Decentralized and Adaptive Control of Multiple Nonholonomic Robots for Sensing Coverage

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***Abstract*—This work deals with optimal coverage of a given area for sensing purposes using multiple nonholonomic mobile sensors. We assume a density function over the region to be covered which can be viewed as a probability density of the phenomena to be sensed. The density function is unknown but assumed to be linearly parameterized with unknown parameter weights. We consider a second order dynamic model for the mobile agents and derive decentralized adaptive control laws so as to achieve coverage of the region. We then consider the case where the dynamic model of the agents are not fully known, and then derive an adaptive controller to achieve the coverage objective.**

***Index Terms*—Coverage control; Multi-agent systems; Adaptive Control; Nonholonomic robots; Decentralized control**

I. INTRODUCTION

Cooperative control has become a popular research theme where multiple agents work together to achieve an objective autonomously. Some of the cooperative tasks which have been investigated include rendezvous where the agents try to converge to a common state, formation control where the agents try to maintain a given spatial formation and coverage control where the agents are deployed to cover a given region of interest. ([1], [2], [3], [4], [5], [6], [7], [8]) The applications include surveillance, patrolling, environmental monitoring and sensing etc. using multiple mobile robots or UAVs.

The agents are assumed to be capable of communicating with other agents. The communication topology is described in terms of a graph where the nodes correspond to the agents and two nodes are connected if the two corresponding agents can communicate with each other. In most cases, communication graphs correspond to proximity graphs meaning that two agents communicate if they are close to each other. This also motivates the use of decentralized or distributed control strategies for efficient solution of multiagent problems where the control laws of individual agents are determined by the information exchange with their neighbouring agents ([2], [3], [6]).

We consider the problem of optimally covering a given region using multiple agents to sense a phenomena/event of interest. The event of interest is described by a *density function* over the region. The density function can be thought of as giving the probability of the particular event to be sensed. For example, in case of mobile agents deployed to sense nuclear radiation over a region, the density function could be the intensity of radiation over the region. In this case, we would like the mobile agents which are deployed starting at some initial position to converge to some optimal configuration for sensing purpose. The density function is typically not completely known and therefore we will look at adaptive algorithms to learn the unknown density function in real-time. The agents are assumed to have nonholonomic (differential-drive vehicle) dynamics.

The coverage problem we investigate was formulated in [5], where n agents are deployed to cover a convex region $\mathcal{Q} \subset \mathbb{R}^q$. The problem was solved for agents with single integrator dynamics and known density function. In [9] and [10], the authors extend the algorithm of [5] using adaptive control for the case where the density function is not fully known. In [11], the authors consider robots with nonholonomic kinematic models and develop algorithms for coverage. They also consider time-varying density functions which can be parameterized using time-varying parameters and show uniform ultimate boundedness of trajectories when the time variation of the parameters are small. The current work is an extension of the adaptive coverage control algorithm for the case where agents dynamics are nonholonomic. We consider both the kinematics as well as the dynamics of the mobile robots. We first treat the case where the agent dynamics is fully known and then show that the algorithm can be extended to the case with unknown parameters in agent nonholonomic dynamics. We derive stabilizing distributed coverage control laws for the same using adaptive control. The adaptive control extension relies on a novel technique for computing time-derivative of area-integrals.

We start with the mathematical description of the problem in section II. In section III, we solve the problem for the case where agent dynamics are fully known. In section IV, we consider the case where the agent dynamics are not fully known. In section V, we provide simulation results to validate the algorithms proposed. Finally, we conclude the paper with section VI.

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II. THE PROBLEM DESCRIPTION

We consider n agents which are to be deployed over a bounded convex polytope $\mathcal{Q} \subset \mathbb{R}^q$ for sensing purposes. We assume that the phenomena to be sensed is described using a density function $\phi : \mathcal{Q} \to \mathbb{R}_+$ which can be thought of as the probability density of the phenomena to be sensed. From the agent's point of view, the regions where ϕ has higher values are more important than the regions with lower values of ϕ and in the optimal coverage configuration, the agents should cover the region in proportion to the value of ϕ. We formulate the problem as in [5]. The position of each agent is denoted by $p_i \in \mathbb{R}^q$ and the corresponding velocities are denoted by $\dot{p}_i$ for each $i \in \{1, 2, \ldots, n\}$.
The voronoi partitions generated by a set of points $\{p_1, p_2, \ldots p_n\}$ is defined as

$$\mathcal{V}_i = \{q : \ ||q - p_i|| \le ||q - p_j|| \quad \forall j \in \{1, 2 \ldots n\}, j \ne i\} \tag{1}$$

We can then formulate the cost function (see [5],[6]):

$$\mathcal{H}(p_1, \ldots, p_n) = \sum_{i=1}^{n} \int_{\mathcal{V}_i} ||q - p_i||^2 \phi(q) dq \tag{2}$$

We look at the term $||q - p_i||^2$ as the unreliability of sensing the phenomena at point q by agent i positioned at p_i. The optimal coverage configuration achieved by the agents is one that minimizes the above cost function (2). It can also be shown that the gradient of $\mathcal{H}$ with respect to the agent positions p_i (see [12],[5],[13]) is given by

$$\frac{\partial \mathcal{H}}{\partial p_i} = -M_{\mathcal{V}_i}(C_{\mathcal{V}_i} - p_i), \tag{3}$$

where

$$C_{\mathcal{V}_i} = \frac{L_{\mathcal{V}_i}}{M_{\mathcal{V}_i}}, \quad L_{\mathcal{V}_i} = \int_{\mathcal{V}_i} q\phi(q)dq, \quad M_{\mathcal{V}_i} = \int_{\mathcal{V}_i} \phi(q)dq \tag{4}$$

$M_{\mathcal{V}_i}$ is called the mass of $\mathcal{V}_i$, $L_{\mathcal{V}_i}$ is the first mass-moment of $\mathcal{V}_i$ and $C_{\mathcal{V}_i}$ is the centroid of $\mathcal{V}_i$. From equation (3), we see that the stationary points of the cost function (2) correspond to $p_i = C_{\mathcal{V}_i}$, i.e. location of each agent corresponds to the centroid of its voronoi partition. We call such a configuration the *centroidal voronoi configuration*. Thus we seek to obtain control laws for the agents to cover the region $\mathcal{Q}$ optimally by making the agents converge to a minimum of the cost function which is a centrodial voronoi configuration.

