In[89]:=

```
Clear["Global`*"];
ffCartPendulumGeneral[n_, τ_, τ1_, A_] :=
 Module[{x, xdot, f, θ, θdot, λ1, λ2, λ3, λ4, Δt, bcs, eqns, sv, froot, xff,
   xdotff, xff0, xdotff0, θff0, θdotff0, uff0, θff, θdotff, uff}, Δt = τ/n;
  f[{x_, xdot_, θ_, θdot_, λ1_, λ2_, λ3_, λ4_}] :=
   {xdot, 1/(1 - A Cos[θ]^2) (A θdot^2 Sin[θ] + 1/(1 - A Cos[θ]^2) (λ4 Cos[θ] - λ2) + A Cos[θ] Sin[θ]), θdot,
    1/(1 - A Cos[θ]^2) (-1/(1 - A Cos[θ]^2) (-λ2 Cos[θ] + λ4 Cos[θ]^2) - Sin[θ] - A θdot^2 Cos[θ] Sin[θ]),
    0, -λ1, 2/(A Cos[2 θ] + A - 2)^3 (Cos[θ] (4 Sin[θ] (A λ4^2 Cos[2 θ] + 4 A λ2^2 + (A + 2) λ4^2) -
          (A Cos[2 θ] - 3 A + 2) (A Cos[2 θ] + A - 2) (A θdot^2 λ2 - λ4)) + A ((A - 2) Cos[2 θ] + A)
        (A Cos[2 θ] + A - 2) (λ2 - θdot^2 λ4) - 4 λ2 λ4 Sin[θ] (3 A Cos[2 θ] + 3 A + 2)),
    4/(A Cos[2 θ] + A - 2) (A θdot Sin[θ] (λ2 - λ4 Cos[θ])) - λ3};
  bcs = {x_0 == xdot_0 == x_n == xdot_n == θ_0 == θdot_0 == θdot_n == 0, θ_n == π};
  eqns = Flatten[Join[bcs, Table[
      Thread[{x_i, xdot_i, θ_i, θdot_i, λ1_i, λ2_i, λ3_i, λ4_i} == 1/2 Δt (f[{x_(i-1), xdot_(i-1), θ_(i-1), θdot_(i-1),
              λ1_(i-1), λ2_(i-1), λ3_(i-1), λ4_(i-1)}] + f[{x_i, xdot_i, θ_i, θdot_i, λ1_i, λ2_i, λ3_i, λ4_i}]) +
         {x_(i-1), xdot_(i-1), θ_(i-1), θdot_(i-1), λ1_(i-1), λ2_(i-1), λ3_(i-1), λ4_(i-1)}], {i, 1, n}]]];
  sv = Flatten[Table[{{x_i, 0}, {xdot_i, 0}, {θ_i, 0}, {θdot_i, 0}, {λ1_i, 0},
      {λ2_i, 0}, {λ3_i, 0}, {λ4_i, 0}}, {i, 0, n}], 1];
  froot = FindRoot[eqns, sv];
  xff0 = ListInterpolation[Table[x_i, {i, 0, n}] /. froot, {0, τ}, InterpolationOrder → 1];
  xdotff0 =
   ListInterpolation[Table[xdot_i, {i, 0, n}] /. froot, {0, τ}, InterpolationOrder → 1];
  θff0 = ListInterpolation[Table[θ_i, {i, 0, n}] /. froot, {0, τ}, InterpolationOrder → 1];
  θdotff0 =
   ListInterpolation[Table[θdot_i, {i, 0, n}] /. froot, {0, τ}, InterpolationOrder → 1];
  uff0 = ListInterpolation[Table[1/(1 - A Cos[θ_i]^2) (λ4_i Cos[θ_i] - λ2_i), {i, 0, n}] /. froot,
    {0, τ}, InterpolationOrder → 1];

  xff[t_] := Piecewise[{{xff0[t], 0 ≤ t ≤ τ}}, 0];
  xdotff[t_] := Piecewise[{{xdotff0[t], 0 ≤ t ≤ τ}}, 0];
  θff[t_] := Piecewise[{{θff0[t], 0 ≤ t ≤ τ}}, π];
  θdotff[t_] := Piecewise[{{θdotff0[t], 0 ≤ t ≤ τ}}, 0];
  uff[t_] := Piecewise[{{uff0[t], 0 ≤ t ≤ τ}}, 0];

  {xff, xdotff, θff, θdotff, uff}]
```

