Group Discussion on Quantum Measurement

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The EPR "Paradox"

I will discuss on Bohm's variant on the original paradox which was proposed by Einstein, Podolsky and Rosen in their very famous 1935 paper.
Say we have prepared a electron-positron pair coming from a single source. These would be entangled in a way where if we measure the spin of one of the particles, the other particle will simultaneously collapse to the opposite spin.

$$|\psi\rangle = \frac{|{\uparrow_e}{\downarrow_p}\rangle + |{\downarrow_e}{\downarrow_p}\rangle}{\sqrt{2}}$$

The EPR "Paradox"

So the main issue which Einstein had was that this means that measurement on one particle directly affects the other no matter how far it is. This makes it a non local effect which clearly does not make sense in a world which was believed to follow local realism.
But does it truly break locality in the sense that there is a faster than light communication?
If one thinks about it more deeply, you would realize that the two agents doing the measurement have no way to communicate using this pair so no faster than light shenanigans.

Bell's inequalities

A very important result which was derived in response to the EPR paradox ware the Bell inequalities [1]. To summarise their violation implies that local hidden variable theories for quantum mechanics cannot exist. This also validated the Bohm's interpretation [2],[3] which was a **non-local** hidden variable theory. Interestingly the inequalities were derived using probability theory and no quantum mechanics and hence have even found a use in classical coherence for light.

Wigner's Friend

Let us first begin with the simple form of the experiment that was stated originally by Wigner himself.

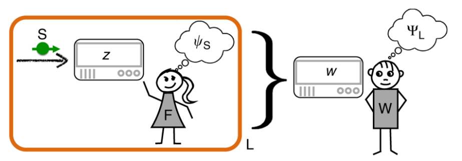

Figure: The Wigner's Friend experiment

The spin of the electron is in state $|\psi\rangle_S = \dfrac{|\uparrow\rangle_S + |\downarrow\rangle_S}{\sqrt{2}}$ and the friend measures it and stores this value.

Wigner's Friend

The lab is perfectly isolated from it's surroundings so while Wigner knows a measurement is done, he doesn't know the result and so for him the lab is in the state

$$|\psi\rangle_L = \frac{|\uparrow\rangle_S |\uparrow\rangle_F + |\downarrow\rangle_S |\downarrow\rangle_F}{\sqrt{2}}$$

Here we treat the result noted as part of the description of the lab. Now Wigner goes for a while and returns and measures the lab.
While the result of F and W will match, when did the actual collapse happen and why does there exist a time when their results do not match?
What are some flaws which you think are very apparent here?

Wigner's Friend

Once collapse has occurred, the friend has noted down the result which "creates" information and so entropy has been radiated away making this irreversible.
Can you think of how other interpretations solve this problem?

Wigner's Extended Friends

So Wigner has decided to socialize a little more and has brought three other individuals into his wacky thought experiments.

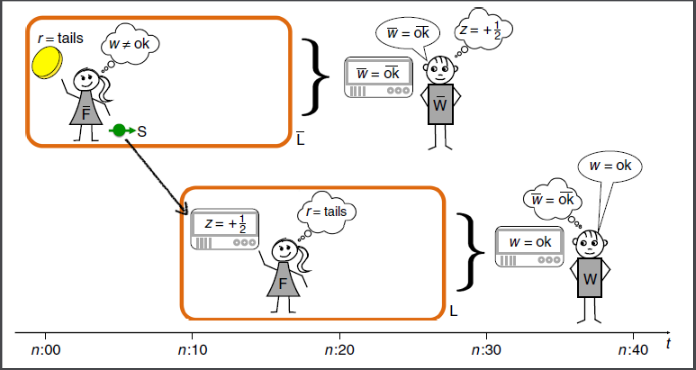

Figure: Wigner with more friends

Wigner's Extended Friends

This thought experiment was proposed in [4]. In here we take three important assumptions throughout.

- Quantum theory is correct (Q)
- Each agent must have consistent predictions (C)
- A measurement only yields one outcome (S)

Assumption (Q) means that the born probability rule holds.
Do you feel any of these assumptions have issues and also does every quantum mechanical interpretation actually follow all of these?

