NOTES FOR LITERATURE REVIEW IN MEASUREMENT IN QUANTUM MECHANICS

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Contents

ABSTRACT

This is a collection of short notes which I have written based on my understanding of the research papers that I have read under the guidance of Professor Amber Jain for my SURP. This is in no way an exhaustive summary of the papers that we have read but merely serves the purpose to summarise key points and important understanding we got from each paper.

1 EPR experiment and Bell inequalities

So we started off with the famous EPR experiment. To boil down the details of this experiment we start off with a state describing two entangled particles. We label the state as $\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2)u_n(x_1)$ here we merely regard $\psi_n(x_2)$ as coefficients for Ψ when expanded in the $u_n(x_2)$ basis for some operator A for which those are the eigenvectors. However if we pick a different operator B which has we can get a different expansion with respect to it's eigenvalues which lets suppose to be $v_s(x_1)$ so we get $\Psi(x_1, x_2) = \sum_{s=1}^{\infty} \phi_s(x_2)v_s(x_1)$. Now the real meat is when we take A and B to be non commutative. The example in this paper is taking $P = (h/2\pi\iota)\partial/\partial x_2$ and $Q = x_2$ which do not commute so in the case where we choose either of these measurements, we affect the other particle in a way where we can change it's state just by our choice hence this goes against local realism and is the spooky action at distance.

While there have been multiple responses to this, the most important one is the one discussed in Bell's paper. First we will start by considering the EPR argument using spin particles. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma_1}$ and $\vec{\sigma_2}$, If measurement of the component $\vec{\sigma_1} \cdot a$, where a is some unit vector, yields the value +1 then, according to quantum mechanics, measurement of $\vec{\sigma_2} \cdot a$ must yield the value -1 and vice versa. We now define some set of parameters λ which is taken as continuous which give us a complete description of the state. The result A of measuring $\vec{\sigma_1} \cdot a$ is then determined by a and λ, and the result B of measuring $\vec{\sigma_2} \cdot b$ in the same instance is determined by b and λ.

$$A(\vec{a}, \lambda) = \pm 1, B(\vec{b}, \lambda) = \pm 1 \tag{1}$$

We now take $\rho(\lambda)$ as the probability distribution of λ and we will now try to find the expectation value of of the product of the two components $\vec{\sigma_1} \cdot a$ and $\vec{\sigma_2} \cdot b$.

$$P(\vec{a}\vec{b}) = \int d\lambda \rho\lambda A(\vec{a}, \lambda)B(\vec{b}, \lambda) \tag{2}$$

The quantum mechanical expectation value of this is the following

$$\langle \vec{\sigma_1} \cdot a \vec{\sigma_2} \cdot b \rangle = -\vec{a} \cdot \vec{b} \tag{3}$$

However it turns out that eq: 2 doesn't give us the same result as 3. So first off we know that we have a normalized distribution.

$$\int d\lambda \rho(\lambda) = 1 \tag{4}$$

if we take $\vec{a} = \vec{b}$ the P in eq: 2 can reach -1.

$$A(\vec{a}, \lambda) = -B(\vec{a}, \lambda) \tag{5}$$

Knowing this we can rewrite eq: 2 into the following

$$P(\vec{a}\vec{b}) = -\int d\lambda \rho\lambda A(\vec{a}, \lambda)A(\vec{b}, \lambda) \tag{6}$$

$$P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) = \int d\lambda \rho\lambda A(\vec{a}, \lambda)A(\vec{b}, \lambda)[A(\vec{b}, \lambda)A(\vec{c}, \lambda) - 1] \tag{7}$$

from eq: 1 we can write this

$$|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq \int d\lambda \rho\lambda A(\vec{a}, \lambda)A(\vec{b}, \lambda)[A(\vec{b}, \lambda)A(\vec{c}, \lambda) - 1] \tag{8}$$

The second term on the right is just $P(\vec{b}, \vec{c})$ so we get

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \tag{9}$$

Now we define a $\overline{P}(\vec{a}, \vec{b})$ and a $\overline{-\vec{a} \cdot \vec{b}}$ which essentially are the averages over vectors differing from a small angle from $\vec{a}$ and $\vec{b}$ of the quantities under the bar. Let's suppose the following holds for some ϵ

$$|\overline{P}(\vec{a}, \vec{b}) + \overline{\vec{a} \cdot \vec{b}}| \leq \epsilon \tag{10}$$

We cannot make ϵ arbitrarily small and this can be proved by using normal algebra (refer the paper) and the inequality in eq: 9 since we can see that unless we take P to be a constant function we can see that the $P(\vec{b}, \vec{c})$ function will not be stationary at the minima of $\vec{b} = \vec{c}$ since for small $|\vec{b} - \vec{c}|$ the order of rhs in eq: 10 would be of $|\vec{b} - \vec{c}|$.