Density function

We assume that the density function $\phi(q)$ is not fully known to the agents since it is unreasonable that the agents have complete information before-hand of the phenomena to be sensed. We will assume that the true density function can be expressed as (see [9], [10])

$$\phi(q) = \mathcal{K}(q)^\top a, \tag{5}$$

where $\mathcal{K} : \mathbb{R}^q \to \mathbb{R}^m_+$ and $a \in \mathbb{R}^m_+$ is a constant vector which we will call the parameter vector. The functions $\mathcal{K}(q)$ are assumed to be known to all the agents whereas a is unknown. $\mathcal{K}(q)^\top = [\mathcal{K}_1(q), \mathcal{K}_2(q), \ldots, \mathcal{K}_m(q)]$ can be interpreted as a set of basis functions whose weighted combination gives the density function $\phi(q)$. The parameter vector a is assumed to be lower bounded and satisfies

$$a(i) \geq a_{\text{min}} \qquad i = 1, 2, \ldots, m, \tag{6}$$

where $a(i)$ is the $i-$th component of a.

The estimate of a computed by agent i is denoted by $\hat{a}_i$. Each agent computes $\hat{a}_i$ using an adaptation law which will be derived in the subsequent sections. Corresponding to $\hat{a}_i$, we also define the corresponding estimated quantities: $\hat{\phi}_i(q) = \mathcal{K}(q)^\top \hat{a}_i$ which is the agent i's estimate of the density function, $\hat{M}_{\mathcal{V}_i} = \int_{\mathcal{V}_i} \hat{\phi}_i(q)dq$ which is the agent i's estimate of the mass of $\mathcal{V}_i$, $\hat{L}_{\mathcal{V}_i} = \int_{\mathcal{V}_i} q\hat{\phi}_i(q)dq$ which is the agent i's estimate of $L_{\mathcal{V}_i}$ and $\hat{C}_{\mathcal{V}_i} = \frac{\hat{L}_{\mathcal{V}_i}}{\hat{M}_{\mathcal{V}_i}}$ which is agent i's estimate of the centroid of $\mathcal{V}_i$. We also assume that agent i can measure the value of density function $\phi(q)$ at its current location p_i.

Agent Dynamics

We consider the agents to be planar mobile robots and use a second order dynamical model for the agents. The general model for mobile robots obtained using the Euler-Lagrange equation is (see [14])

$$M_i(q_i)\ddot{q}_i + V_{mi}(q, \dot{q}_i)\dot{q}_i + F_i(\dot{q}_i) + G(q_i) = B(q_i)\tau_i - A_i^\top(q_i)\eta_i, \tag{7}$$

where the subscript i denotes the $i-$th robot, $q_i \in \mathbb{R}^q$ gives the generalized coordinates, $M_i(q_i) \in \mathbb{R}^{q\times q}$ is inertia matrix (symmetric, positive definite), $V_{mi}(q_i, \dot{q}_i) \in \mathbb{R}^{q\times q}$ is the centripetal and coriolis matrix, $F_i(\dot{q}_i) \in \mathbb{R}^q$ is the vector representing the surface friction, $G_i(q_i) \in \mathbb{R}^q$ is the vector representing the gravitational force, $B_i(q_i) \in \mathbb{R}^{q\times r}$ is the input transformation matrix, $\tau_i \in \mathbb{R}^r$ is the input vector, $A_i(q_i) \in \mathbb{R}^{m\times q}$ is the constraint matrix and $\eta_i \in \mathbb{R}^m$ is the vector of constraint forces. The nonholonlomic constraints are given by

$$A_i(q_i)\dot{q}_i = 0 \tag{8}$$

We also define a matrix $S_i(q_i) \in \mathbb{R}^{q\times(q-m)}$ whose columns span the null space of $A_i(q_i)$,

$$A_i(q_i)S_i(q_i) = 0 \tag{9}$$

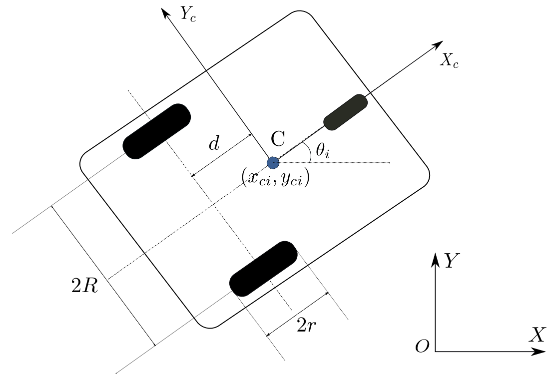


Fig. 1: Mobile robot

Now we consider the dynamic model for a nonholonomic planar mobile robot with two actuated wheels which also integrates the kinematic model as presented in [14]. We will assume that the robots are identical, with mass m, radius of wheels equal to r, length of the axle between the two wheels equal to $2R$ and the distance between the centre of mass and the axle equal to d (see figure 1). In this case $q_i = [x_{ci}\ y_{ci}\ \theta_i]^\top$ where (x_{ci}, y_{ci}) gives the coordinates of the centre of mass of the $i-$th robot and θ_i gives the orientation of the $i-$th robot. The nonholonomic constraints are given by (assuming no-slip condition)

$$\dot{y}_{ci}\cos(\theta_i) - \dot{x}_{ci}\sin(\theta_i) - d\dot{\theta}_i = 0 \tag{10}$$

which gives

$$A_i(q_i) = [-\sin(\theta_i)\ \cos(\theta_i)\ \ -d]$$

and $S_i(q_i)$ is given by

$$S_i(q_i) = \begin{pmatrix} \cos(\theta_i) & -d\sin(\theta_i) \\ \sin(\theta_i) & d\cos(\theta_i) \\ 0 & 1 \end{pmatrix} \tag{11}$$

