201C-MATH177-1 Final Exam

DHRUV SANCHETY

TOTAL POINTS

91 / 100

QUESTION 1

1 Academic Honesty Disclaimer **0 / 0**

✓ **- 0 pts Correct**

QUESTION 2

2 Question 1 **10 / 10**

✓ **- 0 pts Correct**

QUESTION 3

3 Question 2 **10 / 10**

✓ **- 0 pts Correct**

QUESTION 4

4 Question 3 **7 / 10**

✓ **- 3 pts if used p.v. instead of a.v.**

QUESTION 5

5 Question 4 **5 / 5**

✓ **- 0 pts Correct**

QUESTION 6

Question 5 10 pts

6.1 Part (a) **5 / 5**

✓ **- 0 pts Correct**

6.2 Part (b) **5 / 5**

✓ **- 0 pts Correct**

QUESTION 7

7 Question 6 **10 / 10**

✓ **- 0 pts Correct**

QUESTION 8

8 Question 7 **10 / 10**

✓ **- 0 pts Correct**

QUESTION 9

Question 8 20 pts

9.1 Part (a) **5 / 5**

✓ **- 0 pts Correct**

9.2 Part (b) **5 / 5**

✓ **- 0 pts Correct**

9.3 Part (c) **4 / 10**

✓ **- 6 pts asset levels that do not provide Full Immunization**

1 It is not possible with these, but it is possible with other levels!

QUESTION 10

Question 9 15 pts

10.1 Part (a) **5 / 5**

✓ **- 0 pts Correct**

10.2 Part (b) **5 / 5**

✓ **- 0 pts Correct**

10.3 Part (c) **5 / 5**

✓ **- 0 pts Correct**

I hereby acknowledge that I am aware I may use my textbook, notes, recordings during the exam and swear on my honor as a Bruin all the answers I present belong solely to me, in thought, and in writing

1 Academic Honesty Disclaimer 0 / 0

✓ **- 0 pts** Correct

1. Simple interest $\rightarrow A(t) = A(0)(1+i\times t)$, Compound Interest $\rightarrow A(t) = A(0)(1+i)^t$

$A'(t) = A(0)(i)$ $\qquad$ $A'(t) = A(0)(1+i)^t \ln(1+i)$

$\delta_t = \frac{i}{1+it}$ $\qquad$ $\delta_t = \ln(1+i)$

$$\frac{0.08}{1+0.08t} = \ln(1+0.05), \; t = 8$$

YSDS $\Rightarrow$ $\cancel{A(0)(1+}$ $A(0)(1+0.05\cancel{\times 8})^8 = \cancel{14774}$. 14744.55444 , LLTE $\Rightarrow (1+0.08\times 8)\times 10{,}000$ $= 16{,}400$

Difference $= \cancel{1655.44556}$ 1625.44562, and LLTE will be greater.

2 Question 1 **10 / 10**

✓ **- 0 pts** Correct

$1630.31 = \cancel{(1000x)\frac{(1-(1+j))}{j}} + 1000\,\cancel{(1+j)}^{-60}$

2. Value of payments at ~~t=~~ 60^{th} birthday $= x(1.06)^{0}(1.08)^{40} + \dots x(1.06)^{39}(1.08)^{1} = Y$

Value of perpetuity at his 60^{th} birthday $\Rightarrow \frac{48,000}{12} = 4{,}000.$ $(x)^{12} = 1.08$

~~$12\log_2 x = \log$~~ $12\ln x = \ln 1.08$, $x = 1.00643403$.

~~$4{,}000(x)^0 + 4000(x)^1 + \dots$~~ $\frac{4000}{x^0} + \frac{4000}{x^1} + \frac{4000}{x^2} + \dots = \frac{4000}{1-\frac{1}{x}} = Y$

$Y = 625694.3347 = x(1.08)^{40} \Big/ \frac{\left(\left(\frac{1.06}{1.08}\right)^{40} - 1\right)}{\frac{1.06}{1.08} - 1}$

$x = \cancel{533.3572998}$ 1012.949638 = Answer

3 Question 2 10 / 10

✓ **- 0 pts** Correct

3. $X(1.1)^{20} - k(1.1)^{19} - k(1.1)^{18} - \dots k(1.1)^{0} = X(1.1)^{20} - \frac{k(1.1^{20}-1)}{1.1-1} = 0$