So these inequalities do not hold implying that some assumptions that we have taken happen to not work together and that happen to be local determinism and hidden variables. So any hidden variable theory by nature itself is non local.

2 Interpretations of Quantum Mechanics

2.1 Bohmian Mechanics

This is one of the most important and popular hidden variable theory of quantum mechanics. This happens to be a non local theory which is deterministic in nature. So we essentially look at the wavefunction in a different manner in this interpretation as we take our

$$\psi = R\exp(\iota S/\hbar) \tag{11}$$

and we define a new quantum mechanical potential

$$U(x) = \frac{-\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{12}$$

and also we get the velocity as $v = \nabla S/m$. Now here's where things get interesting. This is not the actual "velocity" of the particle in question here even though the original paper kinda calls it that. The actual velocity would be obtained from a combination of this and the quantum mechanical potential converted to a kinetic energy. A thing to note is that when you have a classical system your quantum mechanical potential's contribution would in fact be close to zero because this would mean $\lim \hbar \to 0$ so this is why the Bohmian wave equation actually turns to the classical hamilton jacobi equation at the classical limit and $v = \nabla S/m$ makes sense.
Also the equations for probability (refer eq. 5 in paper 1) actually are similar to that of a modelung flow and its pretty interesting how we can see that the particle is sort of being flown or guided by something and there were experiments which were done using droplets guided by a Faraday instability on the liquid surface however these actually have a source term in their wave term which is not used in Bohmian mechanics and it can be seen that they have a breaking point in a modified Schrödinger equation. This was explored in ()
The potential $U(x)$ actually provides a lot of explanations like if the wave function vanishes then this potential approaches infinity and so it's quite obvious why we wont find the particle there and it balances out energy at points where if particle were to be found would imply a negative kinetic energy and so explains tunneling phenomenon. This also gives this interpretation non locality since a change in ψ would change this potential and can affect things that are much further like in the case of EPR experiment. Also it explains the double slit experiment by saying that the pilot wave would interfere and since they also have effect on the particle its only on measurement that things would change(see fig: 1).
This interpretation also uses hidden variables which actually do not contribute much in the understanding since it treats any measurable as a hidden variable beforhand. The collapse is explained by apparatus and object interaction citing the example of Von neumann where a rapidly acting Hamiltonian causes the states to decohere and the particle would end up in the space of one of these states depending on the initial position which remains hidden. We cannot observe this since it happens far to rapidly and the equations have been demonstrated to describe chaotic motions which is what causes the collapse. In the second paper Bohm also demonstrates the formulation of photoelectric and compton effect using his interpretation and they seem to be successfully explained using this interpretation.
All in all while providing a fairly solid interpretation, it has been critiqued for its inelegant mathematical formulation (take a look at Appendix A of paper 2 and you will see why) and also it doesn't really offer a method of actually making use of the "hidden variables" at play here and also it can be taken as just a different mathematical formulation of the same theory. Another criticism it has received is that it is pretty much equivalent to the Everettian Many worlds interpretation with the only difference being that the latter doesn't use hidden variables and leaves the idea to randomness. While on close examination these theories are actually quite different, they do use the same decoherence principles.

2.2 The many worlds interpretation

This is one of the more popular interpretations of quantum mechanics currently and is possibly the most clean one for explanation of quantum phenomena. So essentially according to this interpretation, the whole of quantum mechanics follows only the deterministic evolution which is described by the Schrödinger evolution and in the act of measurement all the results would actually be obtained and will exist simultaneously but would be in different worlds causing us to only experience one of the results. This splitting of worlds on measurement is explained using the Von neumann process of the very large Hamiltonian interaction of the measurement apparatus. This splitting is essentially a decoherence and its noted that it takes an actual non zero amount of time.
The original paper uses the terminology of a "relative quantum state" where a state is essentially defined relative to a basis in a certain Hilbert space. Essentially equivalent to how we just simply write the state of two systems by their multiplication. The relative state formulation is pretty much the same as how we take a partial trace for density matrices. The interesting approach of this paper is that it treats measurement as a splitting of worlds due to decoherence of the interaction of apparatus to observed system. The branching mechanism actually arises due to the way the wave

