16-720 Computer Vision: Homework 5

3D Reconstruction

Instructor: Srinivasa Narasimhan
TAs: Arun Venkatraman, Aayush Bansal, Yiying Li

Due: Tuesday, April 14th, 2015 11:59 p.m.

Make sure to start early!

Part I

Theory

Before implementing our own 3D reconstruction, let's take a look at some simple theory questions that may arise. The answers to the below questions should be relatively short, consisting of a few lines of math and text (maybe a diagram if it helps your understanding).

Q1.1 **(5 points)** Suppose two cameras fixate on a point P (see figure 1) in space such that their principal axes intersect at that point. Show that if the image coordinates are normalized so that the coordinate origin $(0, 0)$ coincides with the principal point, the $\mathbf{F}_{33}$ element of the fundamental matrix is zero.

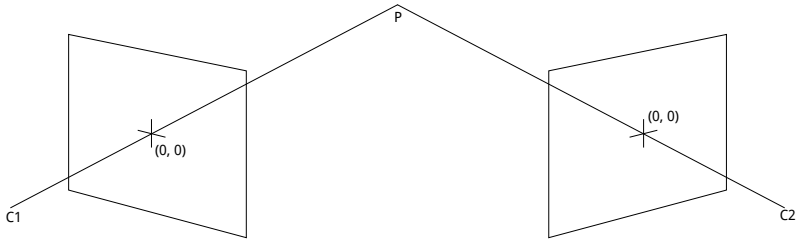


Figure 1: Figure for Q1.1. $C1$ and $C2$ are the optical centers. The principal axes intersect at point P.

Q1.2 **(10 points)** Consider the case of two cameras viewing an object such that the second camera differs from the first by a *pure translation* that is parallel to the x-axis. Show that the epipolar lines in the two cameras are also parallel to the x-axis.

Q1.3 **(10 points)** Suppose that a camera views an object and its reflection in a plane mirror. Show that this situation is equivalent to having two images of the object which are related by a skew-symmetric fundamental matrix. (*Hint:* draw the relevant vectors)

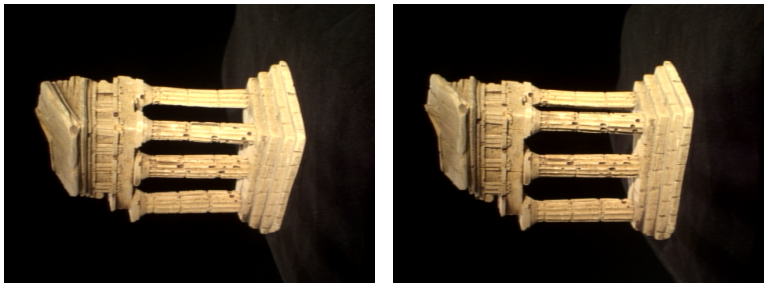


Figure 2: `Temple` images for this assignment

Part II
Practice

1 Overview

In this part you will begin by implementing the two different methods seen in class to estimate the fundamental matrix from corresponding points in two images (Section 2). Then, given the fundamental matrix and calibrated intrinsics (which will be provided) you will compute the essential matrix and use this to compute a 3D metric reconstruction from 2D correspondences using triangulation (Section 3). Finally, you will implement a method to automatically match points taking advantage of epipolar constraints and make a 3D visualization of the results (Section 4).

2 Fundamental matrix estimation

In this section you will explore different methods of estimating the fundamental matrix given a pair of images. Then, you will compute the camera matrices and triangulate the 2D points to obtain the 3D scene structure. In the `data/` directory, you will find two images (see Figure 2) from the Middlebury multi-view dataset[1], which is used to evaluate the performance of modern 3D reconstruction algorithms.

The Eight Point Algorithm

The 8-point algorithm (discussed in class, and outlined in Section 10.1 of Forsyth & Ponce) is arguably the simplest method for estimating the fundamental matrix. For this section, you may manually select point correspondences in an image pair using `cpselect`, or use provided correspondences you can find in `data/some_corresp.mat`.

