Practical: Matrix Multiplication Algorithms

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Introduction

Matrix multiplication is a fundamental operation in computer science and numerical computing. We will study three approaches:

- 1. Classical Iterative Matrix Multiplication
- 2. Recursive Matrix Multiplication
- 3. Strassen's Algorithm

The time taken for multiplication will be measured for different matrix sizes n.

Problem 1. 3(a) Write a program in C language to multiply two square matrices using the iterative approach. Compare the execution time for different matrix sizes.

1. Iterative Matrix Multiplication

The classical method runs three nested loops and computes:

$$C[i][j] = \sum_{k=0}^{n-1} A[i][k] \cdot B[k][j]$$

Algorithm 1 Iterative Matrix Multiplication

```
Require: Matrices A, B of size n \times n

Ensure: Matrix C = A \times B

1: for i \leftarrow 0 to n - 1 do

2: for j \leftarrow 0 to n - 1 do

3: C[i][j] \leftarrow 0

4: for k \leftarrow 0 to n - 1 do

5: C[i][j] \leftarrow C[i][j] + A[i][k] \times B[k][j]

6: end for

7: end for

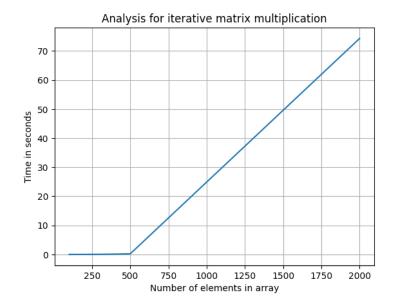
8: end for
```

C Code:

```
#include <stdio.h>
   #include <stdlib.h>
   #include <time.h>
   int mat_mul(int n, int A[n][n], int B[n][n], int C[n][n])
       for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
{</pre>
                C[i][j] = 0;
10
                 for (int k = 0; k < n; k++)
11
12
                     C[i][j] += A[i][k] * B[k][j];
13
14
            }
15
       }
16
17
   void fill_mat(int n, int Mat[n][n])
18
19
20
21
22
                Mat[i][j] = rand() \% 10;
25
       }
26
```

```
}
  int main()
   {
29
       srand(time(NULL));
30
       int size[] = {100, 200, 300, 400,500};
31
       int num = sizeof(size) / sizeof(size[0]);
32
       for (int i = 0; i < num; i++)
33
       {
            int n = size[i];
35
            int (*A)[n] = malloc(size of(int[n][n]));
36
            int (*B)[n] = malloc(sizeof(int[n][n]));
37
            int (*C)[n] = malloc(size of(int[n][n]));
38
39
           if (A == NULL || B == NULL || C == NULL)
41
                printf("Memory allocation failed for size %d\n", n);
42
                return(1);
43
           }
44
           fill_mat(n, A);
45
           fill_mat(n, B);
47
           clock_t start = clock();
48
           mat_mul(n, A, B, C);
49
           clock_t = end = clock();
50
51
           double time_taken = (double)(end - start) / CLOCKS_PER_SEC;
52
53
           printf("Matrix size: %d x %d | Execution time: %f seconds \n", n, n,
54
55
           free(A);
56
           free(B);
57
           free(C);
       }
59
60
       return 0;
61
```

Graph:



Analysis

$$T(n) = O(n^3), \quad S(n) = O(n^2)$$

Problem 2. 3(b) Write a program in C language to multiply two square matrices using the . Compare the execution time for different matrix sizes.

.

2. Recursive Matrix Multiplication

The matrix is divided into four submatrices and the formula

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}, \dots$$

is applied recursively.

```
Algorithm 2 Recursive Matrix Multiplication
```

```
Require: A, B of size n \times n, n > 1

Ensure: C = A \times B

if n = 1 then
C[0][0] \leftarrow A[0][0] \times B[0][0]
else
Split A and B into submatrices of size n/2

Compute submatrices of C recursively
end if
```