Interest accumulated = ~~$k(1.1)^3 + k(1.1)^2 + k(1.1)^1 - 3k = 8606$~~

At time = 3 years, the sinking fund has $k(1.1)^2 + k(1.1) + k$

At ~~During~~ year 4th year before payment $1.1(k(1.1)^2 + k(1.1) + k) = k(1.1)^3 + k(1.1)^2 + k(1.1)$

Interest accumulated = $k(1.1)^3 - k = 86.06$, $k = 260$

$X(1.1)^{20} = \frac{k(1.1^{20}-1)}{1.1-1}$, $X = 2213.526567$ = Answer

4 Question 3 7 / 10

✓ **- 3 pts** if used p.v. instead of a.v.

4.

$F = 1000,\ C = 1000,\ j = \frac{x}{2},\ P = 1630.31,\ \alpha = 2x,\ m = 2,\ r = x$

$$1630.31 = 1000x \frac{\left(1 - \left(1 + \frac{x}{2}\right)^{-100}\right)}{x/2} + 1000\left(1 + \frac{x}{2}\right)^{-100}$$

$$1630.31 = 2000 - 2000\left(1 + \frac{x}{2}\right)^{-100} + 1000\left(1 + \frac{x}{2}\right)^{-100}$$

$$\cancel{= \left(\left\{\frac{1630.31 - 2000}{-1000}\right\}^{100} - 1\right) \times 2 = x}$$

$$\underbrace{\frac{1630.31\ \cancel{1630} - 2000}{-1000}}_{k} = \cancel{1000}\left(1 + \frac{x}{2}\right)^{-100},\quad \ln k = -100\,\cancel{ft}\,\ln\left(1 + \frac{x}{2}\right)$$

$$\left(e^{\ln k/-100} - 1\right)\cancel{\times 2 = x} \times 2 = k$$

$x = 0.02000115902$ = Answer

5 Question 4 **5 / 5**

✓ **- 0 pts** Correct

5 (a) $OB_1 = 10,000(0.97)$, $OB_{10} = 10000(0.97)^{10}$

$OB_{10} = K a_{\overline{10}|0.1} = K\dfrac{(1-(1+0.1)^{-10})}{0.1}$, $K = 1200.123807$ = Answer

~~99508~~

~~1125~~ ~~1490~~

(b)

$P_{12} = \cancel{K\dfrac{(1-(1+0.1)^{-8})}{0.1} + K(1+0.1)^{-8}} = \dfrac{K}{1.1} + \cancel{K} \dots \dfrac{K}{1.1^8}$

$\dfrac{K}{1.1}\left(\dfrac{\frac{1}{1.1}^{8}-1}{\frac{1}{1.1}-1}\right) = 6402.57194$

$P_{12+0.5} = P_{12}(1+0.1)^{0.5} = 6715.074102$

dirty price

$P^{clean}_{12+0.5} = P_{12+0.5} - 0.5K = 6115.012198$ = Answer

6.1 Part (a) **5 / 5**

✓ **- 0 pts** Correct

5 (a) $OB_1 = 10{,}000\,(0.97)$, $OB_{10} = 10000(0.97)^{10}$

$$OB_{10} = K a_{\overline{10}|0.1} = K\,\frac{(1-(1+0.1)^{-10})}{0.1}, \quad K = 1200.123807 = \text{Answer}$$

~~99508~~

~~1125~~ ~~1190~~

(b)

$$P_{12} = \cancel{K\frac{(1-(1+0.1)^{-8})}{0.1} + K(1+0.1)^{-8}} = \frac{K}{1.1} + \cancel{K} \ldots \frac{K}{1.1^8}$$

$$\frac{K}{1.1}\left(\frac{\frac{1}{1.1}^{8}-1}{\frac{1}{1.1}-1}\right) = 6402.57194$$