Test the approximate solution on the open-loop dynamics

In[91]:=
```
TestSwingUpGeneral[τ_, τ1_, uff0_, A_] :=
 Module[{eq, init, x, xdot, θ, θdot, xs, xdots, θs, θdots, t},
  eq = {x'[t] == xdot[t], xdot'[t] == 1/(1 - A Cos[θ[t]]^2)
      (uff0[t] + A θdot[t]^2 Sin[θ[t]] + A Cos[θ[t]] Sin[θ[t]]), θ'[t] == θdot[t],
    θdot'[t] == 1/(1 - A Cos[θ[t]]^2) (-Sin[θ[t]] - Cos[θ[t]] (uff0[t] + A θdot[t]^2 Sin[θ[t]]))};
  init = {x[0] == xdot[0] == θ[0] == θdot[0] == 0};
  {xs, xdots, θs, θdots} = NDSolveValue[{eq, init},
    {x, xdot, θ, θdot}, {t, 0, τ1}, Method → {"DiscontinuityProcessing" → None}];
  {xs, θs}]
```

Show that linear feedback can stabilize against “bad” numerics

In[92]:=
```
TestSwingUpGeneralFB[τ_, τ1_, xff0_, xdotff0_, θff0_, θdotff0_, uff0_, A_] :=
 Module[{eq, init, θ, θdot, θff, θdotff, x, xdot, xff,
   xdotff, uff, t, κ1, κ2, κ3, κ4, ufb, u, θs, θdots, xs, xdots, us},
  κ1 = κ2 = 3;  (* lqr for q=r for balancing pendulum *)
  κ3 = -0.1; κ4 = -0.65;
  xff[t_] := Piecewise[{{xff0[t], 0 ≤ t ≤ τ}}, 0];
  xdotff[t_] := Piecewise[{{xdotff0[t], 0 ≤ t ≤ τ}}, 0];
  θff[t_] := Piecewise[{{θff0[t], 0 ≤ t ≤ τ}}, π];
  θdotff[t_] := Piecewise[{{θdotff0[t], 0 ≤ t ≤ τ}}, 0];
  uff[t_] := Piecewise[{{uff0[t], 0 ≤ t ≤ τ}}, 0];
  ufb[t_] := Piecewise[{{0, 0 ≤ t ≤ τ}}, κ1 (θff[t] - θ[t]) +
     κ2 (θdotff[t] - θdot[t]) + κ3 (xff[t] - x[t]) + κ4 (xdotff[t] - xdot[t])];
  u[t_] := uff[t] + ufb[t];
  eq = {x'[t] == xdot[t], xdot'[t] ==
     1/(1 - A Cos[θ[t]]^2) (u[t] + A θdot[t]^2 Sin[θ[t]] + A Cos[θ[t]] Sin[θ[t]]), θ'[t] == θdot[t],
    θdot'[t] == 1/(1 - A Cos[θ[t]]^2) (-Sin[θ[t]] - Cos[θ[t]] (u[t] + A θdot[t]^2 Sin[θ[t]]))};
  init = {x[0] == xdot[0] == θ[0] == θdot[0] == 0};
  {xs, xdots, θs, θdots} = NDSolveValue[{eq, init},
    {x, xdot, θ, θdot}, {t, 0, τ1}, Method → {"DiscontinuityProcessing" → None}];
  us[t_] := uff[t] + κ1 (θff[t] - θs[t]) +
    κ2 (θdotff[t] - θdots[t]) + κ3 (xff[t] - xs[t]) + κ4 (xdotff[t] - xdots[t]);
  {xs, θs, us}]
```