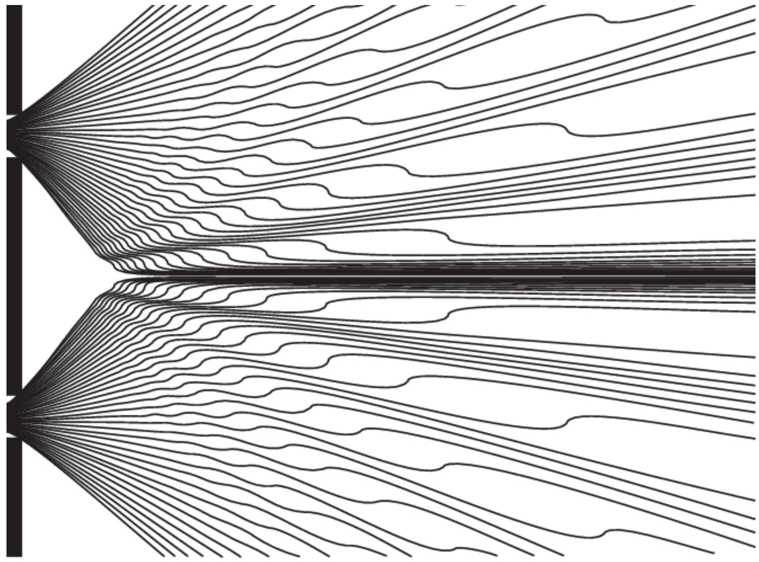

Figure 1: Possible Bohmian double slit trajectories

equation is now represented.
So we start off by having a wave function ψ and a memory register. Our wave function defines a system of multiple particles and as we measure each state, the relative state for that basis is where the change occurs and we then randomly arrive on one of the eigen states. Thing is by this formulation, the actual ψ doesn't change but rather the memory register does since we are simply expanding our wave function in each measurement basis. So in short, the observer can only see one of the results but the rest of them do exist but are outside our memory register. A clean idea but a rather strong one at that.
One of the easiest problems here to point out is that why does it have to expand in that very basis? The choice of basis would easily define the whole observed wave function so why is this basis special. One way to see this is that simply put, its the result of an observation and the eigen functions which define that basis are the wave packets after the decoherence but this still doesn't convince everyone. The next thing this interpretation tackles is the born probability rule. It does so by assuming that this function would have to follow certain rules like a modulo 1 rule and multiplication for independent systems and that way the only possibility of the probabilities of an observation come from the weight of that eigen function since its proven to be the only possible function.
This again is an actively debated topic since many people are not convinced that this would actually be the probability and it actually is a fair doubt since what would probability mean over here exactly? Given that all the states are present their probabilities is a pretty meaningless thing to discuss but this probability states the statistical chance that your memory has that value. In 1999 Deutsch argued that these probabilities must be assigned as the branch weights and that actually makes more sense. Then again the whole concept of observation and memories actually lead to plenty of discussion but by far this is the one formulation which involves the least rigor and is preferred for explanation of most cosmological phenomena cause a relativistic variant of this can be developed.

2.3 QBism

This interpretation is a very nascent one and in our discussions we have not explored this one all too well. This interpretation actually has a lot of philosophy in it and I honestly found the original paper to be quite thought provoking. To put simply this interpretation treats the wave function to be just a meaningless quantity. The observation of a system simply updates the wave function based on the statistical prediction one obtains from it which makes collapse seem to be an obvious route. The probabilities used are bayesian probabilities and also the projection operators $|\psi\rangle$ defined for some d dimensional hilbert space are defined by this relation

$$|\langle\psi_i|\,|\psi_j\rangle|^2 = \frac{1}{d+1} \text{ for } i \neq j \tag{13}$$

These are called the "symmetric informationally complete positive-operator valued measure," or SIC. Now there does happen to be a very properly explained method for why this makes sense but the math was far beyond my level of understanding so I cant say I understood this theory in complete sense.
However on boiling down, this theory explains all phenomena in a very obvious sense but does leave a lot of questions.

