Problem Set: Maximum Flows

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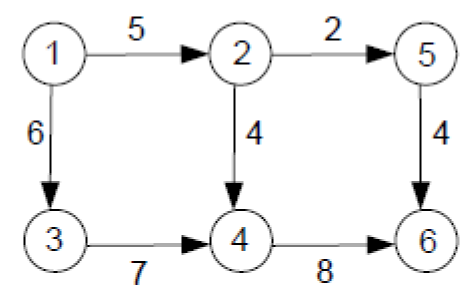
Problem 1: Show the final version of the flow graph that includes the network flow $\{x_{i,j} s\}$ and the labels on the vertices that were obtained in the final iteration of the shortest augmenting path algorithm. Clearly specify the value of the flow and the cut that shows the flow is maximum.

Problems 2 and 3: Clearly specify the flow networks in your drawings with the nodes clearly labeled by what they represent and the edges by their capacities. You may hand draw the diagrams but if it is not neat and easily read it will not be graded. All symbols must be clearly defined and consistent with the above on what you hand in!

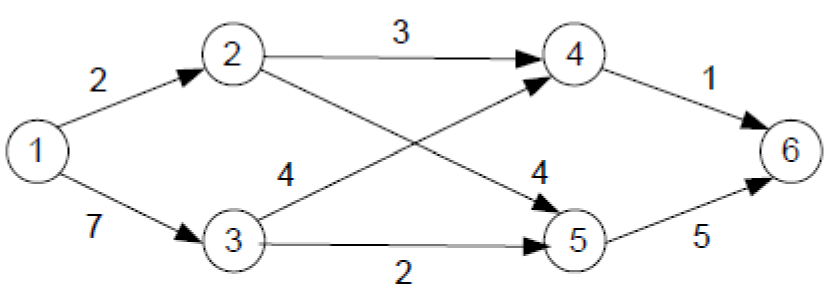
Warm-ups:

1. Apply the shortest-augmenting path algorithm to find a maximum flow and a minimum cut in the following networks:

A.



B.



2. Dining problem Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to find a seating arrangement that meets this objective (or prove that no such arrangement exists) by using a maximum flow problem. You should draw a picture of the network flow problem using the notation below. Then explain how running the Ford-Fulkerson shortest augmenting path algorithm will enable you to find a solution or determine if no solution exists.
   Assume that the dinner contingent has **p families** and that the **ith family has a_i members** and that **q tables** are available and the **jth table has a seating capacity of b_j.**
   [From Network Flows: Theory, Algorithms, and Applications. Prentice Hall 1993]

3. IT department holiday scheduling

Congratulations! You have been promoted to be manager of IT organization of your favorite company. However every position has its headaches. Since it is imperative that the companies systems be available every day of the year, you must ensure that there is a (exactly one) supervisor at the data center every day of the year including all the holidays. Holidays are single days including days like Thursday, Friday, Saturday, and Sunday of Thanksgiving week, every day during Christmas week, etc.

Some holidays occur during holiday periods. For example the Thanksgiving holiday period might be Thursday, Friday, Saturday, and Sunday of Thanksgiving week for your company. For the purposes of this problem we will assume there are **k** holiday periods. Some of these will have a single holiday but many will have more than one day, e.g. Thanksgiving holiday period has 4 holidays. Let **D_j** be the number of holidays in period **j**.

Luckily, there are sufficient supervisors to cover all these days even with the following constraints.

1. There are exactly **n** supervisors, each supervisor offers to work exactly **i** of the holidays. Each supervisor has submitted a list of the holidays they would be willing to work. Exactly one supervisor should be assigned to each holiday and that holiday must be on that supervisors list.
2. No supervisor should be assigned to cover more than **C** holiday days in a year. (i>C)
3. No supervisor should work more than one holiday in each of the holiday periods

So to try to make everyone happy you ask each supervisor to provide you with a list of the "holidays" that they would be willing to work.

You think to yourself, I could write an algorithm to **determine if** there is an assignment of supervisors to holidays that satisfies the above constraints where the supervisors only work on the days from their list - BUT I don't have time. **So you cleverly remember that the maximum flow algorithm your learned at Cal Poly is good at solving matching problems.** Thus you decide to try to model the problem as a maximum flow problem. Then you can just use the shortest augmenting path algorithm to solve the problem!

A. First do the problem **without constraint #3**. That is the supervisors may work more than one day in a holiday period. Doing the problem means determining a flow network that models the problem. G = (V, E, u) and draw a picture of it. **Use labels for the vertices and edges that represent the different things you are trying to model, namely**
Supervisors = { S_1 , S_2 , …. , S_n }
Holidays = { H_1 , H_2 , …. , H_m } where m is the total number of holidays.
Holiday Periods = {HP_1 , HP_2 , …. , HP_k } **(note that the sum of the D_j is m)**
Any symbols you use must be clearly defined before the diagram.

B. Then develop the model to solve the problem with all three constraints. Again determine a flow network that models the problem. G = (V, E, u) and draw a picture of it. Any symbols you use must be clearly defined before the diagram.

For each diagram (A and B)

1. Explain why the maximum flow in the flow network solves the problem in a short paragraph. Namely why does the maximum flow obtained represent a solution to the problem; Satisfies the contraints etc.
2. Explain how to tell if there is no solution to the problem given the list of holidays that each supervisor is willing to work.
3. Explain how to tell what holidays are assigned to what supervisor.

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