Add Feedback Numerically

In[93]:=
```
CalculateGains[xff0_, xdotff0_, θff0_, θdotff0_, uff0_, A_] :=
 Module[{x, L, RHS, xdot, θ, θdot, u, K, S, soltn, i, j, s11, s12, s13, s14, s22,
```

```
 s23, s24, s33, s34, s44, Af, Bf, Q, fx, xState, ric, R, Mf, x2dot, θ2dot},
xState = {x, xdot, θ, θdot};
x2dot = 1/(1 - A Cos[θ]^2) (u + A θdot^2 Sin[θ] + A Cos[θ] Sin[θ]);
θ2dot = 1/(1 - A Cos[θ]^2) (-Sin[θ] - Cos[θ] (u + A θdot^2 Sin[θ]));
fx = {xdot, x2dot, θdot, θ2dot};
L = 1 / 2 * u^2;
Af = Grad[fx, xState]; (* For nD stuff use Grad*)
Bf = D[fx, u] ; (*For 1D stuff use D*)
Q = Grad[Grad[L, xState], xState];
Mf = Grad[D[L, u], xState];
R = D[L, {u, 2}];
S = ( s11 s12 s13 s14
      s12 s22 s23 s24
      s13 s23 s33 s34
      s14 s24 s34 s44 );
ric = Q + Af^T.S + S.Af - Outer[Times, S.Bf, Bf^T.S ] ;
(* This is the Syntax for calculating Outer Products *)(*Q = I, M = 0, R = 1*)
RHS = Table[0, {i, 4}, {j, 4}];
x = xff0;
xdot = xdotff0;
θ = θff0;
θdot = θdotff0;
u = uff0; (* Entering State Values *)
soltn = NMinimize[{1, ric == RHS}, {s11, s12, s13, s14, s22, s23, s24, s33, s34, s44}]〚2〛;
S = S /. soltn;
K = Bf^T.S ;
K]
TestSwingUpGeneralFBNumeric[τ_, τ1_, xff0_, xdotff0_, θff0_, θdotff0_,
 uff0_, A_, KTable_, n_] := Module[{K1, K2, K3, K4, eq, init, θ, θdot, θff,
  θdotff, x, xdot, xff, xdotff, uff, t, ufb, u, θs, θdots, xs, xdots, us},
 K1[t_] :=
  Piecewise[Table[{KTable〚i〛〚1〛, (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}], KTable〚n〛〚1〛];
 K2[t_] := Piecewise[Table[{KTable〚i〛〚2〛, (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}],
   KTable〚n〛〚2〛];
 K3[t_] := Piecewise[Table[{KTable〚i〛〚3〛, (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}],
   KTable〚n〛〚3〛];
 K4[t_] := Piecewise[Table[{KTable〚i〛〚4〛, (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}],
   KTable〚n〛〚4〛];
 xff[t_] := Piecewise[{{x1a[t], 0 ≤ t ≤ τ}}, 0];
 xdotff[t_] := Piecewise[{{xdot1a[t], 0 ≤ t ≤ τ}}, 0];
 θff[t_] := Piecewise[{{θ1a[t], 0 ≤ t ≤ τ}}, π];
 θdotff[t_] := Piecewise[{{θdot1a[t], 0 ≤ t ≤ τ}}, 0];
 uff[t_] := Piecewise[{{u1a[t], 0 ≤ t ≤ τ}}, 0];
```

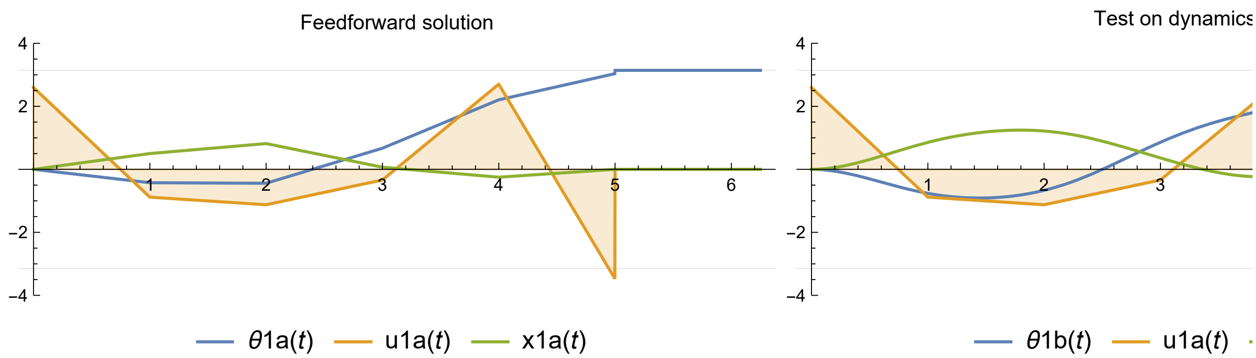
```
ufb[t_] := K3[t] * (θff[t] - θ[t]) + K4[t] * (θdotff[t] - θdot[t]) +
  K1[t] * (xff[t] - x[t]) + K2[t] * (xdotff[t] - xdot[t]);
u[t_] := uff[t] + ufb[t];
eq = {x'[t] == xdot[t], xdot'[t] ==
    1/(1 - A Cos[θ[t]]^2) (u[t] + A θdot[t]^2 Sin[θ[t]] + A Cos[θ[t]] Sin[θ[t]]), θ'[t] == θdot[t],
  θdot'[t] == 1/(1 - A Cos[θ[t]]^2) (-Sin[θ[t]] - Cos[θ[t]] (u[t] + A θdot[t]^2 Sin[θ[t]]))};
init = {x[0] == xdot[0] == θ[0] == θdot[0] == 0};
{xs, xdots, θs, θdots} = NDSolveValue[{eq, init},
  {x, xdot, θ, θdot}, {t, 0, τ1}, Method → {"DiscontinuityProcessing" → None}];
us[t_] := uff[t] + K3[t] (θff[t] - θs[t]) + K4[t] (θdotff[t] - θdots[t]) +
  K1[t] (xff[t] - xs[t]) + K2[t] (xdotff[t] - xdots[t]);
{xs, θs, us, K1, K2, K3, K4}]
```

