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Guidance Laws Against Defended Aerial Targets

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The three body engagement scenario is considered where an aerial missile, homing onto a target aircraft, encounters a defender missile launched by the aircraft. A given set of defender missile guidance laws, namely, proportional navigation and line-of-sight guidance are considered and analysis is carried out for proportional navigation and pure pursuit attacking missile strategies. Analytic capture zones and lateral accelerations ratios for the four resulting scenarios are derived. Closed form expression for attacking missile initial position and launch angles are derived for a successful evasion from the defender. Extensive numerical simulations are carried out which comply with the analytical findings.

Nomenclature

$v_t,\ v_d,\ v_m$	Target, defender, and missile speeds, respectively
$\gamma_t,\ \gamma_d,\ \gamma_m$	Target, defender, and missile headings, respectively
$\gamma_{t0},\ \gamma_{d0},\ \gamma_{m0}$	Target, defender, and missile initial headings, respectively
$\dot{\gamma}_t,\ \dot{\gamma}_d,\ \dot{\gamma}_m$	Target, defender, and missile turning rates, respectively
$R_d,\ R_m,\ R_{dm}$	Target-defender, target-missile, and defender-missile closing ranges, respectively
$R_{d0},\ R_{m0},\ R_{dm0}$	Target-defender, target-missile, and defender-missile initial closing ranges, respectively
$\dot{R}_d,\ \dot{R}_m,\ \dot{R}_{dm}$	Target-defender, target-missile, and defender-missile closing range rates, respectively
$v_{cdm},\ v_{cm}$	Defender-missile and target-missile closing speeds, respectively
$v_{cdm0},\ v_{cm0}$	Defender-missile and target-missile initial closing speeds, respectively
$\lambda_d,\ \lambda_m,\ \lambda_{dm}$	Target-defender, target-missile, and defender-missile line of sight angles, respectively
$\lambda_{d0},\ \lambda_{m0},\ \lambda_{dm0}$	Target-defender, target-missile, and defender-missile initial line of sight angles, respectively
$\dot{\lambda}_d,\ \dot{\lambda}_m,\ \dot{\lambda}_{dm}$	Target-defender, target-missile, and defender-missile line of sight angle rates, respectively
$a_d,\ a_m$	Defender and missile lateral accelerations, respectively

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λ	Line of sight angle
$a_{dplos},\ a_{mplos}$	Defender, and missile accelerations, normal to the line of sight, respectively
$N_d,\ N_m$	Defender and missile navigation constants, respectively
$a_{dmax},\ a_{mmax}$	Defender and missile maximum lateral accelerations, respectively
a_{mpp}	Missile pure pursuit acceleration
a_{mppmax}	Missile maximum pure pursuit acceleration
k	Gain for pure pursuit command
$t_{f1},\ t_{f2}$	Defender-missile and target-missile interception time, respectively
$t_{go1},\ t_{go2}$	Defender-missile and target-missile time-to-go, respectively

I. Introduction

Conventional air to air missiles encounter passive aircraft evasive measures like flares and chaff. A three-body engagement scenario emerges if the aircraft launches a defender antimissile as an active response. The missile now has to evade the defender antimissile and also achieve the prime objective of intercepting the aircraft. In such an engagement it is expected that the missile and the antimissile are similar class of vehicles with no decisive speed or acceleration advantage over the other. In the resulting engagement the mission outcome depends mainly on the guidance laws used by the adversaries.

Boyell[1] presented the first results on kinematics of the three-body problem where closed form relations were derived for constant bearing collision courses. The ratio of launch to interception range was derived as a function of speed ratios. Shneydor[2] in his comments on Boyell's work, simplified one of the collision conditions. In a later work under the same assumptions, Boyell[3] obtained a closed form expression for the intercept point in target centered coordinates. Conditions for the speed ratios were also derived for different attack geometries and desired interception point. The studies with constant bearings provide valuable insight into the kinematics of the problem. However, in realistic engagements the assumptions of constant bearings are not valid as vehicles are expected to maneuver in order to achieve their respective objectives.

Shinar and Silberman[4] solved the three-body problem as a three player two team game. A discrete linearized planar pursuit-evasion model was used for a preliminary investigation of the problem. The missile was assumed not to have any information about the defender and the closing speeds were assumed to be constant. Also the target (a ship) was assumed stationary. The study suggests that the missile should use maximum lateral acceleration for evasion near its interception by the defender. Rusnak presented a solution approach by defining the problem as a two team dynamic game using a linearized model and a quadratic criterion.[5] A numerical test-case was presented showing that the defender acceleration is lower than the one expected should the defender have used proportional navigation guidance (PNG) law.

Recently, the three-body engagement problem received considerable attention. Perelman et al.[6] obtained an analytical solution to the linear quadratic differential game and studied the conditions for the existence of a saddle point solution. Nonlinear two dimensional simulations were used to validate the theoretical analysis. Shaferman and Shima[7] presented a cooperative multiple model adaptive guidance and estimation scheme applicable for the three-body problem. The filter used was a nonlinear adaptation of a multiple model adaptive estimator, in which each model represents a possible guidance law and guidance parameters of the attacking missile. It utilizes cooperation between the aerial target and the defender, as the defender knows the future evasive maneuvers to be performed by the protected target. Shima[8] derived optimal-control-based cooperative evasion and pursuit strategies for the aircraft and its defending missile for the case where the attacking missile uses a known linear guidance strategy. The optimal one-on-one target evasion strategy from such guidance laws was derived as well. Assuming no cooperation between the target aircraft and its defender missile, and that the target is non-maneuvering, Yamasaki and Balakrishnan[9] proposed a guidance law where the defender maximizes the angle contained by its position vectors with respect to the target and the missile. Six-degrees-of-freedom simulation results exemplified better defender performance, as compared to PNG.

By its very nature the line-of-sight (LOS) guidance is a three-point guidance strategy. Ratnoo and Shima[10] highlighted LOS guidance law as a prospective defender missile (hereafter known as defender) strategy protecting an aircraft (hereafter known as target) from an incoming missile (hereafter known as missile) attack. Kinematics of LOS guidance with a moving/maneuvering launch platform were derived and the application of command to LOS (CLOS) guidance as a prospective defender guidance strategy in the target-missile-defender interception scenario was studied. CLOS defender guidance resulted in much improved performance against missiles using PNG law.