We get the kinematic equations as

$$q_i = S_i(q_i)v_i \tag{12}$$

where $v_i := [u_i\ \omega_i]^\top$. Here u_i is the forward speed of the centre of mass and ω_i is the angular velocity. The other dynamical quantities are given by

$$M(q) = \begin{pmatrix} m & 0 & md\sin(\theta_i) \\ 0 & m & -md\cos(\theta_i) \\ md\sin(\theta_i) & -md\cos(\theta_i) & I \end{pmatrix},$$

$$V_i(q_i, \dot{q}_i) = \begin{pmatrix} md\dot{\theta}_i^2\cos(\theta_i) \\ md\dot{\theta}_i^2\sin(\theta_i) \\ 0 \end{pmatrix}, \quad G_i(q_i) = 0,$$

$$B_i(q_i) = \frac{1}{r}\begin{pmatrix} \cos(\theta_i) & \cos(\theta_i) \\ \sin(\theta_i) & \sin(\theta_i) \\ R & -R \end{pmatrix}, \quad \tau_i = \begin{pmatrix} \tau_{ir} \\ \tau_{il} \end{pmatrix},$$

where τ_{ir} and τ_{il} are the torque inputs to the right and left wheels of the $i-$th robot respectively. We can convert the above dynamical model given by (7) into the following form as presented in [14].

$$\dot{q}_i = S_i(q_i)v_i \tag{13}$$

$$\bar{M}_i(q_i)\dot{v}_i = \bar{B}_i\tau_i - \bar{V}_{mi}(q_i, \dot{q}_i)v_i - \bar{F}(v_i), \tag{14}$$

where $\bar{M}_i = S_i^\top M_i S_i$ is a symmetric and positive definite inertia matrix, $\bar{V}_{mi} = S_i^\top(M_i\dot{S}_i + V_{mi}S_i)$ is the centripetal and coriolis matrix, $\bar{F}_i(v_i)$ is the surface friction term, $\bar{B}_i = S_i^\top B_i$ is a constant non-singular matrix. We will use the model given by equations (13) and (14) in what follows. Also, we will assume that all the parameters and other quantities in the model equations are fully known for deriving the coverage control.

It can be shown that the matrix $\dot{\bar{M}} - 2\bar{V}_{mi}$ is skew symmetric (see [14]), i.e.

$$\frac{1}{2}\zeta^\top(\dot{\bar{M}} - 2\bar{V}_{mi})\zeta = 0 \quad \forall \zeta \in \mathbb{R}^2 \tag{15}$$

We also note that $q_i = [x_{ci}\ y_{ci}\ \theta_i]^\top = [p_i^\top\ \theta_i]^\top$ where $p_i = [x_{ci}\ y_{ci}]^\top$ is the location of the $i-$th robot. Correspondingly, we partition the $S_i(q_i)$ matrix as

$$\begin{bmatrix}\dot{p}_i \\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}S_i^1(q_i) \\ S_i^2(q_i)\end{bmatrix} v_i \tag{16}$$

III. Coverage Control with Known Dynamics

In this section, we derive the control laws for decentralized adaptive control of the agents assuming that their dynamics are fully known. We first define the following quantities (see [9], [10]):

$$\Lambda_i(t) = \int_0^t e^{-\alpha(t-\tau)}\mathcal{K}_i(\tau)\mathcal{K}_i(\tau)^\top d\tau \tag{17}$$

$$\lambda_i(t) = \int_0^t e^{-\alpha(t-\tau)}\mathcal{K}_i(\tau)\phi_i(\tau)d\tau, \tag{18}$$

where $\mathcal{K}_i(\tau) := \mathcal{K}(p_i(t))$, $\phi_i = \phi(p_i(t))$ which corresponds to agent i's measurement of the density function $\phi(q)$, α and γ are positive constants. $\Lambda_i(t)$ and $\lambda_i(t)$ can be obtained using the following filter equations with zero initial conditions.

$$\dot{\Lambda}_i = -\alpha\Lambda_i + \mathcal{K}_i\mathcal{K}_i^\top$$
$$\dot{\lambda}_i = -\alpha\lambda_i + \mathcal{K}_i\phi_i$$

We now state a theorem which provides the control and adaptation law. The derivation has similar features as [9], [10] but with modifications to account for the second order agent model.

Theorem 1. *Consider N agents with dynamics given by (13) and (14). With the control law for the $i-$th agent given by*

$$\tau_i = \bar{B}_i^{-1}\left\{-k_1\hat{M}_{\mathcal{V}_i}(S_i^1)^\top(p_i - \hat{C}_{\mathcal{V}_i}) - k_2 v_i + \bar{F}_i\right\} \tag{19}$$

and the adaptation law for $\hat{a}_i$ given by

$$\dot{\hat{a}}_i = \Gamma(b_i - I_{\beta_i}b_i), \tag{20}$$

with

$$b_i = -k_1\int_{\mathcal{V}_i}\mathcal{K}(q)(q-p_i)^\top dq\, S_i^1 v_i - \gamma(\Lambda_i\hat{a}_i - \lambda_i) \tag{21}$$

$$I_{\beta_i} = \begin{cases} 0 & \text{for } \hat{a}_i(j) > a_{min} \\ 0 & \text{for } \hat{a}_i(j) = a_{min}\ \&\ b_i(j) \geq 0 \\ 1 & \text{otherwise}\end{cases} \tag{22}$$

$\Gamma > 0$ a positive definite gain matrix, Λ_i given by equation (17) and λ_i given by equation (18), the following holds:
(a) $\lim_{t\to\infty}(p_i - \hat{C}_{\mathcal{V}_i}) = 0 \quad \forall i \in \{1, 2, \ldots, n\}$.
(b) $\lim_{t\to\infty} v_i = 0 \quad \forall i \in \{1, 2, \ldots, n\}$.
(c) $\lim_{t\to\infty}\mathcal{K}_i(\tau)\tilde{a}_i(t) = 0 \quad \forall\tau$ *s.t* $0 \leq \tau \leq t$ *and* $\forall i \in \{1, 2, \ldots, n\}$.

