EE360C: Algorithms
The University of Texas at Austin — Quiz #6
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Quiz #6

Problem 1: MST [5 points]

Let G be a connected undirected graph with weighted edges, and suppose the weight of every edge of G is distinct. Could there be multiple different minimum spanning trees (i.e., there are some edges in one but not in the other)? Either give an example G with two different minimum spanning trees, or prove that there is exactly one minimum spanning tree in this case.
Hint: Consider two distinct MST T_1, T_2 and use e which is the maximum weight edge in the set $A = (T_1 \setminus T_2) \cup (T_2 \setminus T_1)$.

Solution

Proof by Contradiction :
Let T_1 and T_2 be two distinct minimum spanning trees. Since they are distinct, there are some edges that appear in exactly one of T_1 or T_2. Consider the edge of *maximum* weight among these edges. Without loss of generality, this edge appears only in T_1, and we can call it e_1.
Suppose we take T_1 and remove e_1. This splits the tree into two connected components. Since e_1 is not in T_2, there must be another edge e_2 in T_2 but not in T_1 such that adding it would connect these two components. Thus taking T_1, removing e_1, and adding e_2 forms a new spanning tree T_1'. Since e_2 is an edge different from e_1 and is contained in exactly one of T_1 or T_2, it must be that $w(e_1) < w(e_2)$. Since T_1' is the same as T_1 except for removing e_1 and adding e_2, the total weight of T_1' is smaller than the total weight of T_1. But this is a contradiction, since we have supposed that T_1 is a minimum spanning tree.