Most of the work in the literature on the three-body problem deals with guidance strategies for the aircraft defending team. As the contributions of this work, we study prospective guidance strategies from the missile's point of view against a given set of defender guidance laws whilst the target aircraft follows a non-maneuvering level flight. Closed form analytic solutions for missile initial conditions for a guaranteed escape from the defender is the main thrust of the work. PNG and LOS guidance defender strategies are analyzed against PNG and pure pursuit (PP) missile strategies. Acceleration ratios and launch envelopes are derived analytically for the four combinations of the adversary strategies. Based on the analysis, closed form expressions for launch range and headings are presented for a successful missile mission.Extensive simulation studies support the analytical results.

The remainder of the paper is arranged as follows: The three-body guidance scenario is described in Section II followed by analytical studies on the prospective guidance laws in Section III. Numerical simulation studies are presented in Section IV. Section V contains concluding remarks.

II. Three Body Guidance Problem

Consider the three-body engagement scenario as shown in Fig. 1 where the missile homes on to the target while evading the defender. We assume the three vehicles to be constant speed point masses moving in a plane. Missile and defender are assumed to be from a similar class of vehicles with their speeds higher

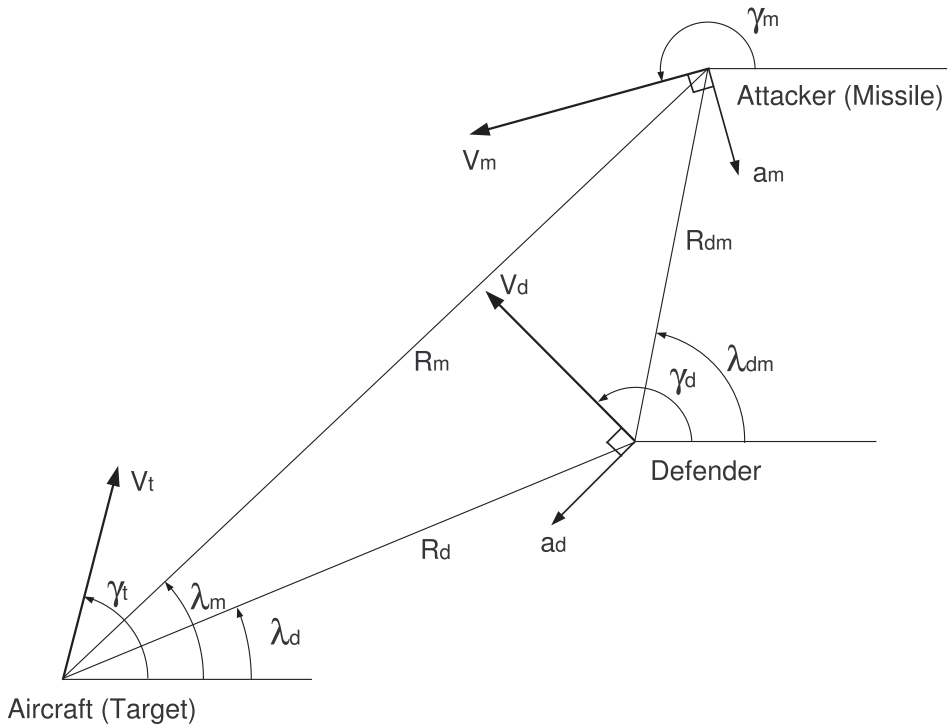


Figure 1. Three-Body Engagement Scenario

than that of the target, that is

$$v_d = v_m > v_t \tag{1}$$

The range rate and LOS rate for missile-target, defender-target, and missile-defender engagement are given as

$$\dot{R}_m = v_m \cos(\gamma_m - \lambda_m) - v_t \cos(\gamma_t - \lambda_m) \tag{2}$$

$$\dot{R}_d = v_d \cos(\gamma_d - \lambda_d) - v_t \cos(\gamma_t - \lambda_d) \tag{3}$$

$$\dot{R}_{dm} = v_m \cos(\gamma_m - \lambda_{dm}) - v_d \cos(\gamma_{dm} - \lambda_{dm}) \tag{4}$$

and

$$\dot{\lambda}_m = \frac{v_m \sin(\gamma_m - \lambda_m) - v_t \sin(\gamma_t - \lambda_m)}{R_m} \tag{5}$$

$$\dot{\lambda}_d = \frac{v_d \sin(\gamma_d - \lambda_d) - v_t \sin(\gamma_t - \lambda_d)}{R_d} \tag{6}$$

$$\dot{\lambda}_{dm} = \frac{v_m \sin(\gamma_m - \lambda_{dm}) - v_d \sin(\gamma_d - \lambda_{dm})}{R_{dm}} \tag{7}$$

respectively. Here γ_m, γ_t, and γ_d represent missile, target, and defender headings, respectively. And their rates are given as

$$\dot{\gamma}_m = \frac{a_m}{v_m} \tag{8}$$

$$\dot{\gamma}_d = \frac{a_d}{v_d} \tag{9}$$

where a_m and a_d are the missile and defender lateral accelerations, respectively. The acceleration capabilities for the two adversaries are limited as

$$|a_m| \leq a_{mmax} \tag{10}$$

$$|a_d| \leq a_{dmax} \tag{11}$$

Also, the target is assumed to be non-maneuvering. Defender-missile and missile-target interception times are represented by t_{f1} and t_{f2}, respectively. The defender-missile engagement closes faster and we assume

$$t_{f1} < t_{f2} \tag{12}$$

The three-body interception problem is an extension of the classical one-on-one interception scenario. Each of the three vehicles has dual guidance objectives. The missile's objective is to intercept the target while successfully evading the defender. The defender has an objective of facilitating target's evasion and intercepting the missile. The target has the dual objective of facilitating the missile-defender interception and successfully evading the missile. If the defender misses the missile then the guidance problem becomes the one-on-one type where the missile pursues and the target evades it.

III. Guidance laws

Owing to its simple structure, ease of implementation, and high performance PN is a widely used missile guidance law. The motivation is to derive in closed form, the missile launch parameters for an escape from the defender in an engagement where both the defender and the missile use PNG law. Also prospective missile strategies need to be investigated for the case when the defender uses LOS guidance resulting in sharp deterioration[10] of PNG missile strategy. Considering PNG and PP guidance as missile strategies against a defender using PNG and LOS guidance, respectively, we have the four engagement scenarios tagged A,B,C, and D as listed in Table 1.

