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Safe Control with Control Barrier Function for Euler-Lagrange Systems Facing Position Constraint

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Abstract: This paper provides a control barrier function (CBF)-based control for general Euler-Lagrange (EL) systems to ensure safety. The objective is to guarantee the forward invariance of the safe set. In the presence of constraints, directly applying the nominal control may make the system violate constraints. It could deteriorate the system stability and cause hardware failure in the worst case. To address this problem, we propose a control scheme and formulate the corresponding control barrier function. Furthermore, we present control of constrained 2-DOF manipulators as an example to show how to implement our proposed solution to EL systems. The effectiveness of the proposed solution is demonstrated through simulations.

Keywords: Safety Control, Euler-Lagrange Systems, Control Barrier Function.

1. INTRODUCTION

Considering Euler-Lagrange (EL) systems as the agent model is important in view of the fact that a variety of robot dynamics are modeled as EL systems. EL systems capture a large class of modern engineering problems, especially some of which are intractable with linear control tools [1]. Most of the plants in the real application such as spacecraft [2], wheeled robots [3], and manipulators [4] have a motion that can be derived via the EL equation [5].

The constraint problem is inevitable, and it has been one of the most typical challenges in control theory and application. Several control techniques have been proposed to deal with constraints for EL systems. In [6], avoidance control were provided for Lagrangian dynamics. Similarly, [7] proposed a passivity-based reference governor for constrained EL systems. With a different method, Barrier Lyapunov function based output-constrained control for EL systems was proposed by [8]. For multiple EL systems, [9] investigated distributed error constrained tracking control. All of them ensure the fulfillment of constraints indirectly through energy and Lyapunov consideration. On the contrary, this paper proposes the control algorithm by considering the sub-level set of a function directly corresponding to the constraint. We ensure the fulfillment of constraints by using a control barrier function (CBF)-based control.

Theory and applications of CBF have been deeply presented in [10]. Numerous constraint problems have been solved by utilizing CBF. In [11], a safety control of a robotic swarm is proposed. However, only a simple integrator model was considered there. This paper upgrades [11] to the context of EL systems. Furthermore, unlike [12], that proposed a CBF for EL systems with input constraints, this paper proposes a CBF for EL systems facing the position constraint problem. The main contribution of this paper is to provide a CBF-based control to ensure obstacle avoidance/position constraint fulfillment for general EL systems, and prove that the safe set is forward invariant. As a comparison to our proposed method, a similar method called Barrier Lyapunov Function (BLF)-based control was presented by [13]. There, the safety conditions are met by designing the control input directly through a computation considering BLF. However, here we provide the control input by solving an optimization problem that minimizes the value between the nominal input and the applied input, subject to the conditions necessary to ensure the constraint fulfillment.

This paper is organized as follows: Section 2 covers the problem setting of constrained general EL systems. Section 3 covers our proposed solution containing the CBF formulation and the control scheme. Section 4 presents the problem and solution of manipulators as an example of how to implement the formulation in Section 3. Finally, Section 5 provides simulation results.

2. PROBLEM SETTING

2.1. Control Goal

Suppose the dynamics of the EL system is represented by

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q}) + g(q) &= \tau, \\ y &= f(q), \end{aligned} \tag{1}$$

where the vector of generalized coordinates $q \in \mathbb{R}^N$, the symmetric positive definite inertia matrix $M(q) \in \mathbb{R}^{N\times N}$, the vector of control input torque $\tau \in \mathbb{R}^N$, the vector of Coriolis and centrifugal forces $C(q,\dot{q}) \in \mathbb{R}^{N\times N}$, the vector of gravitational force $g(q) \in \mathbb{R}^N$, the output $y \in \mathbb{R}^P$, and $f : \mathbb{R}^N \to \mathbb{R}^P$.

In this paper, we face constraint that the system output should lie in the superlevel set of a function $h(y)$. We denote the safe set as

$$S := \{y \mid h(y) \geq 0\}. \tag{2}$$

Practically, in the scenario of this paper, the output y must avoid an obstacle $y_o \in \mathbb{R}^P$ with a certain distance $\delta_o > 0$.