Remark 1. *The adaptation law consists of the term b_i in addition to a projection defined by (22) to make sure that the updated parameter value is always greater than the minimum value a_{min}.*

Remark 2. *Statement (c) of the theorem implies that*

$$\lim_{t\to\infty}\mathcal{K}_i(\tau)^\top\hat{a}_i(t) = \mathcal{K}_i(\tau)^\top a,$$

$\forall\tau$ s.t $0 \leq \tau \leq t$ and $\forall i \in \{1, 2, \ldots, n\}$ where a is the true parameter. i.e. the estimated density function $\hat{\phi}$ converges to the true value for all points which the robot has traversed. Since the agents converge only towards the estimated centroids, we will call such a configuration near optimal configuration. See also [9], [10]. In [10], it is shown that if $\lim_{t\to\infty}\Lambda_i(t)$ is positive definite, then $\lim_{t\to\infty}\tilde{a}_i(t) = 0$ and thus the estimated centroid $\hat{C}_{\mathcal{V}_i}$ converges to the true centroid $C_{\mathcal{V}_i}$ which corresponds to the optimal coverage configuration.

We now give the proof of theorem 1.

Proof. We consider the following Lyapunov function:

$$V(t) = k_1\mathcal{H} + \frac{1}{2}\sum_{i=1}^n \tilde{a}_i^\top\Gamma^{-1}\tilde{a}_i + \frac{1}{2}\sum_{i=1}^n v_i^\top\bar{M}_i v_i \tag{23}$$

Note that $V(t)$ depends on the positions of agents p_i but not on the orientations θ_i. This is because we are not trying to specifically control θ_i as the system is underactuated.
Taking the derivative of $V(t)$,

$$\begin{aligned}\dot{V} &= \sum_{i=1}^n\left\{k_1\frac{\partial\mathcal{H}}{\partial p_i}\dot{p}_i + \tilde{a}_i^\top\Gamma^{-1}\dot{\hat{a}}_i + v_i^\top\bar{M}_i\dot{v}_i + \frac{1}{2}v_i^\top\dot{\bar{M}}_i v_i\right\} \\ &= \sum_{i=1}^n\left\{-k_1\int_{\mathcal{V}_i}(q-p_i)^\top\phi(q)dq\, S_i^1 v_i + \tilde{a}_i^\top\Gamma^{-1}\dot{\hat{a}}_i\right. \\ &\qquad \left. + v_i^\top(\bar{B}_i\tau_i - \bar{V}_{mi}v_i - \bar{F}_i) + \frac{1}{2}v_i^\top\dot{\bar{M}}_i v_i\right\}\end{aligned}$$

Using the skew symmetric property (15), the control τ_i given by (19) and the adaptation law (20), we get

$$\dot{V} = \sum_{i=1}^n\left\{-\gamma\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau - \tilde{a}_i^\top I_{\beta_i}b_i - k_2\|v_i\|^2\right\}$$

It can be shown that all three terms in the above expression are non-positive which means that $\dot{V}$ is non-increasing (see [15], [10]). Since V is non-negative (bounded below by zero) and its time derivative $\dot{V}$ is non-positive, it follows that $\lim_{t\to\infty}V(t) < \infty$ (is finite). This implies that $\dot{V}$ is integrable and $\lim_{t\to\infty}\int_0^t\dot{V}dt < \infty$ (is finite). This together with the fact that v_i, $\dot{v}_i$ are bounded allows us to conclude that

$$\lim_{t\to\infty} v_i = 0 \tag{24}$$

using Barbalat's lemma. In a similar manner, we can show that

$$\lim_{t\to\infty}\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau = 0 \tag{25}$$

Equation (24) along with the fact that $\dot{v}_i$ is uniformly continuous (this is because each term of $\ddot{v}_i = \frac{d}{dt}\left[\bar{M}_i^{-1}\left(-k_1\hat{M}_{\mathcal{V}_i}(S_i^1)^\top(p_i - \hat{C}_{\mathcal{V}_i}) - k_2v_i - \bar{V}_{mi}(q_i, \dot{q}_i)v_i\right)\right]$ is bounded; also see Lemma 1 and 2 in the appendix of [10]) implies that $\lim_{t\to\infty}\dot{v}_i = 0$ using Barbalat's lemma. This along with the fact that the closed loop dynamics are given by $\bar{M}_i\dot{v}_i = -k_1\hat{M}_{\mathcal{V}_i}(S_i^1)^\top(p_i - \hat{C}_{\mathcal{V}_i}) - k_2v_i - \bar{V}_{mi}(q_i, \dot{q}_i)v_i$ allows us to conclude that

$$\lim_{t\to\infty}(p_i - \hat{C}_{\mathcal{V}_i}) = 0 \tag{26}$$

since S_i^1 is non-singular. Now consider equation (25). The integrand is always non-negative which implies that the integral can be zero only if the integrand converges to zero. This in turn implies that

$$\lim_{t\to\infty}\mathcal{K}_i(\tau)\tilde{a}_i(t) = 0 \quad \forall\tau \text{ s.t } 0 \leq \tau \leq t \tag{27}$$

for $\forall i \in \{1, 2, \ldots, n\}$. Thus equations (24), (26), (27) hold and the statements (a), (b) and (c) of the theorem is proved. ☐

Remark 3. *Regarding the orientation of the agents, we can only conclude from theorem 1 that $\lim_{t\to\infty}\dot{\theta}_i = 0$.*

IV. Coverage Control with Unknown Dynamics

In this section, we look at the case where the dynamics of the agents are not fully known. We will however assume that the kinematics are fully known. We will also make the following assumption:

Assumption 1.

$$\bar{M}_i(q_i)\dot{z} + \bar{V}_{mi}(q_i,\dot{q}_i)z + \bar{F}(v) := W_i(q,\dot{q},v,z,\dot{z})^\top \eta \quad \forall z \in \mathbb{R}^2,$$

where $\eta \in \mathbb{R}^p$ is a constant vector.