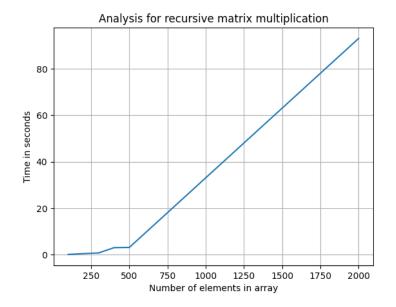
C Code:

```
#include <stdio.h>
  #include <stdlib.h>
  #include <time.h>
  void\ mat_mul(int\ n,int\ A[n][n],int\ B[n][n],\ int\ C[n][n])
  {
      if (n < 2) {
6
           C[0][0] += A[0][0] * B[0][0];
           return;
       }
       int k = n / 2;
12
       int (*A11)[k] = malloc(sizeof(int[k][k]));
13
           (*A12)[k] = malloc(sizeof(int[k][k]));
14
       int (*A21)[k] = malloc(size of(int[k][k]));
15
       int \ (*A22)[k] = malloc(size of(int[k][k]));
17
       int (*B11)[k] = malloc(sizeof(int[k][k]));
18
       int (*B12)[k] = malloc(size of(int[k][k]));
19
       int (*B21)[k] = malloc(sizeof(int[k][k]));
20
       int (*B22)[k] = malloc(sizeof(int[k][k]));
21
       int (*C11)[k] = malloc(size of(int[k][k]));
23
       int \ (*C12)[k] = malloc(size of(int[k][k]));
24
       int (*C21)[k] = malloc(size of(int[k][k]));
25
       int \ (*C22)[k] = malloc(size of(int[k][k]));
26
```

```
28
       for (int i = 0; i < k; i++)
29
           for (int j = 0; j < k; j++) {
30
                C11[i][j] = 0;
31
                C12[i][j] = 0;
32
                C21[i][j] = 0;
33
                C22[i][j] = 0;
34
           }
35
36
       // Divide A and B into 4 parts
37
       for (int i = 0; i < k; i++) {
38
           for (int j = 0; j < k; j++) {
39
                A11[i][j] = A[i][j];
40
                A12[i][j] = A[i][j + k];
41
                A21[i][j] = A[i + k][j];
42
                A22[i][j] = A[i + k][j + k];
43
44
                B11[i][j] = B[i][j];
45
                B12[i][j] = B[i][j + k];
46
                B21[i][j] = B[i + k][j];
47
                B22[i][j] = B[i + k][j + k];
48
           }
49
       }
50
51
52
       mat_mul(k, A11, B11, C11);
53
       mat_mul(k, A12, B21, C11);
54
55
       mat_mul(k, A11, B12, C12);
56
       mat_mul(k, A12, B22, C12);
57
58
       mat_mul(k, A21, B11, C21);
       mat_mul(k, A22, B21, C21);
60
61
       mat_mul(k, A21, B12, C22);
62
       mat_mul(k, A22, B22, C22);
63
64
       for (int i = 0; i < k; i++) {
65
            for (int j = 0; j < k; j++) {
66
                C[i][j] = C11[i][j];
67
                C[i][j + k] = C12[i][j];
68
                C[i + k][j] = C21[i][j];
69
                C[i + k][j + k] = C22[i][j];
70
           }
71
       }
72
73
       free(A11); free(A12); free(A21); free(A22);
74
       free(B11); free(B12); free(B21); free(B22);
75
       free(C11); free(C12); free(C21); free(C22);
  }
77
78
```

```
79
   void fill_mat(int n, int Mat[n][n])
   {
81
       for (int i = 0; i < n; i++)
82
83
           for (int j = 0; j < n; j++)
84
85
                Mat[i][j] = rand() \% 10;
87
       }
88
   }
89
   int main()
90
   {
91
       srand(time(NULL));
92
       int size[] = {100, 200, 300, 400,500};
93
       int num = sizeof(size) / sizeof(size[0]);
94
       for (int i = 0; i < num; i++)
95
       {
96
            int n = size[i];
97
            int (*A)[n] = malloc(size of(int[n][n]));
            int (*B)[n] = malloc(size of(int[n][n]));
99
            int (*C)[n] = malloc(size of(int[n][n]));
100
101
            if (A == NULL || B == NULL || C == NULL)
102
103
                printf("Memory allocation failed for size %d\n", n);
104
                return (1);
105
106
           fill_mat(n, A);
107
           fill_mat(n, B);
108
109
           clock_t start = clock();
110
           mat_mul(n, A, B, C);
111
           clock_t = end = clock();
112
113
           double time_taken = (double)(end - start) / CLOCKS_PER_SEC;
114
115
           116
117
           free(A);
118
           free(B);
119
           free(C);
120
       }
121
122
       return 0;
123
124
```

