Introduction

In the world of finance and investments, making informed decisions is crucial to achieving your financial goals. Harry Markowitz's groundbreaking work on mean-variance portfolio theory revolutionized the way we approach portfolio construction. This theory provides a systematic approach to diversification, enabling investors to optimize their risk-return trade-offs. In this blog post, we'll delve into the core concepts of Markowitz's theory and demonstrate its practical application using Python.

What is a Portfolio?

A portfolio is a collection of financial assets aimed at achieving specific financial goals, like capital appreciation and risk diversification. It includes various assets such as stocks, bonds, real estate, and commodities, with the allocation determined by the investor's risk tolerance and objectives. Key aspects include diversification to reduce risk, balancing risk and return, asset allocation, and periodic rebalancing. Portfolios are managed with a long-term perspective and can be actively or passively managed. They are utilized by individuals and institutions to grow and manage wealth, aligning resources with long-term financial goals.

A well-diversified portfolio typically includes a mix of different asset classes to balance risk and potential returns. Common asset classes include:

Equities (stocks): Represent ownership in companies and offer potential for long-term growth.

Fixed Income (bonds): Debt securities that pay interest over a specified period, providing income and stability.

Cash and Cash Equivalents: Highly liquid and low-risk assets, like money market funds or short-term government bonds.

Real Estate: Physical properties or real estate investment trusts (REITs) that can provide income and capital appreciation.

Commodities: Physical goods like gold, oil, or agricultural products, which can act as a hedge against inflation.

Alternative Investments: Hedge funds, private equity, or venture capital, offering diversification and unique risk-return profiles.

The specific allocation depends on individual goals, risk tolerance, and investment time horizon.

Portfolio Optimization

Portfolio optimization primarily involves evaluating historical returns rather than forecasting. It focuses on optimizing the weights of financial instruments in a portfolio, not on selecting them. Historical returns are commonly used to estimate expected returns, risks, and correlations among assets. This data is used in approaches like mean-variance optimization. Forward-looking optimization incorporates forecasting of future returns, risks, and correlations, allowing for more dynamic strategies. Some techniques combine both historical and forward-looking data for a balanced approach. However, investors must be aware that forecasting introduces uncertainties, making continuous review and adjustment crucial for successful portfolio optimization.

Markowitz’s Mean-Variance Theory

Harry Markowitz's Mean-Variance Portfolio Theory (MVP) is a fundamental concept in finance. It was introduced by Harry Markowitz in his seminal paper "Portfolio Selection" published in 1952, which later earned him the Nobel Prize in Economics in 1990. It suggests that investors can optimize their portfolios by considering both expected returns and risk. By diversifying across assets with different risk-return profiles, they can achieve an efficient frontier, representing the best risk-reward trade-off. The theory introduced the idea of a risk-free asset and a utility function to quantify an investor's risk aversion. Markowitz's work revolutionized modern portfolio management and remains a widely used tool for constructing diversified portfolios aligned with investors' goals and risk preferences.

Case: Sample Portfolio

Our sample portfolio consists of daily returns of 6 hedge funds/PE funds (Blackstone, Blackrock, Lazard, KKR, Icahn Enterprises, Invesco), 1 stock (Berkshire Hathaway), 1 bond (US Treasury 10 year), 1 commodity (Gold), and 1 REIT (American Towers), from 2011 to 2023. This diverse selection of assets, including hedge funds, PE funds, and traditional instruments, offers an exciting analysis as it tracks a wide range of asset classes and financial instruments, especially in the alternative investments asset class.

Preparing Data for Analysis

After plotting the normalized daily closing prices of the portfolio assets over a twelve-year period (2011-2023), Blackstone emerged as the clear winner, showing the highest growth. American Towers and BlackRock followed in second and third place, respectively, with a notable margin. Icahn Enterprises and Invesco lagged behind all other holdings. All assets rebounded after an initial downturn during the pandemic's onset, but faced an inflection and steady decline post-2022.

Logarithmic Returns

Average Returns

When analyzing daily stock returns over time, we often follow a three-step process:

Logarithm of Returns: Take the logarithm of the daily returns. This transformation helps in converting multiplicative returns to additive returns and stabilizes the variance of the data.

Mean of Log Returns: Calculate the mean of the log returns over the given period. This step provides an estimate of the average daily return during that time.

Annualization: Annualize the mean by using one of two methods:

Linear Method: Multiply the mean of log returns by the number of trading days in a year (typically 252) to obtain the annualized return.

Compounding Method: Raise the mean of log returns to the power of the number of trading days in a year (252) to account for compounding effects and obtain the annualized return.

Note: If the time frame is shorter than a year, use the number of days for which returns are available for the annualization step instead of 252.

Below we see a plot of annualized mean returns. Blackstone is leading with a gain of 33%, followed by Berkshire Hathaway with 26% gains. On the other hand, Invesco shows about a 9% reduction, while Icahn demonstrates an 8% reduction. The bar plot provides a visual representation of the performance differences among these assets over the analyzed period.

When assets perform highly relative to their peers, it is reasonable to expect an accompanying high level of risk. In the next section, we will delve into examining the volatility of these assets to better understand the risk profile associated with their performance.

