201C-MATH177-1 Midterm 1

DHRUV SANCHETY

TOTAL POINTS

100 / 100

QUESTION 1

1 Academic Honesty Disclaimer **0 / 0**

✓ **- 0 pts** Correct

QUESTION 2

2 Question 1 **20 / 20**

✓ **- 0 pts** Correct

QUESTION 3

3 Question 2 **20 / 20**

✓ **- 0 pts** Correct

QUESTION 4

Question 3 20 pts

4.1 Part (a) **4 / 4**

✓ **- 0 pts** Correct

4.2 Part (b) **16 / 16**

✓ **- 0 pts** Correct

QUESTION 5

5 Question 4 **20 / 20**

✓ **- 0 pts** Correct

QUESTION 6

6 Question 5 **20 / 20**

✓ **- 0 pts** Correct

I agree hereby acknowledge that I am aware I may use my textbook lecture notes, and recordings during the exam and swear on my honor as a Bruin all the answers I present belong solely to me in thought and writing.

Dhruv 8/13/2020

1 Academic Honesty Disclaimer **0 / 0**

✓ **- 0 pts** Correct

(1) ~~(a)~~

$\frac{24}{12} = 2$

(i) $1000(1.02)^{12\times 4} = 2587.07$. Interest paid $= 2587.07 - 1000 = 1587.07$

2

(i) $\cancel{(1000)(1.02)}$ $0.02 \times 1000 \times 48 + 1000 = 1960$, Interest paid $= 1960 - 1000 = 960$

(iii) $1000 \times (1.02)^{48} - k(1.02)^{24} - k = 0$

$1000(1.02)^{48} = k(1.02^{24} + 1)$

$k = \cancel{991.81}$ $\frac{1000(1.02)^{48}}{1.02^{24}+1}$

$2k = \frac{2000(1.02)^{48}}{1.02^{24}+1}$ ~~Chandler,~~ $= 1983.62$

$1983.62 - 1000$ = Interest paid $= 983.62$

, ~~Chandler, Joey~~

Chandler, Phoebe, Joey from smallest to largest in interest paid.

2 Question 1 20 / 20

✓ **- 0 pts** Correct

2.

$$1000 \times \exp\left[\int_0^2 0.03t^2\,dt\right]$$

$$1000 \times \exp\left[\frac{0.03t^3}{3}\right]_0^2$$

$$1000 \times \exp(0.01(2)^3 - 0.01(0)^3)$$

$$1000 \times \exp(0.08)$$

$$1000\,e^{0.08}$$

$$1000e^{0.08}\exp\left[\int_2^6 0.02t\,dt\right]$$

$$1000e^{0.08}\exp\left[\frac{0.02t^2}{2}\right]_2^6$$

$$1000\,e^{0.08}\exp(0.01\times 6^2 - 0.01\times 2^2)$$

$$1000e^{0.08}\exp(0.36 - 0.04)$$

$$1000\,e^{0.08}\,e^{0.32}$$

$1000\,e^{0.4}$ $\cancel{= 1491}$ ≈ 1491.82 (Rounded up to 2 decimal places)

3 Question 2 **20 / 20**

✓ **- 0 pts** Correct

3 (a) Yearly the additional interest accumulated is withdrawn, and is, therefore not relevant in this calculation.

$1000 - 100n = 0$

$\frac{1000}{100} = n$

$10 = n$

$\therefore$ Denethor's funds ~~with~~ at the Bank of Minas Tirith deplete after 10 years

(b) ~~100~~ Accumulated value of 100 investments:

~~100(100(100+100~~

$100(1.1)^9 + 100(1.1)^8 + \dots 100 = 1593.74246.$

Accumulated value of additional ~~to~~ interest that was invested

$0.08 \times 1000 \times 1.1^9 + 0.08 \times 900 \times 1.1^8 + 0.08 \times 800 \times 1.1^7 + 0.08 \times 700 \times 1.1^6$
$+ 0.08 \times \cancel{100} \, 600 \times 1.1^5 + 0.08 \times 500 \times 1.1^4 + 0.08 \times 400 \times 1.1^3 + 0.08 \times 300 \times 1.1^2$
$+ 0.08 \times 200 \times 1.1 + 0.08 \times 100 = 800$

$=$

$\therefore$ Accumulated value at Bank of Dol Amroth =

$800 + 1593.742\cancel{8}46 = 2393.74246$

4.1 Part (a) **4 / 4**

✓ **- 0 pts** Correct

3 (a) Yearly the additional interest accumulated is withdrawn, and is, therefore not relevant in this calculation.