Graph:



Analysis

$$T(n) = O(n^3), \quad S(n) = O(n^2)$$

Problem 3. 3(c) Given two square matrices A and B of size $n \times n$ (n is a power of 2), write a C code to multiply them using, which reduces the number of recursive multiplications from 8 to 7 by introducing additional addition/subtraction operations. Compare the execution time for different matrix sizes

3. Strassen's Algorithm

Strassen reduces the number of recursive multiplications to 7 using clever combinations.

$$T(n) = 7T\left(\frac{n}{2}\right) + O(n^2)$$

By Master Theorem:

$$T(n) = O(n^{\log_2 7}) \approx O(n^{2.81})$$

```
Algorithm 3 Strassen's Algorithm
```

```
Require: Matrices A, B of size n \times n, n is power of 2

Ensure: C = A \times B

if n = 1 then

Multiply directly

else

Split A and B into quadrants

Compute M_1, M_2, \ldots, M_7

Form C_{11}, C_{12}, C_{21}, C_{22}

end if
```

C Code:

```
#include <stdio.h>
   #include <stdlib.h>
   #include <time.h>
   void fill_matrix(int n, int **M) {
       for (int i = 0; i < n; i++)
6
           for (int j = 0; j < n; j++)
                M[i][j] = rand() \% 10;
  }
9
10
   void print_matrix(int n, int **M) {
11
       for (int i = 0; i < n; i++) {
12
           for (int j = 0; j < n; j++)
13
                printf("%4d ", M[i][j]);
           printf(" \setminus n");
15
16
       printf("\n");
17
   }
18
19
   int **alloc_matrix(int n) {
20
       int **M = malloc(n * sizeof(int *));
21
       for (int i = 0; i < n; i++)
22
           M[i] = calloc(n, sizeof(int)); // initialise to 0
23
```