Defender(→) Missile(↓)	PNG	LOSG
PNG	Case 1	Case 3
PP	Case 2	Case 4

Table 1. Guidance law scenarios

A. Case 1: PNG Defender against PNG missile

The PNG command for the defender can be written as

$$a_d = N_d v_{cdm} \dot{\lambda}_{dm} \tag{13}$$

where v_{cdm} is the defender missile closing speed defined as

$$v_{cdm} = -\dot{R}_{dm} \tag{14}$$

Now consider a missile using PNG law against the target. The missile lateral acceleration is expressed as

$$a_m = N_m v_{cm} \dot{\lambda}_m \tag{15}$$

where v_{cm} is the missile-target closing speed given by

$$v_{cm} = -\dot{R}_m \tag{16}$$

The closed form solution[11] for the missile PNG lateral acceleration command of (15) against a non-maneuvering target aircraft is given as,

$$a_m = \frac{N_m v_{cm0} \sin(\gamma_{m0} - \lambda_{m0})}{t_{f2}} \left(\frac{t_{go2}}{t_{f2}}\right)^{N_m - 2} \tag{17}$$

where the defender-missile and missile-target interception times t_{f1} and t_{f2}, respectively, are approximated as

$$t_{f1} \simeq \frac{R_{m0}}{v_{cdm0}}, \; t_{f2} \simeq \frac{R_{m0}}{v_{cm0}} \tag{18}$$

Using (18) in (17), we get

$$a_m = \frac{N_m v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{R_{m0}} \left(\frac{t_{go2}}{t_{f2}}\right)^{N_m - 2} \tag{19}$$

The missile engages a non-maneuvering target and the defender engages the maneuvering missile. At the defender-missile interception time t_{f1}, the defender lateral acceleration against a maneuvering missile satisfies[11]

$$\frac{a_d}{a_m}(t_{f1}) = \frac{N_d}{N_d - 2} \tag{20}$$

Also, at the defender-missile interception we have,

$$t_{go2} = t_{f2} - t = t_{f2} - t_{f1} \tag{21}$$

From (20) it can be seen that the defender requires a terminal lateral acceleration three times that of the missile for $N_d = 3$. Substituting a_m from (19) into (20) and using (21), we have

$$a_d(t_{f1}) = \frac{N_m N_d v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{(N_d - 2) R_{m0}} \left(\frac{t_{f2} - t_{f1}}{t_{f2}}\right)^{N_m - 2} \tag{22}$$

$$\Rightarrow a_d(t_{f1}) = \frac{N_m N_d v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{(N_d - 2) R_{m0}} \left(\frac{v_{cdm0} - v_{cm0}}{v_{cdm0}}\right)^{N_m - 2} \quad \text{(Using (18))} \tag{23}$$

Theorem 1 *Assuming that both the missile and the defender use PNG law, the defender lateral acceleration saturates near defender-missile interception for all missile initial engagement parameters satisfying*

$$\frac{\sin(\gamma_{m0} - \lambda_{m0})}{R_{m0}} > \frac{N_d - 2}{N_m N_d v_{cm0}^2} \left(\frac{v_{cdm0}}{v_{cdm0} - v_{cm0}}\right)^{N_m - 2} a_{dmax} \tag{24}$$

Proof: For the defender lateral acceleration to saturate near defender-missile interception, we have

$$a_d(t_{f1}) \quad > \quad a_{dmax} \tag{25}$$

Using (23) in (25), we get

$$\frac{N_m N_d v_{cm0}^2 \sin(\gamma_{m0}-\lambda_{m0})}{(N_d-2)R_{m0}}\left(\frac{v_{cdm0}-v_{cm0}}{v_{cdm0}}\right)^{N_m-2} \quad > \quad a_{dmax} \tag{26}$$

$$\Rightarrow \frac{\sin(\gamma_{m0}-\lambda_{m0})}{R_{m0}} \quad > \quad \frac{N_d-2}{N_m N_d v_{cm0}^2}\left(\frac{v_{cdm0}}{v_{cdm0}-v_{cm0}}\right)^{N_m-2} a_{dmax} \tag{27}$$

□

From Theorem 1 we see that a missile using PNG law against the aircraft can evade the defender using PNG law by controlling its initial heading angle or the launch range.

Theorem 2 *A defender using PNG law will always have an unsaturated lateral acceleration demand while intercepting a missile using PNG law against the aircraft with a launch range R_{m0} satisfying*

$$R_{m0} > \frac{N_m N_d v_{cm0}^2}{(N_d-2)a_{dmax}}\left(\frac{v_{cdm0}-v_{cm0}}{v_{cdm0}}\right)^{N_m-2} \tag{28}$$

Proof: From (23) we have,

$$a_d(t_{f1}) \quad = \quad \frac{N_m N_d v_{cm0}^2 \sin(\gamma_{m0}-\lambda_{m0})}{(N_d-2)R_{m0}}\left(\frac{v_{cdm0}-v_{cm0}}{v_{cdm0}}\right)^{N_m-2} < a_{dmax} \tag{29}$$

$$\text{if } R_{m0} \quad > \quad \frac{N_m N_d v_{cm0}^2 \sin(\gamma_{m0}-\lambda_{m0})}{(N_d-2)a_{dmax}}\left(\frac{v_{cdm0}-v_{cm0}}{v_{cdm0}}\right)^{N_m-2} \tag{30}$$

$$\Rightarrow \text{if } R_{m0} \quad > \quad \frac{N_m N_d v_{cm0}^2}{(N_d-2)a_{dmax}}\left(\frac{v_{cdm0}-v_{cm0}}{v_{cdm0}}\right)^{N_m-2} \qquad \forall(\gamma_{m0}-\lambda_{m0}) \in (-\pi,\pi) \tag{31}$$

□

B. Case 2: PNG Defender against PP missile

Consider the missile using PP guidance against the target with the defender using PNG. The pure pursuit lateral acceleration command against a non-maneuvering target is given as[11]

$$a_{mpp} = \frac{v_m v_t \tan\left(\frac{\gamma_{t0}-\lambda_{m0}}{2}\right)^{\frac{v_m}{v_t}}}{R_{m0}\sin(\gamma_{t0}-\lambda_{m0})}\frac{\sin(\gamma_t-\lambda_m)^2}{\tan\left(\frac{\gamma_t-\lambda_m}{2}\right)^{\frac{v_m}{v_t}}} \tag{32}$$

Proposition 1 *A defender using PNG law will always have an unsaturated lateral acceleration demand while intercepting a missile using PP law against the aircraft if*

$$R_{m0} > \frac{N_d v_m v_t}{(N_d-2)a_{dmax}\tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{33}$$

