



## Brief Paper

Global tracking control of underactuated ships by  
Lyapunov's direct method<sup>☆</sup>Zhong-Ping Jiang<sup>\*</sup>*Department of Electrical and Computer Engineering, Polytechnic University, Six Metrotech Center, Brooklyn, NY 11201, USA*

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**Abstract**

This paper studies the global tracking problem for an underactuated ship with only two propellers. Under sufficient conditions of persistent excitation, two constructive solutions are proposed by application of Lyapunov's direct method. Both solutions exploit the inherent cascade interconnected structure of the ship dynamics and generate explicit Lyapunov functions. A common feature of the second solution and the recent cascade approach of Lefeber (Tracking control of nonlinear mechanical systems, Ph.D. Thesis, University of Twente, 2000) is that global exponential tracking is achieved, but at the expense of transient performance. Extension to unmeasured thruster dynamics is also considered. Simulation results validate the proposed tracking methodology. © 2001 Elsevier Science Ltd. All rights reserved.

**Keywords:** Ship control; Global tracking; Lyapunov functions; Stability; Nonlinear control

**1. Introduction**

A control system is underactuated if it has fewer number of independent actuators than degrees of freedom to be engineered. Nonlinear mechanical systems with nonintegrable velocity constraints, a class of so-called nonholonomic systems, are typical examples of this kind (see for concrete examples, Murray & Sastry, 1993; Kolmanovsky & McClamroch, 1995; Reyhanoglu, van der Schaft, McClamroch, & Kolmanovsky, 1999; Jiang, 2000b). In recent years, a good number of novel nonlinear design techniques have been developed and interesting solutions are obtained for the stabilization problem for nonholonomic systems. Time-varying (smooth or merely Lipschitz continuous) feedback and discontinuous state-feedback are two commonly admitted strategies (see for a good survey Kolmanovsky & McClamroch, 1995). Besides the stabilization (or parking) problem, the tracking (or stabilization of trajectory) problem has also received some attention (see for more references Kolmanovsky & McClamroch, 1995; Kanayama, Kimura,

Miyazaki, & Noguchi 1990; Jiang & Nijmeijer, 1997, 1999; Jiang, 2000a).

In this paper, we consider an underactuated ship with only two propellers—one is the surge force and the other the yaw moment. This underactuation is quite common in practical surface vessels carrying the tasks of off-shore oil field operations. Globally, exponentially stabilizing controllers are obtained in Reyhanoglu (1997) and Pettersen (1996). As explained in Reyhanoglu (1997) and Pettersen (1996) (see also Pettersen & Nijmeijer, 1998), the underactuated ship has a nonintegrable constraint on the acceleration and is *not* transformable into a (driftless) chained form (Murray & Sastry, 1993). Consequently, the *global* tracking problem to be addressed in this paper cannot be solved using earlier tracking algorithms developed for nonholonomic mechanical systems transformable to a chained form (Jiang & Nijmeijer, 1997, 1999; Jiang, 2000a), (also see Kanayama et al., 1990). It was nevertheless shown in Pettersen and Nijmeijer (1998) that the recursive technique of Jiang and Nijmeijer (1999) can be adapted to solve the (high-gain based) semiglobal tracking problem for an underactuated surface vessel. The seemingly first global tracking result has recently been obtained in Lefeber (2000) on the basis of a neat cascade approach. The stability analysis in Lefeber (2000) relies upon the stability theory of linear time-varying

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<sup>\*</sup> Tel.: +1-718-260-3646; fax: +1-718-260-3906.

E-mail address: zjiang@control.poly.edu (Z.-P. Jiang).

(LTV) systems. Some related independent work includes Toussaint, Basar, and Bullo (2000) on local  $\mathcal{H}_\infty$  tracking control and the recent paper by Behal, Dawson, Dixon, and Fang (2000) on global practical tracking.

The purpose of this paper is to develop new solutions to the global *asymptotic* tracking of underactuated ships by application of Lyapunov's direct method. We advocate Lyapunov's direct method here because it has become a standard method for nonlinear control design. More importantly, unlike Lefeber (2000), our novel designs produce explicit Lyapunov functions whose availability suits the need of robust and adaptive control design. Solutions to the latter issue will be reported elsewhere. Our two constructive tracking solutions rely heavily on the inherent cascade-interconnected structure of the ship dynamics. The first approach exploits the passivity-based  $L_gV$ -type control strategy (Jurdjevic & Quinn, 1979) and gives globally tracking controllers with desirable transient and asymptotic performance. The second approach combines the cascade approach of Lefeber (2000) with our earlier backstepping scheme (Jiang & Nijmeijer, 1999; Jiang, 2000a) to achieve global *exponential* tracking. Again, this marriage allows us to obtain an explicit Lyapunov function for the closed-loop ship system. Unfortunately, the transient performance of the combined cascade-backstepping controllers is not as good as the passivity-based tracking controllers (see Remark 3).

The rest of this paper is arranged as follows. Section 2 formulates the tracking task and introduces the sufficient condition of "persistent excitation (PE)" on the reference angular velocity  $r_d$  for the underactuated ship. Section 3 presents our novel, constructive tracking designs and states the main results. As an advantage of our Lyapunov design over the cascade approach of Lefeber (2000), an extension to the case of unmeasured thruster dynamics is briefly discussed in the Section 3.3. Section 4 describes some simulation results that serve the purpose of demonstrating and validating the proposed tracking methodology. Section 5 summarizes the main contributions of this paper and pinpoints future work that continues the current effort.

**Notation and notions:** The notations used in this article are quite standard. The vertical bars  $|\cdot|$  mean the Euclidean norm of a vector. The double bars  $\|\cdot\|$  stand for the  $\mathcal{L}_\infty$ -norm of a function  $u : \mathcal{I} \rightarrow \mathbb{R}^n$ , with  $\mathcal{I}$  a subinterval of  $[0, \infty)$  i.e.,  $\|u\| = \sup_{t \in \mathcal{I}} |u(t)|$ . A class  $\mathcal{K}$ -function is a continuous function  $\gamma : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  that is (strictly) increasing and vanishes at the origin. A function  $W : \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be radially unbounded if  $W(x) \rightarrow \infty$  if  $|x| \rightarrow \infty$ .  $W : \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be positive definite if  $W(0) = 0$  and  $W(x) > 0$  for all  $x \neq 0$ .  $V : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be radially unbounded (resp. positive definite) if there is a radially unbounded (resp. positive definite) function  $W(x)$  such that  $V(t, x) \geq W(x)$  for all  $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$ .