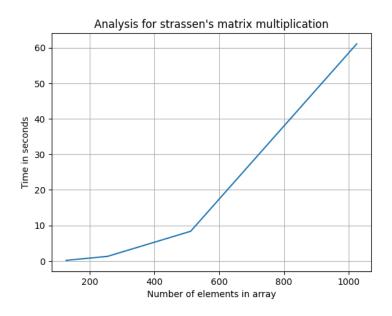
```
return M;
  }
25
26
  void free_matrix(int n, int **M) {
27
       for (int i = 0; i < n; i++) free(M[i]);
28
       free(M);
29
30
31
  void add_matrix(int n, int **A, int **B, int **C) {
32
       for (int i = 0; i < n; i++)
33
           for (int j = 0; j < n; j++)
34
                C[i][j] = A[i][j] + B[i][j];
35
  }
36
37
  void sub_matrix(int n, int **A, int **B, int **C) {
38
       for (int i = 0; i < n; i++)
39
           for (int j = 0; j < n; j++)
40
                C[i][j] = A[i][j] - B[i][j];
41
  }
42
43
  void naive_mul(int n, int **A, int **B, int **C) {
44
       for (int i = 0; i < n; i++)
45
           for (int j = 0; j < n; j++) {
46
                C[i][j] = 0;
47
                for (int k = 0; k < n; k++)
48
                    C[i][j] += A[i][k] * B[k][j];
           }
50
  }
51
52
53
   void strassen(int n, int **A, int **B, int **C) {
54
       if (n \le 2) { // base case: use naive multiplication
           naive_mul(n, A, B, C);
56
           return;
57
       }
58
59
       int k = n / 2;
61
62
       int **A11 = alloc_matrix(k), **A12 = alloc_matrix(k);
63
       int **A21 = alloc_matrix(k), **A22 = alloc_matrix(k);
64
       int **B11 = alloc_matrix(k), **B12 = alloc_matrix(k);
65
       int **B21 = alloc_matrix(k), **B22 = alloc_matrix(k);
       int **C11 = alloc_matrix(k), **C12 = alloc_matrix(k);
67
       int **C21 = alloc_matrix(k), **C22 = alloc_matrix(k);
68
69
       int **M1 = alloc_matrix(k), **M2 = alloc_matrix(k), **M3 = alloc_matrix(k)
70
       int **M4 = alloc matrix(k), **M5 = alloc matrix(k), **M6 = alloc matrix(k)
71
       int **T1 = alloc_matrix(k), **T2 = alloc_matrix(k);
73
       // split A and B into 4 parts
74
```

```
for (int i = 0; i < k; i++) {
75
            for (int j = 0; j < k; j++) {
76
                A11[i][j] = A[i][j];
77
                A12[i][j] = A[i][j + k];
78
                A21[i][j] = A[i + k][j];
79
                A22[i][j] = A[i + k][j + k];
80
81
                B11[i][j] = B[i][j];
82
                B12[i][j] = B[i][j + k];
                B21[i][j] = B[i + k][j];
84
                B22[i][j] = B[i + k][j + k];
85
            }
86
       }
87
        // M1 = (A11 + A22) * (B11 + B22)
89
        add_matrix(k, A11, A22, T1);
90
        add_matrix(k, B11, B22, T2);
91
        strassen(k, T1, T2, M1);
92
93
        // M2 = (A21 + A22) * B11
        add_matrix(k, A21, A22, T1);
95
        strassen(k, T1, B11, M2);
96
97
        // M3 = A11 * (B12 - B22)
98
        sub_matrix(k, B12, B22, T2);
99
        strassen(k, A11, T2, M3);
100
101
       // M4 = A22 * (B21 - B11)
102
        sub_matrix(k, B21, B11, T2);
103
        strassen(k, A22, T2, M4);
104
105
       // M5 = (A11 + A12) * B22
106
        add_matrix(k, A11, A12, T1);
107
        strassen(k, T1, B22, M5);
108
109
        // M6 = (A21 - A11) * (B11 + B12)
110
        sub_matrix(k, A21, A11, T1);
111
        add_matrix(k, B11, B12, T2);
112
        strassen(k, T1, T2, M6);
113
114
        // M7 = (A12 - A22) * (B21 + B22)
115
        sub_matrix(k, A12, A22, T1);
116
        add_matrix(k, B21, B22, T2);
117
        strassen(k, T1, T2, M7);
118
119
        // C11 = M1 + M4 - M5 + M7
120
        for (int i = 0; i < k; i++)
121
            for (int j = 0; j < k; j++)
122
                C11[i][j] = M1[i][j] + M4[i][j] - M5[i][j] + M7[i][j];
123
124
       // C12 = M3 + M5
125
```

```
for (int i = 0; i < k; i++)
126
                                                                  for (int j = 0; j < k; j++)
127
                                                                                         C12[i][j] = M3[i][j] + M5[i][j];
128
129
                                          // C21 = M2 + M4
130
                                          for (int i = 0; i < k; i++)
131
                                                                  for (int j = 0; j < k; j++)
132
                                                                                          C21[i][j] = M2[i][j] + M4[i][j];
133
134
                                          // C22 = M1 - M2 + M3 + M6
135
                                          for (int i = 0; i < k; i++)
136
                                                                  for (int j = 0; j < k; j++)
137
                                                                                          C22[i][j] = M1[i][j] - M2[i][j] + M3[i][j] + M6[i][j];
138
                                         // join C
140
                                          for (int i = 0; i < k; i++) {
141
                                                                  for (int j = 0; j < k; j++) {
142
                                                                                         C[i][j]
                                                                                                                                                                         = C11[i][j];
143
                                                                                         C[i][j + k]
                                                                                                                                                               = C12[i][j];
144
                                                                                         C[i + k][j]
                                                                                                                                                                = C21[i][j];
145
                                                                                         C[i + k][j+k] = C22[i][j];
146
                                                                 }
147
                                         }
148
149
                                         // free memory
150
                                          free\_matrix(k, A11); free\_matrix(k, A12); free\_matrix(k, A21); free\_ma
                                          free_matrix(k, B11); free_matrix(k, B12); free_matrix(k, B21); free_ma
152
                                          free\_matrix(k, C11); free\_matrix(k, C12); free\_matrix(k, C21); free\_ma
153
                                          free_matrix(k, M1);
                                                                                                                                                                    free_matrix(k, M2);
                                                                                                                                                                                                                                                                                                 free_matrix(k, M3);
154
                   free_matrix(k, M4);
                                          free_matrix(k, M5);
                                                                                                                                                                  free_matrix(k, M6);
                                                                                                                                                                                                                                                                                                 free_matrix(k, M7);
155
                                          free_matrix(k, T1);
                                                                                                                                                                  free_matrix(k, T2);
156
                  }
157
158
159
                   int main() {
160
                                           int size[] = {100, 200, 300, 400,500};
161
                                           int num = sizeof(size) / sizeof(size[0]);
162
                                          for (int i = 0; i < num; i++)
163
                                          {
164
                                           srand(time(NULL));
165
                                           int n = size[i];
166
167
                                           int **A = alloc_matrix(n);
168
                                           int **B = alloc_matrix(n);
169
                                          int **C = alloc_matrix(n);
170
171
                                          fill_matrix(n, A);
172
                                          fill_matrix(n, B);
173
174
                                           clock_t start = clock();
175
```

```
strassen(n, A, B, C);
176
        clock_t = end = clock();
178
        double secs = (double)(end - start) / CLOCKS_PER_SEC;
179
        printf("Matrix \ size \ %d \ x \ %d \ / \ Time \ taken: \ %f \ seconds \ n", \ n, \ n, \ secs);
180
181
        // print_matrix(n, C);
182
183
        free_matrix(n, A);
184
        free_matrix(n, B);
185
        free_matrix(n, C);
186
187
        return 0;
188
```