We cannot use the Lyapunov function from section III for deriving the control since it will lead to a detectability obstacle and the desired stability cannot be established. The control law in this case can be designed using the backstepping approach. We consider the kinematic equation (13) with v_i as the control and the Lyapunov function

$$V_1 = k_1\mathcal{H} + \sum_{i=0}^{n}\frac{1}{2}\tilde{a}_i^\top\Gamma_a^{-1}\tilde{a}_i. \tag{28}$$

Its derivative is given by

$$\dot{V}_1 = k_1\sum_{i=1}^{n}\left\{\hat{M}_{\mathcal{V}_i}(p_i - \hat{C}_{\mathcal{V}_i})^\top\dot{p}_i + \tilde{a}_i^\top\int_{\mathcal{V}_i}\mathcal{K}(q)(q-p_i)^\top dq\,\dot{p}_i\right\} + \sum_{i=1}^{n}\tilde{a}_i^\top\Gamma_a^{-1}\dot{\hat{a}}_i \tag{29}$$

Using the adaptation law for $\hat{a}_i$ given by equation (20) and the control law for v_i as

$$v_i = v_{id} := -(S^1(q))^\top(p_i - \hat{C}_{\mathcal{V}_i}) \tag{30}$$

we get

$$\dot{V}_1 = -k_1\hat{M}_{\mathcal{V}_i}\sum_{i=1}^{n}\|(S^1(q))^\top(p_i - \hat{C}_{\mathcal{V}_i})\|^2 - \sum_{i=1}^{n}\left\{\gamma\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau + \tilde{a}_i^\top I_{\beta_i}b_i\right\} \tag{31}$$

The fact that V_1 is lower bounded and $\dot{V}_1$ is non-positive imply that $\lim_{t\to\infty}V_1(t)$ exists. This in turn implies that $\dot{V}_1$ is integrable. This along with the fact that the first term of $\dot{V}_1$ is uniformly continuous and $S^1(q)$ is non-singular allows us to conclude that $\lim_{t\to\infty}(p_i - \hat{C}_{\mathcal{V}_i}) = 0$. Also, the statement (c) of theorem 1 holds in this case.

Now we consider the full dynamical model given by equations (13) and (14). Using assumption 1, we define

$$\bar{M}_i(q_i)\dot{v}_{id} + \bar{V}_{mi}(q_i,\dot{q}_i)v_{id} + \bar{F}(v_i) = W_i(q,\dot{q},v_i,v_{id},\dot{v}_{id})^\top\eta \tag{32}$$

where η is an unknown constant parameter. We will use $\hat{\eta}_i$ to represent an estimate of η_i. We now have the following theorem:

Theorem 2. *Consider N agents with dynamics given by (13) and (14). With the control law for the $i-$th agent given by*

$$\tau_i = \bar{B}_i^{-1}\left\{W_i^\top\hat{\eta}_i - k_1\hat{M}_{\mathcal{V}_i}(S^1(q))^\top(p_i - \hat{C}_{\mathcal{V}_i}) - k_2\tilde{v}_i\right\}, \tag{33}$$

where $\tilde{v}_i = v_i - v_{id}$ with v_{id} given by (30), the adaptation law for $\hat{a}_i$ given by

$$\dot{\hat{a}}_i = \Gamma_a(b_i - I_{\beta_i}b_i), \tag{34}$$

the adaptation law for $\hat{\eta}_i$ given by

$$\dot{\hat{\eta}}_i = \Gamma_\eta W_i\tilde{v}_i, \tag{35}$$

b_i and I_{β_i} given by equations (21) and (22), Γ_a and Γ_η are positive definite gain matrices, Λ_i and λ_i given by equations (17) and (18), the following holds:
(a) $\lim_{t\to\infty}(p_i - \hat{C}_{\mathcal{V}_i}) = 0 \quad \forall i \in \{1,2,\ldots,n\}$.
(b) $\lim_{t\to\infty}v_i = 0 \quad \forall i \in \{1,2,\ldots,n\}$.
(c) $\lim_{t\to\infty}\mathcal{K}_i(\tau)\tilde{a}_i(t) = 0 \quad \forall\tau$ s.t $0 \le \tau \le t$ and $\forall i \in \{1,2,\ldots,n\}$.

Proof. Consider the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{n}\tilde{v}_i^\top\bar{M}\tilde{v}_i + \frac{1}{2}\sum_{i=1}^{n}\tilde{\eta}_i^\top\Gamma_\eta^{-1}\tilde{\eta}_i, \tag{36}$$

where $\tilde{\eta}_i = \hat{\eta}_i - \eta_i$. Computing the derivative of V_2, we have

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \sum_{i=1}^{n}\left[\tilde{v}_i^\top\bar{M}\dot{\tilde{v}}_i + \frac{1}{2}\tilde{v}_i^\top\dot{\bar{M}}\tilde{v}_i\right] + \frac{1}{2}\sum_{i=1}^{n}\tilde{\eta}_i^\top\Gamma_\eta^{-1}\dot{\hat{\eta}}_i\\ &= k_1\sum_{i=1}^{n}\Big\{\hat{M}_{\mathcal{V}_i}(p_i - \hat{C}_{\mathcal{V}_i})^\top S^1(q)v_i + \frac{1}{2}\tilde{\eta}_i^\top\Gamma_\eta^{-1}\dot{\hat{\eta}}_i\\ &\qquad - \gamma\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau - \tilde{a}_i^\top I_{\beta_i}b_i\\ &\qquad + \tilde{v}_i^\top\left[\bar{B}_i\tau_i - \bar{V}_{mi}v_i - F - \bar{M}\dot{v}_{id}\right] + \frac{1}{2}\tilde{v}_i^\top\dot{\bar{M}}\tilde{v}_i\Big\}\end{aligned}$$

Using the skew-symmetry property (15), control law (33) and the adaptation law (35) for $\hat{\eta}_i$, we get

$$\begin{aligned}\dot{V}_2 &= k_1\sum_{i=1}^{n}\hat{M}_{\mathcal{V}_i}v_{id}^\top[S^1(q)]^\top(p_i - \hat{C}_{\mathcal{V}_i}) - k_2\sum_{i=1}^{n}\tilde{v}_i^\top\tilde{v}_i\\ &\quad - \sum_{i=1}^{n}\left\{\gamma\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau + \tilde{a}_i^\top I_{\beta_i}b_i\right\}\\ &= -k_1\hat{M}_{\mathcal{V}_i}\sum_{i=1}^{n}\|(S^1(q))^\top(p_i - \hat{C}_{\mathcal{V}_i})\|^2 - k_2\sum_{i=1}^{n}\|\tilde{v}_i\|^2\\ &\quad - \sum_{i=1}^{n}\left\{\gamma\int_0^t e^{-\alpha(t-\tau)}(\mathcal{K}_i^\top(\tau)\tilde{a}_i(t))^2 d\tau + \tilde{a}_i^\top I_{\beta_i}b_i\right\}\end{aligned}$$