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The main objective of this paper is to achieve

$$y(t) \in S \; \forall t \geq 0. \tag{3}$$

Then, suppose a reference $r \in \mathbb{R}^N$ is given to the system. Subject to (3), we have a soft objective to achieve

$$q \to r. \tag{4}$$

Note that there is the possibility that r could not be reached, depending on the form of (2) which is restricted by the constraint.

2.2. Nominal Control

Suppose that the system (2) is equipped with a local controller to achieve (4). A proportional-derivative (PD) controller as the nominal controller [14]:

$$\tau := M\left(Kp(r-q) - K_d\dot{q}\right), \tag{5}$$

where $K_p > 0$ and $K_d > 0$, is known to ensure the fulfillment of (4). Other existing local controllers might also work. However, the nominal controller is not the main discussion of this paper. Instead, this paper mainly focuses on the constraint fulfillment in the next section.

3. CONTROL STRATEGY

In this section, we will propose a CBF-based controller to ensure the forward invariance of (2).

3.1. Control Barrier Function

First, let us introduce the definition of CBF:

Definition 1: : (from [15]) Consider a control affine system $\dot{x} = F(x) + G(x)u$ together with the set $S \subset R^N$ as the safe set, where x is the state and $u \in U$ is the input. Then, the function h is a Zeroing Control Barrier Function (ZCBF) defined over a set D with $S \subseteq D \subseteq R^N$ if there exists an extended class-$\mathcal{K}$ function $\alpha(h)$ such that for all $x \in D$

$$\sup_{u \in \mathcal{U}} \left[L_F h(x) + L_G h(x)u + \alpha(h(x))\right] \geq 0, \tag{6}$$

where $L_F h(x)$ is the Lie derivative of $h(x)$ along $F(x)$ and $L_G h(x)$ is the Lie derivative of $h(x)$ along $G(x)$.

There are numerous studies providing CBF to ensure obstacle avoidance. As an example, for a system modeled by a simple integrator, facing the same obstacle avoidance problem as this paper, [11] proposed a CBF:

$$h_r(y) = \|y - y_o\|^2 - \delta_o^2. \tag{7}$$

It worked there. However, we see that the obstacle avoidance or position constraint is not trivially treated for EL systems. Since the derivative of (7) does not contain the control input in its expression. To address the issue, suppose the distance to the obstacle boundary $V(y) := \|y - y_o\| - \delta_o$, we propose the candidate ZCBF:

$$\tilde{h}(y, \dot{y}) := V(y) + \tau_d \dot{V}(y, \dot{y}), \tag{8}$$

where $\tau_d = 1.8$ is the time headway. There are numerous formulations of the concept including Time Headway and Time to Collision [16], especially for adaptive cruise control. To determine the value of τ_d, we use the general rule stated in [16]: the minimum distance between two cars is "half the speedometer". Hence, with V in m and $\dot{V}$ in m/s, we set $\tau_d = 1.8$. Due to the structure of equation (8), the forward invariance of the superlevel set constructed by (8) (i.e. the condition of $\tilde{h}(y, \dot{y}) \geq 0$) ensures the fulfillment of the position constraint (i.e. the condition of $V(y) \geq 0$). To explain this, let us first take the expression of $\dot{V}(y)$:

$$\dot{V} = \frac{(y - y_o)^T \dot{y}}{\|y - y_o\|}. \tag{9}$$

From (9), we know that the projected output velocity in the direction away from the obstacle, $\dot{V}$, will increase whenever the output position approaching the obstacle. Moreover, at the critical point of the position constraint, which is $V(y) = 0$, if $\tilde{h}(y, \dot{y}) \geq 0$ is ensured, then $\dot{V}(y, \dot{y}) \geq 0$. It yields the condition $V(y) \geq 0$ is ensured.

