

# 201C-MATH177-1 Midterm 2

DHRUV SANCHETY

TOTAL POINTS

**92 / 100**

QUESTION 1

1 Question 1 20 / 20

✓ - 0 pts Correct

QUESTION 2

2 Question 2 12 / 20

✓ - 8 pts if reported 20th coupon date price as the correct price

💬 P = 87537 would provide a yield rate  $< 5\%$  if bond is allowed to mature ( $n=30$ ).

QUESTION 3

3 Question 3 20 / 20

✓ - 0 pts Correct

QUESTION 4

Question 4 20 pts

4.1 Part (a) 10 / 10

✓ - 0 pts Correct

4.2 Part (b) 10 / 10

✓ - 0 pts Correct

QUESTION 5

5 Question 5 20 / 20

✓ - 0 pts Correct

$$1. \quad P = F \cdot r a_{\overline{n}|j} + C v^n = F r a_{\overline{n}|j} + C (1 - j a_{\overline{n}|j})$$

$$\Rightarrow P = C + (F r - C j) a_{\overline{n}|j}$$

$$1357.24 = 1100 + (1000 \cdot 0.0325 - 1100 \cdot 0.02) a_{\overline{n}|j}$$

$$24.4990462 = a_{\overline{n}|j} = \frac{1 - (1+j)^{-2N}}{j}$$

$$(1+j)^{-2N} = 0.5100190476$$

$$\log_{1.02} 0.5100190476 = -2N$$

$$N = 17 \text{ years.}$$

Amortization by coupon 6:  $P_{t-1} - P_t$

$$(F \cdot r - C \cdot j) (v^{n-t+1})$$

$$(1000 \cdot 0.0325 - 1100 \cdot 0.02) \left( \frac{1}{1 + \frac{34}{100} \cdot 0.02} \right)^{6+1} = 8.$$

$$= \cancel{8.279178344} \quad 5.912679221 = \text{Answer}$$

$$\frac{1 - \cancel{(1+j)^{-2N}}}{j} = \frac{1 - (1+j)^{-2N}}{j}$$

1 Question 1 20 / 20

✓ - 0 pts Correct

2.  $\frac{\text{yield rate}}{\text{nominal}} > .1, F = 100,000, \text{ } \cancel{0.08} = \alpha, m = 2, r = 0.04, C = 100,000$   
 $j = 0.05$  compounded semi-annually.

$$\text{F.v. } a_{\overline{n}|j} + Cv^n$$

$$\frac{100,000 \cdot 0.04 \cdot (1 - (1+j)^{-30})}{j} + 100,000 (1+j)^{-30}$$

① = 84,627.5487

$$\frac{100,000 \cdot 0.04 (1 - (1+j)^{-20})}{j} + 100,000 (1+j)^{-20}$$

② = ~~87.5~~ 87,537.78966.

The Maximum of ① and ② is  
 87,537.78966 = Answer

S r

## 2 Question 2 12 / 20

✓ - 8 pts if reported 20th coupon date price as the correct price

💬 P = 87537 would provide a yield rate  $< 5\%$  if bond is allowed to mature ( $n=30$ ).

3.

$$50,000(1+i) + 24,000(1+\frac{8i}{12})$$

$$- 36,000(1+\frac{3i}{12}) = \text{~~50000~~ } 44,000$$

$$50,000 + 24,000 - 36,000 - 44,000 + i(50,000 + 24,000(\frac{8}{12}) - 36,000(\frac{3}{12})) = 0$$

$$-6000 + i(\frac{57000}{\text{~~50000~~}}) = 0, i = \frac{6000}{\text{~~50000~~ } 57000} = \frac{0.1052631579}{\text{~~0.24~~}}$$

$$\frac{105,000}{100,000}$$

$$\frac{105,000}{100,000}$$

$$\frac{95,000}{95,000} \cdot X$$

$$-1 = \text{~~0.24~~ } 0.1052631579$$

$$X = \text{~~316484.17~~ } 72964.66974$$

$$X = 100,000$$

3 Question 3 20 / 20

✓ - 0 pts Correct

4. a)  $F = 100$   
 $r = 5\% = 0.05$   
 $F \cdot r = 5$   
 $C = 110$

$$\text{Price} = 5 \left( \frac{1 - (1+j)^{-4}}{1 - \frac{1}{1+j}} \right) + 110 (1+j)^{-4} \quad \left. \vphantom{\text{Price}} \right\} j = 0.050745$$

$$= 5 \frac{1 - (1+j)^{-4}}{j} + 110 (1+j)^{-4}$$

Answer  $\approx$  ~~107~~ 107.94

(b)  $5 \times 1.04^{-1} + 5 \times 1.0425^{-2} + 5 \times 1.0445^{-3} + 115 \times (1+x)^{-4} = 107.94$

