Guidance Laws in 3 body pursuit and evasion games

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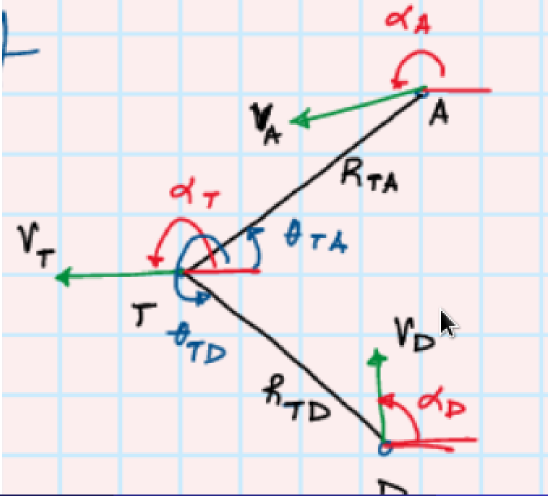
Overview

Problem Statement

The problem consists of three entities: an evading target denoted as T, an attacking missile denoted as A, and a defending missile denoted as D. We assume perfect information and analyze the engagement in two dimensions.



Problem Statement

We have the following state space equations:

$$\begin{bmatrix} \dot{R}_{TA} \\ R_{TA}\dot{\theta}_{TA} \\ \dot{R}_{DT} \\ R_{DT}\dot{\theta}_{DT} \\ \dot{\alpha_A} \\ \dot{\alpha_D} \\ \dot{\alpha_T} \end{bmatrix} = \begin{bmatrix} v_A cos(\alpha_A - \theta_{TA}) - v_T cos(\alpha_A - \theta_{TA}) \\ v_A sin(\alpha_A - \theta_{TA}) - v_T sin(\alpha_A - \theta_{TA}) \\ v_D cos(\alpha_D - \theta_{DT}) - v_T cos(\alpha_T - \theta_{DT}) \\ v_D sin(\alpha_D - \theta_{DT}) - v_T sin(\alpha_D - \theta_{DT}) \\ \frac{a_A}{v_A} u_A \\ \frac{a_D}{v_D} u_D \\ \frac{a_T}{v_T} u_T \end{bmatrix}$$

Thus, $|u_A| \leq 1$, $|u_D| \leq 1$ and $|u_T| \leq 1$. The objective of the Defender team is to bring $R_{AD} < \varepsilon$ while maintaining $R_{TA} > \varepsilon$. Similarly the objective of the Attacker is to bring $R_{TA} < \varepsilon$ while maintaining $R_{AD} > \varepsilon$. Where ε is the minimum safe distance.

Safety Critical Control

Consider the control affine system of the form

$$\dot{x} = f(x) + g(x)u \qquad x \in D, u \in U \tag{1}$$

Define the Safe set by

$$C_h := \{x \in D | h(x) \geq 0\} \tag{2}$$

$$\partial C_h := \{x \in D | h(x) = 0\}$$

$$Int(C_h) := \{x \in D | h(x) > 0\}$$

In order to ensure the safety of the system i.e ensure $Int(C_h)$ is forward invariant we use the ZCBFs.

Zeroing Control Barrier Functions

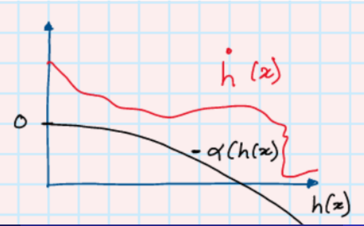
Definition ([1])

Consider 1 and C_h , h defined by 2. A continuously differentiable function $h\colon Int(C_h) \to \mathcal{R}$ is called a Zeroing Control Barrier function (ZCBF) if there exists class κ function α such that $\forall x \in Int(C_h)$

$$\sup_{u\in U}[L_f h(x) + L_g h(x)u + \alpha(h(x))] \geq 0 \tag{3}$$

Define $K_h(x)$ to be the set of admissible controls at x

$$K_h(x) := \{u \in U | L_f h(x) + L_g h(x)u + \alpha(h(x)) \geq 0\} \tag{4}$$



Mediating Safety using Quadratic Programs

One can use QPs when one has a control objective and a safety objective that cannot be simultaneously satisfied to choose a control that mediates between the control objective and safety objective.

$$
\begin{aligned}
\min_{u} \quad & ||u(x) - \hat{u}(x)|| \\
\text{s.t.} \quad & L_f h(x) + L_g h(x) u(x) + \alpha(h(x)) \geq 0, \\
& u(x) \in U
\end{aligned}
$$

where $\hat{u}(x) \in U$ is the nominal control that would ensure that the control objective is satisfied.