Moreover, let us clarify the relation between $h(y)$ and $\tilde{h}(y, \dot{y})$. From (2), the safe set is defined as the superlevel set of $h(y)$. However, with general EL systems modeled by (1), since $h(y)$ is a function of y, the derivative of $h(y)$ does not contain the control input τ. Accordingly, $h(y)$ cannot be a ZCBF candidate to ensure the forward invariance of the safe set. Hence, we propose a ZCBF candidate $\tilde{h}(y, \dot{y})$. Since it is a function of y and $\dot{y}$, the derivative of $\tilde{h}(y, \dot{y})$ contains τ. As we will show later in the proof of Theorem 1, there exists $\alpha(\tilde{h})$ such that (6). Thus, the superlevel set of $\tilde{h}(y, \dot{y})$ is forward invariant. As we revealed earlier that $\tilde{h}(y, \dot{y}) \geq 0$ ensures $V(y) \geq 0$, the condition $\tilde{h}(y, \dot{y}) \geq 0$ ensures $h(y) \geq 0$. Hence, the forward invariance of the safe set (2) is ensured.

Furthermore, let us present the main theorem of this paper.

Theorem 1: Suppose that $\frac{\partial f}{\partial q} \neq 0$ for all q such that $h(y) = h(f(q)) \geq 0$. Then, there exists a Lipshitz continuous controller $u(q, \dot{q})$ such that renders the set $\{(q, \dot{q}) | \tilde{h}(q, \dot{q}) \geq 0\}$ is forward invariant.

Proof: First, let us take the second derivative of y:

$$\ddot{y} = \frac{d}{dt}\left(\frac{\partial f}{\partial q}\right)\dot{q} + \frac{\partial f}{\partial q}\ddot{q}, \tag{10}$$

and the second derivative of V:

$$\ddot{V} = -\frac{\{(y - y_o)^T \dot{y}\}^2}{\|y - y_o\|^3} + \frac{\dot{y}^T \dot{y}}{\|y - y_o\|} + \frac{(y - y_o)^T}{\|y - y_o\|}\ddot{y}. \tag{11}$$

Considering (1) and (8) -(11), we take the derivative of $\tilde{h}$:

$$\begin{aligned} \dot{\tilde{h}} =& \dot{V} + \tau_d \Bigg(-\frac{\{(y - y_o)^T \dot{y}\}^2}{\|y - y_o\|^3} + \frac{\dot{y}^T \dot{y}}{\|y - y_o\|} \\ &+ \frac{(y - y_o)^T}{\|y - y_o\|}\left(\frac{d}{dt}\left(\frac{\partial f}{\partial q}\right)\dot{q} \right. \\ &\left. + \frac{\partial f}{\partial q}\left(M^{-1}\tau - M^{-1}(C(q, \dot{q}) + g(q))\right)\right)\Bigg). \end{aligned} \tag{12}$$

Then, by denoting

$$
\begin{aligned}
A =& \tau_d \frac{(y-y_o)^T}{\|y-y_o\|}\frac{\partial f}{\partial q}M^{-1};\\
b =& \dot{V} + \tau_d \Bigg(-\frac{\{(y-y_o)^T\dot{y}\}^2}{\|y-y_o\|^3} + \frac{\dot{y}^T\dot{y}}{\|y-y_o\|} \\
&+ \frac{(y-y_o)^T}{\|y-y_o\|}\left(\frac{d}{dt}\left(\frac{\partial f}{\partial q}\right)\dot{q}\right.\\
&\left.+\frac{\partial f}{\partial q}\left(-M^{-1}(C(q,\dot{q})+g(q))\right)\right)\Bigg),
\end{aligned}
\tag{13}
$$

we have a linear form of $\dot{\tilde{h}}$: $\dot{\tilde{h}} = A\tau + b$. The condition $\frac{\partial f}{\partial q} \neq 0$ for all q implies $A \neq 0$. Then, applying the feedback control

$$\tau_x := -\frac{A^T}{\|A\|^2}(b + \alpha(h)) \tag{14}$$

provides a specific example of $\tau \in \mathbb{R}^N$ fulfilling (6). Hence, the superlevel set of $\tilde{h}(q,\dot{q})$ is forward invariant. ■

Detailed discussions on the assumption in the theorem are left as future work, but it is ensured at least for the example in Section 4.