Proof: For evaluating the maximum value of a_{mpp} from (35), we have the maximum PP lateral acceleration a_{mppmax} given as

$$\max\left(\frac{\tan\left(\frac{\gamma_{t0}-\lambda_{m0}}{2}\right)^{\frac{v_m}{v_t}}}{\sin(\gamma_{t0}-\lambda_{m0})}\right) = 1 \quad \text{for} \quad (\gamma_{t0}-\lambda_{m0}) = \pm\frac{\pi}{2} \tag{34}$$

that is, the missile is launched with maximum initial heading error. Using (34) in (35), we have

$$a_{mppmax} = \frac{v_m v_t}{R_{m0}} \frac{\sin(\gamma_t - \lambda_m)^2}{\tan\left(\frac{\gamma_t - \lambda_m}{2}\right)^{\frac{v_m}{v_t}}} \tag{35}$$

Assuming that half the initial heading error $(\gamma_{t0} - \lambda_{m0})$ is annulled by $t = t_{f1}$, yields

$$a_{mppmax}(t_{f1}) = \frac{v_m v_t}{R_{m0} \tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{36}$$

Using (20) for an unsaturated defender lateral acceleration command at t_{f1}, we have

$$a_{dmax} > a_d = a_{mppmax}(t_{f1}) \frac{N_d}{N_d - 2} \tag{37}$$

From (37) and (36), we deduce

$$a_{dmax} > \frac{N_d}{N_d - 2} \frac{v_m v_t}{R_{m0} \tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{38}$$

$$\Rightarrow R_{m0} > \frac{N_d v_m v_t}{(N_d - 2) a_{dmax} \tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{39}$$

□

C. Case 3: LOS Defender against PNG missile

This combination of the three-body guidance strategies is presented in detail in Ref.[10]. Therein the defender and missile lateral acceleration components normal to the line of sight represented as a_{dplos} and a_{mplos}, respectively, are related as

$$a_{dplos} = \frac{R_d}{R_m} a_{mplos} + 2\dot{\lambda}\left(\dot{R}_d - \frac{R_d}{R_m}\dot{R}_m\right) \tag{40}$$

where typically at t_{f1}, we have

$$\frac{a_d}{a_m} \simeq \frac{a_{dplos}}{a_{mplos}} = 1/3 \tag{41}$$

Proposition 2 *A defender using LOS guidance law will always have an unsaturated lateral acceleration demand while intercepting a missile using PN law against the aircraft if*

$$R_{m0} > \frac{N_m v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{3a_{dmax}} \left(\frac{v_{cdm0} - v_{cm0}}{v_{cdm0}}\right)^{N_m - 2} \tag{42}$$

Proof: Using (41) for an unsaturated defender lateral acceleration demand at t_{f1}, we have

$$a_{dmax} > a_d = \frac{a_m}{3} \tag{43}$$

Using the PNG command of (19) in (59), together with (18) and (21), we deduce

$$a_{dmax} > \frac{N_m v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{3R_{m0}} \left(\frac{v_{cdm0} - v_{cm0}}{v_{cdm0}}\right)^{N_m - 2} \tag{44}$$

$$\Rightarrow R_{m0} > \frac{N_m v_{cm0}^2 \sin(\gamma_{m0} - \lambda_{m0})}{3a_{dmax}} \left(\frac{v_{cdm0} - v_{cm0}}{v_{cdm0}}\right)^{N_m - 2} \tag{45}$$

□

D. Case 4: LOS Defender vs PP missile

We use results from the detailed kinematics of the three body guidance problem with the defender using LOS guidance Ref[10]. Therein the defender and missile lateral acceleration components normal to the line of sight represented as a_{dplos} and a_{mplos}, respectively, are related as

$$a_{dplos} = \frac{R_d}{R_m} a_{mplos} + 2\dot{\lambda}\left(\dot{R}_d - \frac{R_d}{R_m}\dot{R}_m\right) \tag{46}$$

where λ is the angle the line joining the three vehicles makes with the reference. Near defender-missile interception time t_{f1}, we have

$$\frac{R_d}{R_m} \rightarrow 1 \tag{47}$$

Using (47) in (46), we get

$$a_{dplos} = a_{mplos} + 2\dot{\lambda}\left(\dot{R}_d - \dot{R}_m\right) \tag{48}$$

$$\Rightarrow a_{dplos} = a_{mplos} + 2\dot{\lambda} v_{cdm} \tag{49}$$

Assuming the missile uses pure pursuit guidance with the lateral acceleration command given as

$$a_m = v_m \dot{\lambda} - k(\gamma_m - \lambda) \tag{50}$$

where k is a constant of proportionality. The first term keeps the missile on the pursuit course and the second term annuls the initial heading error. Further, assuming that the initial heading error is annulled by the defender-missile interception time t_{f1}, that is

$$\gamma_m - \lambda \simeq 0 \text{ at } t = t_{f1}$$

we have

$$a_m = v_m \dot{\lambda} \tag{51}$$

and

$$a_{mplos} = a_m \cos(\gamma_m - \lambda) \simeq v_m \dot{\lambda} \tag{52}$$

Using (49) and (52), we deduce

$$\frac{a_{dplos}}{a_{mplos}}(t_{f1}) = 1 + 2\frac{v_{cdm}}{v_m} \tag{53}$$

Theorem 3 *For all positive defender-missile terminal closing speeds, a defender using LOS guidance against a missile using PP guidance will have,*

$$5 > \frac{a_{dplos}}{a_{mplos}}(t_{f1}) > 1 \tag{54}$$

Proof. Its given that the minimum defender-missile interception closing speed is positive, that is,

$$v_{cdm}(t_{f1}) > 0 \tag{55}$$

and the maximum value of the closing speed is bounded by

$$\dot{R}_d - \dot{R}_m < (v_m + v_d) = 2v_m \tag{56}$$

Using (55) and (56) in (53), we have

$$5 > \frac{a_{dplos}}{a_{mplos}}(t_{f1}) > 1 \tag{57}$$

□

Line-of-sight defender guidance law gives an excellent performance against missiles using PNG law for all attack geometries.[10] Here as shown in Theorem 3, a PP guided missile can result in a very high terminal LOS guidance defender lateral acceleration thus resulting in a potential successful missile evasion from the defender.