In the paper the author addresses how this interpretation can be easily mistaken as solipsism and goes on to explain how it is quite different from this but all in all this is an interpretation which cannot be proven wrong, but cannot be proven right either and so we decided that exploring this further may not be of much use to us.

2.4 Objective Collapse theories

There are multiple objective collapse theories and we will describe GRW and CSL.
We will start by describing the GRW Model. To summarise the whole idea of the GRW model is that every component of our state is subject to spontaneous localization which is multiplication by this Gaussian in eq: 14

$$G(q_i, x) = K \exp[-\{1/(2d^2)\}(q_i - x)^2] \tag{14}$$

And the localization occurs over the overall state as shown in 15

$$L_i(q_1, q_2, \ldots, q_N; x) = F(q_1, q_2, \ldots, q_N)G(q_i, x) \tag{15}$$

Now how does this localization occur? It follows a probability distibution which resembles that of a poisson distribution with some frequency λ. We will be taking this λ and the d in eq: 14 as universal constants. With experimental analysis we have obtained limits on these constants and in GRW the proposed values are

$$\lambda = 10^{-16}s^{-1} \text{ and } d = 10^{-7}m \tag{16}$$

From here it is clear that in a single particle system, if left unmeasured it would collapse after a time which is longer than the life of the universe. Now an important factor of this theory is that this frequency scales up with the size of the system so in the case we have a mole of something, we have collapses occurring every 10^{-7} seconds and that makes sense since that would be in the classical realm and of course measurement by itself is just a disturbance so that affects quantum systems by noticeable manners only. Now you may have noticed a big drawback of this theory being that it cannot account for symmetries and anti symmetries which are present in actual quantum systems and more importantly **energy isn't conserved** in this. Large scale systems will have their energy conserved on averaging out but given that an undisturbed state can collapse even if its after a time longer than the life of the universe is something to note but given that all quantum theories face such issues as it is we can ignore this.

Now in the CSL model the Schrödinger equation is supplemented with a nonlinear and stochastic diffusion process driven by a suitably chosen universal noise coupled to the mass-density of the system, which counteracts the quantum spread of the wave function. The equation for the density matrix is written in eq: 17 also called the Lindblad equation.

$$\dot{\rho} = \iota[\rho, H] - \lambda \sum_n ({K_n K_n}^\dagger \rho + \rho {K_n K_n}^\dagger - 2K_n \rho {K_n}^\dagger) \tag{17}$$

The second term is what explains the process of collapse since this effectively collapses the off-diagonal elements of ρ in K's eigenstate representation. A clear thing to see here is that energy will not be conserved but this does offer quite an elegant formulation for collapse. Infact if we use a noise term, we can write Schrödinger equation as described in eq: 18

$$\frac{d\,|\Psi, t\rangle}{dt} = -\iota \int dx 2\lambda N(x, t)\eta(x, t)\,|\Psi, t\rangle \tag{18}$$

And here the $\eta(x, t)$ which is the noise term follows this equation eq: 19

$$\langle \eta(x, t)\eta(x', t')\rangle = \frac{1}{4\lambda} e^{-(x-x')^2/4a^2}\delta(t - t') \tag{19}$$

Which essentially signifies the Gaussian multiplication.

3 Experiments

Now there are multiple thought experiments which were made with the aim of finding a way to break quantum mechanics one of the earliest one can be thought to be EPR experiment but there were plenty of responses to that and it made us realise that quantum mechanics happens to be non local. I will now list some more experiments which explore interesting consequences of quantum mechanics.