Test example

In[95]:=
```
n = 5; τ = 5; τ1 = τ * 1.25 ; n2 = 40;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable = Table[CalculateGains[x1a[τ1 / n2 * i], xdot1a[τ1 / n2 * i],
    θ1a[τ1 / n2 * i], θdot1a[τ1 / n2 * i], u1a[τ1 / n2 * i], A], {i, 0, n2}];
{x1b, θ1b} = TestSwingUpGeneral[τ, τ1, u1a, A];
{x1c, θ1c, u1c, K1, K2, K3, K4} =
  TestSwingUpGeneralFBNumeric[τ, τ1, x1a, xdot1a, θ1a, θdot1a, u1a, A, KTable, n2];
p1a = Plot[{θ1a[t], u1a[t], x1a[t]}, {t, 0, τ1}, Filling → {2 → Axis},
   PlotRange → {-4, 4}, PlotLegends → "Expressions", PlotLabel → "Feedforward solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1b = Plot[{θ1b[t], u1a[t], x1b[t]}, {t, 0, τ1}, PlotRange → {-4, 4},
   Filling → {2 → Axis}, PlotLegends → "Expressions", PlotLabel → "Test on dynamics",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1c = Plot[{θ1c[t], u1c[t], x1c[t]}, {t, 0, τ1}, PlotRange → {-4, 4}, Filling → {2 → Axis},
   PlotLegends → "Expressions", PlotLabel → "Linear feedback solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
Grid[{{p1a, p1b, p1c}}]
```

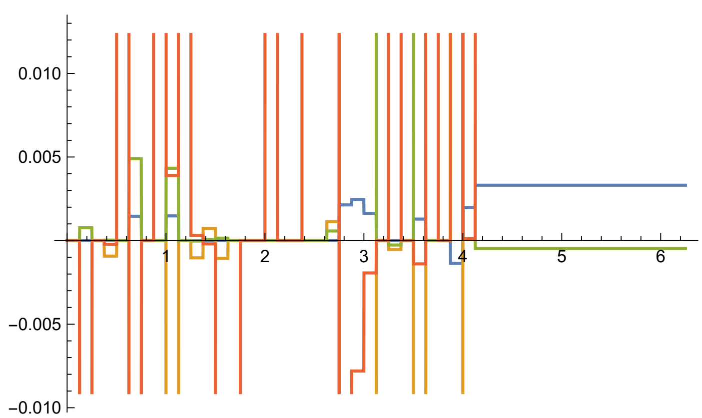
FindRoot: Failed to converge to the requested accuracy or precision within 100 iterations.

Out[104]=



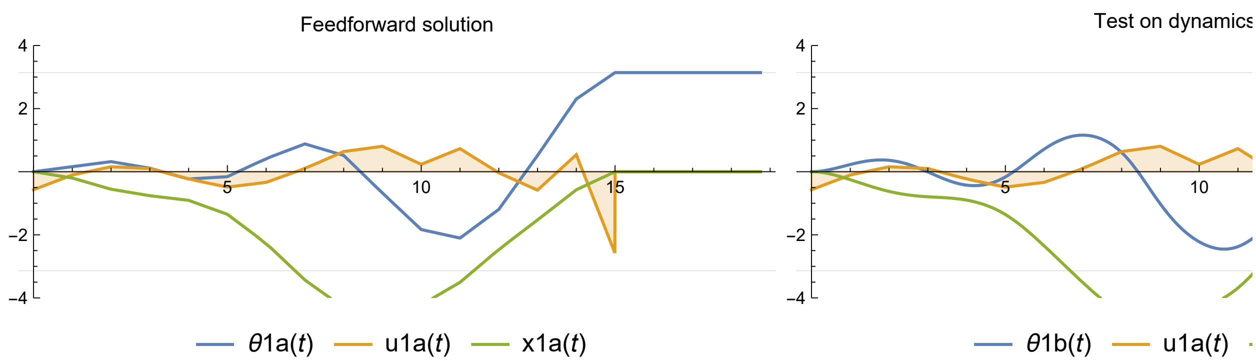
In[105]:=
```
Plot[{K1[t], K2[t], K3[t], K4[t]}, {t, 0, τ1}]
```