Graph:



Analysis

$$T(n) = O(n^{2.81}), \quad S(n) = O(n^2)$$

Problem 4. Rewrite a program that generates random square matrices of order 2^n . Implement using the three methods discussed in class: You need to perform matrix multiplication using all three methods. Record and compare the execution times for each method across varying matrix sizes. Analyze and discuss the observed results with respect to their theoretical time complexities. C Code:

```
#include <stdio.h>
   #include <stdlib.h>
   #include <time.h>
   int **alloc_matrix(int n) {
       int **M = malloc(n * sizeof(int *));
       for (int i = 0; i < n; i++)
           M[i] = calloc(n, size of(int));
       return M;
10
  }
11
12
   void free_matrix(int n, int **M) {
13
       for (int i = 0; i < n; i++)
14
           free(M[i]);
15
       free(M);
16
  }
17
18
   void fill matrix(int n, int **M) {
19
       for (int i = 0; i < n; i++)
20
           for (int j = 0; j < n; j++)
21
                M[i][j] = rand() \% 10;
22
  }
23
24
   void iterative_mul(int n, int **A, int **B, int **C) {
25
       for (int i = 0; i < n; i++)
26
           for (int j = 0; j < n; j++) {
27
                C[i][j] = 0;
28
                for (int k = 0; k < n; k++)
29
                    C[i][j] += A[i][k] * B[k][j];
30
           }
31
32
33
   void\ recursive\_mul(int\ n,\ int\ **A,\ int\ **B,\ int\ **C)\ \{
34
       if (n == 1) {
35
           C[0][0] += A[0][0] * B[0][0];
36
            return;
37
38
       int k = n / 2;
       int **A11 = alloc_matrix(k), **A12 = alloc_matrix(k);
40
       int **A21 = alloc_matrix(k), **A22 = alloc_matrix(k);
41
       int **B11 = alloc_matrix(k), **B12 = alloc_matrix(k);
42
       int **B21 = alloc_matrix(k), **B22 = alloc_matrix(k);
43
       int **C11 = alloc_matrix(k), **C12 = alloc_matrix(k);
       int **C21 = alloc_matrix(k), **C22 = alloc_matrix(k);
45
       for (int i = 0; i < k; i++) {
46
```