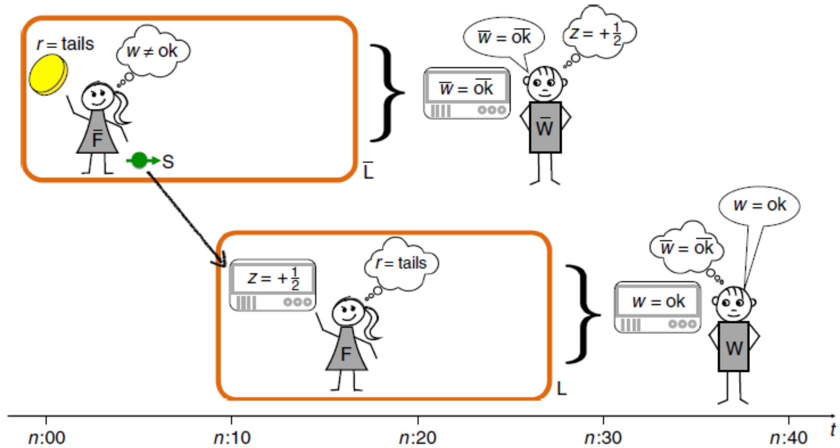

Figure 2: Illustration for the extended Wigner's friend experiment

3.1 Wigner's Friend Paradox

This essentially is set up like this. We have Wigner(W) and his friend(F). F is in a lab that is isolated from outside and he has an electron which is in equal superposition of spin up $|0\rangle_s$ and spin down $|1\rangle_s$ states so it is $(|0\rangle_s + |1\rangle_s)/\sqrt{2}$. Now F does a measurement over this electron. For W the state is now $(|0\rangle_s \otimes |0\rangle_F + |1\rangle_s \otimes |1\rangle_F)/\sqrt{2}$ since W only knows that F has performed a measurement but doesn't know the result. Now if after a long time he breaks into the room and measures his friend he finds the result. Now obviously the result F got on his measurement and W's result must match but then when did the actual collapse happen then? One of the most straightforward answer to this is that collapse happened when F did their measurement and W was simply at a lack of information. In many worlds interpretation since there is no collapse ever happening, W will measure any possibility but will simply be in the world where F has the exact same measurement in their memory since the state has to be defined objectively instead of subjectively. All in all this is a clearly solved problem.

3.2 Wigner's Friend Paradox Extended

This is a thought experiment which claimed that it proved that quantum mechanics cannot consistently describe itself however we will see ahead why the argument in this paper happens to be flawed. You can see in fig: 2 the way this experiment works. We have $\overline{W}$ and his friend $\overline{F}$ who is in an isolated lab $\overline{L}$ and we have W and his friend F who is again in an isolated lab L. Do note that the fact that this lab is completely isolated is of great importance since this would infer that if any result were to be obtained in this lab, until communicated the outside world will consider that state to be in superposition. The steps of the experiment were described in the original paper as follows:

1. **At time n00:** $\overline{F}$ has a coin r which gives heads with probability $1/3$ and tails with probability $2/3$. If the output is heads $\overline{F}$ sets the electron S in state $|\downarrow\rangle_S$ and if the output is heads, S is set to $|\rightarrow\rangle_S$. This S is sent to F
2. **At time n10:** F measures S in the $\{|\uparrow\rangle_S, |\downarrow\rangle_S\}$ basis and records the outcome as $z \in \{-\frac{1}{2}, \frac{1}{2}\}$.
3. **At time n20:** $\overline{W}$ measures $\overline{L}$ for being in state $|\overline{ok}\rangle = (|h_R h_{\overline{F}}\rangle - |t_R t_{\overline{F}}\rangle)/\sqrt{2}$ and if it is in this state, he declares $\overline{w} = \overline{ok}$ or he will declare $\overline{w} = \overline{fail}$.
4. **At time n30:** W measures L for being in state $|ok\rangle = (|\downarrow_S \downarrow_F\rangle - |\uparrow_S \uparrow_F\rangle)/\sqrt{2}$ and if it is then he declares $w = ok$ else he declares $w = fail$.
5. **At time n40:** If $\overline{w} = \overline{ok}$ and $w = ok$ the experiment halts otherwise it will reset.

Now a few points to be made about this, because of the fact that in the first step thereis communication from a lab that is isolated from outside, this causes certain effects that make quantum mechanics contradict itself as per the original paper.
Let us take this sequence of events, We get r to be tails and following this it turns out that $z = \frac{1}{2}$, in this situation we can see that F can conclude that $r =$ tails. Now when $\overline{F}$ got tails he concludes that W will get $w = fail$ because the

second lab has received the state $|{\rightarrow}\rangle_S$ so it will definitely give $w = fail$.
Now lets look into another sequence of events. Say that $\overline{W}$ gets the result $\overline{w} = \overline{ok}$, then he concludes that $z = \frac{1}{2}$ from the fact that at this time we can write the state of the two labs as $|\psi\rangle_{\overline{L}\otimes L} = \sqrt{\frac{2}{3}}\,|fail\rangle\,|{\downarrow_S\downarrow_F}\rangle + \sqrt{\frac{1}{6}}\,|fail\rangle\,|{\uparrow_S\uparrow_F}\rangle - \sqrt{\frac{1}{6}}\,|ok\rangle\,|{\uparrow_S\uparrow_F}\rangle$.
Another statement can be made by W when he measures $w = ok$ that there will be a round where even $\overline{w} = \overline{ok}$ and this comes from representing the final state $|\psi\rangle_{\overline{L}\otimes L}$ in terms of the ok, $fail$ and $\overline{ok}$, $\overline{fail}$ and the case of both being ok has probability of $1/12$.
Now let's suppose that $w = ok$ and also $\overline{w} = \overline{ok}$ too then this will imply that $z = \frac{1}{2}$ which then implies that $r =$ tails which would actually imply that $w = fail$ and that's a contradiction.