Wigner's Extended Friends

The exact setup is a five step process which goes as follows.

1. **At time** $t = t_0$**:** $\overline{F}$ tosses a coin R with $|\psi\rangle_R = \sqrt{\frac{1}{3}}\,|h\rangle_R + \sqrt{\frac{2}{3}}\,|t\rangle_R$. If heads $\overline{F}$ sets the electron S as $|{\downarrow}\rangle_S$ and for tails, S is set to $|{\rightarrow}\rangle_S$. This S is sent to F.
2. **At time** $t = t_1$**:** F measures S in the $\{|{\uparrow}\rangle_S, |{\downarrow}\rangle_S\}$ basis and records the outcome.
3. **At time** $t = t_2$**:** $\overline{W}$ measures $\overline{L}$ for being in state $|\overline{ok}\rangle$ or $|\overline{fail}\rangle$.
4. **At time** $t = t_3$**:** W measures L for being in state $|ok\rangle$ or $|fail\rangle$.
5. **At time** $t = t_4$**:** If $\overline{w} = \overline{ok}$ and $w = ok$ the experiment halts otherwise it will reset.

Wigner's Extended Friends

The measurements of $\overline{W}$ are done in the basis of $\left|\overline{ok}\right\rangle = (\left|h_R h_{\overline{F}}\right\rangle - \left|t_R t_{\overline{F}}\right\rangle)/\sqrt{2}$ and $\left|\overline{fail}\right\rangle = (\left|h_R h_{\overline{F}}\right\rangle + \left|t_R t_{\overline{F}}\right\rangle)/\sqrt{2}$.
The measurements of W are done in the basis of $|ok\rangle = (|{\downarrow_S}{\downarrow_F}\rangle - |{\uparrow_S}{\uparrow_F}\rangle)/\sqrt{2}$ and $|fail\rangle = (|{\downarrow_S}{\downarrow_F}\rangle + |{\uparrow_S}{\uparrow_F}\rangle)/\sqrt{2}$.
Now with this long setup, the paper proposed that there is in fact a contradiction we arrive to if we assumed (Q), (S) and (C) to be true

Wigner's Extended Friends: The contradiction

Let us make a few statements regarding conclusions that some agents can draw after they do their respective measurements.

- If $\overline{F}$ gets tails, they conclude that W gets $|fail\rangle$ since F would receive the spin state $|\rightarrow\rangle_S$.
- If F measures and gets $|\uparrow\rangle_S$ then they conclude that the coin must have gotten tails otherwise spin up is not possible.
- If $\overline{W}$ gets the result $|\overline{ok}\rangle$, they conclude that F must have measured $|\uparrow\rangle_S$ (why?).
- If W gets the result $|ok\rangle$, there is a non zero probability that $\overline{W}$ got the result $|\overline{ok}\rangle$.

The last two statements can be proven by writing the expression of the state $|\psi\rangle_{\overline{L}\otimes L}$ at time $t = t_1$.

Wigner's Extended Friends: The contradiction

Once the coin has been tossed and F measures S, the state of the two labs as seen from outside would appear to be

$$|\psi\rangle^{t=t_1}_{\overline{L}\otimes L} = \sqrt{\frac{1}{3}}\,|h_R h_{\overline{F}}\rangle\,|{\downarrow_S}{\downarrow_F}\rangle + \sqrt{\frac{2}{3}}\,|t_R t_{\overline{F}}\rangle \left(\frac{|{\downarrow_S}{\downarrow_F}\rangle + |{\uparrow_S}{\uparrow_F}\rangle}{\sqrt{2}}\right)$$

This can be rewritten in the $\overline{W}$ measurement basis as

$$|\psi\rangle^{t=t_1}_{\overline{L}\otimes L} = \sqrt{\frac{2}{3}}\,|\overline{fail}\rangle\,|{\downarrow_S}{\downarrow_F}\rangle - \sqrt{\frac{1}{6}}\,|\overline{ok}\rangle\,|{\uparrow_S}{\uparrow_F}\rangle + \sqrt{\frac{1}{6}}\,|\overline{fail}\rangle\,|{\uparrow_S}{\uparrow_F}\rangle$$

So one can directly see that once $\overline{W}$ measures $|\overline{ok}\rangle$ the electron would have to be in the up spin state.

