EE360C: Algorithms
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Quiz #10
Due: April 19, 2018 (*in class*)

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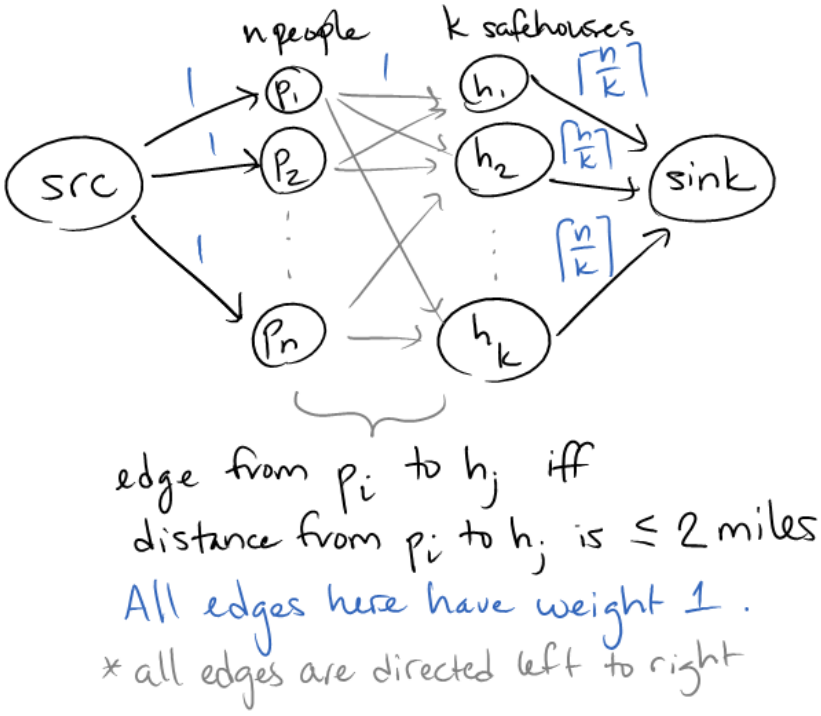
Problem 1: Zombie Invasion: Evacuation

It's the zombie apocalypse. Unfortunately, the zombie invasion has occurred while you're the watch captain for your neighborhood. It's your job to see that everyone in your neighborhood evacuates safely. There are k safe houses have been set up at various locations around the city, and there are n people that must be evacuated from your neighborhood. Each safe house will accept no more than $\lceil n/k \rceil$ of the residents of your neighborhood.

Because of how fast zombies move, you need to be sure to send each evacuee to a safe house that is at most 2 miles from the evacuee's location. Formulate this as a network flow algorithm that takes the given information about the residents needing evacuation (and their locations) and determines whether complete evacuation is possible. In cases where it is possible, demonstrate how to determine a workable allocation of evacuees to safe houses.

Solution



Constructing Flow Graph

Construct a flow graph G as follows: create a node for each person, p_i where $1 \leq i \leq n$, and create a node for each safehouse, h_j where $1 \leq j \leq k$. Create a source and a sink. Connect the source to each person node by an edge with weight 1. Connect each safehouse to the ssink by an edge with capacity $\left\lceil \frac{n}{k} \right\rceil$. For every pair of nodes (p_i, k_j), if and only if the distance from p_i to h_j is ≤ 2 miles,

connect these nodes by an edge with capacity 1. (See the figure above for how this graph might look.)

Solution

Having constructed this flow graph G, run FORD-FULKERSON(G), which returns a number M which is the value of the max-flow. If $M = n$ then that means that it is possible to safely evacuate all of the people. If $M < n$ then some people get left behind.

In order to find the evacuation plan, we must examine the flow graph G'. We look at all the edges between people and safehouses and note that an edge (p_i, h_j) which has a flow of 1 on it means that person p_i should evacuate to safehouse h_j. This may seem trivial, but it is significant that we actually get this information for free as a side effect of running FORD-FULKERSON.[1]

[1]This step is important because just knowing that it is possible to evacuate all the people doesn't mean that it is trivial to know where to send them. If, for instance, we simply tell everyone "RUN TO THE NEAREST SAFEHOUSE NOW!!" then maybe that house will be over capacity and it is too late for them to get to the next farthest one. Instead we want to be able to tell every person exactly where to go so that we save as many people as possible.