Now this has had a huge number of responses and it has been fairly accepted that this argument is flawed. So essentially the original paper takes three assumptions and they can be simplified as assuming quantum theory is correct, there is consistency in all predictions and measurement can only give single outcomes. Different interpretations actually drop one of these assumptions in their theory as per the paper but on some discussion it seems that this may not actually be the case.
One of the first inconsistencies we can spot is that in the situation where a $z = -\frac{1}{2}$ this actually makes the state of the coin appear to be as $(|heads\rangle + |tails\rangle)/\sqrt{2}$ from outside of $\overline{L}$ and if consistency has to be followed absolutely, this would imply that the state of the coin changes and would actually change all the calculations involved since it assumes the coin state to be fixed. Now one may argue that this result doesn't make sense since we have a macrorealism issue then and this state cannot be assumed since this is like the original Wigner's friend where there is a lack of knowledge but if that was the case then the fact that this is an isolated lab also adds more issues into it.

3.3 Experimental tests for MWI and Bohmian mechanics

An experiment for MWI was proposed in [1990 paper] and essentially offers a method to test whether the many worlds interpretation is actually correct by sort of creating a communication between two ”worlds” while they are in the process of splitting.
The experiment it poses is as follows, we have some two level equal superposition state of say up and down spin with us which we then proceed to measure, and while this result is obtained we will excite the state of some other system S' if we measure up and wont excite if we measure down. The crux of the argument is that in the many worlds interpretation, this situation will lead into the world with down actually having a possible excited state in S' because of incomplete decoherence which occurs between the worlds if this process is carried out fast enough since we can only consider the system of our superposition to be decohered after the whole thing is done. This argument is actually flawed because as per many worlds, this excitation will occur in a seperate world hence eliminating any possibility of a possible ”communication” between these ”worlds”.
An interesting experiment was done in [double slit 2018] where a special double slit experiment was carried out. As per the paper they had set up an apparatus which they use bosons (in this case photons) in a way that two identical bosons are sent at the same time to the two different slits using an optical condenser and as per their references, this situation in Bohmian mechanics would result in a definite case of no coincidences being measured on the final screen which would otherwise be observed in standard quantum mechanics. In our discussion, we argued that if we can actually guarantee that photons are being passed individually to both the slits, that is not short of an act of measurement and so this experiment would not actually be able to distinguish between these two interpretations. Of course this paper is still up to debate but the whole idea still remains slightly unclear so we cannot take this as a solid proof against Bohmian mechanics.
Another experiment proposed in [] claimed a method to differentiate between standard quantum mechanics and Bohmian mechanics using a special construction of a 3d infinite potential well however despite actually offering a proper setup, it based it's premise over a fallacy that momentum in Bohmian mechanics is actually ∇S whereas this is not the actual momentum and that is also addressed in the original paper.
All in all we could not find any reputable paper which could actually give a breaking point for MWI or Bohmian mechanics. This doesn't imply that such an experiment is impossible but it surely would be hard to come by.

3.4 Experiments for CSL parameters

The parameters for the objective collapse model are one of quite some debate over time. In the original GRW paper the parameters were set by a vague intuition with no actual experimental evidence directly backing it up. The CSL paper similarly set it's bounds fairly vaguely however the λ bound has limits which have been verified experimentally

however the parameter for d from eq: 16 is something which we could not find the exact origin of other than a vague intuition that it is in the scale of an atom.
All the multiple experiments and plans for experiments which can test these limits were summarised very well in [big ass 2012 print]. Note that we will refer to the distance parameter d from eq: 16 as r_c in this section.
In the original GRW paper, the parameters proposed were $\lambda = 10^{-16}s^{-1}$ and $r_c = 10^{-7}m$ however Adler proposed that $\lambda = 10^{-8\pm 2}s^{-1}$ and r_c was the same. Using matter wave interferometry the upper bound for λ is obtained as $10^{-5}s^{-1}$. The experiment can be boiled down to observing a double slit experiment with macromolecules with the idea of finding the quantum to classical limit. As per eq: 17 we have the density matrix evolution as follows