Wigner's Extended Friends: The contradiction

$$|\psi\rangle_{\overline{L}\otimes L}^{t=t_1} = \sqrt{\frac{2}{3}}\left|\overline{fail}\right\rangle|{\downarrow_S}{\downarrow_F}\rangle - \sqrt{\frac{1}{6}}\left|\overline{ok}\right\rangle|{\uparrow_S}{\uparrow_F}\rangle + \sqrt{\frac{1}{6}}\left|\overline{fail}\right\rangle|{\uparrow_S}{\uparrow_F}\rangle$$

Now on closely inspecting this expression we can also see that $|\langle ok|\left\langle\overline{ok}\middle|\psi\right\rangle_{\overline{L}\otimes L}^{t=t_1}|^2 = \frac{1}{12}$ and so all the four statements *seem* to be right since this means that getting the final result of $\left|\overline{ok}\right\rangle \otimes |ok\rangle$ is possible.

Wigner's Extended Friends: The contradiction

Egad! We seem to have stumbled on an issue now. If we do get the state $|\overline{ok}\rangle \otimes |ok\rangle$ finally, that would mean that F measured a spin up. From this F had concluded that $\overline{F}$ must have gotten tails. When $\overline{F}$ gets tails they conclude that W measures $|fail\rangle$ which clearly is a contradictory viewpoint!

So where did we go wrong?

Wigner's Extended Friends: The fallacy

The paper [4] more specifically makes a no-go theorem which states that interpretations which can solve this contradiction must let go of at least one of (Q), (C) and (S).
Here wavefunctions are described subjectively since measurement results are not known to all agents and most interpretations have objective descriptions for wavefunctions and so such a contradiction actually stems only when one does not have an objective wavefunction.

But is there more to this?

Wigner's Extended Friends: The fallacy

An important step that is not so much accounted here and was discussed in [5] is that in the statement regarding conclusion of result of W when $\overline{F}$ gets tails is that there will be a measurement done by $\overline{W}$ in the middle of this.
Measuring the lab $\overline{L}$ leaves it in a **coherent superposition** of heads and tails but it is only when the state is purely tails that we have measurement of W limited to $|ok\rangle$ which clearly would not be the case once $\overline{W}$ has performed their measurement.

Wigner's Extended Friends: The fallacy

In Bohmian mechanics, collapse is described by interaction of measurement apparatus with the state which is an extremely fast and chaotic evolution.
More importantly it decoheres the wavefunction into seperate eigenstates in a certain space leaving just one component of the wavefunction but the rest of the eigenstates are present as empty waves.
One can think of the measurement by $\overline{W}$ awakening this empty wave since it prepares a coherent superposition. So there we have it, Bohmian mechanics has violated the no-go theorem pretty nicely.

A meme I liked

Figure: An interesting take on this meme

Another meme

Figure: >tfw you make a thought experiment to ridicule Copenhagen but people end up using it to explain that very interpretation.

References

- Bell, J. S. On the Einstein Podolsky Rosen paradox. *Physics Physique Fizika* **1,** 195–200. https://link.aps.org/doi/10.1103/PhysicsPhysiqueFizika.1.195 (3 Nov. 1964).
- Bohm, D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I. *Phys. Rev.* **85,** 166–179. https://link.aps.org/doi/10.1103/PhysRev.85.166 (2 Jan. 1952).
- Bohm, D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. II. *Phys. Rev.* **85,** 180–193. https://link.aps.org/doi/10.1103/PhysRev.85.180 (2 Jan. 1952).
- Frauchiger, D. & Renner, R. Quantum theory cannot consistently describe the use of itself. *Nature Communications* **9.** ISSN: 2041-1723. http://dx.doi.org/10.1038/s41467-018-05739-8 (Sept. 2018).
- Lazarovici, D. & Hubert, M. How Quantum Mechanics can consistently describe the use of itself. *Scientific Reports* **9,** 470. ISSN: 2045-2322.

Thank You !