A general construction for the ZCBF

Define

$$h(x(t); \rho, \gamma) := \inf_{\tau \in [0,\infty]} \rho(\hat{x}(t+\tau)) \tag{5}$$

where ρ is the safety function i.e $C := \{x \in D | \rho(x) > 0\}$ and γ is some nominal safety control i.e the system can use γ to ensure its safety.

$$\hat{x}(t) = x(t) + \int_0^{\tau} \gamma(\hat{x}(s)) ds$$

We can see that $h(x(t); \rho, \gamma)$ is a valid ZCBF.

$$\begin{aligned}
\dot{h} &= L_f h(x) + L_g h(x) \gamma(x) \\
&= \lim_{a \to 0^+} \frac{1}{a} [\inf_{\tau \in [a,\infty]} \rho(\hat{x}(t+\tau)) - \inf_{\tau \in [0,\infty]} \rho(\hat{x}(t+\tau))] \\
&\geq 0
\end{aligned}$$

Hence $\gamma(x) \in K_h(x) \quad \forall \quad x \in C_h$ which implies that h is a valid ZCBF.

Approach 1: TAD as a Safety Critical QP

Let $\hat{u}_A(x)$ be some guidance law that enables pure-pursuit of the target by the attacker.
Let $h(x)$ be an appropriately defined ZCBF.

$$\begin{aligned} \min_{u} \quad & ||u(x) - \hat{u}(x)|| \\ \text{s.t.} \quad & L_f h(x) + L_g h(x) u(x) + \alpha(h(x)) > 0, \\ & |u(x)| \leq 1 \end{aligned}$$

Note: This is a pointwise optimisation problem.

$$h(x(t); \rho, \gamma) := \inf_{\tau \in [0, \infty]} \rho(\hat{x}(t + \tau))$$

we need to chose ρ and γ appropriately.

Designing γ given ρ

One strategy is to chose γ in a greedy way that drives the system towards the direction of maximum increase in $h(x)$ i.e in the direction of $\nabla h(x)$.

A way to do this would be to point the tangent vector of the system in the direction closest to that of $\nabla h(x)$.

However in our case, the tangent vector has the form $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_A \\ 0 \\ 0 \end{bmatrix}$

Clearly this wont give good results as the control does not directly affect the states but does it through some integrator layers.

We need a different approach to choose γ.

We use two choices of ρ in our simulations.

1] $\rho(x) = ((x_A - x_D)^2 + (y_A - y_D)^2)^{1/2}$

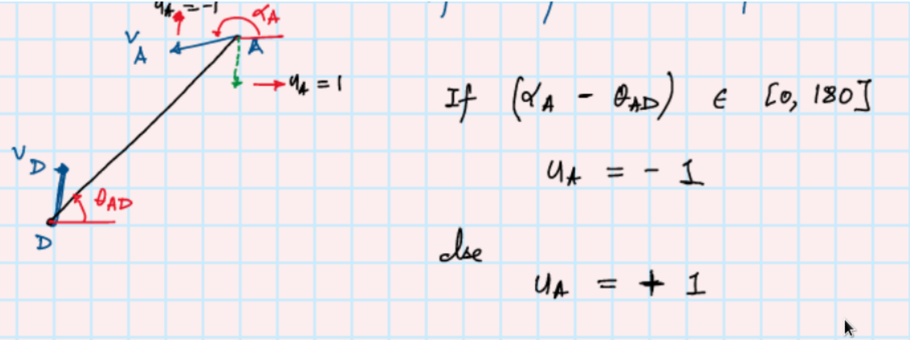


Figure: Choice of γ

2] $\rho(x) = \Delta\theta(x) = |\theta_{TA} - \theta_{DT}|$

As shown in [4] if the defender is able to position itself on the LOS joining A and T, then the defender has won. Thus, this safety function tries to ensure that $\Delta\theta(x)$ which is the angle between TA and TD does not fall below a threshold.