Out[105]=



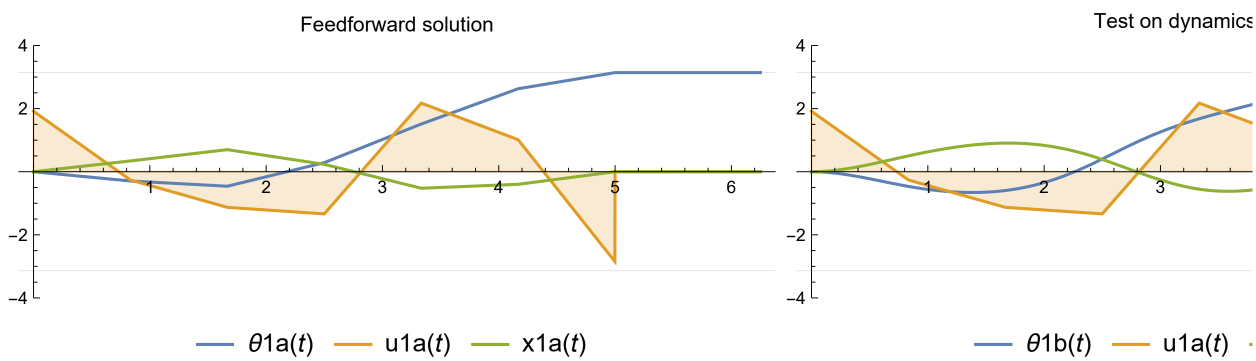
In[106]:=
```
n = 15; τ = 15; τ1 = τ * 1.25 ; n2 = 40;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable = Table[CalculateGains[x1a[τ1 / n2 * i], xdot1a[τ1 / n2 * i],
    θ1a[τ1 / n2 * i], θdot1a[τ1 / n2 * i], u1a[τ1 / n2 * i], A], {i, 0, n2}];
{x1b, θ1b} = TestSwingUpGeneral[τ, τ1, u1a, A];
{x1c, θ1c, u1c} =
  TestSwingUpGeneralFBNumeric[τ, τ1, x1a, xdot1a, θ1a, θdot1a, u1a, A, KTable, n2];
p1a = Plot[{θ1a[t], u1a[t], x1a[t]}, {t, 0, τ1}, Filling → {2 → Axis},
   PlotRange → {-4, 4}, PlotLegends → "Expressions", PlotLabel → "Feedforward solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1b = Plot[{θ1b[t], u1a[t], x1b[t]}, {t, 0, τ1}, PlotRange → {-4, 4},
   Filling → {2 → Axis}, PlotLegends → "Expressions", PlotLabel → "Test on dynamics",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1c = Plot[{θ1c[t], u1c[t], x1c[t]}, {t, 0, τ1}, PlotRange → {-4, 4}, Filling → {2 → Axis},
   PlotLegends → "Expressions", PlotLabel → "Linear feedback solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
Grid[{{p1a, p1b, p1c}}]
```

Out[115]=



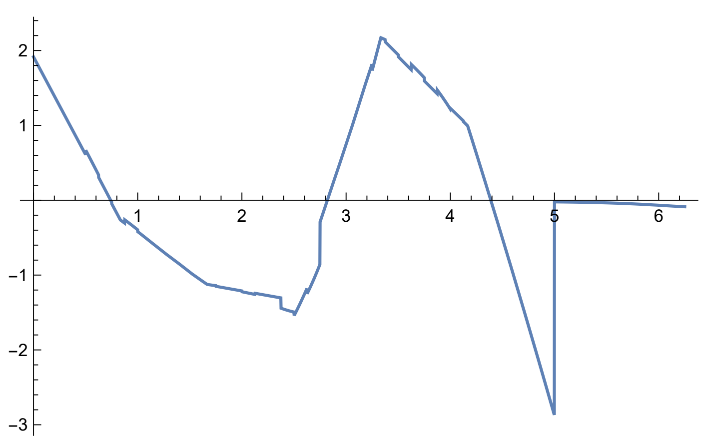
In[116]:=
```
n = 6; τ = 5; τ1 = τ * 1.25 ; n2 = 40;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable = Table[CalculateGains[x1a[τ1 / n2 * i], xdot1a[τ1 / n2 * i],
    θ1a[τ1 / n2 * i], θdot1a[τ1 / n2 * i], u1a[τ1 / n2 * i], A], {i, 0, n2}];
{x1b, θ1b} = TestSwingUpGeneral[τ, τ1, u1a, A];
{x1c, θ1c, u1c} =
  TestSwingUpGeneralFBNumeric[τ, τ1, x1a, xdot1a, θ1a, θdot1a, u1a, A, KTable, n2];
p1a = Plot[{θ1a[t], u1a[t], x1a[t]}, {t, 0, τ1}, Filling → {2 → Axis},
   PlotRange → {-4, 4}, PlotLegends → "Expressions", PlotLabel → "Feedforward solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1b = Plot[{θ1b[t], u1a[t], x1b[t]}, {t, 0, τ1}, PlotRange → {-4, 4},
   Filling → {2 → Axis}, PlotLegends → "Expressions", PlotLabel → "Test on dynamics",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1c = Plot[{θ1c[t], u1c[t], x1c[t]}, {t, 0, τ1}, PlotRange → {-4, 4}, Filling → {2 → Axis},
   PlotLegends → "Expressions", PlotLabel → "Linear feedback solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
Grid[{{p1a, p1b, p1c}}]
```