## 2. Statement of the problem

In this paper, we consider a surface ship that is operated under a failure mode, i.e., that the side force does not work or is not implemented because of economical and weight considerations. The only two propellers are the force in surge and the control torque in yaw. Under this realistic assumption, the motion of the ship dynamics is described by the following ordinary differential equations (see for details, Fossen & Strand, 1999; Pettersen, 1996):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix}, \quad (1)$$

$$\dot{u} = \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u + \frac{1}{m_{11}}\tau_s,$$

$$\dot{v} = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v,$$

$$\dot{r} = \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}\tau_y, \quad (2)$$

where  $(x, y)$  denotes the coordinates of the surface vessel in the earth-fixed frame,  $\psi$  is the heading angle of the surface vessel, and  $u, v$  and  $r$  denote the velocity in surge, sway and yaw respectively, the surge force  $\tau_s$  and the yaw torque  $\tau_y$  are considered as the control inputs. The parameters  $m_{ii}$  and  $d_{ii}$  are positive that are assumed to be constant and are given by the ship inertia and damping matrices.

Clearly, the surface vessel is underactuated because the sway force is missing in the  $v$ -equation (2). As a consequence of this underactuation, there is no continuous (nor discontinuous) state-feedback control law that (locally) asymptotically stabilizes the ship to the origin (see, e.g., Pettersen, 1996, Proposition 2.3). Time-varying (smooth or continuous) state feedback is a nonstandard control approach that can successfully accommodate this technical challenge.

In this paper, we move away from the stabilization problem and concentrate on the *tracking* issue. The control task is formulated as follows: given a suitably defined trajectory, find a feedback controller that forces the ship to asymptotically follow the desired trajectory from *any* initial conditions. As said in Section 1, the seemingly first global (state-feedback) solution has been presented in Lefeber (2000) using a cascade approach and thanks to the observation that the tracking error dynamics of the ship can be decomposed as a cascade interconnection of two linear time-varying (LTV) systems. This observation turns out to be crucial for previous work in nonholonomic chained systems (Sørdalen & Egeland, 1995; Jiang, 1996). Our contributions in the present paper include the development of two novel solutions to the *global tracking* problem. One of these solutions is

based on a passivity argument and the other combines the cascade approach (Lefeber, 2000) and the recursive backstepping technique (Jiang, 2000a; Jiang & Nijmeijer, 1999). With the help of these novel approaches, we are able to surmount the hardship in testing the uniform complete controllability for a LTV system (Lefeber, 2000) and to introduce more standard conditions on the reference signals than the “persistent excitation (PE)” condition in Lefeber (2000). In contrast to the cascade approach of Lefeber (2000), it is shown in Section 3.3 that our Lyapunov trackers can be easily made robust against (unmeasured) thruster dynamics. To date, we are not aware of any existing result to this *robust* global tracking problem.

For future uses, our desired trajectory is generated by the dynamic equations of a *virtual* underactuated ship:

$$\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\psi}_d \end{bmatrix} = \begin{bmatrix} \cos \psi_d & -\sin \psi_d & 0 \\ \sin \psi_d & \cos \psi_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_d \\ v_d \\ r_d \end{bmatrix}, \quad (3)$$

$$\begin{aligned} \dot{u}_d &= \frac{m_{22}}{m_{11}} v_d r_d - \frac{d_{11}}{m_{11}} u_d + \frac{1}{m_{11}} \tau_{sd}, \\ \dot{v}_d &= -\frac{m_{11}}{m_{22}} u_d r_d - \frac{d_{22}}{m_{22}} v_d, \\ \dot{r}_d &= \frac{m_{11} - m_{22}}{m_{33}} u_d v_d - \frac{d_{33}}{m_{33}} r_d + \frac{1}{m_{33}} \tau_{yd}, \end{aligned} \quad (4)$$

where  $(x_d, y_d, \psi_d)$  denote the (desired) position and orientation of the virtual ship,  $(u_d, v_d, r_d)$  stand for the (desired) velocities, and  $(\tau_{sd}, \tau_{yd})$  are the reference inputs in surge and yaw. Throughout the remainder of the paper, it is assumed that the reference signals  $(x_d, y_d, u_d, v_d, r_d)$  (except the reference angle  $\psi_d$ !) and  $(\tau_{sd}, \tau_{yd})$  are bounded over  $[0, \infty)$ . This assumption is realistic from the physics of the problem. Furthermore, a standard PE condition is needed on the reference angular velocity  $r_d(t)$ :

(H) There is a constant  $\sigma_r > 0$  such that, for any pair of  $(t_0, t)$ ,  $0 \leq t_0 \leq t < \infty$ ,

$$\int_{t_0}^t r_d^2(\tau) d\tau \geq \sigma_r(t - t_0). \quad (5)$$

The above-formulated tracking task can be translated into the issue of designing a feedback control law for  $(\tau_s, \tau_y)$  so that system (1), (2) (globally) synchronizes with the reference model (3), (4).