~~$\log_{1+x} 0.818642553 = -4$~~

$(1+x)^{-4} = 0.818642553$

~~$\times 4 \log_2 (1+x)^{-4} = 0.818642$~~   $\frac{\log_2 0.818642553}{-4}$

$x = 0.05129941107$

$X = 5.129941107$



4.1 Part (a) 10 / 10

✓ - 0 pts Correct

4. a)  $F = 100$   
 $r = 5\% \pm 0.05$

$F \cdot r = 5$

$C = 110$

$$\text{Price} = 5 \left( \frac{1 - (1+j)^{-4}}{1 - \frac{1}{1+j}} \right) + 110 (1+j)^{-4} \quad \left. \vphantom{\text{Price}} \right\} j = 0.050745$$

$$= 5 \left( \frac{1 - (1+j)^{-4}}{j} \right) + 110 (1+j)^{-4}$$

Answer  $\approx$  \$107.94

(b)  $5 \times 1.04^{-1} + 5 \times 1.0425^{-2} + 5 \times 1.0445^{-3} + 115 \times (1+x)^{-4} = 107.94$

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$x = 0.05129941107$

$X = 5.129941107$

4.2 Part (b) 10 / 10

✓ - 0 pts Correct

S.

$$\left(1 + f_{[3,4]}\right)^1 = \frac{(1 + 0.1 + 0.03 \times 4)^4}{(1 + 0.1 + 0.03 \times 3)^3}$$

$$\Rightarrow f_{[3,4]} = \frac{[1.22]^4}{[1.19^3]} - 1 = 0.3146145616.$$

~~100,000~~

Since interest is always repaid, the initial money at 3 is 1,000

$$\therefore \text{Answer} = 1000(0.3146145616 - 0.2) = 114.6145616.$$

5 Question 5 20 / 20

✓ - 0 pts Correct

Math 177  
Summer 2020  
Midterm 2

08/26/2019

Time Limit: 24 Hours

Name: \_\_\_\_\_

UID: \_\_\_\_\_

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This exam contains 7 pages (including this cover page) and 5 questions. Total of points is 100. Make sure to write your answers in full detail, so that you may get the maximum possible points.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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- Please type the following statement in your handwriting, then sign and date below:

“I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor as a Bruin *all* the answers I present belong solely to me, in thought and in writing.”

1. (20 points) A bond with 1000 face value, 6.5% nominal yearly coupon rate, semiannual coupons and a redemption amount of 1100 at the end of  $N$  years is purchased for 1357.24 to yield a nominal rate of 4% compounded semiannually. Calculate the amount for amortization of premium in the 6th coupon.

2. (20 points) Robert California is an investor, who bought a 15-year bond with a face value of 100,000 and 8% nominal yearly coupon rate and semiannual coupons. The bond is callable at par on any coupon date beginning with the 20th coupon. Robert wants to make sure his yield rate (yearly, nominal, compounded semiannually) is at least 10%. What's the maximum price he should be willing to pay?



3. (20 points) A businessman named Varrick has two potential funds he can invest in: Fund Aang and Fund Korra. His CFO, Zhu Li provides the following data (from last year) on these two funds:

FUND AANG			
Date	Withdrawal	Deposit	Value of the Fund Before Action
January 1			50,000
March 1			55,000
May 1		24,000	50,000
October 1	36,000		78,000
December 31			44,000

FUND KORRA			
Date	Withdrawal	Deposit	Value of the Fund Before Action
January 1			100,000
July 1	10,000		105,000
December 31			$X$

Zhu Li further reports that Fund Korra's time-weighted yield rate is equal to Fund Aang's dollar-weighted yield rate. Based on these information, calculate  $X$ .

4. (20 points) In order to guarantee the salary payment of the city guard for the next 4 years, Master of Coins, Petyr Baelish purchases a 4-year bond with 100 face value and 5% annual coupons that matures at 110 (this is the redemption amount). Petyr buys this bond to produce an annual yield rate of 5.0745%.
- (a) (10 points) What is the price Petyr has to pay to purchase this bond?

(b) (10 points) For the valuation of bond, the term structure in effect has the following spot rates:

- $r_1 = 4\%$
- $r_2 = 4.25\%$
- $r_3 = 4.45\%$
- $r_4 = X\%$ .

Calculate  $X$ .

5. (20 points) Consider the term structure  $\{r_t\}_{t \geq 0}$  with  $r_t = 0.1 + 0.03t$ . Boromir borrows 1000 from Legolas for 10 years, making interest-only payments at the end of each year and returning the whole principal at the end of the term. The interest-only payments at the end of a year are based on the floating rate available at the beginning of that year.

Boromir is not happy with this arrangement, however, as he cannot anticipate the future rates. Gimli agrees to act as a financial intermediary, guaranteeing Boromir's payments in the sense that they will always be based on the forward rates determined by the original term structure.

Assuming the floating rate  $u_{[3,4]} = 0.2$ , what is Gimli's net cashflow (income - outgo) at time 4?