Adaptation Law

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$$\dot{x} = f(x) + g(x)\,\theta + h(x)\cdot u$$

Original System : $\longrightarrow$ a, δ : Controls

$$\dot{x} = v \sin\theta \qquad F_{y,f} = C_y \cdot \left(\delta - L_a \cdot \frac{\phi}{v}\right)$$
$$\dot{y} = v \cos\theta$$
$$\dot{v} = \cos\delta \cdot a - \frac{2}{m} F_{y,f} \cdot \sin\delta$$
$$\dot{\theta} = \phi \qquad F_{y,r} = C_y \cdot \frac{L_b \phi}{v}$$
$$\dot{\phi} = \frac{1}{J} L_a \left(m a \sin\delta + 2 F_{y,f} \cos\delta\right) - 2 L_b F_{y,r}$$

Tuning funct^n Method :

$$\dot{x} = f(x) + F(x)\,\theta + g(x)\,u$$

for θ known, $u = \alpha_c(x,\theta)$ s.t.

$$\frac{\partial V_c}{\partial x}\left(f(x) + F(x)\,\theta + g(x)\cdot \alpha_c(x,\theta)\right) \le -W(x,\theta) \quad \forall\ \theta$$

If θ is unkown

$$V = V_c(x,\hat{\theta}) + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

$$\dot{V} = \frac{\partial V_c}{\partial x}\left(f(x) + F(x)\theta + g(x)\cdot u\right) - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}$$

$$\vdots$$

$$\le -W(x,\hat{\theta})$$

Unkown : m, J, C_y

Form of the System :

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{v} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} v \sin\theta \\ v\cos\theta \\ a\cos\delta \\ \phi \\ 0 \end{pmatrix} + \left(\underbrace{\begin{matrix} 0 \\ 0 \\ -\frac{2}{m} F_{y,f} \sin\delta \\ 0 \\ \frac{1}{J} L_a \left(m a \sin\delta + 2 F_{y,f} \cos\delta\right) - 2 L_b \cdot C_y \cdot L_b \frac{\phi}{v} \end{matrix}}_{\downarrow} \right)_{5\times 1}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2\left(\delta - L_a \frac{\phi}{v}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_a \cdot a \cdot \sin\delta & 2 L_a \left(\delta - L_a \frac{\phi}{v}\right) \cos\delta & -2 L_b^2 \frac{\phi}{v} \end{pmatrix}_{5\times 4} \begin{pmatrix} C_y / m \\ m / J \\ C_y / J \\ C_y \end{pmatrix}_{4\times 1}$$

$$\dot{x} = F(x, u_1, u_2) + G(x, u_1, u_2) \cdot \theta$$

We had $V_c(x,\theta)$ — for the known Parameter Case

$$V = V_c(x,\hat{\theta}) + \frac{1}{2}\underset{4\times 1}{\tilde{\theta}^T} \underset{4\times 4}{\Gamma^{-1}} \underset{4\times 1}{\tilde{\theta}} \quad ; \quad \underset{4\times 1}{\dot{\hat{\theta}}} = \underset{4\times 4}{\Gamma}\, \underset{4\times 1}{\tau(x,\hat{\theta})}$$

cancel each other

$$\dot{V} = \underbrace{\frac{\partial V_c}{\partial x}\left(F(x,u_1,u_2) + G(x,u_1,u_2)\,\hat{\theta}\right)}_{\le -W(x,\hat{\theta})} + \underset{1\times 5}{\frac{\partial V_c}{\partial x}} \underset{5\times 4}{G(x,u_1,u_2)} \underset{4\times 1}{\tilde{\theta}} + \cancel{\frac{\partial V_c}{\partial \hat{\theta}}}^{\,0} \dot{\hat{\theta}} - \underbrace{\tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}}_{\tilde{\theta}^T \tau(x,\hat{\theta})}$$

Luckily $\frac{\partial V_c}{\partial \hat{\theta}} = 0 \longrightarrow$ No Terms depend upon $\hat{\theta}$.

$$\boxed{\left(\underset{1\times 5}{\frac{\partial V_c}{\partial x}}\ \underset{5\times 4}{G(x,u_1,u_2)}\right)^T = \tau(x,\hat{\theta})} \longrightarrow \text{Parameter Update Law} \quad \dot{\hat{\theta}} = \Gamma\,\tau(x,\hat{\theta})$$

$$V_c = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$$

$$e_1 = x - x^*$$
$$e_2 = y - y^*$$
$$z_1 = v - v_{des}$$
$$z_2 = \theta - \theta_{des}$$
$$z_3 = \phi - \phi_{des}$$

Compute $\left(\frac{\partial V_c}{\partial x}\right)_{1\times 5}$ Symbolically