$$P_{12+0.5} = P_{12}\,(1+0.1)^{0.5} = 6715.074102$$

dirty price

$$P^{\text{clean}}_{12+0.5} = P_{12+0.5} - 0.5K = 6115.012198 = \text{Answer}$$

6.2 Part (b) 5 / 5

✓ **- 0 pts** Correct

6. ~~6~~ ~~7~~. ~~Evariable payments~~ $P = 1722.25, 0.08 = \alpha, m = 2, r = 0.04, F = x, j = \frac{0.06}{2}$

$$1722.25 = X \cdot r \cdot \left(\frac{1-(1+j)^{-30}}{j}\right) + X \cdot \kappa \cdot (1+j)^{-30}$$

$$x = 1440.003047.$$

$$P = \cancel{1440.003047 \cdot r (} X \cdot r \left(\frac{1-(1+j)^{-40}}{j}\right) + X (1+j)^{-40}$$

$$P = 1772.856468.$$

7 Question 6 **10 / 10**

✓ **- 0 pts** **Correct**

7.

January 1 ↑ inflow 1000 | July 1 ↑ inflow | January 1 ↓ outflow 2,000

$1000(1+i) + x(1+\frac{i}{2}) = 2000$, $x = 870.8133971$.

$\frac{F_1}{\cancel{1000}} \times \frac{\cancel{2000}}{F_1 + x} - 1 = 0.1$

$\frac{F_1}{F_1+x} = 0.55$, $0.55F_1 + 0.55x = F_1$, $0.45F_1 = 0.55x$

$F_1 = 1064.327485$

$(1000)(1+i) = F_1$, $1+i = 1.064327485$

$(1+i)^2 = \cancel{0.1327}\ 1.132792995$

Answer $= 13.2792995\%$

4. $\alpha = 2x$, $r = 2x/2 = x$

8 Question 7 10 / 10

✓ **- 0 pts** Correct

interest.

8. a) ~~Full immunization. If change in interest high enough, Redington immunization may be satisfied but a deficit in but deficit in PV may occur and it won't be fully immunized. immunized.~~

Full immunization because company wants to protect itself from any interest change regardless of the size of the interest change.

(b) Macaulay duration →
$$\frac{3 \times 100{,}000(1+0.08)^{-3} + 5 \times 120{,}000 \times (1+0.08)^{-5}}{100{,}000\,(1+0.08)^{-3} + 120{,}000(1+0.08)^{-5}}$$

$= 4.01419893 =$ Answer.

convexity →
$$\frac{3^2 \times 100000(1+0.08)^{-3} + 5^2 \cdot 120{,}000(1+0.08)^{-5}}{100000(1+0.08)^{-3} + 120000(1+0.08)^{-5}}$$

$= 17.11359026$

9.1 Part (a) 5 / 5

✓ - 0 pts Correct

8.a) ~~Full immunization. If change in interest high enough, Redington immunization may be satisfied but a deficit in PV may occur and it won't be fully immunized.~~

Full immunization because company wants to protect itself from any interest change regardless of the size of the interest change.

(b) Maculay duration →
$$\frac{3 \times 100{,}000(1+0.08)^{-3} + 5 \times 120{,}000 \times (1+0.08)^{-5}}{100{,}000(1+0.08)^{-3} + 120{,}000(1+0.08)^{-5}}$$

$= 4.01419873 = \text{Answer}$

convexity →
$$\frac{3^2 \times 100000(1+0.08)^{-3} + 5^2 \cdot 120{,}000(1+0.08)^{-5}}{100000(1+0.08)^{-3} + 120000(1+0.08)^{-5}}$$

$= 17.11359026$

9.2 Part (b) **5 / 5**

✓ **- 0 pts** **Correct**

c)

1) $\frac{A_2}{1.08^2} + \frac{A_4}{1.08^4} = \frac{100,000}{1.08^3}$

2) $\frac{A_4}{1.08^4} + \frac{A_6}{1.08^6} = \frac{120,000}{1.08^5}$

~~3) $\frac{A_2}{1.08^2} - \frac{A_6}{1.08^6} = \frac{100,000}{1.08^3} - \frac{120,000}{1.08^5}$~~