### 3. Global tracking by state feedback

First of all, we set out to simplify the kinematic equations (1). First, as in Pettersen (1996), apply the transformation of coordinates  $\Phi: (x, y, \psi) \mapsto (z_1, z_2, z_3) = (x \cos \psi + y \sin \psi, -x \sin \psi + y \cos \psi, \psi)$  to

both system (1) and reference system (3). Then, introduce the tracking error variables  $z_{ie} = z_i - z_{id}$  for  $1 \leq i \leq 3$ ,  $u_e = u - u_d$ ,  $v_e = v - v_d$  and  $r_e = r - r_d$ . Direct computation gives that the tracking error dynamics obey the following differential equations:

$$\begin{aligned} \dot{z}_{1e} &= u_e + z_{2e} r_d + z_{2e} r_e, \\ \dot{z}_{2e} &= v_e - z_{1e} r_d - z_{1e} r_e, \\ \dot{z}_{3e} &= r_e, \\ \dot{u}_e &= \frac{m_{22}}{m_{11}} (v r - v_d r_d) - \frac{d_{11}}{m_{11}} u_e + \frac{1}{m_{11}} (\tau_s - \tau_{sd}) := u_1, \\ \dot{v}_e &= -\frac{m_{11}}{m_{22}} u_e r_d - \frac{d_{22}}{m_{22}} v_e - \frac{m_{11}}{m_{22}} u r_e, \\ \dot{r}_e &= \frac{m_{11} - m_{22}}{m_{33}} (u v - u_d v_d) - \frac{d_{33}}{m_{33}} r_e + \frac{1}{m_{33}} (\tau_y - \tau_{yd}) \\ &:= u_2. \end{aligned} \quad (6)$$

Clearly, the tracking problem for the original ship model (1), (2) has now been converted into a *stabilization* problem for the new system (6) with  $(\tau_s, \tau_y)$ , or, equivalently,  $(u_1, u_2)$  as the inputs. In the rest of this section, we propose two different stabilization schemes for (6).

#### 3.1. A passivity approach

We begin with the passivity-based design scheme. Throughout the controller design procedure, we will exploit the passivity properties hidden in system (6), and, in particular, the popular  $L_g V$ -type control strategy (Jurdjevic & Quinn, 1979).

To guide the reader smoothly through the rather involved design procedure, we split the scheme into two steps starting with the  $(z_{1e}, z_{2e}, u_e, v_e)$ -subsystem of (6). It is worth noting that the  $(z_{1e}, z_{2e}, u_e, v_e)$ -system is LTV if  $r_e$  is considered as a time-varying signal (i.e., not a state) and  $u_1$  as the input.

*Step 1 (Design of the surge force  $\tau_s$ ):* Motivated by our previous nonholonomic algorithms in Jiang (1996, 2000a) introduce the quadratic function

$$V_0 = \frac{1}{2} (z_{1e} - \lambda_1 z_{2e} r_d)^2 + \frac{1}{2} z_{2e}^2 + \frac{\lambda_0}{2} v_e^2, \quad (7)$$

where  $\lambda_0, \lambda_1 > 0$  are design parameters to be determined later.

Differentiating  $V_0$  along the solutions of (6) yields

$$\begin{aligned} \dot{V}_0 &= (z_{1e} - \lambda_1 z_{2e} r_d) (u_e + z_{2e} r_d + z_{2e} r_e - \lambda_1 z_{2e} \dot{r}_d \\ &\quad - \lambda_1 r_d (v_e - z_{1e} r_d - z_{1e} r_e)) \\ &\quad + z_{2e} (v_e - z_{1e} r_d - z_{1e} r_e) + \lambda_0 v_e \\ &\quad \times \left( -\frac{m_{11}}{m_{22}} u_e r_d - \frac{d_{22}}{m_{22}} v_e - \frac{m_{11}}{m_{22}} u r_e \right). \end{aligned} \quad (8)$$

Collecting the terms containing  $r_e$  and decomposing the terms  $-z_{1e}r_d$  in (8) as  $-\lambda_1 z_{2e}r_d^2 - (z_{1e} - \lambda_1 z_{2e}r_d)r_d$ , we obtain

$$\begin{aligned} \dot{V}_0 = & (z_{1e} - \lambda_1 z_{2e}r_d)[u_e + \lambda_1 r_d^2(z_{1e} - \lambda_1 z_{2e}r_d) - \lambda_1 \dot{r}_d z_{2e} \\ & - \lambda_1 r_d(v_e - \lambda_1 z_{2e}r_d^2)] - \lambda_1 r_d^2 z_{2e}^2 + z_{2e}v_e \\ & + \lambda_0 v_e \left( -\frac{m_{11}}{m_{22}} u_e r_d - \frac{d_{22}}{m_{22}} v_e \right) + \left[ (z_{1e} - \lambda_1 z_{2e}r_d) \right. \\ & \times (z_2 + \lambda_1 r_d z_1) - z_{2e}z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u \left. \right] r_e. \end{aligned} \quad (9)$$

Unlike the standard application of backstepping, in order to reduce the complexity of the controller expressions, we will choose a simpler virtual control law  $\alpha_0$  for  $u_e$ . That is, without cancelling the known terms in the first square brackets of (9),  $\alpha_0$  is selected as

$$\alpha_0 = -\lambda_2(z_{1e} - \lambda_1 z_{2e}r_d) \quad (10)$$

with  $\lambda_2 > 0$  a design parameter to be determined later. Introducing the new variable  $\bar{u}_e = u_e - \alpha_0$ , (9) gives

$$\begin{aligned} \dot{V}_0 = & -(\lambda_2 - \lambda_1 r_d^2)(z_{1e} - \lambda_1 z_{2e}r_d)^2 - \lambda_1 r_d^2 z_{2e}^2 - \frac{\lambda_0 d_{22}}{m_{22}} v_e^2 \\ & + \left[ (z_{1e} - \lambda_1 z_{2e}r_d) - \frac{\lambda_0 m_{11}}{m_{22}} r_d v_e \right] \bar{u}_e \\ & - (z_{1e} - \lambda_1 z_{2e}r_d)[\lambda_1 \dot{r}_d z_{2e} + \lambda_1 r_d(v_e - \lambda_1 z_{2e}r_d^2)] \\ & + z_{2e}v_e + \lambda_0 \lambda_2 \frac{m_{11}}{m_{22}} r_d(z_{1e} - \lambda_1 z_{2e}r_d)v_e \\ & + \left[ (z_{1e} - \lambda_1 z_{2e}r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ & \left. - z_{2e}z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u \right] r_e. \end{aligned} \quad (11)$$