3.2. Control Scheme

In this section, we will provide the control scheme utilizing the CBF we have formulated in the previous section to achieve the objective. The control scheme is illustrated in Fig. 1. The nominal control provides the control

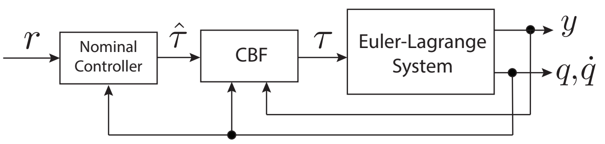

Fig. 1 Control Scheme

command to achieve (4). Then, by solving a quadratic programming problem, the CBF-based control acts like a filter that ensures the applied control of the plant is guaranteed to be safe.

The CBF-based control filters the nominal control $\hat{\tau}$ so that the input becomes a safe input to be applied to the plant. Besides that, we want to minimize the difference value between $\hat{\tau}$ and τ. This problem is regarded as a constrained optimization problem. Finally, τ is calculated through solving the following quadratic programming (QP) problem:

$$
\begin{aligned}
&\min_{\tau} \ \|\tau - \hat{\tau}\|^2\\
&\text{subject to } \ \dot{\tilde{h}} + \alpha(\tilde{h}) \geq 0.
\end{aligned}
\tag{15}
$$

In the next section, we will present an example to show how our proposed solution is utilized in the real application such as manipulators.

4. CONSTRAINED MANIPULATORS

In this section, we present a case of constrained manipulators as an example of an EL system facing a constraint problem. The structure of the manipulator in this scenario of this paper is illustrated in Fig. 2.

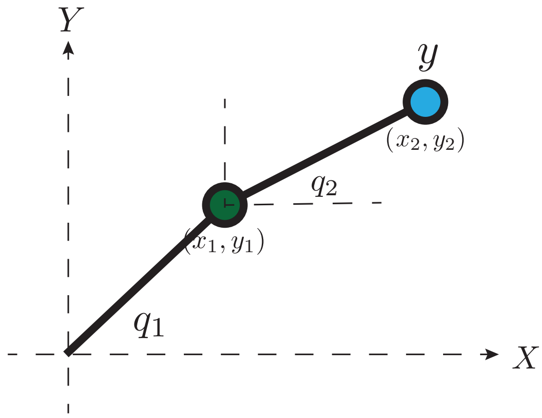

Fig. 2 Position constraint of the end-effector

4.1. Manipulator Structure and Motion

In this section, we provide the expressions of necessary terms needed for constructing the CBF of manipulators. First, we briefly introduce the manipulator structure in the scenario of this paper. For simplicity, we suppose a 2-DOF manipulator. We assume the mass of each link is located at each edge of the link (at the joint). We denote the mass of link-1 and link-2 by M_1 and M_2, respectively, and the length of link-1 and link-2 by L_1 and L_2, respectively. The dynamics of the manipulator is expressed as (1).

We define the position of the end-effector as the output of the manipulator. To eliminate complications in trigonometry terms, we denote the following:

$$
\begin{aligned}
S_1 := \sin(q_1);\ S_2 := \sin(q_2);\\
C_1 := \cos(q_1);\ C_2 := \cos(q_2);\\
S_{12} = \sin(q_1+q_2);\ C_{12} = \cos(q_1+q_2).
\end{aligned}
\tag{16}
$$

We set the following transformation from the angles of joints to the position in XY-plane:

$$x_1 = L_1C_1;\ y_1 = L_1S_2; \tag{17}$$

$$x_2 = L_1C_1 + L_2C_{12};\ y_2 = L_1S_1 + L_2S_{12}. \tag{18}$$

From Fig. 2, we calculate the output which is the position of the end-effector as

$$y(q) := \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} L_1C_1 + L_2C_{12} \\ L_1S_1 + L_2S_{12} \end{bmatrix}. \tag{19}$$

4.2. Control Barrier Function

In this scenario, we consider that the manipulator faces a position constraint problem as introduced in Section 2, which is a typical problem in the real situation. The manipulator must perform the obstacle avoidance so that the position of the end-effector avoids the obstacle.