Out[125]=



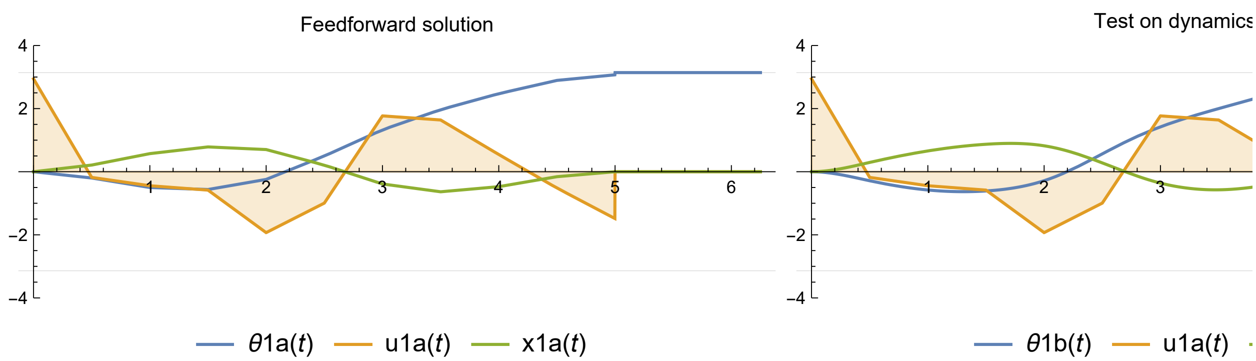
In[126]:=
```
Plot[u1c[t], {t, 0, τ1}]
```

Out[126]=



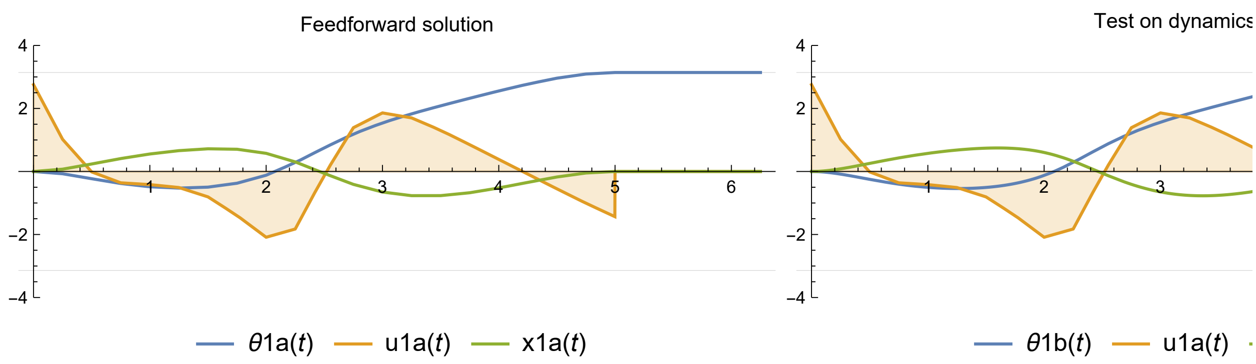
In[127]:=
```
n = 10; τ = 5; τ1 =  τ * 1.25 ;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable  = Table[CalculateGains[x1a[τ1 / n2 * i], xdot1a[τ1 / n2 * i],
     θ1a[τ1 / n2 * i], θdot1a[τ1 / n2 * i], u1a[τ1 / n2 * i], A], {i, 0, n2}];
{x1b, θ1b} = TestSwingUpGeneral[τ, τ1, u1a, A];
{x1c, θ1c, u1c} =
  TestSwingUpGeneralFBNumeric[τ, τ1, x1a, xdot1a, θ1a, θdot1a, u1a, A, KTable];
p1a = Plot[{θ1a[t], u1a[t], x1a[t]}, {t, 0, τ1}, Filling → {2 → Axis},
   PlotRange → {-4, 4}, PlotLegends → "Expressions", PlotLabel → "Feedforward solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1b = Plot[{θ1b[t], u1a[t], x1b[t]}, {t, 0, τ1}, PlotRange → {-4, 4},
   Filling → {2 → Axis}, PlotLegends → "Expressions", PlotLabel → "Test on dynamics",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1c = Plot[{θ1c[t], u1c[t], x1c[t]}, {t, 0, τ1}, PlotRange → {-4, 4}, Filling → {2 → Axis},
   PlotLegends → "Expressions", PlotLabel → "Linear feedback solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
Grid[{{p1a, p1b, p1c}}]
```

