Backstepping known parameters

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→ Idea: Aim $x_1 \to 0$

$\dot{x}_1 = f_1(x_1, x_2)$

Put $x_2 = x_{2d}$ so that

$V_1 = \frac{1}{2} x_1^2$

$\dot{V}_1 = x_1 f(x_1, x_{2d}) < 0 \longrightarrow$ States $\equiv x_1$

but, $\dot{x}_2 = f_2(x_1, x_2, x_3)$

$z_2 = x_2 - x_{2d}$

$\dot{z}_2 = f_2(x_1, x_2, x_3) - \dot{x}_{2d}$

choose $x_3 = x_{3d}$ so that

$V_2 = V_1 + \frac{1}{2} z_2^2$

$\dot{V}_2 = \dot{V}_1 + z_2 \left(f_2(x_1, x_2, x_3) - \dot{x}_{2d} \right)$

$\dot{x} = f_0(x) + g_0(x) \cdot z_1$

$\dot{z}_1 = f_1(x, z_1) + g_1(x, z_1) \cdot z_2$

$\dot{z}_2 = \ldots$

Also, $f_0, f_1, \ldots$ vanish at the origin

How to Control this System:

$\dot{x} = v \sin\theta$

$\dot{y} = v \cos\theta$

$\dot{\theta} = \omega \longrightarrow$ Control - 2

$\dot{v} = a \longrightarrow$ Control - 1

Our System: Aim $(e_1 \to 0,\ e_2 \to 0)$

$\dot{e}_1 = v \sin\theta - \dot{x}^*$

$\dot{e}_2 = v \cos\theta - \dot{y}^*$

$\dot{v} = \cos\delta \cdot a - \frac{2}{m} F_{y,f} \sin\delta = \alpha$

$\dot{\theta} = \phi$

$\dot{\phi} = \frac{1}{J} \cdot L_a \cdot (m a \sin\delta + 2F_{yf} \cos\delta) - 2L_b F_{y,r} = \frac{L_a}{J} \cdot \beta - 2L_b F_{y,r}$

$\cancel{\dot{\delta} = \omega}$

$\left.\begin{aligned} v_{des} &= \ldots \\ \theta_{des} &= \ldots \end{aligned}\right\} \Rightarrow \begin{aligned} v_{des} \cdot \sin\theta_{des} &= -k_1 e_1 + \dot{x}^* \\ v_{des} \cdot \cos\theta_{des} &= -k_2 e_2 + \dot{y}^* \end{aligned}$

$\left.\begin{aligned} z_1 &= v - v_{des} \\ z_2 &= \theta - \theta_{des} \end{aligned}\right\} \Rightarrow \begin{aligned} \dot{z}_1 &= \alpha - \dot{v}_{des} \\ \dot{z}_2 &= \phi - \dot{\theta}_{des} \end{aligned}$

$z_3 = \phi - \phi_{des} \Rightarrow \dot{z}_3 = \frac{L_a}{J} \beta - 2L_b F_{y,r} - \dot{\phi}_{des}$

$V_1 = e_1^2 + e_2^2 + z_1^2 + z_2^2 \longrightarrow \alpha, \phi$ are controls

$\dot{V}_1 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + z_1 \dot{z}_1 + z_2 \dot{z}_2$

$\dot{e}_1 = v \sin\theta - \dot{x}^*$

$= v_{des} \cdot \sin\underbrace{\theta}_{\theta_{des} + z_2} + \underbrace{z_1}_{v - v_{des}} \cdot \sin\theta - \dot{x}^*$

$= v_{des} \cdot \sin\theta_{des} \cdot \underbrace{\cos z_2}_{1 + \cos z_2 - 1} + v_{des} \cdot \cos\theta_{des} \cdot \sin z_2 + z_1 \cdot \sin\theta - \dot{x}^*$

$\dot{e}_2 = \ldots + z_1 \cos\theta - \dot{y}^*$

Will have to approximate here

$\sin z_1 \longrightarrow z_1$

$\sin z_2 \longrightarrow z_2$

$\cos z_1 \longrightarrow 1$

$\cos z_2 \longrightarrow 1$

$\dot{e}_1 = v_{des} \cdot \sin\theta_{des} + v_{des} \cos\theta_{des} \cdot z_2 + z_1 \cdot \sin\theta - \dot{x}^*$

$\cong -k_1 e_1 + v_{des} \cdot \cos\theta_{des} \cdot z_2 + z_1 \sin\theta$

$\dot{e}_2 = v \cos\theta - \dot{y}^*$

$= v_{des} \cos(\underbrace{\theta}_{z_2 + \theta_{des}}) + z_1 \cos\theta - \dot{y}^*$

$= v_{des} \cdot \cos\theta_{des} \cdot \cos z_2 - v_{des} \sin\theta_{des} \cdot \sin z_2 + z_1 \cos\theta - \dot{y}^*$

$\cong -k_2 e_2 + z_1 \cos\theta - z_2 \cdot v_{des} \sin\theta_{des}$

$\dot{V}_1 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + z_1 \dot{z}_1 + z_2 \dot{z}_2$

$= -k_1 e_1^2 - k_2 e_2^2 + z_1 \underbrace{(\dot{z}_1 + e_1 \sin\theta + e_2 \cos\theta)}_{\mathrm{I}} + z_2 \underbrace{(\dot{z}_2 + e_1 v_{des} \cos\theta_{des} - e_2 v_{des} \sin\theta_{des})}_{\mathrm{II}}$

$\dot{z}_1 = \alpha - \dot{v}_{des}$

$\Rightarrow \boxed{\alpha = -k_3 z_1 + \dot{v}_{des} - e_1 \sin\theta - e_2 \cos\theta}$

$\dot{z}_2 = \phi - \dot{\theta}_{des}$

$\Rightarrow \phi = \phi_{des} = -k_4 z_2 + \dot{\theta}_{des} - e_1 v_{des} \cos\theta_{des} + e_2 v_{des} \sin\theta_{des}$

Moving another Layer Deep: $z_3 = \phi - \phi_{des} \Rightarrow \dot{z}_3 = \frac{L_a}{J} \beta - 2L_b F_{y,r} - \dot{\phi}_{des}$

$V_{final} = e_1^2 + e_2^2 + z_1^2 + z_2^2 + z_3^2 \longrightarrow \alpha, \beta$ are Controls

$\dot{V}_{final} = \ldots + -k_3 z_1^2 + z_2(-k_4 z_2 + z_3) + z_3 \dot{z}_3$

$= -k_1 e_1^2 - k_2 e_2^2 - k_3 z_1^2 - k_4 z_2^2 + z_3(\dot{z}_3 + z_2)$

Choose β:

$\frac{L_a}{J} \beta - 2L_b F_{y,r} - \dot{\phi}_{des} = -k_5 z_3 - z_2$

$$\boxed{\beta = \frac{J}{L_a} \cdot \left(2L_b F_{y,r} + \dot{\phi}_{des} - k_5 z_3 - z_2 \right)}$$

Use (α, β) to get (a, δ)