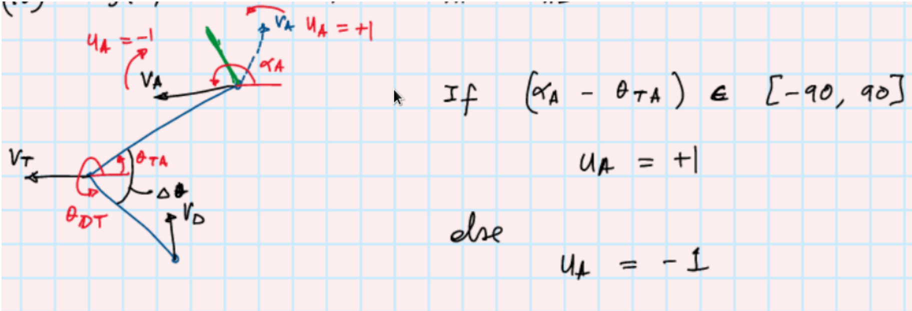


Figure: Choice of γ

Simulations

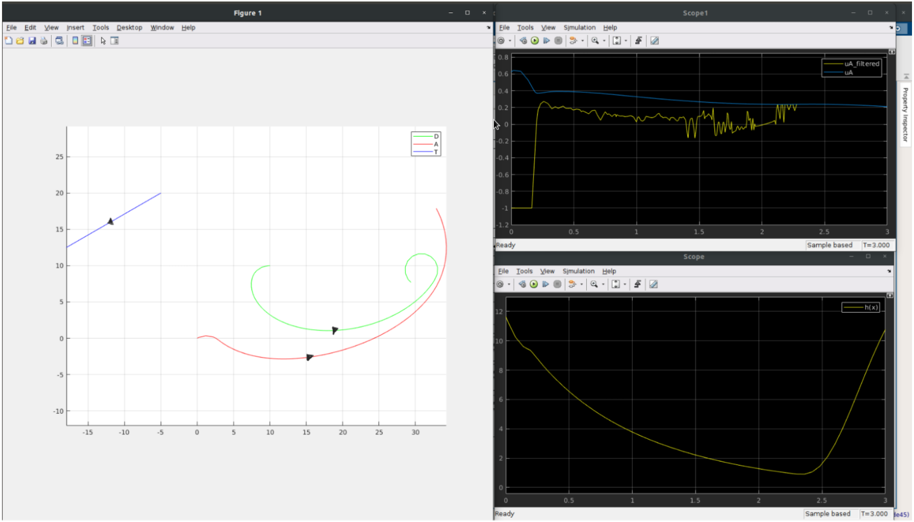


Figure: Attacker is successfully able to evade the Defender while pursuing the Target using 1st safety function

Simulations

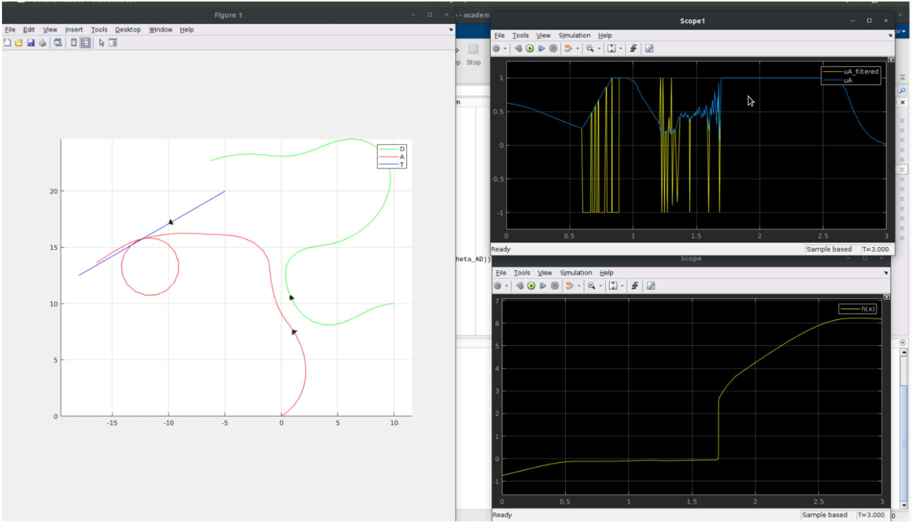


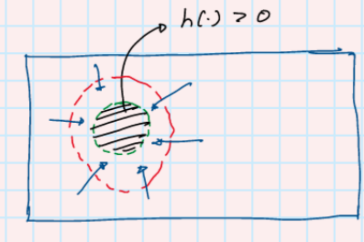
Figure: Attacker is successfully able to evade the Defender while pursuing the Target using 2nd safety function

Approach 2 : Using Time of Engagement

Let $T(x)$ be the time required for the attacker to capture the target with the initial state being x.

$$h(x(t); \rho, \gamma) := \inf_{\tau \in [0, T(x)]} \rho(\hat{x}(t + \tau)) \tag{6}$$

where γ is some guidance law where the attacker pursues the target ignoring the defender and h is the minimum separation between the attacker and defender under γ over the finite time horizon.



Case 1] $h(x(t); \rho, \gamma) > \varepsilon$

Capture is ensured and we are done

Case 2] $h(x(t); \rho, \gamma) \leq \varepsilon$

1. In this case we need to design a maneuver say $E(x)$ for the attacker to move in a way that increases the value of h(x) until $h(x) > \varepsilon$.
2. One way to do this would be to steer the system using the tangent vector in the direction of $\nabla h(x)$. However as described above, the system under consideration is not steerable using the tangent vector.
3. As done in the previous case (using the physical understanding of h(x)), one could use a guidance law where the attacker purely evades the defender. However there is no guarantee that this is the optimal choice. Thus, this remains a further area of research.

Future Work : Numerical Studies

1. Under Approach 1, simulations need to be conducted with different choices of nominal guidance strategies and varying initial conditions.
2. Furthermore, these simulations need to be compared with the existing literature for the TAD problem (Specifically those implementing cooperative guidance against the attacker).
3. Under Approach 2, various heuristic based maneuvers E(x) need to be evaluated using simulations and their performance compared with that of Approach 1.

Future Work: Theoretical Investigations

1. An open challenge to obtain a closed-form expression of the barrier function for the system under consideration still prevails. One could also characterize the capture regions numerically if a closed-form expression for a ZCBF is available.
2. Designing the maneuver $E(x)$ with a theoretically sound justification remains to be explored. One could use ideas from backstepping to steer the system in the direction of $\nabla h(x)$ or explore something completely different.

References

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Github Repository

The End