We propose a CBF for constrained manipulators which has the same form as (8). For the CBF calculation for manipulators, we derive the following terms:

$$
\begin{aligned}
\frac{\partial f}{\partial q} &= \begin{bmatrix} -L_1S_1 - L_2S_{12} & -L_2S_{12} \\ L_1C_1 + L_2C_{12} & L_2C_{12} \end{bmatrix} \in \mathbb{R}^{2\times 2}; \\
\frac{d}{dt}\left(\frac{\partial f}{\partial q}\right) &:= \begin{bmatrix} d_1(q,\dot{q}) \\ d_2(q,\dot{q}) \end{bmatrix} \in \mathbb{R}^2, \\
d_1(q,\dot{q}) &:= (-L_1C_1 - L_2C_{12})(\dot{q}_1)^2 + (-L_2C_{12})\dot{q}_1\dot{q}_2 \\
&\quad + (-L_2C_{12})(\dot{q}_2)^2 \\
d_2(q,\dot{q}) &:= (-L_1S_1 - L_2S_{12})(\dot{q}_1)^2 + (-L_2S_{12})\dot{q}_1\dot{q}_2 \\
&\quad + (L_2S_{12})(\dot{q}_2)^2.
\end{aligned} \tag{20}
$$

For the nominal control $\hat{\tau}$, we use a PD controller (5). Finally, we obtain τ through solving (15).

5. SIMULATIONS

In this section, we will present the simulation results. First, we conduct the simulation without the safe control strategy. Then, we conduct the simulation with the safe control strategy.

5.1. Simulation without the safe control strategy

The structure of constrained manipulator in this simulation is illustrated in Fig. 3. For the mass and length

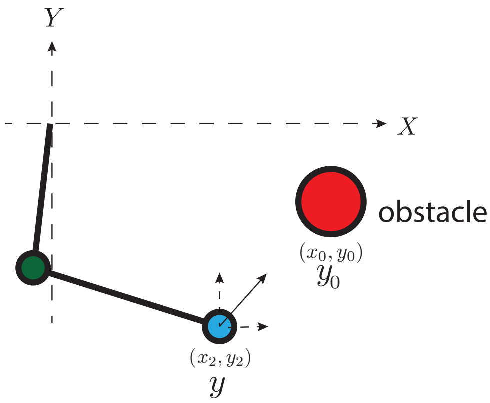

Fig. 3 Position constraint of the end-effector

of the manipulator, $M_1 = 1$ kg, $M_2 = 1$ kg, $L_1 = 1$ m, and $L_2 = 1$ m. The initial joint angle and velocity were $q(0) = [(-\frac{\pi}{2} - \frac{1}{4}) \ -\frac{\pi}{8}]$ rad, and $\dot{q}(0) = [0 \ 0]$ rad/s, respectively. The reference angle was $r = [-\frac{\pi}{2} \ 0]$ m. To control the joint angles, we used a PD controller with $K_p = 0.1$ and $K_d = 0.05$. For the obstacle, $y_o = [0 \ -2]^T$ m, and $\delta_o = 0.2$ m. We ran the simulation for 60 s.

The simulation result without the safe control strategy can be seen in Figs. 4–7. From Fig. 4, we see that the manipulator started to move toward the reference angle $r = [-\frac{\pi}{2} \ 0]$ m. From Fig. 5, we observe that q actually converged to r. Also, Fig. 6 confirms that τ converged to 0, indicating the convergence of the joint angle to the reference. Hence, the nominal controller successfully made the joint angle of the manipulator converge to the reference. Furthermore, since we did not employ any obstacle

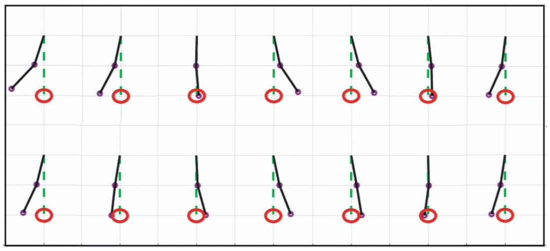

Fig. 4 Sequences of the manipulator movement without the safe control strategy (from the upper left to the lower right, green: reference, and red: obstacle).