[1] `http://vision.middlebury.edu/mview/data/`

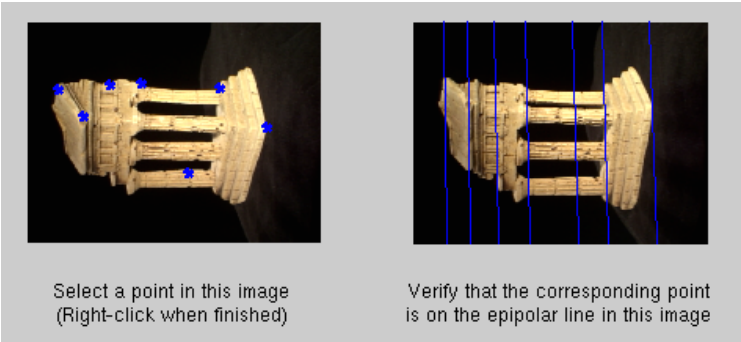


Figure 3: `displayEpipolarF.m` creates a GUI for visualizing epipolar lines

Q2.1 **(10 points)** Submit a function with the following signature for this portion of the assignment:

```
function F = eightpoint(pts1, pts2, M);
```

where `pts1` and `pts2` are $N \times 2$ matrices corresponding to the (x, y) coordinates of the N points in the first and second image repectively (the format returned by `cpselect`). `M` is a scale parameter.

- You should scale the data as was discussed in class, by dividing each coordinate by M (the maximum of the image's width and height). After computing $\mathbf{F}$, you will have to "unscale" the fundamental matrix.
  Hint: If $x_{normalized} = Tx$, then $F_{unnormalized} = T^T \mathbf{F} T$.
  You must enforce the singularity condition of the $\mathbf{F}$ before unscaling.

- You may find it helpful to refine the solution by using local minimization. This probably won't fix a completely broken solution, but may make a good solution better by locally minimizing a geometric cost function.
  For this we have provided `refineF` (takes in F and the two sets of points), which you can call from `eightpoint` before unscaling `F`.

- Remember that the x-coordinate of a point in the image is its column entry, and y-coordinate is the row entry. Also note that eight-point is just a figurative name, it just means that you need at least 8 points; your algorithm should use an over-determined system ($N > 8$ points).

- To visualize the correctness of your estimated $\mathbf{F}$, use the supplied function in `displayEpipolarF.m` (takes in F, and the two images). This GUI lets you select a point in one of the images and visualize the corresponding epipolar line in the other image (Figure 3).

- **Output:** Save your matrix `F`, scale `M`, 2D points `pts1` and `pts2` to the file `q2_1.mat`. **In your answer sheet:** Write your recovered $\mathbf{F}$ and include an

image of some example output of `displayEpipolarF`.
Hint: In LaTeX, the `verbatim` environment may be useful for copy-pasting the matrix displayed in Matlab

The Seven Point Algorithm

Q2.2 **(20 points)** Since the fundamental matrix only has seven degrees of freedom, it is possible to calculate **F** using only seven point correspondences. This requires solving a polynomial equation. In the section, you will implement the Seven-point algorithm described in class, and outlined in Section 15.6 of Forsyth and Ponce. Use manually selected points with `cpselect` to recover a fundamental matrix **F**. The function should have the signature:

```
function F = sevenpoint(pts1, pts2, M)
```

where `pts1` and `pts2` are 7×2 matrices containing the correspondences and `M` is the normalizer (use the maximum of the images' height and width), and `F` is a cell array of length either 1 or 3 containing Fundamental matrix/matrices. Use `M` to normalize the point values between $[0, 1]$ and remember to "unnormalize" your computed **F** afterwards.

- Use `displayEpipolarF` to visualize **F** and pick the correct one.
- **Output:** Save your matrix `F`, scale `M`, 2D points `pts1` and `pts2` to the file `q2_2.mat`. **In your answer sheet:** Write your recovered **F** and print an output of `displayEpipolarF`.
- *Hints:* You can use Matlab's `roots()` and `conv()` The epipolar lines may not match exactly due to imperfectly selected correspondences, and the algorithm is sensitive to small changes in the point correspondences. You may want to try with different sets of matches.

Extra credit: Automatic Computation of F

Q2X **(15 points)** In some real world applications, manually determining correspondences is infeasible and often there will be noisy coorespondances. Fortunately, the RANSAC method seen in class can be applied to the problem of fundamental matrix estimation.

Implement the above algorithm with the signature:

```
function [F] = ransacF(pts1, pts2, M)
```

We have provided some noisy coorespondances in `some_corresp_noisy.mat` in which around 75% of the points are inliers. Compare the result of RANSAC with the result of the eightpoint when ran on the noisy coorespondances. Briefly explain the error metrics you used, how you decided which points were inliers and any other optimizations you may have made.

- *Hints:* Use the seven point to compute the fundamental matrix from the minimal set of points. Then compute the inliers, and refine your estimate using all the inliers.

