EE 324 Controls System Lab

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Problem Sheet 1

1 Question 1

The transfer functions of all cases are given below

1. **Cascade system**

$$\frac{C(s)}{R(s)} = G_1 \cdot G_2$$

2. **Parallel system**

$$\frac{C(s)}{R(s)} = G_1 + G_2$$

3. **Feedback system**

$$\frac{C(s)}{R(s)} = \frac{G_1}{1 + G_1 \cdot G_2}$$

Code for MATLAB is provided below.

```
%% Q1 - Transfer Functions for Different Connections

G1 = tf([10],[1 2 10]);
G2 = tf([5],[1 5]);

cascade_tf = G1*G2;
parallel_tf = (G1 + G2);
feedback_tf = G1 / (1 + G1*G2);

step(G1,'r');
```

Output

```
cascade_tf =

            50
  ----------------------
  s^3 + 7 s^2 + 20 s + 50
```

```
parallel_tf =

     5 s^2 + 20 s + 100
  ----------------------
  s^3 + 7 s^2 + 20 s + 50


feedback_tf =

          10 s^3 + 70 s^2 + 200 s + 500
  ---------------------------------------------
  s^5 + 9 s^4 + 44 s^3 + 210 s^2 + 400 s + 1000
```

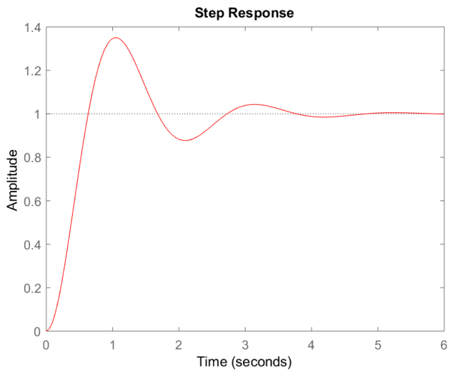

Figure 1: Unit step response to G_1

2 Question 2

The zeroes of a transfer function are the roots of its numerator while the poles are the roots of its denominator.
Code for MATLAB:

```
%% Q2 Poles and Zeroes of Above Transfer Functions
% We use 2 commands of matlab to find the poles and zeros
% tfdata - returns array polynomials for the numerator and denominator
% roots - given an array of polynomial coefficients it returns the roots o

[num,den] = tfdata(cascade_tf,'v'); % Gets the num and deno of a tf
zeros_cascade = roots(num);
poles_cascade = roots(den);

[num,den] = tfdata(parallel_tf,'v');
zeros_parallel = roots(num);
poles_parallel = roots(den);

[num,den] = tfdata(feedback_tf,'v');
zeros_feedback = roots(num);
poles_feedback = roots(den);
```

Output

```
zeros_cascade =
  empty

poles_cascade =
  -5.0000 + 0.0000i
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

zeros_parallel =
  -2.0000 + 4.0000i
  -2.0000 - 4.0000i

poles_parallel =
  -5.0000 + 0.0000i
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

zeros_feedback =
  -5.0000 + 0.0000i
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

poles_feedback =
  -6.3348 + 0.0000i
  -0.3326 + 3.9592i
  -0.3326 - 3.9592i
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
```

3 Question 3

The mesh equations in matrix form are,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1-s & -2 & \frac{1}{1+s}+6+s \\ 2(1+s)+\frac{1}{1+s} & -\frac{1}{1+s} & -1-s \\ -\frac{1}{1+s} & \frac{1}{1+s}+3+s & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Code for MATLAB

```
%% Mesh Analysis of a Circuit
% Here we write down the mesh equations in matrix form and
% hardcode them in Matlab
% We will be using the symbolic variable s to write down
% the Laplace Transform of the KCL KVL equations
% Note : We will be assuming zero initial conditions
% while writing the equations

syms s % These are the circuit parameters defined as symbolic variables

Z_s = [ -1-s -2 (1/(1+s))+6+s;
        2*(1+s)+(1/(1+s)) -1/(1+s) -1-s ;
        -1/(1+s) (1/(1+s))+3+s -2 ];
I = inv(Z_s) * [1 ; 0 ; 0];
simplify(I)
```

Output

```
ans =

((s+1)*(s^3+5*s^2+8*s+6))/(s^5+17*s^4+74*s^3+147*s^2+144*s+57)
((s+1)*(4*s^2+9*s+7))/(s^5+17*s^4+74*s^3+147*s^2+144*s+57)
((s+1)*(2*s^3+10*s^2+17*s+11))/(s^5+17*s^4+74*s^3+147*s^2+144*s+57)
```

So we can conclude that

$$\frac{I_1(s)}{V_1(s)} = \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5}$$
$$\frac{I_2(s)}{V_2(s)} = \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5}$$
$$\frac{I_3(s)}{V_3(s)} = \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5}$$