If the design parameters  $\lambda_1$  and  $\lambda_2$  are chosen such that

$$\lambda_2 - \lambda_1 \|r_d\|^2 > 0, \quad (12)$$

$$\begin{aligned} & -(z_{1e} - \lambda_1 z_{2e}r_d)[\lambda_1 \dot{r}_d z_{2e} + \lambda_1 r_d(v_e - \lambda_1 z_{2e}r_d^2)] + z_{2e}v_e \\ & + \lambda_0 \lambda_2 \frac{m_{11}}{m_{22}} r_d(z_{1e} - \lambda_1 z_{2e}r_d)v_e - (1 - \varepsilon)(\lambda_2 - \lambda_1 r_d^2) \\ & \times (z_{1e} - \lambda_1 z_{2e}r_d)^2 - \frac{m_{22}}{(1 - \varepsilon)d_{22}\lambda_0} z_{2e}^2 \\ & - \frac{(1 - \varepsilon)\lambda_0 d_{22}}{m_{22}} v_e^2 \leq 0 \end{aligned} \quad (13)$$

for some constant  $0 < \varepsilon < 1$ , then (11) implies

$$\begin{aligned} \dot{V}_0 \leq & -\varepsilon(\lambda_2 - \lambda_1 r_d^2)(z_{1e} - \lambda_1 z_{2e}r_d)^2 \\ & - \left( \lambda_1 r_d^2 - \frac{m_{22}}{(1 - \varepsilon)\lambda_0 d_{22}} \right) z_{2e}^2 - \frac{\varepsilon \lambda_0 d_{22}}{m_{22}} v_e^2 \end{aligned}$$

$$\begin{aligned} & + \left[ (z_{1e} - \lambda_1 z_{2e}r_d) - \frac{\lambda_0 m_{11}}{m_{22}} r_d v_e \right] \bar{u}_e \\ & + \left[ (z_{1e} - \lambda_1 z_{2e}r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ & \left. - z_{2e}z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u \right] r_e. \end{aligned} \quad (14)$$

By completing the squares, it is not hard to show that (12) and (13) can always be fulfilled as long as  $\|r_d\|$  is small, i.e.,  $\|r_d\| := \sup_{t \geq 0} |r_d(t)| \leq r^*$  for small  $r^*$ . For example, (12) and (13) hold under the following sufficient conditions:

$$\begin{aligned} \lambda_2 & > \lambda_1 \|r_d\|^2, \\ (1 - \varepsilon)(\lambda_2 - \lambda_1 r_d^2) - \left( \lambda_0 \lambda_2 \frac{m_{11}}{m_{22}} r_d - \lambda_1 r_d \right)^2 & \frac{m_{22}}{2(1 - \varepsilon)\lambda_0 d_{22}} \\ & \geq \frac{(1 - \varepsilon)\lambda_0 d_{22}}{2m_{22}} (\lambda_1^2 r_d^3 - \lambda_1 \dot{r}_d)^2. \end{aligned} \quad (15)$$

Now, we consider the following Lyapunov function candidate for the  $(z_{1e}, z_{2e}, u_e, v_e)$ -subsystem of (6) with  $u_1$  as the control input and  $r_e$  as the disturbance input:

$$\begin{aligned} V_1 = & V_0 + \frac{1}{2} \bar{u}_e^2 = \frac{1}{2} (z_{1e} - \lambda_1 z_{2e}r_d)^2 + \frac{1}{2} z_{2e}^2 + \frac{\lambda_0}{2} v_e^2 \\ & + \frac{1}{2} (u_e + \lambda_2(z_{1e} - \lambda_1 z_{2e}r_d))^2. \end{aligned} \quad (16)$$

When we select the surge force as

$$\begin{aligned} \tau_s = & \tau_{sd} + m_{11} \left[ -\frac{m_{22}}{m_{11}} (vr - v_d r_d) + \frac{d_{11}}{m_{11}} u_e \right. \\ & - c_1 (u_e + \lambda_2(z_{1e} - \lambda_1 z_{2e}r_d)) \\ & - \left( (z_{1e} - \lambda_1 z_{2e}r_d) - \frac{\lambda_0 m_{11}}{m_{22}} r_d v_e \right) \\ & - \lambda_2 (u_e + z_{2e}r_d + z_2 r_e) + \lambda_1 \lambda_2 \dot{r}_d z_{2e} + \lambda_1 \lambda_2 r_d \\ & \left. \times (v_e - z_{1e}r_d - z_1 r_e) \right] \end{aligned} \quad (17)$$

with  $c_1 > 0$  a design parameter, the time derivative of  $V_1$  along the solutions of (6) satisfies

$$\begin{aligned} \dot{V}_1 \leq & -c(t)V_1 + \left[ (z_{1e} - \lambda_1 z_{2e}r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ & \left. - z_{2e}z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u \right] r_e, \end{aligned} \quad (18)$$

where  $c(t)$  is a time-varying signal defined by

$$c(t) = \min \left\{ 2\varepsilon(\lambda_2 - \lambda_1 r_d(t)^2), \right. \\ \left. 2 \left( \lambda_1 r_d^2(t) - \frac{m_{22}}{(1 - \varepsilon)\lambda_0 d_{22}} \right), 2\varepsilon \frac{d_{22}}{m_{22}}, 2c_1 \right\} \quad (19)$$

*Step 2 (Design of the yaw moment  $\tau_y$ ):* We first apply the traditional  $L_g V$ -control strategy to design a

stabilizing, virtual control law for  $r_e$ . Then, a straightforward application of the backstepping approach (Krstić, Kanellakopoulos, & Kokotović, 1995) completes the design procedure for  $\tau_y$ .

