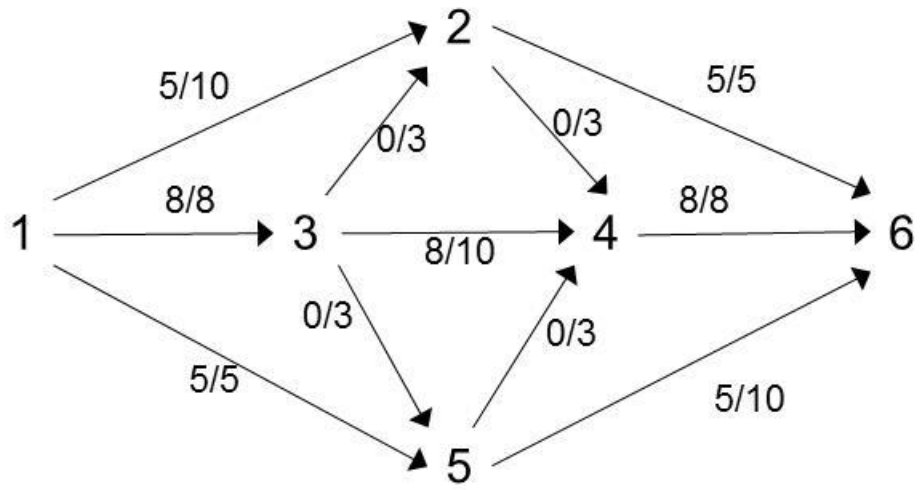
Lab: Flow Networks

1. **You are given the following graph that shows a flow network with the capacity and flow associated with each edge. It is not the graph that would necessarily result from the algorithm discussed in class.**



a) What is the value of this flow?

b) Use the algorithm discussed in class to find the next augmenting path? What is the augmenting path?

c) Find the resulting flow? Draw a picture of the resulting graph with the edges labeled with their capacities and flows.

d) In trying to find the next augmenting path, the algorithm stops without reaching the sink. What is the value of maximal flow?

e) What is the set of labeled vertices?

f) What is the set of unlabeled vertices?

g) What is the cut?

h) What is the capacity of this cut? How does this compare with the value found in d)

Problem 2

a) Explain how a maximum flow problem for a network with several sources and sinks can be transformed into an equivalent problem maximum flow problem with a single source and a single sink that will give the correct answer to the original problem.

b) Suppose that in addition to the usual constraints a flow network has capacity constraints on the flow that can go through each intermediate vertex. Explain how a maximum flow problem for such a network can be transformed into an equivalent (will give an answer to the original problem) maximum flow problem.

Problem 3

Consider a flow network that is a rooted tree, with the root as its source, the leaves of the tree as its sinks, and all the edges directed in the direction of the paths from the root to the leaves. Design and give pseudo code for an efficient algorithm, $\Theta(|V|+|E|)$, for finding a maximum flow in such a network. Give pseudo code for your algorithm. Why is its efficiency $\Theta(|V|+|E|)$?