3) $\frac{2.A_2}{1.08^2} + \frac{4.A_4}{1.08^4} = \frac{3.100,000}{1.08^3}$

4) $\frac{4.A_4}{1.08^4} + \frac{6A_6}{1.086} = \frac{5.120,000}{1.08^5}$

5) $\frac{2A_2}{1.08^2} - \frac{6A_6}{1.08^6} = -\frac{5.120,000}{1.08^5} + \frac{3.100,000}{1.08^3}$

$\frac{A_2}{1.08^2} \cancel{- \frac{3A_6}{1.08^6} = \frac{5.60,000}{1.08^5}}$

$\cancel{\frac{-A_6}{1.08^6} - \frac{3A_6}{1.08^6} = 100,000}$

$\frac{-6A_6}{1.086} + \frac{2(+1)A_6}{1.086} = \frac{3.100000}{1.08^3} - \frac{5.120,000}{1.08^5} - \frac{200000}{1.08^3} + \frac{240000}{1.08^5}$

$\frac{-4A_6}{1.08^6} = \frac{100000}{1.08^3} - \frac{360000}{1.08^5}$

$+A_6 = \frac{1.08^3.100000}{-4} - \frac{360000.1.08}{-4}$, $A_6 = 65707.2$

$$\cancel{A_2 = 22814.73861}$$

$$\cancel{A_4 = 81388.88889}$$

$$A_4 = 54000$$

$$A_2 = \cancel{[illegible]}\ 46296.2963$$

$$A_6 = 66614.4$$

$$\frac{5^2 \cdot 120{,}000}{1.08^5} \overset{?}{>} \frac{4^2 \cdot \overbrace{54000}^{A_4}}{1.08^4} + \frac{6^2 \cdot \overbrace{66614.4}^{A_6}}{1.08^6}$$

$$\frac{3^2 \cdot 100000}{1.08^3} \overset{?}{>} \frac{2^2 \cdot A_2}{1.08^2} + \frac{4^2 \cdot A_4}{1.08^4}$$

Left hand side is smaller than right hand side in both equations, ∴ not satisfied and no full immunization

9.3 Part (c) 4 / 10

✓ **- 6 pts asset levels that do not provide Full Immunization**

1. It is not possible with these, but it is possible with other levels!

(a) 7. $f[0,1] = \cancel{(0.08+0.02)}^1 \quad \frac{(1+r_1)^1}{(1+r_0)^0} - 1 = r_1 = 0.08 + 0.02(1)$
$= \cancel{0.08}\ 0.10$

$$f[1,2] = \frac{(1+r_2)^2}{(1+r_1)^1} - 1 = \frac{(1+0.08+0.02\cdot 2)^2}{\cancel{0}(1+0.08+0.02)^1} - 1 = 0.1403636364$$

$$f[2,3] = \frac{(1+\cancel{00}\,0.08+0.02\cdot 3)^3}{(1+0.08+0.02\cdot 2)^2} - 1 = 0.1810778061$$

$$f[3,4] = \frac{(1+0.08+0.02\cdot 4)^4}{(1+0.08+0.02\cdot 3)^3} - 1 = 0.2221249941$$

$$f[4,5] = \frac{(1+0.08+0.02\cdot 5)^5}{(1+0.08+0.02\cdot 4)^4} - 1 = 0.2635082432.$$

$$\sum_{i=0}^{i=4} f(0+i, 1+i)\cdot 1{,}000{,}000 = 1{,}000{,}000(j)\cdot 5$$

$\cancel{1000(f[0,1] - k\} + 1000}$

$\cancel{j = \{f[0,1] + f[1,2]} + f\,\cancel{[2,3] + f[3,4]} + \cancel{f[4,5]}\}/5$

$\cancel{j = 0.1816159936.}$

$$\sum_{t=1} \cancel{\frac{}{(1+r_t)^t}} \qquad \frac{}{(1+r_t)^t}$$

(a) $$\sum_{t=1}^{5} \frac{f\,[t-1,\,t]}{(1+r_t)^t} \Big/ \sum_{t=1}^{5} \frac{1}{(1+r_t)^t} = 0.1669973681$$

10.1 Part (a) **5 / 5**

✓ **- 0 pts** Correct

(b) $1,000,000\left(\sum_{t=1}^{3} \frac{f[t-1,t]}{(1+r_t)^t} \Big/ \cancel{\sum_{t=1}^{3} \frac{1}{(1+r_t)^t}}\right) + 2,000,000\left(\sum_{t=4}^{5} \frac{f[t-1,t]}{(1+r_t)^t}\right)$