Proposition 3 *A defender using LOS guidance law will always have an unsaturated lateral acceleration demand while intercepting a missile using PP law against the aircraft if*

$$R_{m0} > \frac{5v_m v_t}{a_{dmax}\tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{58}$$

Proof: Using (57) for an unsaturated defender lateral acceleration demand at t_{f1}, we have

$$a_{dmax} > a_d \simeq a_{dplos} = 5a_{mplos} \simeq 5a_{mppmax} \tag{59}$$

Using (36) and (59), we deduce

$$R_{m0} > \frac{5v_m v_t}{a_{dmax}\tan\left(\frac{\pi}{8}\right)^{\frac{v_m}{v_t}}} \tag{60}$$

□

IV. Simulations

We consider a planar engagement scenario with constant speed point mass vehicles. The speeds of the adversaries are chosen as $v_d = v_m = 600$ m/sec and $v_t = 300$ m/sec. The defender and the missile have a lateral acceleration capability bounded by $\pm 20\ g$. Vehicles have lag free dynamics and use perfect information. Case study numbers for the four combinations of adversary guidance laws are the same as listed in Table 1.

A. Case 1: PNG Missile against PNG defender)

We consider the engagement parameters as shown in Table 2. This set of initial parameters satisfies the missile evading condition as given in Theorem 1. The trajectories are shown in Fig. 2 (a) where the missile

$Parameter$	$Value$
λ_{m0}	0 deg
$R_d(0) = R_{dm}(0)$	5 km
γ_{m0}	220 deg
γ_{d0}	0 deg
γ_{t0}	90 deg
N_m	3
N_d	3

Table 2. Engagement parameters

successfully evades the defender and subsequently intercepts the target. The defender-missile line of sight rotates at a very high rate as $t \rightarrow t_{f1}$ thus resulting in defender lateral acceleration saturation as shown in Fig. 2 (b). Fig. 2 (c) shows the missile lateral acceleration history which is a typical PNG profile. The ratio of defender to missile lateral accelerations is plotted in Fig. 2 (d) showing a higher defender terminal lateral acceleration requirement. The sample run shows that by suitably choosing the initial conditions satisfying Theorem 1, a PNG guided missile can evade an identical defender and intercept the target.

B. Case 2: PP Missile against PNG defender

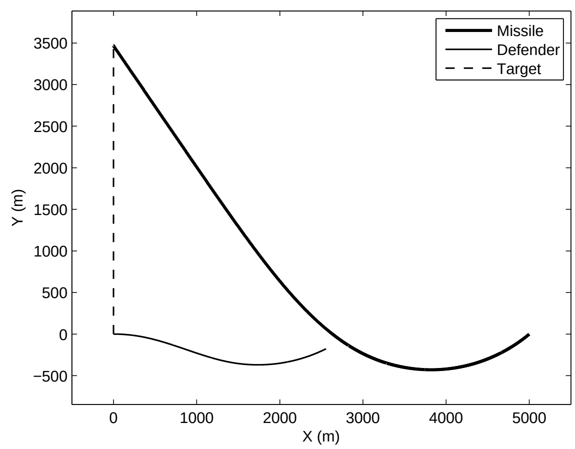
The engagement parameters are the same as in Case 1. The trajectories are shown in Fig. 3 (a) where the missile successfully evades the defender and subsequently intercepts the target. Defender lateral acceleration, as shown in Fig. 3 (b), is high initially owing to a high defender pure pursuit heading error. This high magnitude missile maneuver causes the typical PNG defender lateral acceleration to saturate as plotted in Fig. 3 (c). The ratio of defender to missile lateral accelerations is plotted in Fig. 2 (d) where the magnitude increases to more than unity and finally assumes a near constant value as the defender lateral acceleration response saturates.

C. Case 3: PNG Missile against LOS defender

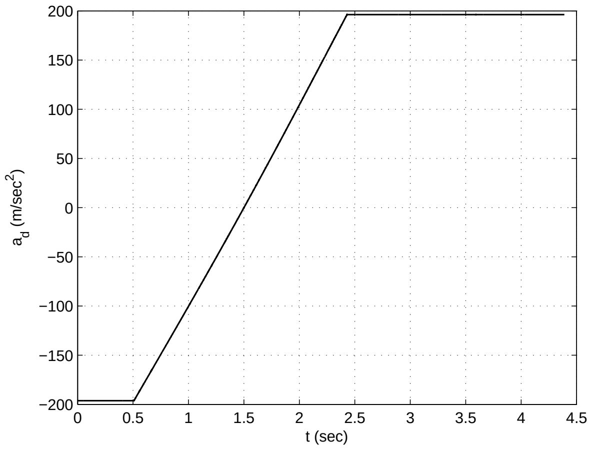
A scenario with same engagement parameters as for Case 1 and Case 2 is considered for a defender using LOS guidance against a missile using PNG law. The trajectories are plotted in Fig. 4 (a) with a successful defender-missile interception. The missile follows the typical PNG command as shown in Fig. 4 (b) with initial saturations because of the heading error. The resulting defender CLOS lateral acceleration, as shown in Fig. 4 (c), is relatively lower. Also, as plotted in Fig. 4 (d), the defender to missile lateral acceleration ratio is less than unity and goes to the value less than 0.33 as claimed in Ref. [10].

D. Case 4: PP Missile against LOS defender

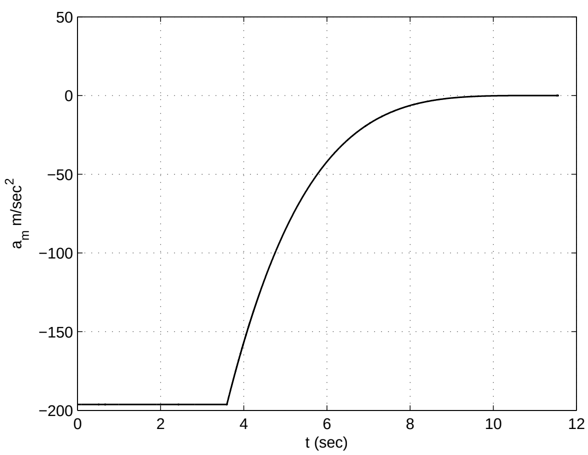
The same scenario as for the previous case is considered with the missile using PP law. The trajectories are plotted in Fig. 5 (a) with a 29.8 m miss distance between the defender and the missile and a successful interception of the target. The defender lateral acceleration, as shown in Fig. 5 (b), increases from an initial high value and saturates in the endgame. The missile follows a typical PP command as shown in Fig. 5 (c).