3 Metric Reconstruction

To obtain the Euclidean scene structure, first convert the fundamental matrix $\mathbf{F}$ to an essential matrix $\mathbf{E}$. Examine the lecture notes and the textbook to find out how to do this when the internal camera calibration matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ are known; these are provided in `data/intrinsics.mat`.

Q2.3 **(5 points)** Write a function to compute the essential matrix $\mathbf{E}$ given $\mathbf{F}$ and $\mathbf{K}_1$ and $\mathbf{K}_2$ with the signature:

```
function E = essentialMatrix(F, K1, K2)
```

In your answer sheet: Write your estimated $\mathbf{E}$ using $\mathbf{F}$ from the eight-point algorithm.
Given an essential matrix, it is possible to retrieve the projective camera matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ from it. Assuming $\mathbf{M}_1$ is fixed at $[\mathbf{I}, 0]$, $\mathbf{M}_2$ can be retrieved up to a scale and four-fold rotation ambiguity. For details on recovering $\mathbf{M}_2$, see section 7.2 in Szeliski. We have provided you with the function in `matlab/camera2.m` to recover the four possible $\mathbf{M}_2$ matrices given $\mathbf{E}$ and $\mathbf{K}_2$.

Q2.4 **(10 points)** Using the above, write a function to triangulate a set of 2D coordinates in the image to a set of 3D points with the signature:

```
function P = triangulate(M1, p1, M2, p2)
```

where `p1` and `p2` are the $N \times 2$ matrices with the 2D image coordinates and `P` is an $N \times 3$ matrix with the corresponding 3D points per row. `M1` and `M2` are the 3×4 camera matrices. Remember that you will need to multiply the given intrinsics matrices with your solution for the canonical camera matrices to obtain the final camera matrices. Various methods exist for triangulation - probably the most familiar for you is based on least squares, but feel free to implement any method. See Szeliski Chapter 7.

Q2.5 **(10 points)** Write a script `findM2.m` to obtain the correct `M2` from `M2s` by testing the four solutions through triangulations. You can use you own correspondences or the correspondences from `data/some_corresp.mat`. Save the correct `M2`, 2D points `p1`, `p2` and 3D points `P` to `q2_5.mat`.

4 3D Visualization

You will now create a 3D visualization of the temple images. By treating our two images as a stereo-pair, we can triangulate corresponding points in each image, and render their 3D locations.

Q2.6 **(10 points)** Implement a function with the signature:

```
function [x2, y2] = epipolarCorrespondence(im1, im2, F, x1, y1)
```

This function takes in the `x` and `y` coordinates of a pixel on `im1` and your fundamental matrix $\mathbf{F}$, and returns the coordinates of the pixel on `im2` which correspond to the input point. The match is obtained by computing the similarity of a small window around the (x_1, y_1) coordinates in `im1` to various windows around possible matches in the `im2` and returning the closest.

Instead of searching for the matching point at every possible location in `im2`, we can use $\mathbf{F}$ and simply search over the set of pixels that lie along the epipolar line (recall that the epipolar line passes through a single point in `im2` which corresponds to the point (x_1, y_1) in `im1`).

There are various possible ways to compute the window similarity. For this assignment, simple methods such as the Euclidean or Manhattan distances between the intensity of the pixels should suffice. See Szeliski Chapter 11, on stereo matching, for a brief overview of these and other methods.
Implementation hints:

- Experiment with various window sizes.
- It may help to use a Gaussian weighting of the window, so that the center has greater influence than the periphery.
- Since the two images only differ by a small amount, it might be beneficial to consider matches for which the distance from (x_1, y_1) to (x_2, y_2) is small.

To help you test your `epipolarCorrespondence`, we have included a GUI:

```
function [coordsIM1, coordsIM2] = epipolarMatchGUI(im1, im2, F)
```

that can be found in `matlab/epipolarMatchGUI.m`.

This script allows you to click on a point in `im1`, and will use your function to display the corresponding point in `im2`. The process repeats until you right-click in the figure, and the sets of matches will be returned. See Figure 4.

It's not necessary for your matcher to get *every* possible point right, but it should get easy points (such as those with distinctive, corner-like windows). It should also be good enough to render an intelligible representation in the next question.