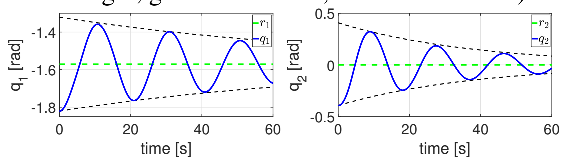

Fig. 5 Plot of the joint angle without the safe control strategy

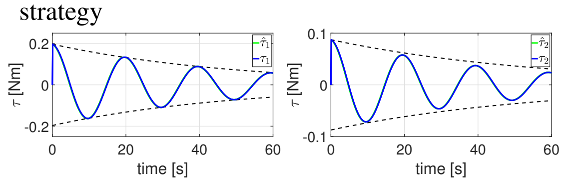

Fig. 6 Plot of the control command without the safe control strategy

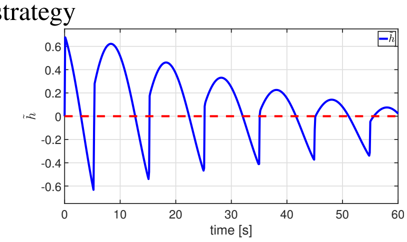

Fig. 7 Plot of $\tilde{h}$ value without the safe control strategy

avoidance mechanism here, it violated the constraint. In Fig. 4, it passed the obstacle. Also, in Fig. 7, the value of $\tilde{h}$ went to below 0. In summary, without the safe control strategy, the joint angle of the manipulator successfully converged to the reference, but it violated the constraint.

5.2. Simulation with the safe control strategy

The employ the same simulation setup and apply the safe control strategy with CBF. The simulation result with the safe control strategy can be seen in Fig. 8–11. From this simulation, we see that the nominal control made the manipulator move toward the reference. Moreover, from Fig. 10, we see that the safe control strategy suitably modified the control command to ensure safety. Namely, it successfully avoided the obstacle as illustrated in Fig. 8. Also, from Fig. 11, we see that the $\tilde{h}$ value always remained to be greater than 0 during this simulation, indicating the non-violation of constraint.

6. CONCLUSION

In this paper, we have proposed a control strategy for general EL systems subject to position constraints. We provided the control scheme and the formulation of CBF. Furthermore, we also provided a specific example of con-

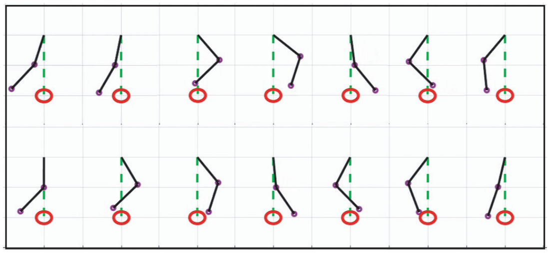

Fig. 8 Sequences of the manipulator movement with the safe control strategy (from the upper left to the lower right, green: reference, and red: obstacle).

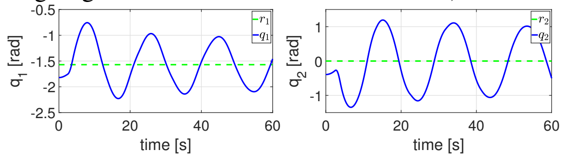

Fig. 9 Plot of the joint angle with the safe control strategy

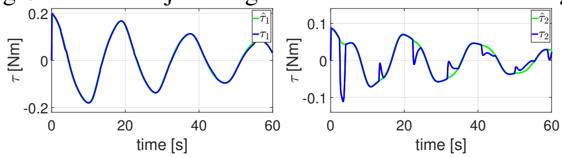

Fig. 10 Plot of the control command with the safe control strategy

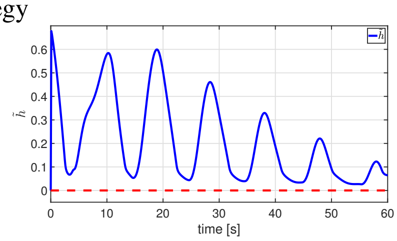

Fig. 11 Plot of $\tilde{h}$ value with the safe control strategy

strained EL systems. We showed how our proposed formulation of CBF is implemented for a manipulator to deal with a position constraint. We have conducted simulations to show the effectiveness of our proposed solution. The results showed that the nominal control drove the manipulator toward the joint angle reference and our designed CBF successfully ensured the fulfillment of the constraint. For future works, we could consider other forms of constraints and provide another example of the CBF implementation.

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