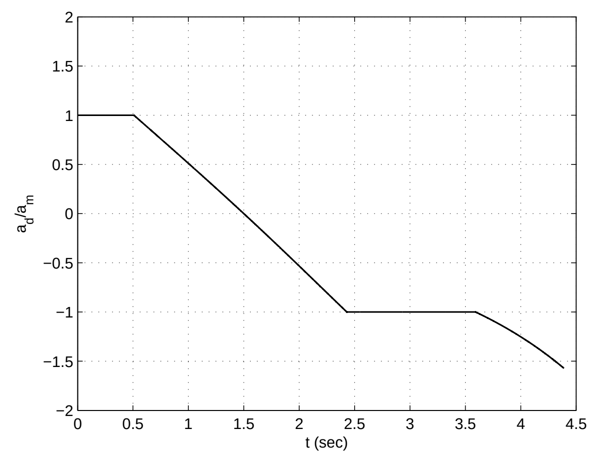
(a) Trajectories



(b) Defender lateral acceleration

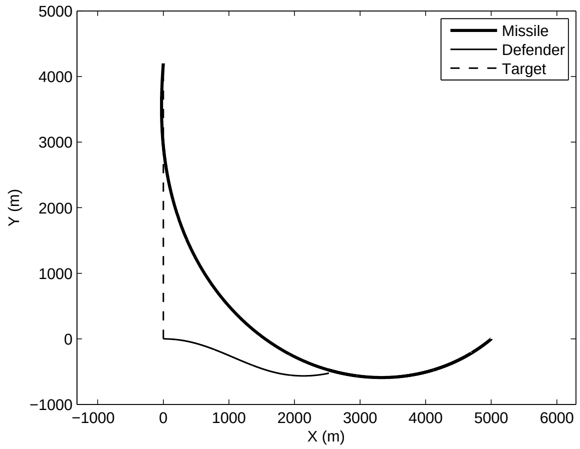


(c) Missile lateral acceleration

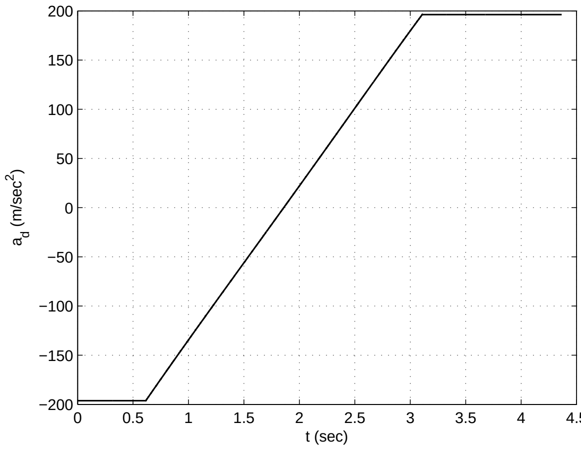


(d) Defender to missile lateral acceleration ratio

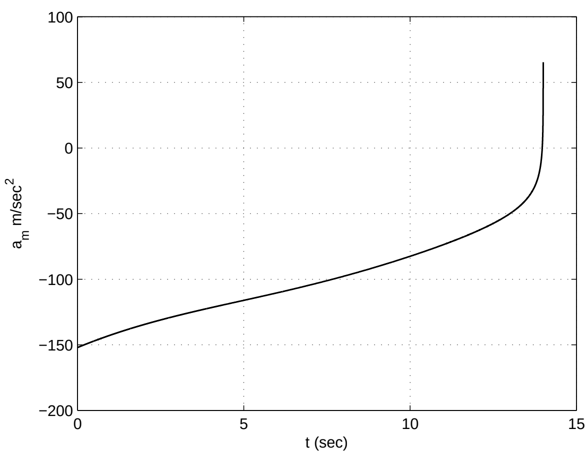
Figure 2. Missile (PNG) against a defender (PNG): Missile mission successful