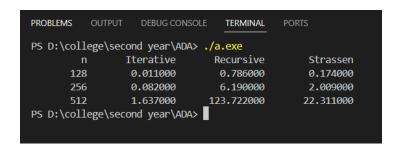
```
for (int j = 0; j < k; j++) {
47
                A11[i][j] = A[i][j];
                A12[i][j] = A[i][j + k];
49
                A21[i][j] = A[i + k][j];
50
                A22[i][j] = A[i + k][j + k];
51
                B11[i][j] = B[i][j];
52
                B12[i][j] = B[i][j + k];
53
                B21[i][j] = B[i + k][j];
54
                B22[i][j] = B[i + k][j + k];
55
           }
56
       }
57
       recursive_mul(k, A11, B11, C11);
58
       recursive_mul(k, A12, B21, C11);
59
       recursive_mul(k, A11, B12, C12);
       recursive_mul(k, A12, B22, C12);
61
       recursive_mul(k, A21, B11, C21);
62
       recursive_mul(k, A22, B21, C21);
63
       recursive_mul(k, A21, B12, C22);
64
       recursive_mul(k, A22, B22, C22);
65
       for (int i = 0; i < k; i++) {
           for (int j = 0; j < k; j++) {
67
                C[i][j] = C11[i][j];
68
                C[i][j + k] = C12[i][j];
69
                C[i + k][j] = C21[i][j];
70
                C[i + k][j + k] = C22[i][j];
71
           }
73
       free_matrix(k, A11);
74
       free_matrix(k, A12);
75
       free_matrix(k, A21);
76
       free\_matrix(k, A22);
77
       free_matrix(k, B11);
       free_matrix(k, B12);
79
       free_matrix(k, B21);
80
       free_matrix(k, B22);
81
       free_matrix(k, C11);
82
       free_matrix(k, C12);
       free_matrix(k, C21);
       free_matrix(k,C22);
85
  }
86
87
   void add_matrix(int n, int **A, int **B, int **C) {
88
       for (int i = 0; i < n; i++)
89
           for (int j = 0; j < n; j++)
90
                C[i][j] = A[i][j] + B[i][j];
91
  }
92
93
   void sub_matrix(int n, int **A, int **B, int **C) {
94
       for (int i = 0; i < n; i++)
           for (int j = 0; j < n; j++)
96
                C[i][j] = A[i][j] - B[i][j];
97
```

```
}
98
   void naive_mul(int n, int **A, int **B, int **C) {
100
       for (int i = 0; i < n; i++)
101
            for (int j = 0; j < n; j++) {
102
                C[i][j] = 0;
103
                for (int k = 0; k < n; k++)
104
                     C[i][j] += A[i][k] * B[k][j];
105
            }
106
   }
107
108
   void strassen(int n, int **A, int **B, int **C) {
109
        if (n <= 2) {
110
            naive_mul(n, A, B, C);
            return;
112
       }
113
        int k = n / 2;
114
        int **A11 = alloc_matrix(k), **A12 = alloc_matrix(k);
115
        int **A21 = alloc_matrix(k), **A22 = alloc_matrix(k);
116
        int **B11 = alloc_matrix(k), **B12 = alloc_matrix(k);
117
        int **B21 = alloc_matrix(k), **B22 = alloc_matrix(k);
118
        int **C11 = alloc_matrix(k), **C12 = alloc_matrix(k);
119
        int **C21 = alloc_matrix(k), **C22 = alloc_matrix(k);
120
        int **M1 = alloc_matrix(k), **M2 = alloc_matrix(k),
121
        **M3 = alloc_matrix(k);
122
        int **M4 = alloc_matrix(k), **M5 = alloc_matrix(k),
123
        **M6 = alloc_matrix(k), **M7 = alloc_matrix(k);
124
        int **T1 = alloc_matrix(k), **T2 = alloc_matrix(k);
125
       for (int i = 0; i < k; i++) {
126
            for (int j = 0; j < k; j++) {
127
                A11[i][j] = A[i][j];
                A12[i][j] = A[i][j + k];
129
                A21[i][j] = A[i + k][j];
130
                A22[i][j] = A[i + k][j + k];
131
                B11[i][j] = B[i][j];
132
                B12[i][j] = B[i][j + k];
133
                B21[i][j] = B[i + k][j];
134
                B22[i][j] = B[i + k][j + k];
135
            }
136
       }
137
        add_{matrix}(k, A11, A22, T1);
138
        add_matrix(k, B11, B22, T2);
139
        strassen(k, T1, T2, M1);
140
        add_matrix(k, A21, A22, T1);
141
        strassen(k, T1, B11, M2);
142
        sub_matrix(k, B12, B22, T2);
143
        strassen(k, A11, T2, M3);
144
        sub_matrix(k, B21, B11, T2);
145
        strassen(k, A22, T2, M4);
146
        add_matrix(k, A11, A12, T1);
147
        strassen(k, T1, B22, M5);
148
```

```
sub_matrix(k, A21, A11, T1);
149
                           add_matrix(k, B11, B12, T2);
                          strassen(k, T1, T2, M6);
151
                          sub_matrix(k, A12, A22, T1);
152
                          add_matrix(k, B21, B22, T2);
153
                          strassen(k, T1, T2, M7);
154
                          for (int i = 0; i < k; i++)
155
                                         for (int j = 0; j < k; j++)
156
                                                       C11[i][j] = M1[i][j] + M4[i][j] - M5[i][j] + M7[i][j];
157
                          for (int i = 0; i < k; i++)