V_2 is non-negative and $\dot{V}_2$ is non-increasing. Using a similar analysis to that in the proof of theorem 1, if follows that $\dot{V}_2$ is integrable and using Barbalat's Lemma, for each $i \in \{1,2,\ldots,n\}$,

$$\lim_{t\to\infty}\|(S^1(q))^\top(p_i - \hat{C}_{\mathcal{V}_i})\| = 0 \tag{37}$$

$$\lim_{t\to\infty}\|\tilde{v}_i\| = \lim_{t\to\infty}\|v_i - v_{id}\| = 0 \tag{38}$$

$$\lim_{t\to\infty}\mathcal{K}_i(\tau)\tilde{a}_i(t) = 0, \quad \forall\tau \text{ s.t } 0 \le \tau \le t \tag{39}$$

Statement (a) of the theorem follows from equation (37) and the fact that $S^1(q)$ is non-singular. Equations (37) and (38) imply statement (b) of the theorem. Statement (c) of the theorem follows from equation (39). □

Remark 4. *Computing the adaptive control law (33) requires computing $\dot{v}_{id}$ which in turn requires the knowledge of $\dot{\hat{C}}_{\mathcal{V}_i}$. Computation of $\dot{\hat{C}}_{\mathcal{V}_i}$ requires computing the time derivative of spatial integrals whose area of integration depends on time and as such a non-trivial task. We look at the computation of $\dot{\hat{C}}_{\mathcal{V}_i}$ for planar robots below.*

Computing $\dot{\hat{C}}_{\mathcal{V}_i}$

We have

$$\hat{C}_{\mathcal{V}_i} = \frac{\hat{L}_{\mathcal{V}_i}}{\hat{M}_{\mathcal{V}_i}}, \tag{40}$$

where $\hat{L}_{\mathcal{V}_i} = \int_{\mathcal{V}_i} q\hat{\phi}(q)dq$ and $\hat{M}_{\mathcal{V}_i} = \int_{\mathcal{V}_i}\hat{\phi}(q)dq$. Then,

$$\dot{\hat{C}}_{\mathcal{V}_i} = \frac{\hat{M}_{\mathcal{V}_i}\dot{\hat{L}}_{\mathcal{V}_i} - \hat{L}_{\mathcal{V}_i}\dot{\hat{M}}_{\mathcal{V}_i}}{\hat{M}_{\mathcal{V}_i}^2} \tag{41}$$

To compute $\dot{\hat{C}}_{\mathcal{V}_i}$, we thus need to compute the time derivatives $\dot{\hat{L}}_{\mathcal{V}_i}$ and $\dot{\hat{M}}_{\mathcal{V}_i}$. Both $\hat{L}_{\mathcal{V}_i}$ and $\hat{M}_{\mathcal{V}_i}$ are of the form

$$I_i := \int_{\mathcal{V}_i} f(q,t)dq \quad \text{with } f(q,t) = \begin{cases}\phi(q) & \text{for } \hat{M}_{\mathcal{V}_i}\\ q\,\phi(q) & \text{for } \hat{L}_{\mathcal{V}_i}\end{cases},$$

where the region of integration $\mathcal{V}_i$ is a function of agent positions p_i's. The derivatives can be computed as

$$\dot{\hat{L}}_{\mathcal{V}_i} = \frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial t} + \sum_{j=1}^{n} \frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial p_j}\dot{p}_j; \; \dot{\hat{M}}_{\mathcal{V}_i} = \frac{\partial \hat{M}_{\mathcal{V}_i}}{\partial t} + \sum_{j=1}^{n} \frac{\partial \hat{M}_{\mathcal{V}_i}}{\partial p_j}\dot{p}_j, \quad (42)$$

where

$$\frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial t} = \int_{\mathcal{V}_i} \mathcal{K}(q)^\top \dot{\hat{a}}_i dq; \quad \frac{\partial \hat{M}_{\mathcal{V}_i}}{\partial t} = \int_{\mathcal{V}_i} q\mathcal{K}(q)^\top \dot{\hat{a}}_i dq \quad (43)$$

It remains to compute $\frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial p_j}$ and $\frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial p_j}$. Let $\mathcal{N}_i$ denote the set of voronoi neighbours of agent i. For $j \notin \mathcal{N}_i \cup \{i\}$,

$$\frac{\partial \hat{L}_{\mathcal{V}_i}}{\partial p_j} = \frac{\partial \hat{M}_{\mathcal{V}_i}}{\partial p_j} = 0 \quad (44)$$

since the voronoi region $\mathcal{V}_i$ does not depend on p_j. For $j \in \mathcal{N}_i \cup \{i\}$,

$$\frac{\partial I_i}{\partial p_j} = \begin{bmatrix} \frac{\partial I_i}{\partial p_{j_x}} & \frac{\partial I_i}{\partial p_{j_y}} \end{bmatrix}$$

where (see [16])

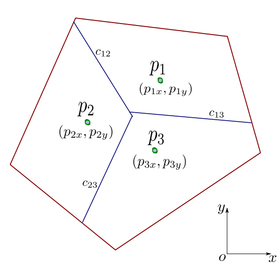
$$\frac{\partial I_i}{\partial p_{j_x}} = \int_{\partial \mathcal{V}_i} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{j_x}} dq$$
$$\frac{\partial I_i}{\partial p_{j_y}} = \int_{\partial \mathcal{V}_i} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{j_y}} dq$$

where $\partial\mathcal{V}_i$ is the boundary of $\mathcal{V}_i$ and n_q is the normal to $\partial\mathcal{V}_i$ at the point q. This can be further simplified as

$$\frac{\partial I_i}{\partial p_{j_x}} = \begin{cases} \displaystyle\int_{c_{ij}} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{j_x}} dq & j \neq i, \\ \displaystyle\sum_{k \in \mathcal{N}_i} \int_{c_{ik}} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{i_x}} dq & j = i, \end{cases} \quad (45)$$

$$\frac{\partial I_i}{\partial p_{j_y}} = \begin{cases} \displaystyle\int_{c_{ij}} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{j_y}} dq & j \neq i, \\ \displaystyle\sum_{k \in \mathcal{N}_i} \int_{c_{ik}} f(q,t)\, n_q^\top \frac{\partial q}{\partial p_{i_y}} dq & j = i, \end{cases} \quad (46)$$

where c_{ij} is the segment of $\partial\mathcal{V}_i$ that is shared between agents i and j. The terms $n_q^\top \frac{\partial q}{\partial p_{j_x}}$ and $n_q^\top \frac{\partial q}{\partial p_{j_y}}$ can be computed to be (see Lemma 2.2 in [17])