$\cancel{\sum_{t=4}^{5} \frac{1}{(1+r_t)^t}}$

$= \cancel{R1}\ 1,000,000R\sum_{t=1}^{3} \frac{1}{(1+r_t)^t} + 2,000,000R\sum_{t=4}^{5} \frac{1}{(1+r_t)^t}$

$R = \cancel{0.3611996929}\ 0.183656566$

(c) r_0, $r_1^* = 0.\overset{14}{\cancel{05}}$, $r_2^* = 0.\overset{18}{\cancel{09}}$, $r_3^* = 0.\overset{22}{\cancel{18}}$, $r_4 = 0.\overset{26}{\cancel{1}}$, $r_0^* = 0.\overset{1}{\cancel{01}}$

$f[0^*, 1^*] = 0.14$

$f[1^*, 2^*] = \frac{(1+0.18)^2}{1+0.14} - 1 = 0.22$

$f[2^*, 3^*] = \frac{(1+0.22)^3}{(1+0.18)^2} - 1 = 0.304$

$f[3^*, 4^*] = \frac{(1+0.26)^4}{(1+0.22)^3} - 1 = 0.3888$

~~$f[4^*, 5^*]$~~

PV_2 (future fixed payments) $= 1,000,000 \times 0.183656566 \times 1/1.14 + 2,000,000 \times 0.183656566 \times \frac{1}{1.18^2} + 2,000,000 \times 0.183656566 / 1.22^3$

$= \cancel{1285595.962}$

$= 627183.0157$

PV_2 (future floating payments)

$= \frac{1,000,000}{1.14} \cancel{\times 0.14} \times 0.14 + 2,000000\left(\frac{0.22}{1.18^2} + \frac{0.304}{1.22^3}\right)$

$= 775779.2077$

MV_2 ~~[illegible]~~ $= 148596.192$

Vlad has advantage and can sell the swap agreement.

10.2 Part (b) 5 / 5

✓ - 0 pts Correct

(b) $1,000,000\left(\sum_{t=1}^{3} \frac{f[t-1,t]}{(1+r_t)^t} \Big/ \cancel{\sum_{t=1}^{3} \frac{1}{(1+r_t)^t}}\right) + 2,000,000\left(\sum_{t=4}^{5} \frac{f[t-1,t]}{(1+r_t)^t}\right)$

$= \cancel{R\,1}\ 1,000,000R\sum_{t=1}^{3} \frac{1}{(1+r_t)^t} + 2,000,000R\sum_{t=4}^{5} \frac{1}{(1+r_t)^t}$

$R = \cancel{0.3611996929}\ 0.183656566$

(c) r_0, $r_1^* = 0.14$, $r_2^* = 0.18$, $r_3 = 0.22$, $r_4 = 0.26$, $r_0^* = 0.1$

$f[0^*, 1^*] = 0.14$

$f[1^*, 2^*] = \frac{(1+0.18)^2}{1+0.14} - 1 = 0.22$

$f[2^*, 3^*] = \frac{(1+0.22)^3}{(1+0.18)^2} - 1 = 0.304$

$f[3^*, 4^*] = \frac{(1+0.26)^4}{(1+0.22)^3} - 1 = 0.3888$

~~$f[4^*, 5^*]$~~

PV_2 (future fixed payments) $= 1,000,000 \times 0.183656566 \times 1/1.14 + 2,000,000 \times 0.183656566 \times \frac{1}{1.18^2} + 2,000,000 \times 0.183656566/1.22^3$

$= \cancel{1285595.962}$

$= 627183.0157$

PV_2 (future floating payments)

$= \frac{1,000,000}{1.14} \times 0.14 + 2,000000\left(\frac{0.22}{1.18^2} + \frac{0.304}{1.22^3}\right)$

$= 775779.2077$

$MV_2 = 148596.192$

Vlad has advantage and can sell the swap agreement.