$$1000 - 100n = 0$$
$$\frac{1000}{100} = n$$
$$10 = n$$

$\therefore$ Denethor's funds ~~will~~ at the Bank of Minas Tirith deplete after 10 years

(b) ~~100~~ Accumulated value of 100 investments:

~~100(100(100+100~~

$$100(1.1)^9 + 100(1.1)^8 + \dots 100 = 1593.74246.$$

Accumulated value of additional ~~to~~ interest that was invested

$$0.08 \times 1000 \times 1.1^9 + 0.08 \times 900 \times 1.1^8 + 0.08 \times 800 \times 1.1^7 + 0.08 \times 700 \times 1.1^6$$
$$+ 0.08 \times \cancel{100}\, 600 \times 1.1^5 + 0.08 \times 500 \times 1.1^4 + 0.08 \times 400 \times 1.1^3 + 0.08 \times 300 \times 1.1^2$$
$$+ 0.08 \times 200 \times 1.1 + 0.08 \times 100 = 800$$
$$=$$

$\therefore$ Accumulated value at Bank of Dol Amroth =

$$800 + 1593.742\cancel{8}46 = 2393.74246$$

4.2 Part (b) **16 / 16**

✓ **- 0 pts** Correct

5.6.

Monthly effective growth $\rightarrow (1+i)^{\frac{1}{12}}$, since yearly effective growth ~~rate~~ $= (1+i)$

$\therefore$ value @ time $n = 50 + 50(1+i)^{\frac{1}{12}} \ldots + 50(1+i)^{\frac{1}{12}(12n)-\frac{1}{12}}$

This is a geometric sum $\rightarrow \dfrac{50\left((1+i)^{\frac{1}{12}(12n)} - 1\right)}{\cancel{(1+i)^{12n}} \; (1+i)^{\cancel{44}\,1/12} - 1}$

$$Y \rightarrow \frac{50((1+i)^n - 1)}{(1+i)^{1/12} - 1} = \frac{50(10.894481 - 1)}{(1+i)^{1/12} - 1} = \frac{494.72405}{(1+i)^{1/12} - 1}$$

Equation 1: $\dfrac{494.72405}{(1+i)^{1/12} - 1} = \overset{x}{\cancel{50}}\,\ddot{a}_{\overline{\infty}|i} = \overset{x}{\cancel{50}}(8.884259)$ where x is yearly pension payment.

Finding Equation 2:

$$a^{(m)}_{\overline{n}|i} = a_{\overline{n}|i} \frac{i}{i^{(m)}}$$

$$a^{(12)}_{\overline{n}|i} = \cancel{a_{\overline{n}|i} \frac{i}{12 i^{1/12}}} = \cancel{\frac{(a_{\overline{n}|i})(i^{1/12})}{12}} \quad a_{\overline{n}|i} \frac{i}{12((1+i)^{1/12} - 1)}$$

$\cancel{\dfrac{90.81942}{i^{1/12}} = a_{\overline{n}|i} = \dfrac{1-v^n}{i}}$

$\cancel{90.81942\, i^{\frac{1}{12}} = a_{\overline{n}|i} = 1 - v^n = 1 - \frac{1}{(1+i)^n}} = \cancel{\frac{(1+i)^n - 1}{(1+i)^n}}$

$\cancel{90.81942((1+i)^{1/12} - 1)}$ $\quad -\dfrac{1}{1+i} + 1 = d, \ \dfrac{1+i-1}{1+i} = d, \ \dfrac{i}{1+i} = d$

$\cancel{\dfrac{1}{i} = a_{\overline{\infty}|i}}$ $\quad \ddot{a}_{\overline{\infty}|i} = \dfrac{1}{d} = \dfrac{1+i}{i} = 8.884259$, $i = 0.1268350012$

$\cancel{\dfrac{x}{i} = Y = x(8.884259), \ i = 0.1125586276}$

$\cancel{\dfrac{494.72405}{(1+i)^{1/12} - 1} / 8.884259}$

$$\frac{90.81942((1+i)^{1/12} - 1)}{i} = a_{\overline{n}|i} = \frac{1 - \frac{1}{(1+i)^n}}{i}$$