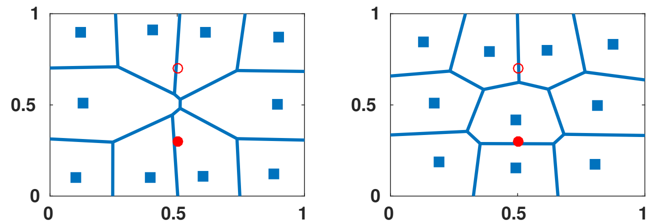


$$n_q^\top \frac{\partial q}{\partial p_{j_x}} = \frac{\mathrm{c}(\theta_q)}{2} + \mathrm{s}(\theta_q)\frac{d_{qc}}{d_{ij}}, \; n_q^\top \frac{\partial q}{\partial p_{i_x}} = \frac{\mathrm{c}(\theta_q)}{2} - \mathrm{s}(\theta_q)\frac{d_{qc}}{d_{ik}},$$
$$n_q^\top \frac{\partial q}{\partial p_{j_y}} = \frac{\mathrm{s}(\theta_q)}{2} + \mathrm{c}(\theta_q)\frac{d_{qc}}{d_{ij}}, \; n_q^\top \frac{\partial q}{\partial p_{i_y}} = \frac{\mathrm{s}(\theta_q)}{2} - \mathrm{c}(\theta_q)\frac{d_{qc}}{d_{ik}},$$

where θ_q is the angle (in radians) between the x-axis and n_q, $\mathrm{s}(\theta_q) = \sin(\theta_q)$, $\mathrm{c}(\theta_q) = \cos(\theta_q)$, $d_{ij} = \|p_i - p_j\|$ is the distance between agents i and j, d_{qc} is the distance between point q of $\partial\mathcal{V}_i$ and the point $c = \frac{(p_i+p_j)}{2}$.

V. Simulations

Now we look at some simulation results obtained using the algorithms presented in this paper. We consider the unit square $\mathcal{Q}$ with 10 agents. The density function $\phi(.)$ is a combination of two gaussians and a constant term: $\phi(q) = \mathcal{K}(q)^\top a$ where $\mathcal{K}(q) = [\mathcal{K}_1(q), \mathcal{K}_2(q), \mathcal{K}_3(q)]^\top$ and $\mathcal{K}_i(q) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{(q-\mu_i)^\top(q-\mu_i)}{2\sigma^2}\right)$ for $i = 1, 2$, $\mathcal{K}_3(q) = 1$. The values of various constants used are given in table I. One of the gaussians is weighted more than the other and forms the major component of the density function $\phi(.)$.



(a) Initial Positions **(b)** Final Positions

Fig. 2: Simulation results for the known dynamics case: The blue squares indicate the robot positions. The filled red circle indicates the mean of the Gaussian with the higher weight and the unfilled red circle indicates the mean of the other Gaussian component.

A. Coverage with known dynamics

Simulations for the case with known dynamics have also been presented in [15]. The simulation results are shown in figure 3. The averaged position error ($p_i - \hat{C}_{\mathcal{V}_i}$) and velocity ($v_i$) are plotted in figures 3a and 3b respectively. The initial and final positions of the robots and the corresponding voronoi regions are shown in figures 2a and 2b respectively. The averaged integrated parameter error which is given by $\int_0^t \mathrm{e}^{-\alpha(t-\tau)} \left(\mathcal{K}_i^\top \tilde{a}_i(t)\right)^2 d\tau$ is plotted in figure 3c. This quantity is a weighted integral of the error in the density function estimate at time t along the path the agent has traversed till time t, and corresponds to assertion (c) in theorems 1 and 2. It can be seen that the quantities approach zero with time. The orientations of the agents are plotted in figure 3d. The orientations approach some constant value.

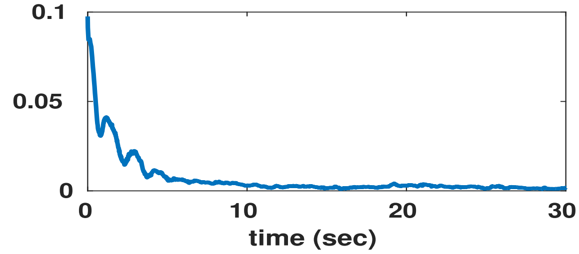
TABLE I: Simulation parameters: **I** represents the identity matrix

Parameter	Value	Parameter	Value
n	10	k_1	5
m	3	k_2	10
μ_1	[0.5, 0.3]	Γ_η	10**I**
μ_2	[0.7, 0.5]	m	10 kg
σ^2	0.2	d	0.25 m
a	$[100, 0.3, 0.3]^\top$	r	0.05 m
Γ_a	**I**	R	0.5 m
γ	300	I	5 kg.m^2
α	1	k_f	0.3 N.s.m^{-1}

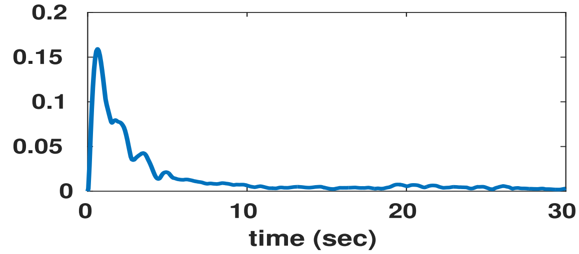
B. Coverage with unknown dynamics

We assume that the friction term $\bar{F}(v)$ is given by $\bar{F}(v) = k_f \mathrm{sgn}(v)$ where k_f is unknown. For simplicity, the other parameters in the agent dynamics are assumed to be known. The true value of k_f is given in table I. The plots for average position error and average velocity is given in figures 5a and 5b respectively. The initial and final positions of the robots with the corresponding voronoi regions are shown in figures 4a and 4b respectively. The average integrated parameter error is shown in figure 5c and the orientations of the agents are shown in figure 5d.