10.3 Part (c) **5 / 5**

✓ **- 0 pts** Correct

Math 177
Summer 2020
Final Exam
09/09/2020
Time Limit: 24 Hours

Name: ____________________

UID: ____________________

This exam contains 16 pages (including this cover page) and 9 questions. Total of points is 100. Make sure to write your answers in full detail, so that you may get the maximum possible points.

Question	Points	Score
1	10	
2	10	
3	10	
4	5	
5	10	
6	10	
7	10	
8	20	
9	15	
Total:	100	

- Please type the following statement in your handwriting, then sign and date below:

  "I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor as a Bruin *all* the answers I present belong solely to me, in thought and in writing."

1. (10 points) Carloman and Charles are two brothers. Brother's finance advisor, Desiderius arranged two funds for these brothers, *Fund YSDS* (stands for "You Shall Die Soon") for Carloman and *Fund LLTE* (stands for "Long Live the Emperor") for Charles. At the beginning, both make deposits of $10,000$. Fund YSDS has a compound accumulation at an annual effective interest rate of 5%, whereas Fund LLTE has a simple accumulation with a yearly effective rate of 8%.

   t years into the future, the forces of interest on the two funds are equal. At this time t, find the difference between funds YSDS and LLTE, specifying Carloman/YSDS or Charles/LLTE will be *greater* (pun intended).

   (**Fun fact: Charlemagne's brother name was Carloman or Karlmann, which is a name derived from the same root!**)

2. (10 points) Evariste Galois, aged exactly 20, purchases a perpetuity-due to act as a pension benefit that starts 40 years later, exactly at his 60th birthday. According to this benefit he will be paid an annual total of $48,000$ with level payments at the start of every month. (The total annual payment of $48,000$ is to be equally distributed to each month.)

   To purchase this benefit, he will be making forty annual premium payments, first payment being at the time of policy purchase, at the 20th birthday. Every payment after the first will increase by 6%. Assuming a flat term structure and annual effective interest rate of 8% for *all* valuations, what should be the amount of initial payment?

   (**I highly suggest reading about the allegedly tragic life of Evariste Galois. He actaully died at the age of 21.**)

3. (10 points) John Sobieski, third of his name, is the King of Poland. Under the threat of a great war with the neighboring Ottoman Empire, he borrows X from a local noble to modernize his cavalry regiments.

   He is to repay the principal by making 20 level annual payments at the end of each year into a sinking fund which earns an annual effective rate of 10%. The interest earned in the sinking fund *during the 4th year* is 86.06. Calculate X.

   (`https://www.quora.com/What-was-the-biggest-cavalry-charge-in-history`)

4. (5 points) A 50-year, $1,000$ par-valued bond with semiannual coupons, with a nominal coupon rate of $2X$. Avicenna purchased it for $1,630.31$ to yield a nominal annual rate of X compounded semiannually. Determine X.

   (**HINT: Being one of the most influential physicians in the history, Avicenna does not need a financial calculator to be able to calculate this rate. Neither do you.**)

5. (10 points) Feanor is in dire need of cash and borrows $10,000$ from Durin. They agree to a 20-year repayment plan, where the payments are to be at the end of each year. Each of the first ten payments is equal to the 130% of the amount of interest due at the time of payment. Each of the last ten payments is a level amount of K.

   (**J.R.R. Tolkien's *Silmarillion* is definitely worth reading, despite the fact that it was published after his death.**)

   (a) (5 points) Assuming that Durin charges Feanor interest at an effective annual rate of 10%, what should K be?

(b) (5 points) After 12 years, suspecting Feanor's motives and ability to continue the payments, Durin wants to sell the rights to the future payments a third actor. To cover his relative's liabilities, Glorfindel steps up and offers to purchase the rights to the future payments from Durin, at the exact time $t = 12.5$. However, Glorfindel states that he would only pay the "clean price", which considers the next payment as a coupon payment of a bond. How much will Glorfindel pay?

6. (10 points) In order to address its huge expenses, the South Sea Company issues a 20-year, callable bond with 8% nominal coupon rate with semiannual coupons at a price of $1,722.25$. Isaac Newton sees this as a lucrative investment opportunity and purchases this bond. The bond can be called at the par value X on any coupon date starting at the end of year 15, up to and including the maturity date. At this price, the lowest possible nominal yearly yield rate Isaac Newton can obtain is 6%, convertible semiannually.