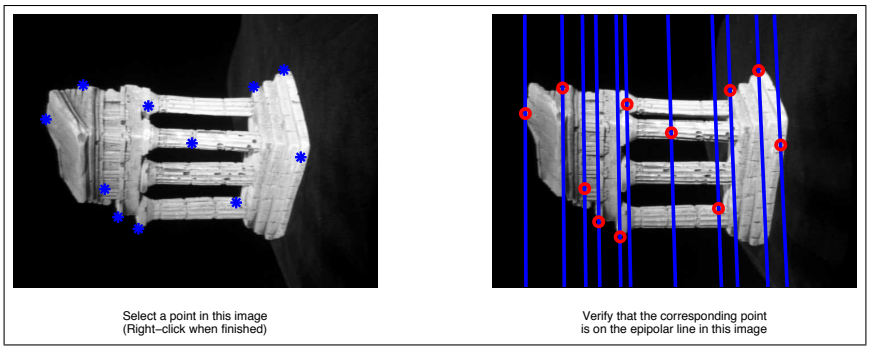


Figure 4: `epipolarMatchGUI` shows the corresponding point found by calling `epipolarCorrespondence`

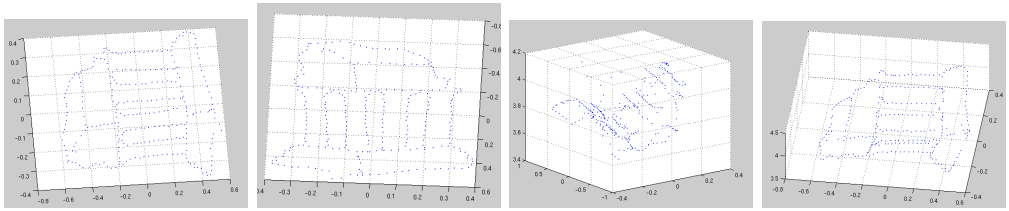


Figure 5: An example point cloud

In you answer sheet: Include a screenshot of `epipolarMatchGUI` with some detected correspondences.
Output: Save the matrix `F`, points `pts1` and `pts2` which you used to generate the screenshot to the file `q2_6.mat`.

Q2.7 **(10 points)** Included in this homework is a file `data/templeCoords.mat` which contains 288 hand-selected points from `im1` saved in the variables `x1` and `y1`.

Now, we can determine the 3D location of these point correspondences using the `triangulate` function. These 3D point locations can then plotted using the MATLAB function `scatter3`. The resulting figure can be rotated using the Rotate 3D tool, which can be accessed through the figure menubar.

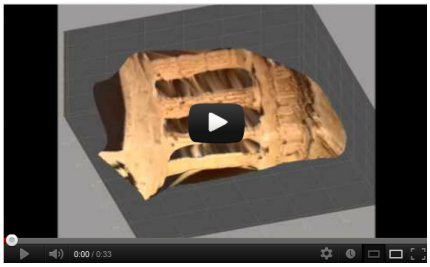
In your submission/writeup: Take a few screenshots of the 3D visualization so that the outline of the temple is clearly visible, and include them with your homework submission. **Output:** Again, save the matrix `F`, matrices `M1` and `M2` which you used to generate the screenshots to the file `q2_7.mat`.

5 Extra credit: awesome visualization

QX **(upto 20 points)** The sparse 3D points from last item are a rather crude visualization of the 3D scene. There are many more ways that interesting 3D visu-

alizations can be generated, including meshes, dense 3D point clouds, depth maps, and so on. Feel free to use third party code; it is not necessary to include it in your submission, just include figures and a brief writeup explaining how they were obtained (and maybe a video!)

For example, here's Arun's visualization when he took the course (click the thumbnail!):



Video of TA's Dense 3D reconstruction is found at `https://youtu.be/WOAqndrcDIw`

Deliverables

If your andrew id is `bovik`, your submission should be a zip file `bovik.zip` containing a folder `bovik/`. This should contain

- ☐ `bovik.pdf`: a pdf containing your answers to all written items, including extra credit.
- ☐ `eightpoint.m`: implementation of the eight-point algorithm.
- ☐ `q2_1.mat`: file with output of Q2.1.
- ☐ `sevenpoint.m`: implementation of the seven-point algorithm.
- ☐ `q2_2.mat`: file with output of Q2.1.
- ☐ `essentialMatrix.m`: function to compute the essential matrix.
- ☐ `triangulate.m`: function to triangulate correspondences.
- ☐ `findM2.m`: script to compute the correct camera matrix.
- ☐ `q2_5.mat`: file with output of Q2.5.
- ☐ `epipolarCorrespondence.m`: function to compute correspondences with epipolar constraints.
- ☐ `q2_6.mat`: file with output of Q2.6.
- ☐ `q2_7.mat`: file with output of Q2.7.