EE360C: Algorithms
The University of Texas at Austin — Quiz #7
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Quiz #7

Problem 1: Divide and Conquer

Algorithm 1 draws a recursive pattern (finite fractal) based on a given image. Let C(n) denote the total number of times that the drawOnce function is called by the invocation draw(im, x, y, n).

Algorithm 1: Recursive Image Generation
Input: An image im, location coordinates x, y, size n.
Output: Displays recursive image based on im of size n n at (x, y)
draw(im, x, y, n)
(1) **if** $n > 0$
(2) **drawOnce**(im, x, y, n)
(3) **draw**(im, $x, y+n$, $\lfloor n/2 \rfloor$)
(4) **draw**(im, $x+n, y$, $\lfloor n/2 \rfloor$)
(5) **draw**(im, $x, y-n$, $\lfloor n/2 \rfloor$)
(6) **draw**(im, $x-n, y$, $\lfloor n/2 \rfloor$)

(a) Write a recurrence relation for C(n).
(b) Solve the recurrence relation to find a big Θ expression for the number of drawOnce calls as a function of n.

Solution

(a) Since each level of the recursion calls drawOnce exactly once, and since no calls are made when n = 0, we have the following:
$C(n) = 1 + 4 * C(\lfloor n/2 \rfloor)$ for $n > 0$
$C(0) = 0$

(b) By the Master Theorem, with $a = 4, b = 2, l = 0$ (note that $l < log_b a$), we find that $C(n) = \Theta(n^{log_b a}) = \Theta(n^2)$.