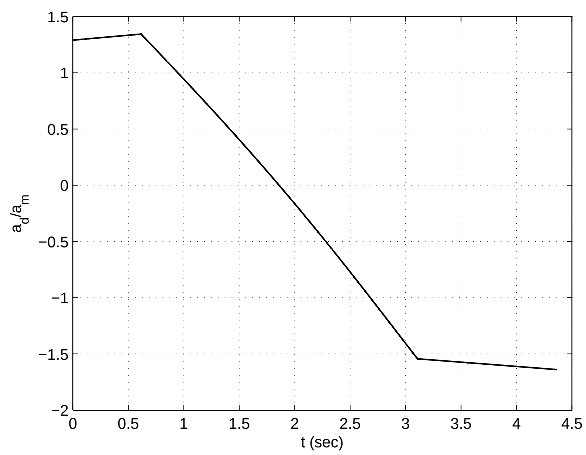
(a) Trajectories



(b) Defender lateral acceleration

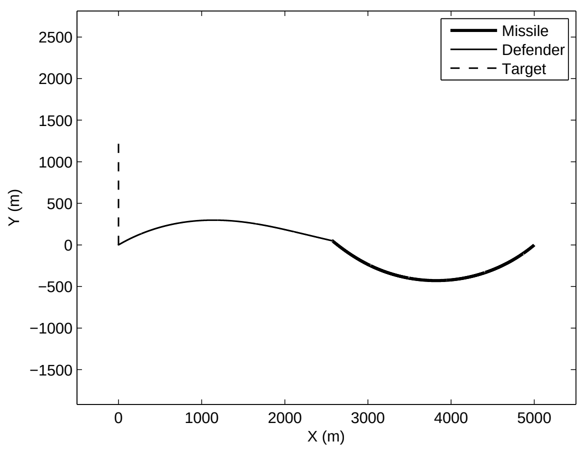


(c) Missile lateral acceleration

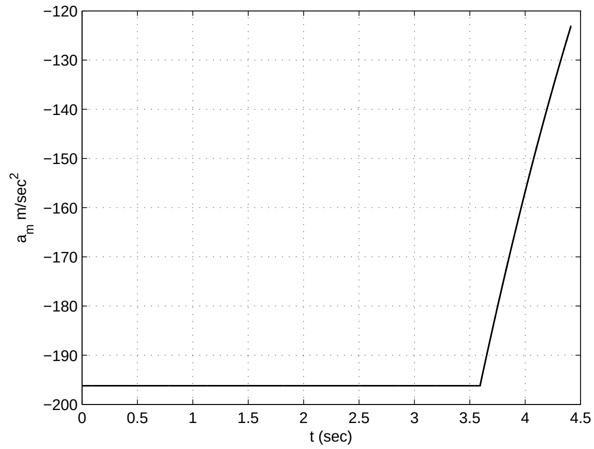


(d) Defender to missile lateral acceleration ratio

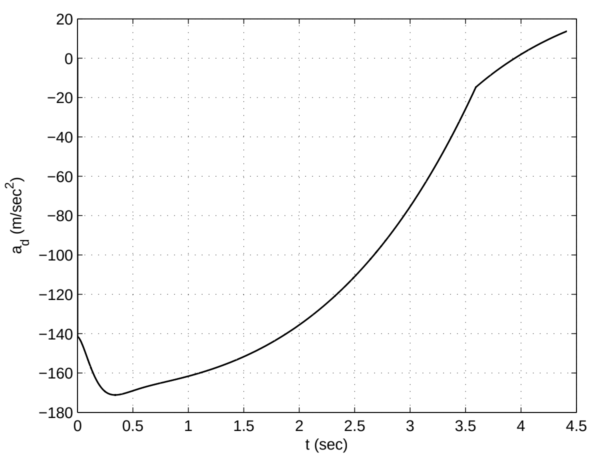
Figure 3. Missile (PP) against a defender (CLOS): Missile mission successful



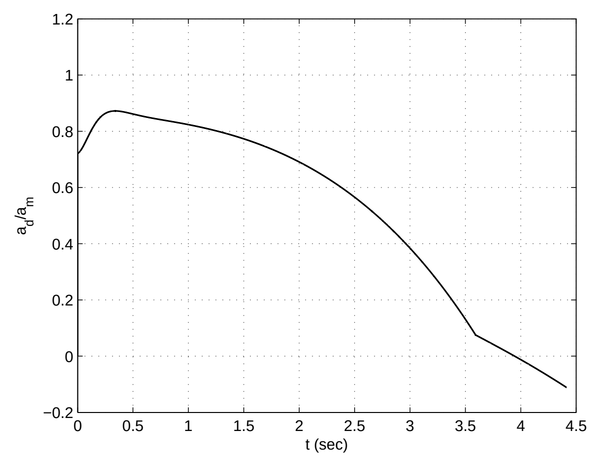
(a) Trajectories



(b) Missile lateral acceleration



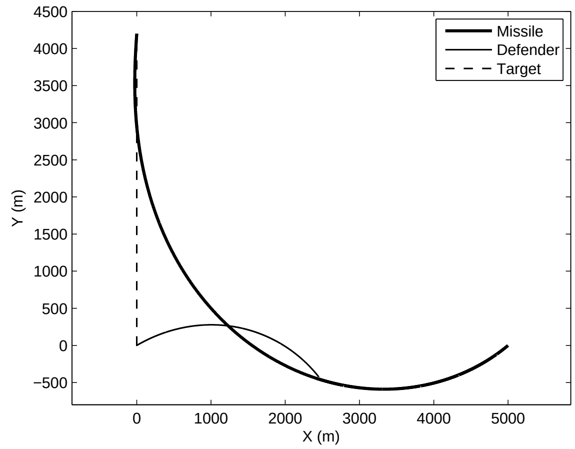
(c) Defender lateral acceleration



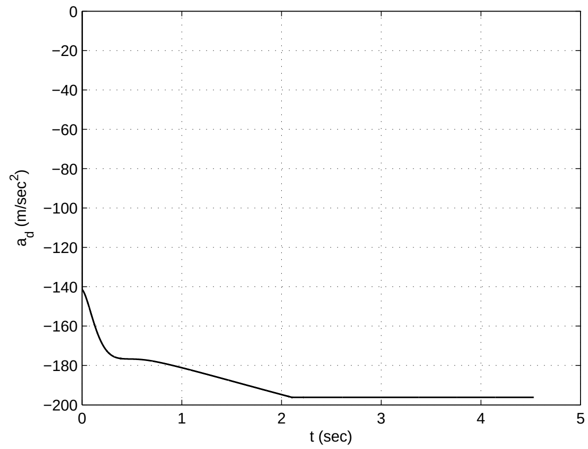
(d) Defender to missile lateral acceleration ratio

Figure 4. Missile (PNG) against a defender (CLOS): Defender mission successful

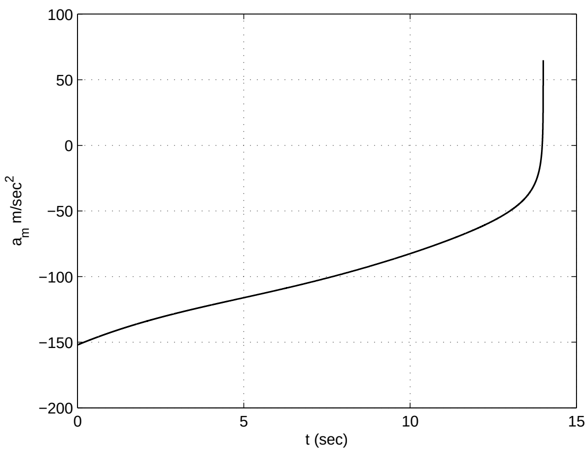
The defender to missile lateral acceleration ratio is plotted in Fig. 5 (d) with the terminal value satisfying Theorem 3.