Volatility (Measure of Risk)

When analyzing stock volatility using daily close prices over time, we follow a three-step process:

Logarithm of Returns: Take the logarithm of the daily returns. This transformation converts multiplicative returns to additive returns and stabilizes the variance of the data.

Standard Deviation of Log Returns: Calculate the standard deviation of the log returns over the given period. This step provides a measure of the volatility or risk associated with the stock's performance.

Annualization: Annualize the standard deviation using one of two methods:

Linear Method: Multiply the standard deviation of log returns by the square root of the number of trading days in a year (typically 252 for daily returns) to obtain the annualized volatility.

Compounding Method: Raise the standard deviation of log returns to the power of the square root of the number of trading days in a year (252 for daily returns) to account for compounding effects and obtain the annualized volatility.

Note: If the time frame is shorter than a year, use the number of days for which returns are available for the annualization step instead of 252.

Below is a plot of annualized volatility. The higher the value, the greater the volatility of the asset. Here, US 10-year Treasury bonds exhibit the highest volatility with a value of 0.64, Invesco follows closely with a volatility of 0.57, and Blackstone has the third-highest volatility at 0.54. The annualized standard deviation or volatility is a measure of risk, with higher values indicating potentially higher levels of price instability and uncertainty in the asset's performance.

The covariance matrix is a valuable tool for measuring volatility within a portfolio as it captures the relationships between different assets. It shows how their returns move together or in opposite directions. A higher covariance between two assets suggests a stronger co-movement, which can lead to higher portfolio volatility. By annualizing the covariance matrix with the number of trading days in a year (252 for daily returns), it scales the measure to reflect a yearly timeframe, allowing investors to better assess and manage the overall risk and diversification of their portfolio over time.

Portfolio Optimization

The next step in our portfolio optimization process is to incorporate weights into the analysis. By multiplying the annualized returns of individual assets with their assigned weights, we obtain the overall returns of the portfolio, allowing us to measure performance over a specific time period.

The primary goal of portfolio optimization is to allocate optimal weights to assets to minimize risk and maximize returns. In this analysis, we'll use the SciPy Python package to minimize risk, which we'll measure through the volatility of the portfolio.

Alternatively, in other instances, we might choose to maximize the Sharpe ratio, which is a measure of risk-adjusted returns. The Sharpe ratio quantifies the excess return of an asset per unit of risk (usually measured as volatility) and helps investors assess the efficiency of a portfolio in generating returns given its risk level.

Special Case: Two Assets (Berkshire Hathaway, Blackstone)

To motivate the selection of an optimal portfolio, let's illustrate a special case using only two assets: Berkshire Hathaway and Blackstone stocks. By simulating 500 weight combinations for these assets, adding up to 1, we can plot the risk-return combinations. This visualization helps us understand the risk-return tradeoff in this simplified scenario, guiding our considerations when dealing with a general case involving a larger basket of assets. Below is the first five rows of a 500 row table containing risk return combinations of the two-asset portfolio:

Plotting the various combinations we have:

In the next section, with all assets included, we'll demonstrate that selecting the optimal portfolio may not be as straightforward or discernible when dealing with 10 assets compared to the simplicity of the two-asset scenario.

Generalized Case: All Instruments

Once again, we'll simulate weights for each asset, totaling 1, but this time generating 2500 combinations. We'll plot the portfolio returns and volatilities resulting from these weight assignments in each of the 2500 scenarios. This analysis will allow us to visualize the distribution of risk and return across the different portfolio allocations, shedding light on the complexity of selecting an optimal portfolio with a larger set of assets.

Below are the first five rows of a 2500 row table containing the portfolio returns and volatility for each set of simulated weights:

In general, identifying a clear trend in the data is challenging. However, we observe an inverse relationship between risk and return in this portfolio, contrary to the Mean-Variance Portfolio Theory, where risk and return are typically expected to have a positive relationship. In the next section, our focus will be on selecting the minimum volatility portfolio among all 2500 weight combinations.

Portfolio Selection: Minimizing Volatility

To achieve the minimum volatility portfolio, we will utilize the minimize function from the SciPy library, using portfolio volatility as our utility function to minimize. This optimization process will help us identify the portfolio with the lowest risk among the 2500 combinations. Below are the results:

Here, 'fun' represents the minimum volatility calculated by the function, while 'x' represents the set of weights used to achieve minimum volatility. Below are the weights recommended by the algorithm:

While the minimization process provides valuable weight recommendations, it serves as a guidepost, not a rigid rule. The current recommendation is 63.1% Gold ETF, 17.8% American Towers REIT, 8.7% Berkshire Hathaway stock, 6.8% US 10-Year Treasury bonds, and 3.7% Icahn Enterprises stock. Minimizing volatility is a good starting point, but our end goal might be maximizing the Sharpe ratio for the optimal portfolio. Additionally, we may consider running more simulations for a more comprehensive analysis.

Conclusion

Harry Markowitz's mean-variance portfolio theory remains a cornerstone in modern finance. By using Python code to implement this theory, investors can efficiently construct portfolios tailored to their risk tolerance and investment objectives. The optimization process helps identify the optimal allocation of assets, ultimately leading to better risk-adjusted returns and long-term financial success.