Consider the quadratic function

$$\begin{aligned} V_2 &= V_1 + \frac{1}{2}z_{3e}^2 \\ &= \frac{1}{2}(z_{1e} - \lambda_1 z_{2e} r_d)^2 + \frac{1}{2}z_{2e}^2 + \frac{\lambda_0}{2}v_e^2 \\ &\quad + \frac{1}{2}(u_e + \lambda_2(z_{1e} - \lambda_1 z_{2e} r_d))^2 + \frac{1}{2}z_{3e}^2. \end{aligned} \quad (20)$$

In view of (6) and (18), the time derivative of  $V_2$  satisfies

$$\begin{aligned} \dot{V}_2 &\leq -c(t)V_1 + \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) - z_{2e} z_1 \right. \\ &\quad \left. - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right) r_e. \end{aligned} \quad (21)$$

By choosing the following virtual  $L_g V_2$ -type stabilizer for  $r_e$

$$\begin{aligned} \alpha_1 &= -c_2 \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) - z_{2e} z_1 \right. \\ &\quad \left. - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right) \end{aligned} \quad (22)$$

with  $c_2 > 0$  a design parameter, and setting  $\bar{r}_e = r_e - \alpha_1$ , it follows from (21) that

$$\begin{aligned} \dot{V}_2 &\leq -c(t)V_1 - c_2 \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ &\quad \left. - z_{2e} z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right)^2 + \left( (z_{1e} - \lambda_1 z_{2e} r_d) \right. \\ &\quad \left. \times (z_2 + \lambda_1 r_d z_1) - z_{2e} z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right) \bar{r}_e. \end{aligned} \quad (23)$$

Now, we consider the Lyapunov function candidate for the entire tracking-error dynamic system (6)

$$V_3 = V_2 + \frac{1}{2}\bar{r}_e^2. \quad (24)$$

Clearly,  $V_3$  is a positive definite and radially unbounded function in  $(t, z_{1e}, z_{2e}, z_{3e}, u_e, v_e, r_e)$ . The time dependence arises from the reference signals.

With (23) and (6), we obtain

$$\begin{aligned} \dot{V}_3 &\leq -c(t)V_1 - c_2 \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ &\quad \left. - z_{2e} z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right)^2 + \bar{r}_e \\ &\quad \left[ \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) - z_{2e} z_1 \right. \right. \\ &\quad \left. \left. - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right) + u_2 - \dot{\alpha}_1 \right], \end{aligned} \quad (25)$$

where  $\dot{\alpha}_1$  stands for the time derivative of  $\alpha_1$ . Consequently, whenever we choose the yaw moment  $\tau_y$  as

$$\begin{aligned} \tau_y &= \tau_{yd} + m_{33} \left[ -\frac{m_{11} - m_{22}}{m_{33}} (uv - u_d v_d) + \frac{d_{33}}{m_{33}} r_e \right. \\ &\quad \left. - c_3 \bar{r}_e + \dot{\alpha}_1 - ((z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ &\quad \left. - z_{2e} z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e}) \right] \end{aligned} \quad (26)$$

with  $c_3 > 0$  a design parameter, it follows that

$$\begin{aligned} \dot{V}_3 &\leq -c(t)V_1 - c_2 \left( (z_{1e} - \lambda_1 z_{2e} r_d)(z_2 + \lambda_1 r_d z_1) \right. \\ &\quad \left. - z_{2e} z_1 - \frac{\lambda_0 m_{11}}{m_{22}} v_e u + z_{3e} \right)^2 - c_3 \bar{r}_e^2. \end{aligned} \quad (27)$$

Therefore, we are in a position to state the first main result of this paper.

**Theorem 1** (Global asymptotic tracking). *Assume that the reference signals  $(x_d, y_d, u_d, v_d)$  and  $(\tau_{sd}, \tau_{yd})$  in (3) and (4) are bounded over  $[0, \infty)$ . If there exist two (sufficiently small) constants  $r_\star$  and  $r^\star$  such that, for each  $t \geq 0$ ,*

$$0 < r_\star \leq |r_d(t)| \leq r^\star, \quad (28)$$

*then the Problem of Global Asymptotic Tracking is solved for the underactuated ship (1) and (2) by the time-varying and Lipschitz continuous state-feedback laws (17) and (26). In particular, the tracking errors  $x(t) - x_d(t)$ ,  $y(t) - y_d(t)$ ,  $\psi(t) - \psi_d(t)$ ,  $u(t) - u_d(t)$ ,  $v(t) - v_d(t)$ ,  $r(t) - r_d(t)$ ,  $\tau_s(t) - \tau_{sd}(t)$  and  $\tau_y(t) - \tau_{yd}(t)$  all converge to zero as  $t \rightarrow \infty$ .*

**Proof.** By assumptions, we can tune the tracking controller parameters  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  in (17) and (26) so that  $c(t) \geq c_\star$  for some constant  $c_\star > 0$ . We first prove the completeness of the closed-loop solutions of (6), (17) and (26). Assume that  $[0, T_c)$ , with  $T_c \leq \infty$ , is the maximal interval of definition of the solutions  $(z_{1e}, z_{2e}, z_{3e}, u_e, v_e, r_e)$ , or equivalently, the closed-loop signals  $(x, y, \psi, u, v, r)$ . By definition of  $V_3$  as in (24), it follows from (27) that  $(z_{1e}, z_{2e}, z_{3e}, u_e, v_e, r_e)$  are bounded on  $[0, T_c)$  and therefore  $T_c = \infty$ . As a result, the control inputs  $\tau_s$  and  $\tau_y$  are also bounded on  $[0, \infty)$ .

Then, consider the time-dependent nonpositive function  $-W(t)$  at the right-hand side of (27) with  $c(t)$  replaced by  $c_\star$ . Since  $W(t)$  is bounded on  $[0, \infty)$ ,  $W(t)$  is uniformly continuous. In addition, (27) yields that  $W$  is integrable over  $[0, \infty)$ . With the aid of Barbalat's lemma (Krstić et al., 1995), these facts imply that  $W(t)$  tends to zero as  $t \rightarrow \infty$ . Finally, the convergence property of the tracking errors follows readily.  $\square$

**Remark 1.** By means of Lyapunov's first method, we can prove (Jiang, 2001) that the closed-loop system (6),

(17) and (26) is (locally) exponentially stable at the zero equilibrium.

It should be mentioned that the restrictive condition (28) is also required in Pettersen and Nijmeijer (1998) for the semiglobal state tracking of a surface vessel.

