Probabilistic Reasoning – Chapter 14.

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Independence and *conditional independence* relationships are very powerful for modeling our world. **Bayesian Networks** introduce a systematic way of representing these relationships.

1 Representing Knowledge in an Uncertain Domain

Full joint probability distributions can answer any question about the domain, however, they become intractable as the number of variables grows. **Bayesian Networks** represent the same dependencies among variables, and give a concise specification of *any* full joint probability distribution.

A Bayesian Network is a DAG where each node is anntated with probability information, under the following specification:

1. A set of random variables makes up the nodes of the network. Variables may be discrete or continuous
2. A set of directed links or arrows connects pairs of nodes. If there is an arrow from node X to node Y, X is said to be a parent of Y.
3. Each Node X_i has a conditional probability distribution $P(X_i|Parents(X_i))$ which quantifies the effect of the parents on the node
4. The graph has no cycles (it is a DAG)

2 The Semantics of Bayesian Networks

2.1 Bayes' nets as Representations of a Full Joint Distribution

A generic query to a full joint distribution (FJD) is the probability of a conjunction of assignments to each variable: $P(X_1 = x_1 \wedge \cdots \wedge X_n = x_n)$. The following formula can be used.

$$P(x_1, \ldots x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

So the Bayesian network can answer any query, simply by summing the relevant joint CPDs for each node.

2.2 Constructing Bayesian Networks

The **chain rule** can be derived from the above formula, using the product rule to reduce the terms.

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | x_{i-1}, \ldots, x_n) \tag{1}$$