$$\rho_t(x,y) = \rho_0(x,y)e^{-\lambda N(x-y)^2 t/2} \tag{20}$$

This just tells us that to measure a collapse we need our states to be highly spatially separated or have a high N (number of clusters). Now we can represent a decay function $\Gamma_{\text{CSL}}(x,y)$ such that we can write eq: 17 as the following.

$$\frac{d}{dt}\rho_t(x,y) = -\frac{\iota}{\hbar}[H, \rho_t(x,y)] - \Gamma_{\text{CSL}}(x,y)\rho_t(x,y) \tag{21}$$

We take $\Gamma_{\text{CSL}}(x,y) = \lambda[1 - e^{-x^2/4r_c^2}]$. This scales according to the number of nuclei involved so if it is n nuclei and we have N clusters the scaling is n^2N. This experiment was setup to take $N = 1$ since all the particle clusters were far and for the performed experiment fullerene atoms and we take macro molecule size to be about $1nm$ so it is lower than the scale of r_c. We will have $x >> r_c$ so our damping factor essentially becomes $\Gamma_{\text{CSL}} \approx n^2\lambda$ and since no interferometry based experiment has detected spontaneous collapse the damping factor $\exp(-\Gamma_{\text{CSL}}t)$ must be very small so we get the limit on λ as

$$\lambda \leq 1/(n^2 t) \tag{22}$$

Here we take t to be duration of experiment since it is the time of decoherence. This is the basis of most experiments for getting the limit on λ. In the paper there is a description given for setup for different interferometers, the TLI (Talbot Lau Interferometer) which has three gratings, the KDTLI (Kapitza Dirac Talbot Lau Interferometer) which replaces the middle grating from the previous one to a laser which makes an optical potential for diffracting the particles. The OTIMA (Optical Time-Domain Ionizing Matter Interferometer) replaces all the three with these lasers. The gratings have a mechanism by which a small de broglie wavelength particle can still have it's wave nature observed and is discussed in depth in [big ass 2012 paper]. There are also experiments using Optical Cantilevers which give an upper bound over λ as $10^{-8}s^{-1}$.
Spontaneous X ray emission from Germanium offers the strongest experimental bound known over the parameter λ as $10^{-11}s^{-1}$ and was done so from the IGEX data of that experiment. In [last paper] the author discusses theoretical prediction from CSL using the known parameters and compares them with the actual experimental predictions of the IGEX experiment with the assumption that the four outermost electrons of Ge can be taken as free electrons and ignoring any relativistic effects. The data matches in certain points however falls off pretty quickly and still leaves points like whether λ is even universal up to debate since they must also be scaled for electrons and protons. This was further discussed in [other oe] where if mass dependence were ignored we would actually have $\lambda \leq 1.4 \times 10^{-17}$ however it must be noted that these emissions are argued as a result of CSL's non conservation of energy and are better explained by various other theories so this is all up for debate.

4 Conclusion

Over the course of this project we have read quite a lot of papers which all stem from the measurement problem of quantum mechanics. While many dismiss the problem since standard quantum mechanics gets the job done over the past 100 years there was always the debate for what really was quantum mechanics. It is kind of amusing to see that one can never really even tell if quantum mechanics is actually random or not since in theory we can never really tell if anything even is random and we often consider anything that's noisy enough to feel "random" and this really makes you rethink how often we even use this term in our daily linguistic exchanges. For every theory which has been proposed there always seems to be as much going for it as much as there is against it. Interestingly CSL theory is the one theory which has experimental differences from the standard quantum theory however the clean idea of Bohmian mechanics has actually held up way better than it was expected to given that there is no solid evidence against it yet. Many worlds is an interpretation which as commented by Deutsch was one which is quite romanticised by people who aren't actually exploring this topic of physics much. QBism is a very interesting interpretation which despite being quite nascent has actually garnered quite a lot of attention and may soon be realized into a possibly quite elegant theory. Either way studying the multiple interpretations of quantum mechanics and seeing their responses does make one see a lot of things in new light.

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