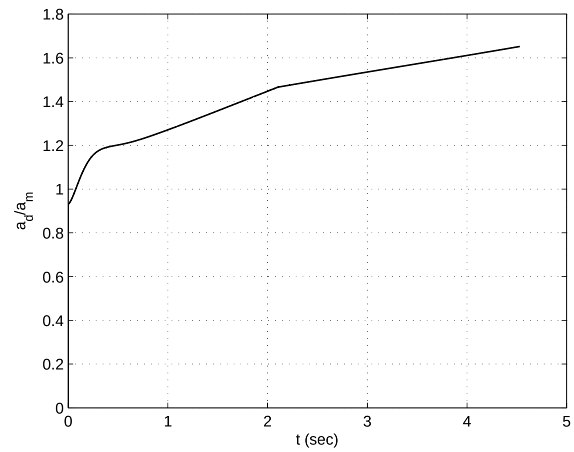
(a) Trajectories



(b) Defender lateral acceleration



(c) Missile lateral acceleration



(d) Defender to missile lateral acceleration ratio

Figure 5. Missile (PP) against a defender (CLOS): Missile mission successful

E. Launch envelopes for the Cases 1-4

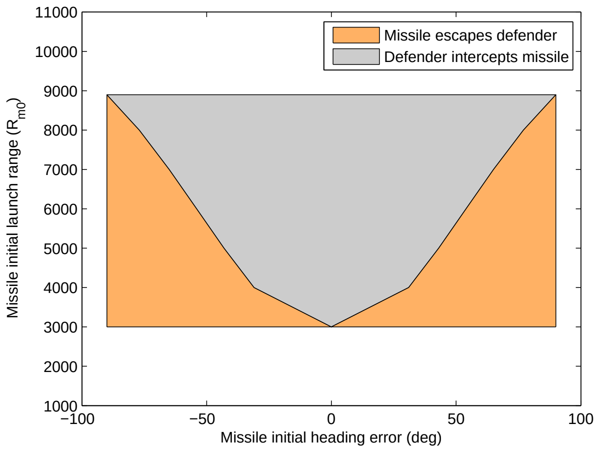
Comprising of various sample runs, we now simulate engagements with different missile launch angles and initial ranges to generate launch envelopes. A miss distance of less than 1 m is considered a successful defender-missile interception. Results are plotted in Fig. 6. Results for Case 1 with both the adversaries using PNG law is shown in Fig. 6 (a) here the capture envelope follows within 5% error the analytic condition of Theorem 1. For a given launch range, the missile maneuver magnitude increases with an increasing heading error. This results is a higher defender acceleration demand to intercept the missile and hence the missile enters the orange region where it escapes the defender by saturating its lateral acceleration response. For a missile launching range less than 3 km from the target, it escapes the defender for all the launch angles. For a launch range greater than 8.9 km the defender always intercepts the missile for any launch angle. This can be credited to the fact that for higher launch ranges the missile acceleration levels reduce considerably at t_{f1} and hence the defender-missile interception. Results for Case 2 are plotted in Fig. 6 (b) where the successful region for the missile reduces slightly as the pure pursuit missile command is relatively lower at t_{f1}. This results in a reduced defender lateral acceleration demand and hence a reduces launch envelope. Results for Case 3 show a drastic performance degradation for a PNG missile against LOS defender and the minimum launch range reduces to 3.5 km. LOS guided defender exploits the line of sight rate based PNG missile homing maneuvers onto the aircraft and guarantees an acceleration demand which is one third[10] of the missile. The best missile performance is obtained in Case 4 where the launch range increases to 10694 m by using PP guidance strategy against a defender using LOS guidance. A lower pure pursuit homing maneuver near t_{f1} results in higher defender LOS acceleration as governed by (57). Results highlight the use of pure pursuit as a prospective strategy against a defender using LOS guidance. The maximum launch ranges for all the four strategy combinations are listed in Table 3. The analytic values match the simulation findings to a very good extent.

	$R_{m0}(m)$ Analytic	$R_{m0}(m)$ Numerical
Defender (PNG) vs Missile (PNG)	9290 (from Theorem 1)	8900
Defender (PNG) vs Missile (PP)	8020 (from Proposition 1)	8100
Defender (CLOS) vs Missile (PNG)	3096 (from Proposition 2)	3500
Defender (CLOS) vs Missile (PP)	10694 (from Proposition 3)	10500

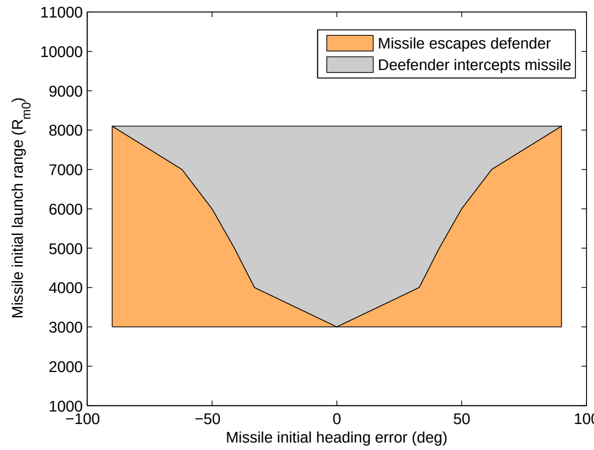
Table 3. Maximum launch range R_{m0} for a successful missile mission

V. Conclusions

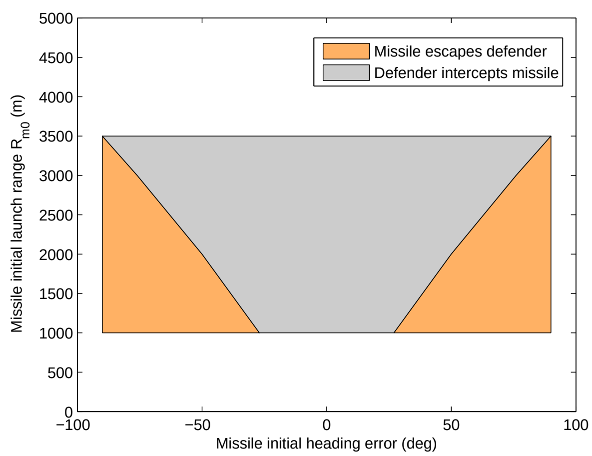
Analysis of proportional navigation and pure pursuit missile strategies against proportional navigation and line-of-sight guidance defender strategies, respectively, in the three body engagement scenario is presented. Lateral acceleration ratios and launch envelopes are derived analytically. Closed form expressions for missile launch conditions for a successful missile evasion from the defender are derived. Extensive numerical simulations are carried in support of the analytical findings. Against the defender using proportional navigation both the missile strategies have a comparable capture zone. Against the defender using line-of-sight guidance, pure pursuit missile strategy results in a drastic improvement in performance.



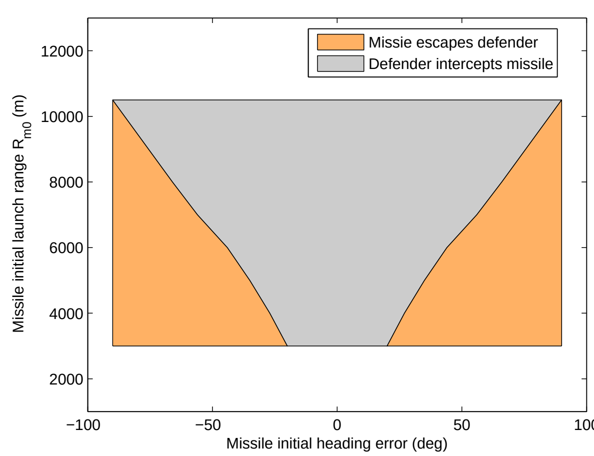
(a) Defender (PNG) vs Missile (PNG)



(b) Defender (PNG) vs Missile (PP)



(c) Defender (CLOS) vs Missile (PNG)



(d) Defender (CLOS) vs Missile (PP)

Figure 6. Launch envelopes

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