Out[136]=



In[137]:=
```
n = 20; τ = 5; τ1 = τ * 1.25 ;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable = Table[CalculateGains[x1a[τ1 / n2 * i], xdot1a[τ1 / n2 * i],
    θ1a[τ1 / n2 * i], θdot1a[τ1 / n2 * i], u1a[τ1 / n2 * i], A], {i, 0, n2}];
{x1b, θ1b} = TestSwingUpGeneral[τ, τ1, u1a, A];
{x1c, θ1c, u1c} =
  TestSwingUpGeneralFBNumeric[τ, τ1, x1a, xdot1a, θ1a, θdot1a, u1a, A, KTable];
p1a = Plot[{θ1a[t], u1a[t], x1a[t]}, {t, 0, τ1}, Filling → {2 → Axis},
   PlotRange → {-4, 4}, PlotLegends → "Expressions", PlotLabel → "Feedforward solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1b = Plot[{θ1b[t], u1a[t], x1b[t]}, {t, 0, τ1}, PlotRange → {-4, 4},
   Filling → {2 → Axis}, PlotLegends → "Expressions", PlotLabel → "Test on dynamics",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
p1c = Plot[{θ1c[t], u1c[t], x1c[t]}, {t, 0, τ1}, PlotRange → {-4, 4}, Filling → {2 → Axis},
   PlotLegends → "Expressions", PlotLabel → "Linear feedback solution",
   AspectRatio → 1 / 3, ImageSize → 400, GridLines → {None, {-π, π}}];
Grid[{{p1a, p1b, p1c}}]
```

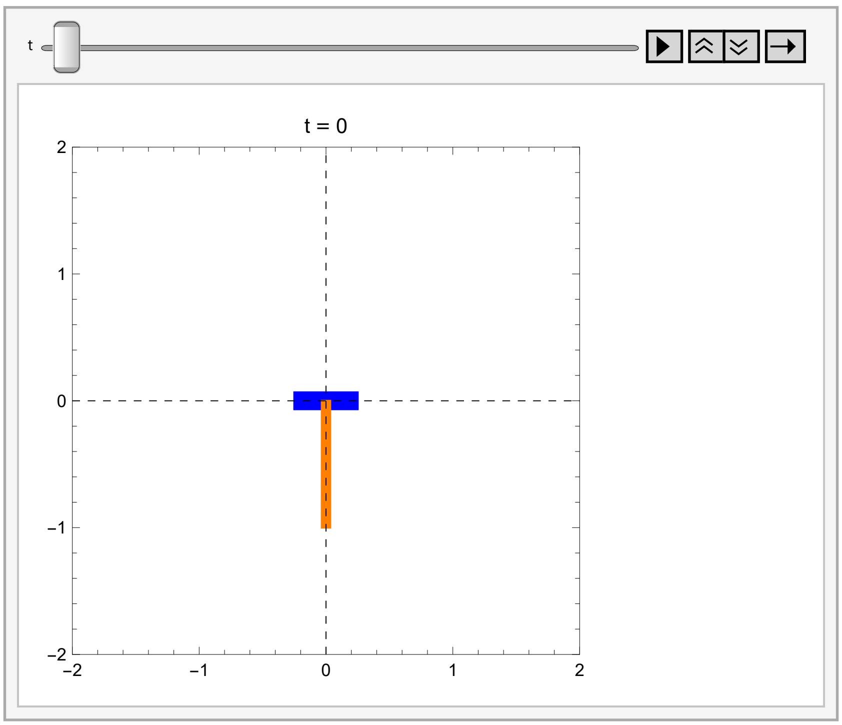
Out[146]=



Animation with General Cart-Pendulum swing up

In[147]:=
```
AnimatePendulum[rules_] :=
 Animate[Evaluate[DrawSinglePendulum[x[t] /. rules, {θ[t] /. rules, 1, 1}, t]],
  {t, 0, τ1}, DefaultDuration → 5, AnimationRunning → False]
DrawSinglePendulum[cart_, {angle1_, length1_, mass1_}, t_] :=
 Module[{width1, density = 30},
  width1 = mass1 / length1 / density;
  Graphics[{
    {Blue, Rectangle[{cart - 1 / 4, -1 / 15}, {cart + 1 / 4, 1 / 15}]},
    {Orange, Translate[Rotate[
       Rectangle[{0, width1}, {length1, -width1}], angle1 - π / 2, {0, 0}], {cart, 0}]}
   },
   PlotRange → 2, ImageSize → 280,
   Frame → True, Axes → True, AxesStyle → Dashed,
   PlotLabel → "t" == NumberForm[t, {4, 2}]]]
anim = Animate[Evaluate[DrawSinglePendulum[x1c[t], {θ1c[t], 1, 1}, t]], {t, 0, τ1},
  DefaultDuration → 5, AnimationRunning → False, AnimationRepetitions → 1]
```