### 3.2. A combined cascade-backstepping approach

As said already, the observation that the tracking-error dynamic system (6) is composed of two LTV systems plays a central role in Lefeber (2000) on the global tracking of a surface vessel. Notice that this decomposition viewpoint has also proved useful in earlier works (Sørdalen & Egeland, 1995; Jiang, 1996) on the global stabilization of nonholonomic chained systems. We show in this subsection that a merge of this observation with our backstepping-based tracking schemes (Jiang & Nijmeijer, 1997, 1999; Jiang, 2000a) leads to a new solution of global exponential tracking under the relaxed PE condition (H). As opposed to Lefeber (2000), our approach does not require the test of uniform complete controllability and observability for LTV systems and, more interestingly, does generate explicit Lyapunov functions.

The combined cascade-backstepping controller design procedure is composed of two steps as in Section 3.1.

*Step 1 (Design of the surge force  $\tau_s$ ):* Same as in Step 1 of Section 3.1.

*Step 2 (Design of the yaw moment  $\tau_y$ ):* We choose the following controller to stabilize the  $(z_{3e}, r_e)$ -subsystem of (6):

$$u_2 = -k_1 z_{3e} - k_2 r_e, \quad k_1, k_2 > 0 \quad (29)$$

which leads to the control law for the yaw moment  $\tau_y$

$$\tau_y = \tau_{yd} + m_{33} \left[ -\frac{m_{11} - m_{22}}{m_{33}}(uv - u_d v_d) + \frac{d_{33}}{m_{33}} r_e - k_1 z_{3e} - k_2 r_e \right]. \quad (30)$$

Clearly, this choice renders the  $(z_{3e}, r_e)$ -subsystem globally exponentially stable at the origin, i.e., for any pair of initial conditions  $(z_{3e}(t_0), r_e(t_0))$  and any initial time instant  $t_0 \geq 0$ , the solution  $(z_{3e}(t), r_e(t))$  exists for each  $t \geq t_0$  and satisfies

$$\begin{bmatrix} z_{3e}(t) \\ r_e(t) \end{bmatrix} \leq p_1 e^{-p_2(t-t_0)} \begin{bmatrix} z_{3e}(t_0) \\ r_e(t_0) \end{bmatrix} \quad (31)$$

for some constants  $p_1, p_2 > 0$ .

Then, by virtue of (18) and (31) and the definition of  $V_1$  as in (7), we can establish our second main result on the global tracking of underactuated ships.

**Theorem 2** (Global exponential tracking). *Under Assumption (H), with (15), the Problem of Global*

*Exponential Tracking is solved for the underactuated ship (1), (2) by the time-varying and Lipschitz continuous state-feedback laws (17) and (30). In particular, letting  $X_e := (z_{1e}, z_{2e}, z_{3e}, u_e, v_e, r_e)^T$ , there exist a class  $\mathcal{K}$ -function  $\gamma$  and a constant  $\sigma > 0$  such that, for any  $t_0 \geq 0$  and any  $X_e(t_0) \in \mathbb{R}^6$ , the solution  $X_e(t)$  exists for each  $t \geq t_0$  and satisfies*

$$|X_e(t)| \leq \gamma(|X_e(t_0)|) e^{-\sigma(t-t_0)}. \quad (32)$$

Before proving Theorem 2, we first give a technical lemma.

**Lemma 1.** *Consider a first-order differential equation of the form*

$$\dot{x} = -(a(t) + f_1(\xi(t)))x + f_2(\xi(t)), \quad (33)$$

where  $x \in \mathbb{R}$ ,  $f_1$  and  $f_2$  are continuous functions, and  $\xi: [0, \infty) \rightarrow \mathbb{R}^m$  is a time-varying vector-valued signal that exponentially converges to zero and, for all  $t \geq t_0 \geq 0$ , satisfies

$$|f_1(\xi(t))| \leq \gamma_1(|\xi(t_0)|) e^{-\sigma_1(t-t_0)}, \quad (34)$$

$$|f_2(\xi(t))| \leq \gamma_2(|\xi(t_0)|) e^{-\sigma_2(t-t_0)}, \quad (35)$$

where  $\sigma_1, \sigma_2 > 0$ , and  $\gamma_1$  and  $\gamma_2$  are class- $\mathcal{K}$  functions. If  $a(t)$  enjoys the property that there is a constant  $\sigma_3 > 0$  such that

$$\int_{t_1}^{t_2} a(\tau) d\tau \geq \sigma_3(t_2 - t_1) \quad \forall t_2 \geq t_1 \geq 0, \quad (36)$$

then there exist a class- $\mathcal{K}$  function  $\gamma$  and a constant  $\sigma > 0$  such that

$$|x(t)| \leq \gamma(|(x(t_0), \xi(t_0))|) e^{-\sigma(t-t_0)}. \quad (37)$$

**Proof.** For any initial time instant  $t_0 \geq 0$ , the solution  $x(t)$  at  $t \geq t_0$  satisfies

$$x(t) = x(t_0) e^{-\int_{t_0}^t (a(\tau) + f_1(\xi(\tau))) d\tau} + \int_{t_0}^t f_2(\xi(\tau)) e^{-\int_{\tau}^t (a(s) + f_1(\xi(s))) ds} d\tau. \quad (38)$$

By assumptions,

$$\begin{aligned} |x(t)| &\leq |x(t_0)| e^{\sigma_1^{-1} \gamma_1(|\xi(t_0)|)} e^{-\sigma_3(t-t_0)} + \sigma_1^{-1} \gamma_1(|\xi(t_0)|) \\ &\quad \times \gamma_2(|\xi(t_0)|) e^{-\sigma_3 t + \sigma_2 t_0} \int_{t_0}^t e^{(\sigma_3 - \sigma_2)\tau} d\tau. \end{aligned} \quad (39)$$

No matter whether  $\sigma_2 = \sigma_3$  or  $\sigma_2 \neq \sigma_3$ , there are always a positive constant  $\sigma_4 < \min\{\sigma_2, \sigma_3\}$  and a positive constant  $\sigma_5$  such that

$$e^{-\sigma_3 t + \sigma_2 t_0} \int_{t_0}^t e^{(\sigma_3 - \sigma_2)\tau} d\tau \leq \sigma_5 e^{-\sigma_4(t-t_0)}. \quad (40)$$

Therefore, combining (39) and (40), (37) follows readily.  $\square$

Now, we return to prove Theorem 2.