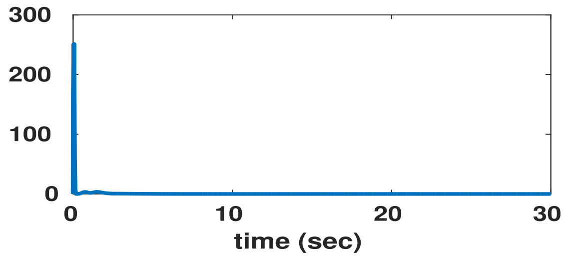
The robots take longer to converge in this case as seen from the plots. The plots of the lyapunov functions with time for both the known as well as unknown dynamics cases are shown in figure 6. It can be seen that lyapunov function decreases at a slower rate in the



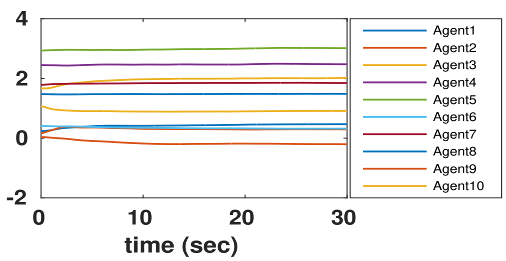
(a) Average Position Error



(b) Average Velocity



(c) Avg Integrated Parameter Error: Final value $(t = 30s) \sim 10^{-4}$



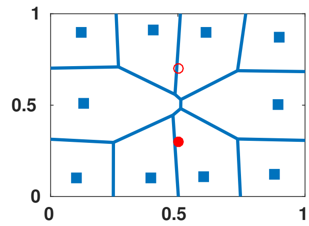
(d) Orientations

Fig. 3: Simulation results for the known dynamics case

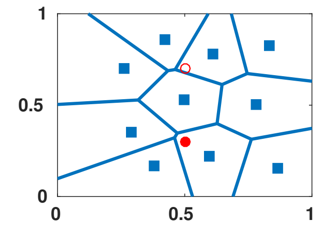
unknown case and thus slower convergence. The final configurations of the robots in the two cases do not match. It should be noted that the centroidal voronoi configurations are not unique [5], and the robots may settle in different configurations with different initial conditions and control laws.

VI. Conclusions

We looked at coverage algorithms for nonholonomic agents deployed on a planar region. Algorithms were developed for the case where the agent dynamics were fully known as well as the case where the dynamics are not fully known. We also presented simulations to validate the algorithms. Further work will include testing the algorithms in hardware. Analysis of the effect of limited communication range between agents have not been considered in this work. This would also form part of the future work.

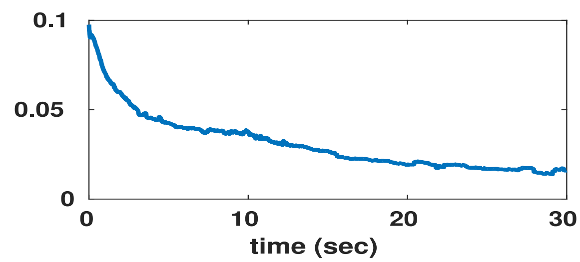


(a) Initial Positions

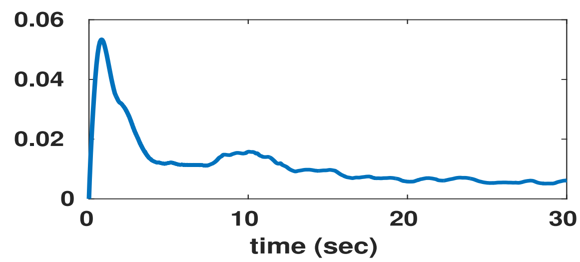


(b) Final Positions

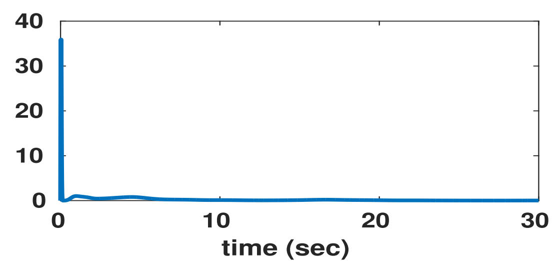
Fig. 4: Simulation results for the unknown dynamics case: The blue squares indicate the robot positions. The filled red circle indicates the mean of the Gaussian with the higher weight and the unfilled red circle indicates the mean of the other Gaussian component.



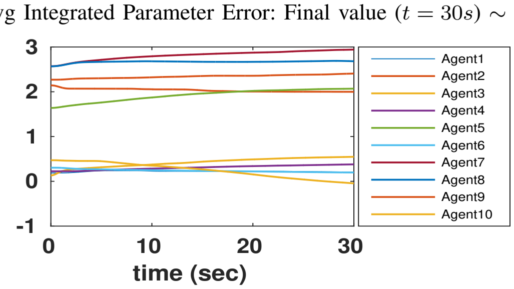
(a) Average Position Error



(b) Average Velocity



(c) Avg Integrated Parameter Error: Final value $(t = 30s) \sim 10^{-2}$



(d) Orientations

Fig. 5: Simulation results for the unknown dynamics case

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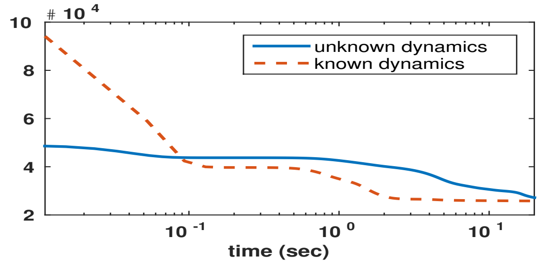

Fig. 6: Lyapunov functions with time: x-axis is shown in log scale to make the plots clearer.

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