Out[149]=



In[150]:=
```
(*SetDirectory["D:\\"];
Export["General_anim.avi",anim];*)
```

Test Feedback function

```
In[151]:= CalculateGains[xff0_, xdotff0_, θff0_, θdotff0_, uff0_, A_] :=
  Module[{x, L, RHS, xdot, θ, θdot, u, K, S, soltn, i, j, s11, s12, s13, s14, s22,
    s23, s24, s33, s34, s44, Af, Bf, Q, fx, xState, ric, R, Mf, x2dot, θ2dot},
   xState = {x, xdot, θ, θdot};
   x2dot = 1/(1 - A Cos[θ]^2) (u + A θdot^2 Sin[θ] + A Cos[θ] Sin[θ]);
   θ2dot = 1/(1 - A Cos[θ]^2) (-Sin[θ] - Cos[θ] (u + A θdot^2 Sin[θ]));
   fx = {xdot, x2dot, θdot, θ2dot};
   L = 1 / 2 * u^2;
   Af = Grad[fx, xState]; (* For nD stuff use Grad*)
   Bf = D[fx, u] ; (*For 1D stuff use D*)
   Q = Grad[Grad[L, xState], xState];
   Mf = Grad[D[L, u], xState];
   R = D[L, {u, 2}];
   S = ( s11 s12 s13 s14
         s12 s22 s23 s24
         s13 s23 s33 s34
         s14 s24 s34 s44 );
   ric = Q + Af^T.S + S.Af - Outer[Times, S.Bf, Bf^T.S ] ;
   (* This is the Syntax for calculating Outer Products *)(*Q = I, M = 0, R = 1*)
   RHS = Table[0, {i, 4}, {j, 4}];
   x = xff0;
   xdot = xdotff0;
   θ = θff0;
   θdot = θdotff0;
   u = uff0; (* Entering State Values *)
   soltn = NMinimize[{1, ric == RHS}, {s11, s12, s13, s14, s22, s23, s24, s33, s34, s44}][[2]];
   S = S /. soltn;
   K = Bf^T.S ;
   K]
```

In[152]:=
```
n = 5; τ = 5; τ1 = τ * 1.25;
A = 0.2;
{x1a, xdot1a, θ1a, θdot1a, u1a} = ffCartPendulumGeneral[n, τ, τ1, A];
KTable = Table[CalculateGains[x1a[τ / n * i], xdot1a[τ / n * i],
    θ1a[τ / n * i], θdot1a[τ / n * i], u1a[τ / n * i], A], {i, 0, n}];
K1[t_] := Piecewise[Table[{KTable[[i]][[1]], (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}], 0];
K2[t_] := Piecewise[Table[{KTable[[i]][[2]], (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}], 0];
K3[t_] := Piecewise[Table[{KTable[[i]][[3]], (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}], 0];
K4[t_] := Piecewise[Table[{KTable[[i]][[4]], (i - 1) * τ / n ≤ t ≤ i * τ / n}, {i, 1, n}], 0];
xff[t_] := Piecewise[{{x1a[t], 0 ≤ t ≤ τ}}, 0];
xdotff[t_] := Piecewise[{{xdot1a[t], 0 ≤ t ≤ τ}}, 0];
θff[t_] := Piecewise[{{θ1a[t], 0 ≤ t ≤ τ}}, π];
θdotff[t_] := Piecewise[{{θdot1a[t], 0 ≤ t ≤ τ}}, 0];
uff[t_] := Piecewise[{{u1a[t], 0 ≤ t ≤ τ}}, 0];
ufb[t_] := Piecewise[{{K3[t] * (θff[t] - θ[t]) + K4[t] * (θdotff[t] - θdot[t]) +
      K1[t] * (xff[t] - x[t]) + K2[t] * (xdotff[t] - xdot[t]), 0 ≤ t ≤ τ}}, 0];
u[t_] := uff[t] + ufb[t];
eq = {x'[t] == xdot[t], xdot'[t] ==
    1/(1 - A Cos[θ[t]]^2) (u[t] + A θdot[t]^2 Sin[θ[t]] + A Cos[θ[t]] Sin[θ[t]]), θ'[t] == θdot[t],
   θdot'[t] == 1/(1 - A Cos[θ[t]]^2) (-Sin[θ[t]] - Cos[θ[t]] (u[t] + A θdot[t]^2 Sin[θ[t]]))};
init = {x[0] == xdot[0] == θ[0] == θdot[0] == 0};
```

FindRoot: Failed to converge to the requested accuracy or precision within 100 iterations.