**Proof of Theorem 2.** From (18) and the definition of  $V_1$  in (7), there exists a constant  $\delta > 0$  such that

$$\dot{V}_1(t) \leq -c(t)V_1(t) + \delta(V_1(t) + \sqrt{V_1(t)})|r_e(t)|. \quad (41)$$

By defining  $\kappa(t) = 2\sqrt{V_1(t)}$ , we have

$$\dot{\kappa}(t) \leq -0.5c(t)\kappa(t) + \delta(1 + 0.5\kappa(t))|r_e(t)|. \quad (42)$$

Thanks to (H), for large values of  $\lambda_1$ ,  $0.5c(t) := a(t)$  meets property (36). By means of the generalized Gronwall inequality (Hale, 1980) and Lemma 1, noticing (31), we obtain

$$2\sqrt{V_1(t)} = \kappa(t) \leq \gamma(|(\kappa(t_0), z_{3e}(t_0), r_e(t_0))|)e^{-\sigma(t-t_0)}. \quad (43)$$

Finally, (43) together with (31) and the definition of  $V_1$  in (7) yields (32). This completes the proof of Theorem 2.  $\square$

**Remark 2.** Since the map  $\Phi : (x, y, \psi) \mapsto (z_1, z_2, z_3)$  is a global diffeomorphism with its inverse  $\Phi^{-1}$  defined by  $\Phi^{-1}(z_1, z_2, z_3) = (z_1 \cos z_3 - z_2 \sin z_3, z_1 \sin z_3 + z_2 \cos z_3, z_3)$ , it is not hard to prove that  $x(t) - x_d(t)$ ,  $y(t) - y_d(t)$  and  $\psi(t) - \psi_d(t)$  also converge to zero at an exponential rate as  $t \rightarrow \infty$ .

**Remark 3.** The main difference between the passivity approach and the cascade-backstepping approach is the way of how they handle the interconnecting terms between two LTV systems. Based on this difference, an advantage of the passivity approach over the combined cascade-backstepping approach is that it may result in smaller settling time and improved transient performance.

**Remark 4.** In general, Theorem 2 does *not* imply that the closed-loop tracking-error system (6), (17) and (30) is globally exponentially stable in the sense of Lyapunov (Hale, 1980), i.e.,  $\gamma$  in (32) must be a linear function. However, as in Remark 1, the local exponential stability of the closed-loop system (6), (17) and (30) is guaranteed.

### 3.3. Robustness to unmeasured thruster dynamics

The main purpose of this subsection is to show that the tracking algorithms proposed previously can be easily extended to a dynamic model of underactuated ships with *unmeasured* thruster dynamics. The latter has often been left out in earlier work (see for details, Pettersen, 1996).

As shown in Pettersen (1996), the thruster dynamics are described by

$$\dot{\tau}_s = -\frac{1}{T_s}(\tau_s - v_s), \quad \dot{\tau}_y = -\frac{1}{T_y}(\tau_y - v_y), \quad (44)$$

where  $T_s$  and  $T_y$  are the time constants for the thrust in surge and yaw,  $v_s$  and  $v_y$  are the controlled thrusters in

surge and yaw, respectively. It is assumed that  $\tau_s$  and  $\tau_y$  are not perfectly measurable.

In order to reconstruct the unmeasured states  $\tau_s$  and  $\tau_y$ , introduce the following observer:

$$\dot{\hat{\tau}}_s = -\frac{1}{T_s}(\hat{\tau}_s - v_s), \quad \dot{\hat{\tau}}_y = -\frac{1}{T_y}(\hat{\tau}_y - v_y). \quad (45)$$

Obviously, (45) is a global exponential observer for system (44). With this observation in mind, the robust global (exponential) tracking problem is solvable for the underactuated ship (1), (2) with unmeasured thruster dynamics (44). Indeed, it suffices to apply the above Lyapunov algorithms directly to the combined controller/observer augmented system composed of (6) and (45). See Jiang (2001) for details.

## 4. Numerical simulations

In this section, we carry out some computer simulations to demonstrate the performance of our tracking controllers and to validate our constructive methodology for underactuated ships. For the purpose of comparisons, we utilize a circle of radius 1 m as the reference trajectory that has been considered in Pettersen and Nijmeijer (1998) and Lefeber (2000).

For simulation uses, we make the following choice of initial conditions for reference system (3), (4):

$$\begin{aligned} (x_d(0), y_d(0), \psi(0), u_d(0), v_d(0), r_d(0)) \\ = (0, 0, 0, 0.1, 0, 0.1) \end{aligned} \quad (46)$$

and impose the following requirement for the reference velocities  $u_d$  and  $r_d$ :  $u_d(t) = r_d(t) = 0.1$  for all  $t \geq 0$ . That is, the virtual ship moves at a constant speed. Thus,  $\tau_{sd} = 0.1$  and  $\tau_{yd} = 0$ . For the true ship, we pick the following initial conditions:

$$(x(0), y(0), \psi(0), u(0), v(0), r(0)) = (1, -1, 1, 0, 0, 0). \quad (47)$$

For the plots in Figs. 1 and 2, we select the design parameter values  $\lambda_1 = 8$ ,  $\lambda_2 = 5$ ,  $d_{22} = 0.2$ ,  $d_{11} = d_{33} = 0$ ,  $m_{11} = m_{22} = m_{33} = 0.1$ , that fulfill condition (15), with  $\lambda_0 = 10$ .

In Fig. 1, we have simulated the underactuated ship with the passivity-based tracking controllers (17) and (26). The plots in Fig. 2 are based on the cascade-backstepping tracking controllers (17) and (30). Clearly, both types of controllers yield satisfactory tracking results. After a closer look at the plots in Figs. 1 and 2, one observes that the passivity-based tracking controllers (17) and (26) yield better *transient* performance while the cascade-backstepping tracking controllers (17) and (30) give rise to slightly better *asymptotic* behavior.