   Seeing the meteoric rise in the South Sea Company's prestige, Daniel Defoe, too, purchases a 20-year bond issued by the company, identical to the one purchased by Isaac Newton, except that it is not callable. Assuming a nominal yearly yield rate of 6%, convertible semiannually, calculate the cost of the bond purchased by Defoe.

   (**The fascinating story of "the South Sea Bubble"**: `https://www.youtube.com/watch?v=k1kndKWJKB8&ab_channel=ExtraCredits`)

   (**Both Defoe and Newton lost huge amounts of money in the bubble.**)

   (**Andrew Odlyzko's writings on the South Sea Bubble:** `http://www.dtc.umn.edu/~odlyzko/doc/mania13b.pdf` and `https://physicstoday.scitation.org/doi/pdf/10.1063/PT.3.4521`)

7. (10 points) Qin Jiushao deposits $1,000$ into a fund on January 1, 2020 and then another deposit of unknown amount into the fund on July 1, 2020. On January 1, 2021, the balance in the fund is $2,000$. Noting that the time-weighted yield rate is 10% and the dollar-weighted yield rate is 9%, calculate the annual effective interest rate earned on the fund during the first six months of 2020.

   (**Qin Jiushao is a Chinese mathematician that provided the general solution of the "Chinese Remainder Theorem" in his work *Shushu Jiuzhang* (Mathematical Treatise in Nine Sections) that was published in 1248. The same generality of a solution of this problem was presented in Europe by Gauss as late as 1801.**)

8. (20 points) The Honorable East India Company must pay two liabilities, $100,000$ in exactly three years ($t = 3$) and $120,000$ in exactly five years ($t = 5$). The current interest rate is 8% effective per year. The company wants to immunize itself from *any* potential change in interest rates, no matter how large, as the amount of silver and gold introduced to Europe by Spanish conquistadors is causing the rates to deviate frantically.

   The company will attempt to achieve this by setting three zero-coupon bonds as assets associated with their liabilities, the bonds maturing at times $2, 4$ and 6.

   (**Just horrifyingly amazing:** `https://en.wikipedia.org/wiki/East_India_Company`)

   (a) (5 points) What form of immunization are they seeking to form: Redington Immunization or Full Immunization? Briefly explain.

   (b) (5 points) Calculate the Macaulay duration and convexity for the liabilities, considered as a single portfolio of liabilities.

(c) (10 points) Is it possible for the company to achieve the form of immunization they would like to have? If it is, find the redemption amounts (A_2, A_4, A_6) for the three and zero-coupon bonds and justify that they immunize the liabilities. If it is not, explain it is not possible to fully immunize.

9. (15 points) Vlad Tepes, Prince-Protector of Wallachia temporarily purchases (or loans), for a term of 5 years, the rights to use the income of the lands of Tirgoviste, which is worth 1,000,000 ducats. At the end of every year, there will be an interest payment with respect to the floating rate available at the beginning of that year. At the end of the term, the lands are returned (similar to capital return). The term structure at the time is as follows:

$$r_k = 0.08 + 0.02k, \text{ for } k = 1, 2, 3, 4, 5.$$

However, as Wallachia has a relatively small economy, Vlad wants no risk and asks Mehmed II, of Ottoman Empire to act as a financial guarantor who ensures Vlad's total cash outflow at each payment period is based on some fixed payments based on a constant swap rate.

(**Vlad is the Dracula.**)

(a) (5 points) Assuming the land value of Tirgoviste stays constant throughout the term, what would be the swap rate corresponding to this agreement.

(b) (5 points) Assume Tirgoviste is worth $1,000,000$ through the first three years, immediately after which, due to great prosperity in this region, is envisioned to increase to $2,000,000$. In the remaining portion of the term this value stays fixed. Find the new appropriate swap rate under this new forecast.

(c) (5 points) Suppose the at the end of the second year we have a new term structure:

$$r_k^* = 0.1 + 0.04k, \text{ for } k = 1, 2, 3, 4.$$

Find the market value (at time 2) of this swap assuming the scenario in part (b) still holds true. Who has the right to sell the swap agreement: Vlad or Mehmed?