

Notation:

n^L = number of neurons in layer L. n^0 is number of inputs

w_{ij}^L = weight into layer L, from neuron j to neuron i

W^L = matrix of weights from L-1 to L, dimension 2

b_i^L = bias for neuron i of layer L

B^L = bias vector for layer L

a_i^L = output i of layer L.

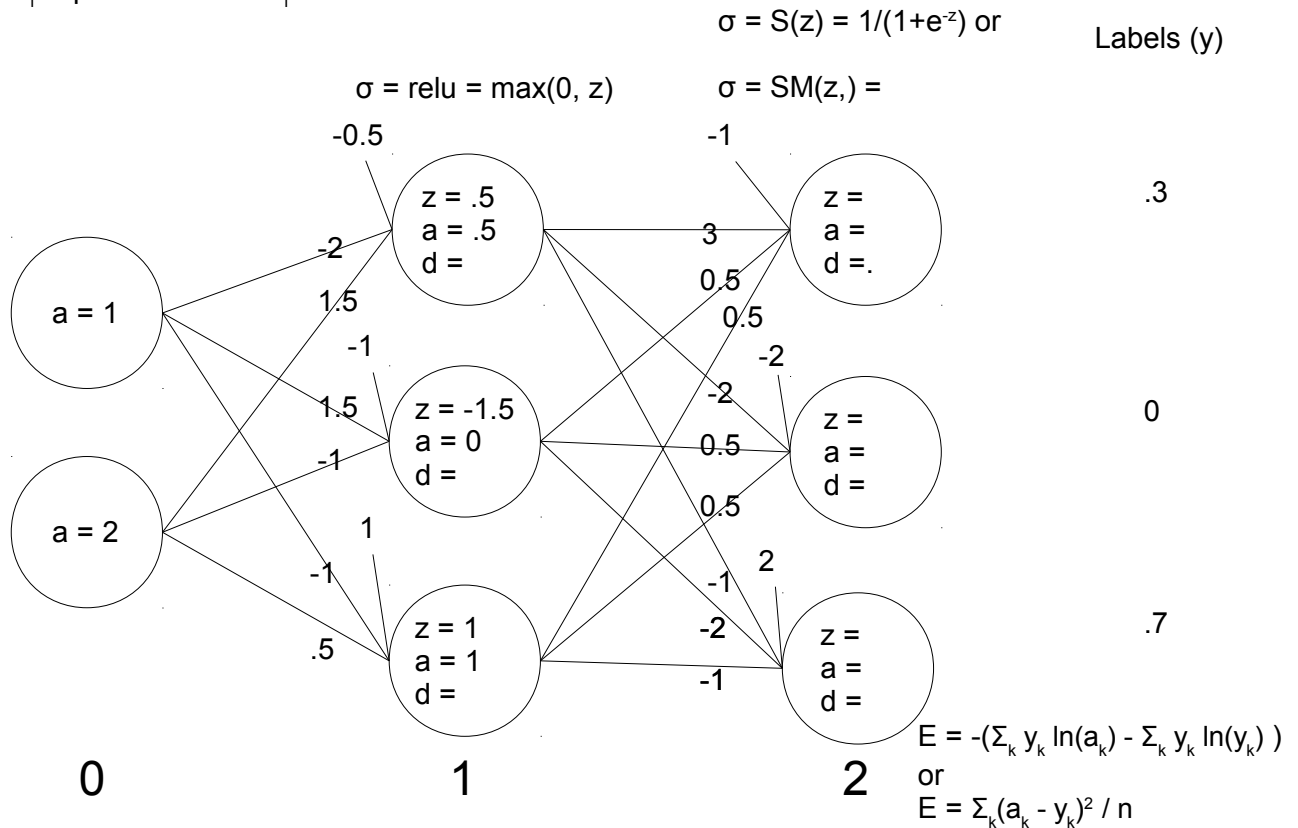
A^L = vector output of layer L. A^0 is input vector

z_i^L = weighted sum of neuron i of layer L incl bias.

σ = activation function. $a_i^L = \sigma(z_i^L)$, or sometimes, $a_i^L = \sigma(Z^L)$ e.g. for softmax

E = error function cost = $E(A^N, Y)$, where N is final layer

d_i^L = partial of E wrt z_i^L



For all w_{ij} :

$$\partial E / \partial w_{ij}^x = a_j^{x-1} \partial E / \partial z_i^x$$

$$\begin{aligned} \partial E / \partial z_1^1 &= \partial a_1^1 / \partial z_1^1 * \sum_k (\partial z_k^2 / \partial a_1^1 * \partial E / \partial z_k^2) \\ &= \partial a_1^1 / \partial z_1^1 * \sum_k (w_{k1}^2 * \partial E / \partial z_k^2) \end{aligned}$$

$$\partial E / \partial z_j = \partial a_j / \partial z_j * \partial E / \partial a_j$$

$$\partial E / \partial a_j = -y_j / a_j$$

For S(z) activation:

$$\partial a / \partial z = e^{-z} / (1+e^{-z})^2$$

More complex for softmax, where da/dz becomes a matrix of da_j/dz_i for all j, i:

$\partial a_j / \partial z_i =$ for $i=j$: $a_j(1-a_j)$, for $i < j$: $a_j(0-a_i)$
or just $\partial a_j / \partial z_i = \sum_j a_j(\delta_{ij} - a_i)$

$$\begin{vmatrix} \partial E / \partial z_1 \\ \partial E / \partial z_2 \\ \partial E / \partial z_3 \end{vmatrix} = \begin{vmatrix} \partial a_1 / \partial z_1 & \partial a_2 / \partial z_1 & \partial a_3 / \partial z_1 \\ \partial a_1 / \partial z_2 & \partial a_2 / \partial z_2 & \partial a_3 / \partial z_2 \\ \partial a_1 / \partial z_3 & \partial a_2 / \partial z_3 & \partial a_3 / \partial z_3 \end{vmatrix} \begin{vmatrix} \partial E / \partial a_1 \\ \partial E / \partial a_2 \\ \partial E / \partial a_3 \end{vmatrix}$$