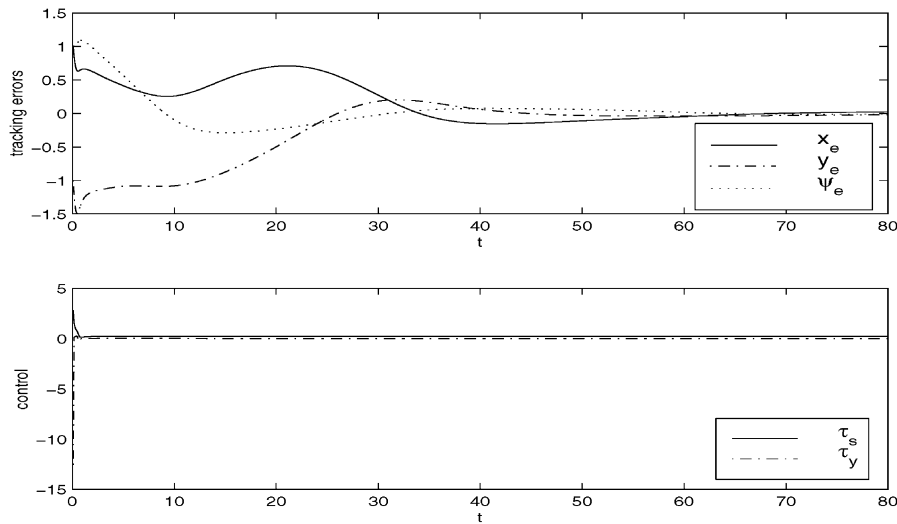


Fig. 1. Global tracking using the passivity-based controllers (17) and (26).

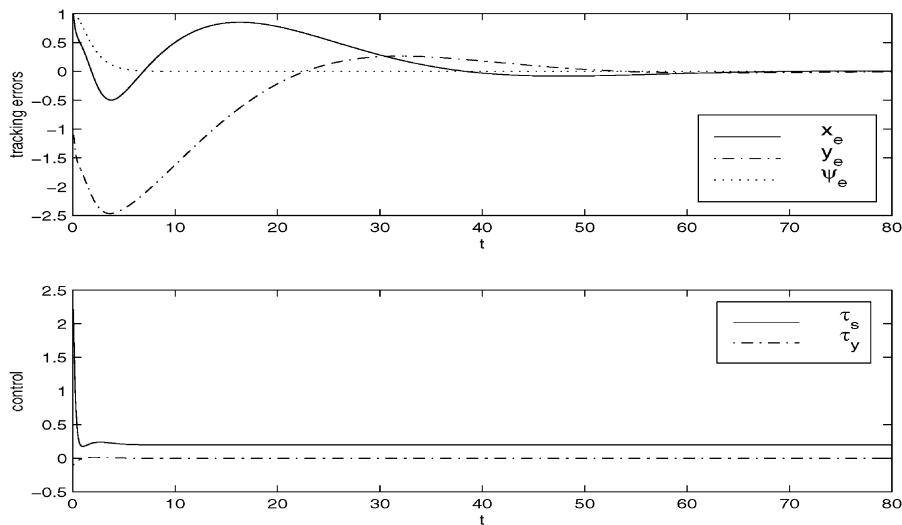


Fig. 2. Global tracking using the cascade-backstepping controllers (17) and (30).

## 5. Summary and future work

The problem of global tracking has been investigated for underactuated ships with only a surge force and a yaw moment. Two systematic tracking controller designs are developed with the aid of Lyapunov's direct method: the passivity approach and the cascade-backstepping approach. As illustrated by simulations, the passivity approach yields tracking controllers with better transient performance while the cascade-backstepping controllers enjoy better asymptotic behavior. More importantly, it has been shown in this paper how to render our tracking controllers robust against unmeasured thruster dynamics, whose presence has often been ignored in previous studies.

Evidently, as in Pettersen (1996), Reyhanoglu (1997) and Pettersen and Nijmeijer (1998), our results in ship

tracking control are directly extendible to the situation where, instead of the absence of the sway torque, the surge force  $\tau_s$  or the yaw moment  $\tau_y$  is absent.

Future research will be directed at the following issues:

- *Output feedback global tracking:* It is quite natural to ask whether we can avoid the use of full-state information. From a practical point of view, it is economically beneficial to avoid the direct measurements of velocities ( $u, v, r$ ). We believe that the passive nonlinear observer design of Fossen and Strand (1999) and our previous output-feedback global tracking results in Jiang (2000a) should be helpful.
- *Robust adaptive tracking:* Since the proposed results depend on the precise knowledge of system parameters, it is of interest to examine the robustness issue when these parameters are either not accurately



measurable or are time varying. As opposed to Lefeber (2000), our tracking designs generate Lyapunov functions. This feature can be exploited in conjunction with abundant Lyapunov design algorithms in past literature to attack the robust adaptive tracking problem for underactuated ships. Some results based on similar techniques are obtained in Godhavn, Fossen, and Berge (1998) for adaptive tracking of fully actuated ships.

- **Experimental work:** It is of great practical interest to validate the presented tracking methodology and compare it with previous tracking controllers in experiments.

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**Zhong-Ping Jiang** received B.Sc. degree in mathematics from University of Wuhan, Wuhan, China, in 1988, M.Sc. degree in statistics from the Université de Paris-sud, Paris, France, in 1989, and Ph.D. degree in automatic control and mathematics from the École des Mines de Paris, Paris, France, in 1993.

From 1993 to 1998, he held visiting researcher positions in several institutions including INRIA (Sophia-Antipolis), France, Department of Systems Engineering in the Australian National University, Canberra and Department of Electrical Engineering in the University of Sydney. In 1998, he also visited several US universities. In January 1999, he joined the Polytechnic University at Brooklyn as an Assistant Professor of Electrical Engineering. His main research interests lie in robust and adaptive nonlinear control, with special emphasis on applications to underactuated mechanical systems and secure communications. He has authored or coauthored over 90 refereed technical papers in these areas.

Currently, Dr. Jiang is an Associate Editor for *Systems & Control Letters*, and the *International Journal of Robust and Nonlinear Control*. He has been a member of the IEEE CSS Conference Editorial Board.

Dr. Jiang is the recipient of a prestigious Queen Elizabeth II Fellowship Award (1998) from the Australian Research Council and a CAREER Award (2000) from the U.S. National Science Foundation.