Uncertanity

- Unconditional Probability → degree of belief in a proposition in the absence of any other evidence.

☆ We deal with Conditional probability in the world of A.I.

- Conditional Probability → degree of belief in a proposition given some evidence that has already been revealed.

$P(a/b)$

Ex = P(rain today / rain yesterday), P(route change / traffic conditions)

$$\boxed{P(a/b) = \frac{P(a \wedge b)}{P(b)}}$$

Q. Red & Blue Dice example:
- Red is set to 6.
- Find the prob. of getting sum of both 12.

$$P(\text{Sum } 12 \wedge \underbrace{⚅}_{\text{Red Dice}}) = 1/36, \quad P(\underbrace{⚅}_{\text{Red Dice}}) = 1/6$$

$$P(a/b) \Rightarrow \frac{P(a \wedge b)}{P(b)} \Rightarrow \frac{1/36}{1/6} \Rightarrow \frac{1 \times \cancel{6}}{3\cancel{6}6}$$

$$= 1/6.$$

$$\text{So, } \boxed{P(\text{sum}12 / \underbrace{⚅}_{\text{Red Dice}}) = 1/6}$$

- Random Variable ⇒ a variable in probability theory with a domain of possible values it can take on.

EX = Roll{1, 2, 3, ...}, Weather{sun, cloud, wind, rain, snow}

- Probability Distribution → It takes a random variable & gives me the probability for each of the possible value in ~~the~~ its domain

Ex = Flight {on time, Delay, Cancel}

P or, P.D(on time) = 0.6, P.D(Delay) = 0.3, P.D(Cancel) = 0.1

→ Sum of all possible ~~worlds~~ P.D for the value of random variable is going to be 1.

- Vector → It is a sequence of values.

$P(\text{Flight}) = \langle 0.6, 0.3, 0.1 \rangle$

- Independence → the knowledge that one event occurs does not affect the probability of the other event

$P(a \wedge b) \Rightarrow P(a)\,P(b|a)$ since it is independent.

$= P(a)\,P(b)$

Baye's rule → Derivation.

$\because$ We know, $P(a \wedge b) = P(b)\,P(a|b)$, $P(a \wedge b) = P(a)\,P(b|a)$

$\Rightarrow P(b)\,P(a|b) = P(a)\,P(b|a) \Rightarrow P(b|a) = \dfrac{P(b)\,P(a|b)}{P(a)}$

The probability of b given a is equal to the probability of b times the prob. of a given b divided by the prob. of a

$$\boxed{\text{Bayes rule, } P(b|a) = \frac{P(b)\,P(a|b)}{P(a)}}$$

Teacher's Signature

Ex= Given clouds in the morning, what's the probability of rain in the afternoon?

- 80% of rainy afternoons starts with cloudy mornings ~~afternoon~~.
- 40% of days have cloudy morning.
- 10% of days have rainy afternoon.

$$P(rain | cloud) = \frac{P(cloud | rain)\, P(rain)}{P(clouds)}$$

$$\Rightarrow \frac{0.8 \times 0.1}{0.4} = 0.2$$

Knowing $\rightarrow$ P(cloudy morning | rainy afternoon)

we can calculate, P(rainy afternoon | cloudy morning)

- Joint Probability $\rightarrow$

C = Cloud	C = ¬Cloud
0.4	0.6

,

R = rain	R = ¬rain
0.1	0.9

$\Downarrow$

	Rain.	¬Rain
Cloud	0.08	0.32
¬Cloud	0.02	0.58

$$P(c | rain) \Rightarrow \frac{P(c, rain)}{P(rain)} \Rightarrow \alpha\, P(c, rain)$$

$$= \alpha \langle 0.08, 0.2 \rangle$$

- Negation $\rightarrow$ $P(\neg a) = 1 - P(a)$

- Inclusion-Exclusion $\rightarrow$ $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

- Marginalization → $P(a) = P(a, b) + P(a, \neg b)$

  $P(X = x_i) = \sum_j (X = x_i, Y = y_j)$

$P(c) = P(c, R) + P(c, \neg R)$

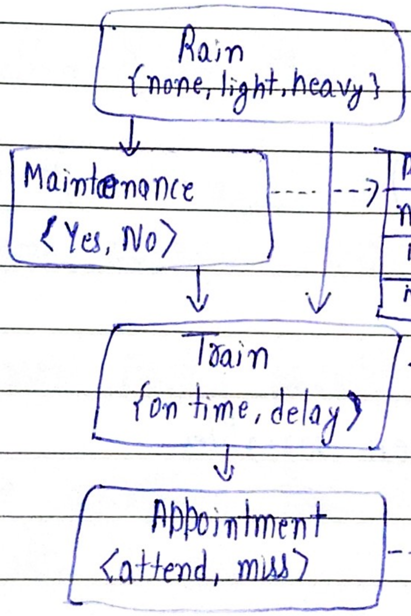
$\Rightarrow 0.08 + 0.32$

$= 0.4$

- Conditioning → $P(a) = P(a|b)P(b) + P(a|\neg b)P(\neg b)$

  $P(x = x_i) = \sum_j P(X = x_i | Y = y_j) P(Y = y_j)$

- Bayesian network → data structure that represents the dependencies among random variables.
  - directed graph
  - each node represents a random variable
  - arrow from X to Y means X is a parent of Y.
  - each node X has prob. distribution $P(X | Parents(X))$



none	light	heavy
0.7	0.2	0.1

R	Yes	No
none	0.4	0.6
light	0.2	0.8
heavy	0.1	0.9

R	M	on time	delay
none	Y	0.8	0.2
"	N	0.9	0.1
light	Y	0.6	0.4
"	N	0.7	0.3
heavy	Y	0.4	0.6
"	N	0.5	0.5

T	attend	miss
on time	0.9	0.1
delayed	0.6	0.4

- Inference

  - Query X: variable for ~~each~~ which to compute distribution
  - Evidence Variables E: observed variable for event e
  - Hidden Variable Y: non-evidence, non-query variable.

  - Goal: Calculate $P(X|e)$

$P(\text{Appointment} \mid \text{light}, \text{no}) \Rightarrow \alpha\, P(\text{Appointment}, \text{light}, \text{no})$

$\Rightarrow \alpha\,[P(\text{Appointment}, \text{light}, \text{no}, \text{ontime}) + P(\text{''}, \text{''}, \text{''}, \text{delay})]$

$$\boxed{P(X|e) \Rightarrow \alpha\, P(X,e) = \alpha \sum_{y} P(X, e, y)}$$

Python library for inference-

(i) pomengrante.

- Inference by Enumeration:

$P(X|e) = \alpha\, P(X, e) = \alpha \sum_{y} P(X, e, y)$

X is the query variable.
e is the evidence.
y ranges over values of hidden variables.
α normalizes the result.

- **Likelihood Weighting:**
    - Start by fixing the values for evidence variables.
    - Sample the non-evidence variable using conditional probabilities in the Bayesian Network.
    - Weight each sample by its likelihood: the probability of all the evidence.

- **Markov assumption:**

the assumption that the current state depends on only a finite fixed number of previous states.

- **Markov chain:**

a sequence of random variables where the distribution of each variable follows the markov assumption.

- **Hidden Markov Model:**

a Markov model for a system with hidden states that generate some observed event.

Sensor Markov assumption:

the assumption that the evidence variable depends only the corresponding state.

Tasks	Definition
Filtering	given observation from start untill now, calculate distribution for current state.
Prediction	given observations from start untill now, Calculate distribution for a future state.
Smoothing	" Calculate distribution for past state.
most-likely explanation	" calculate most-likely sequence of states.