$\cancel{a_{\overline{\infty}|i} = \dfrac{90.81942((1+i)^{1/12} - 1)}{i} = \dfrac{1}{i}, \ i = 0.1404332025}$

$$\frac{494.72405}{(1 + 0.1404332025)^{1/12} - 1} / 8.884259 = X = 5057.32$$

$$\frac{90.81942\left((1+i)^{1/12}-1\right)}{i} = a_{\overline{n}|i} = \frac{1-\frac{1}{(1+i)^n}}{i} = 0.4082104049$$

$$i = \cancel{0.126827419}\ 0.126827419$$

$$\frac{494.72405}{(1+0.12687419)^{1/12}-1} \Big/ 8.884259 = 5568.45$$

5 Question 4 **20 / 20**

✓ **- 0 pts** Correct

5. $\frac{12}{12} = 1\%$

1% monthly interest rate

X is principal

geometric sum with 60 terms, initial value $-1000(1.01)^{59}$ and ratio $0.97/1.01$

$$\cancel{X} \; X(1.01)^{60} - 1000(0.97)^0(1.01)^{59} - 1000(0.97)^1(1.01)^{58} - 1000(0.97)^2(1.01)^{57}$$

$$\ldots - 1000(0.97)^{59}(1.01)^0 = 0$$

Since ~~loan with interest~~ loan is amortized over 5 years.

$$X = 22787.10077$$

$$X(1.01)^{36} - 1000(0.97)^0(1.01)^{35} \ldots - 1000(0.97)^{35}(1.01)^{\cancel{59}\,\cancel{35}\,0}$$

$$OB_{36} \approx \cancel{5186.55} = \text{Answer}$$

$$5184.55$$

geometric sum with ratio $0.97/1.01$, 36 terms, and initial value $-1000[1.01)\wedge 35]$

6 Question 5 **20 / 20**

✓ **- 0 pts** Correct

Math 177 **Name:** ____________________

Summer 2020

Midterm 1

08/12/2019

Time Limit: 24 Hours UID: ____________________

This exam contains 6 pages (including this cover page) and 5 questions. Total of points is 100. Make sure to write your answers in full detail, so that you may get the maximum possible points.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

- Please type the following statement in your handwriting, then sign and date below:

  "I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor as a Bruin *all* the answers I present belong solely to me, in thought and in writing."

1. (20 points) Chandler, Joey and Phoebe each borrow 1000 for four years, at a nominal interest rate of 24%, compounded monthly.

   (i) Joey has interest accumulated over the four years and pays all the interest and principal in a lump sum at the end of four years.

   (ii) Chandler pays interest at the end of every month as it accumulates and the whole principal at the end of four years.

   (iii) Phoebe makes two level payments at the end of the second and fourth years to pay back the loan.

   Calculate the net total amount interest paid by each of these “friends” and compare these interest amounts from smallest to largest.

2. (20 points) Emperor Justinian I of Roman Empire is offered an investment opportunity. Emperor's consort and chief financial advisor Theodora reports that the investment accumulation is calculated according to the following force of interest:

$$\delta_t = \begin{cases} 0.03\,t^2 & 0 \le t \le 2 \\ 0.02\,t & 2 < t \end{cases}$$

Emperor invests 1000 to this investment fund at $t = 0$. To what amount would his investment accumulate to over a period of six years?

3. (20 points) Steward Denethor, son of Ecthelion is a shrewd financial manueverer. He originally deposits 1000 in an account at the Bank of Minas Tirith, earning an annual effective rate of 8%. *At the end of each year*, he withdraws 100 plus the interest accumulated that year to reinvest into an account in the Bank of Dol Amroth, earning an annual effective rate of 10%.

   (a) (4 points) When would Denethor's funds at the Bank of Minas Tirith deplete?

   (b) (16 points) Determine the accumulated value of Denethor's investments at the Bank of Dol Amroth, immediately after he reinvests the last amount of withdrawals from the Bank of Minas Tirith.

4. (20 points) Flavius Belisarius is a famous military commander and close friend of Emperor Justinian I (from Question 2). After retaking the city of Rome for Romans from Ostrogoths, Belisarius returns to Constantinople for a calm life where he can focus on this botany and farming.

   In order to ensure a stable pension when he retires, Belisarius has been paying 50 gold from his monthly salary the instant he receives it (which is at the end of every month) for the past n years.

   Belisarius retires after n years of monthly payments, by purchasing a yearly perpetuity-due that begins right away, granting him an infinite supply of annual salary with payments at the start of each year. We assume $i > 0$ is the underlying annual effective interest rate that governs all the growth in this scenario.

   Given that $a^{(12)}_{\overline{n}|i} = 7.568285$, $(1+i)^n = 10.894481$ and $\ddot{a}_{\overline{\infty}|i} = 8.884259$, how much is Belisarius' yearly pension payment?

5. (20 points) A loan is amortized over five years with monthly payments at a nominal interest rate of 12% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 3% lower than the prior payment. Calculate the outstanding loan balance immediately after the 36th payment is made.