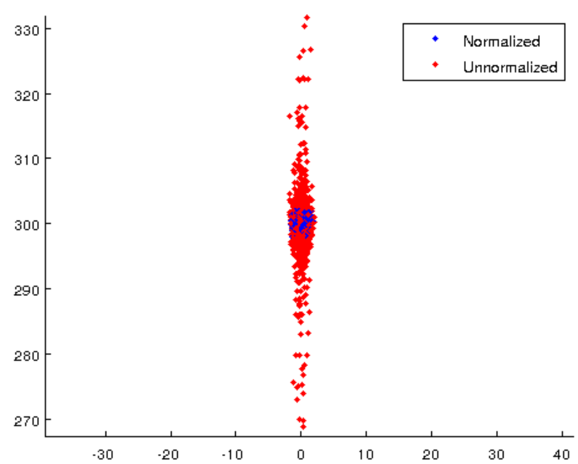

Figure 2: Normalized vs Unnormalized Zoomed In

3.1.c

Running the ratio test once I get a value of 47, meaning the unnormalized version has 47 times the standard deviation of the normalized.

3.1.d

Running it a few times, I get values between 20 and 60, meaning that the normalized version is consistently better. Visually the cluster of points is similar every time to the figures above, a nice clustered ball for the normalized points and a long trailing set for the unnormalized. This tells you that the normalized has a better result, with lower error on average, every time.

4.1

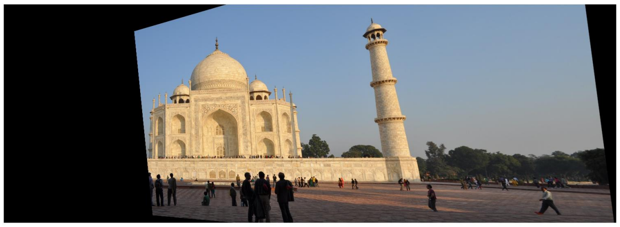

Figure 3: Warped Image

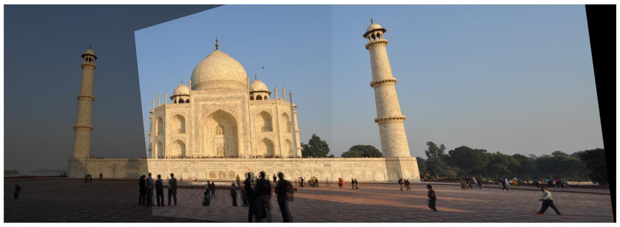

Figure 4: Stitched Image

4.2

Figure 5: Warped and Centered Image

5.2

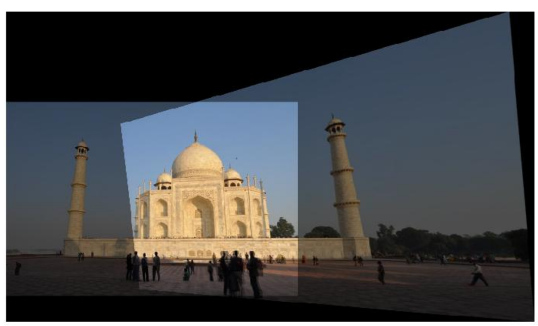

Figure 6: Warped and Centered Image

Extra Credit

This was taken outside Newell Simon Hall

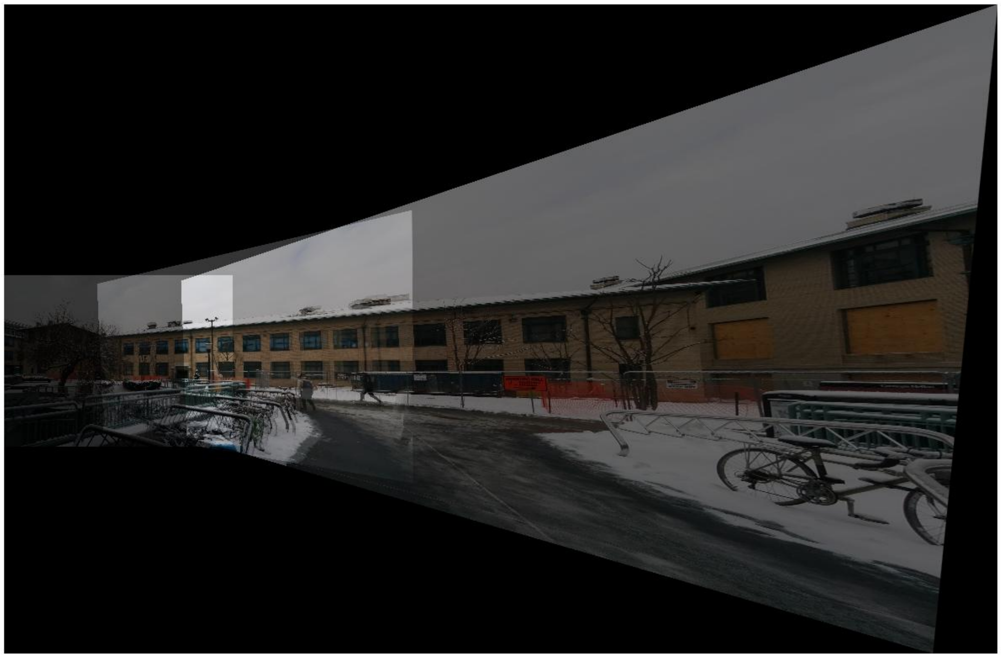

Figure 7: Warped and Centered Image