158
                                         for (int j = 0; j < k; j++)
159
                                                        C12[i][j] = M3[i][j] + M5[i][j];
160
                          for (int i = 0; i < k; i++)
161
                                         for (int j = 0; j < k; j++)
                                                       C21[i][j] = M2[i][j] + M4[i][j];
163
                          for (int i = 0; i < k; i++)
164
                                         for (int j = 0; j < k; j++)
165
                                                        C22[i][j] = M1[i][j] - M2[i][j] + M3[i][j] + M6[i][j];
166
                          for (int i = 0; i < k; i++) {
167
                                         for (int j = 0; j < k; j++) {
168
                                                       C[i][j] = C11[i][j];
169
                                                       C[i][j + k] = C12[i][j];
170
                                                       C[i + k][j] = C21[i][j];
171
                                                       C[i + k][j + k] = C22[i][j];
172
                                         }
173
                          free_matrix(k, A11); free_matrix(k, A12); free_matrix(k, A21); free_matrix(k, A21);
175
                          free\_matrix(k, B11); free\_matrix(k, B12); free\_matrix(k, B21); free\_ma
176
                          free\_matrix(k, C11); free\_matrix(k, C12); free\_matrix(k, C21); free\_ma
177
                          free_matrix(k, M1); free_matrix(k, M2); free_matrix(k, M3); free_matrix(
178
                          free_matrix(k, M5); free_matrix(k, M6); free_matrix(k, M7);
                          free_matrix(k, T1); free_matrix(k, T2);
           }
181
182
            int main() {
183
                          int sizes[] = {128, 256, 512, 1024};
184
                          int num = sizeof(sizes) / sizeof(sizes[0]);
185
                          printf("\%10s \%15s \%15s \%15s \n", "n", "Iterative", "Recursive", "Strassen
186
                          for (int i = 0; i < num; i++) {
187
                                         int n = sizes[i];
188
                                         int **A = alloc matrix(n);
189
                                         int **B = alloc_matrix(n);
190
                                         int **C = alloc_matrix(n);
                                         fill_matrix(n, A);
192
                                         fill_matrix(n, B);
193
194
                                         clock\_t start, end;
195
                                         double t_iter, t_rec, t_strassen;
196
                                         start = clock();
197
                                         iterative_mul(n, A, B, C);
198
                                         end = clock();
199
```

```
t_iter = (double)(end - start) / CLOCKS_PER_SEC;
200
201
            start = clock();
202
            recursive_mul(n, A, B, C);
203
            end = clock();
204
            t_rec = (double)(end - start) / CLOCKS_PER_SEC;
205
206
            start = clock();
207
            strassen(n, A, B, C);
208
            end = clock();
209
            t_strassen = (double)(end - start) / CLOCKS_PER_SEC;
210
211
            printf("%10d %15.6f %15.6f %15.6f n", n, t_iter, t_rec, t_strassen);
212
            free_matrix(n, A);
213
            free_matrix(n, B);
214
            free_matrix(n, C);
215
216
        return 0;
217
   }
```

Comparison:



Practical Notes on Strassen's Algorithm

While Strassen's algorithm has a better theoretical complexity $(O(n^{2.81})$ compared to $O(n^3)$), in practice we observed that it can be slower than the naïve and recursive methods for smaller input sizes. This is due to:

- Overhead of recursive function calls.
- Extra memory allocations for temporary submatrices.
- Increased number of additions/subtractions, which outweigh the savings in multiplications for small n.
- Poorer cache locality compared to the simple iterative method.

Therefore, Strassen's algorithm usually only becomes faster for **very large** n (in the order of thousands). On typical laptops, the cross-over point where Strassen starts to outperform classical methods is usually between n = 512 and n = 2048.

Conclusion

- The classical iterative and recursive approaches both run in $O(n^3)$.
- Strassen's algorithm asymptotically improves runtime to $O(n^{2.81})$.
- For small n, iterative may outperform Strassen due to overhead.