Practical: Matrix Multiplication Algorithms

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Introduction

Matrix multiplication is a fundamental operation in computer science and numerical computing. We will study three approaches:

1. Classical Iterative Matrix Multiplication
2. Recursive Matrix Multiplication
3. Strassen's Algorithm

The time taken for multiplication will be measured for different matrix sizes n.

Problem 1. *3(a) Write a program in C language to multiply two square matrices using the iterative approach. Compare the execution time for different matrix sizes.*

1. Iterative Matrix Multiplication

The classical method runs three nested loops and computes:

$$C[i][j] = \sum_{k=0}^{n-1} A[i][k] \cdot B[k][j]$$

Algorithm 1 Iterative Matrix Multiplication

Require: Matrices A, B of size $n \times n$
Ensure: Matrix $C = A \times B$

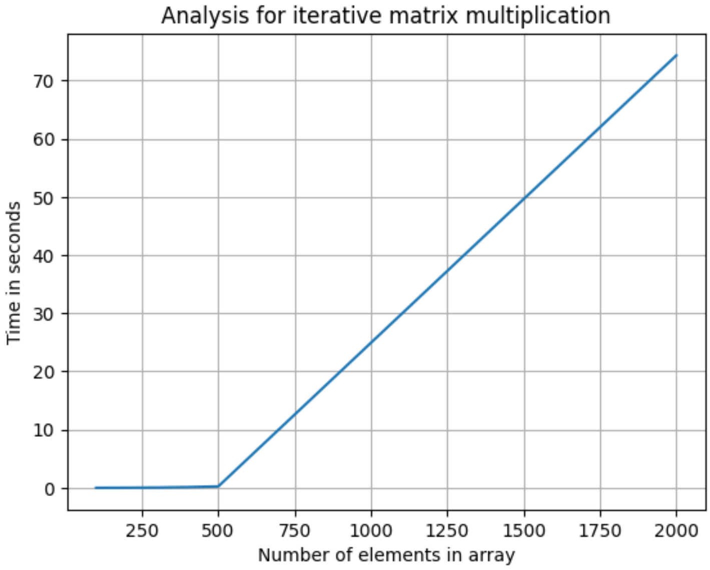
1: **for** $i \leftarrow 0$ to $n-1$ **do**
2:     **for** $j \leftarrow 0$ to $n-1$ **do**
3:         $C[i][j] \leftarrow 0$
4:         **for** $k \leftarrow 0$ to $n-1$ **do**
5:             $C[i][j] \leftarrow C[i][j] + A[i][k] \times B[k][j]$
6:         **end for**
7:     **end for**
8: **end for**

C Code:

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
int mat_mul(int n, int A[n][n], int B[n][n], int C[n][n])
{
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            C[i][j] = 0;
            for (int k = 0; k < n; k++)
            {
                C[i][j] += A[i][k] * B[k][j];
            }
        }
    }
}
void fill_mat(int n, int Mat[n][n])
{
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            Mat[i][j] = rand() % 10;
        }
    }
```

```
}
int main()
{
    srand(time(NULL));
    int size[] = {100, 200, 300, 400,500};
    int num = sizeof(size) / sizeof(size[0]);
    for (int i = 0; i < num; i++)
    {
        int n = size[i];
        int (*A)[n] = malloc(sizeof(int[n][n]));
        int (*B)[n] = malloc(sizeof(int[n][n]));
        int (*C)[n] = malloc(sizeof(int[n][n]));

        if (A == NULL || B == NULL || C == NULL)
        {
            printf("Memory allocation failed for size %d\n", n);
            return(1);
        }
        fill_mat(n, A);
        fill_mat(n, B);

        clock_t start = clock();
        mat_mul(n, A, B, C);
        clock_t end = clock();

        double time_taken = (double)(end - start) / CLOCKS_PER_SEC;

        printf("Matrix size: %d x %d | Execution time: %f seconds\n", n, n,

        free(A);
        free(B);
        free(C);
    }

    return 0;
}
```

Graph:



Analysis

$$T(n) = O(n^3), \quad S(n) = O(n^2)$$

Problem 2. *3(b) Write a program in C language to multiply two square matrices using the . Compare the execution time for different matrix sizes.*

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2. Recursive Matrix Multiplication

The matrix is divided into four submatrices and the formula

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}, \quad \ldots$$

is applied recursively.

Algorithm 2 Recursive Matrix Multiplication

Require: A, B of size $n \times n$, $n > 1$
Ensure: $C = A \times B$
  if $n = 1$ **then**
    $C[0][0] \leftarrow A[0][0] \times B[0][0]$
  else
    Split A and B into submatrices of size $n/2$
    Compute submatrices of C recursively
  end if

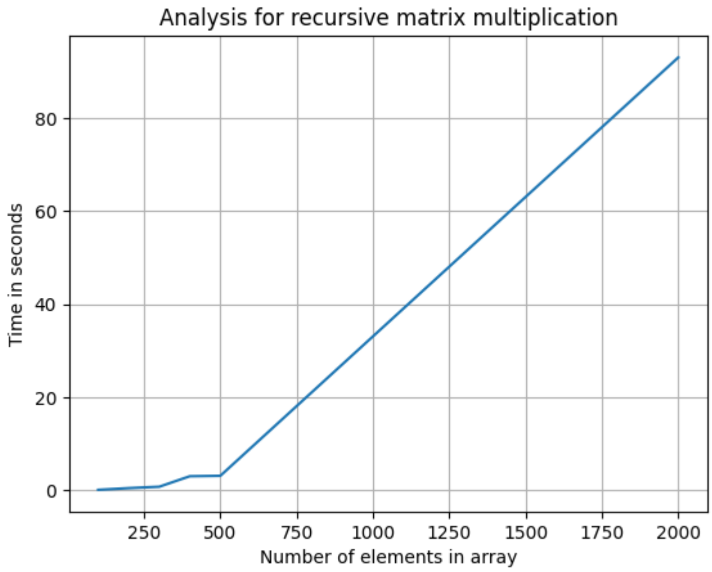
C Code:

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
void mat_mul(int n,int A[n][n],int B[n][n], int C[n][n])
{
   if (n < 2) {
        C[0][0] += A[0][0] * B[0][0];
        return;
    }

    int k = n / 2;

    int (*A11)[k] = malloc(sizeof(int[k][k]));
    int (*A12)[k] = malloc(sizeof(int[k][k]));
    int (*A21)[k] = malloc(sizeof(int[k][k]));
    int (*A22)[k] = malloc(sizeof(int[k][k]));

    int (*B11)[k] = malloc(sizeof(int[k][k]));
    int (*B12)[k] = malloc(sizeof(int[k][k]));
    int (*B21)[k] = malloc(sizeof(int[k][k]));
    int (*B22)[k] = malloc(sizeof(int[k][k]));

    int (*C11)[k] = malloc(sizeof(int[k][k]));
    int (*C12)[k] = malloc(sizeof(int[k][k]));
    int (*C21)[k] = malloc(sizeof(int[k][k]));
    int (*C22)[k] = malloc(sizeof(int[k][k]));

```

```c

    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++) {
            C11[i][j] = 0;
            C12[i][j] = 0;
            C21[i][j] = 0;
            C22[i][j] = 0;
        }

    // Divide A and B into 4 parts
    for (int i = 0; i < k; i++) {
        for (int j = 0; j < k; j++) {
            A11[i][j] = A[i][j];
            A12[i][j] = A[i][j + k];
            A21[i][j] = A[i + k][j];
            A22[i][j] = A[i + k][j + k];

            B11[i][j] = B[i][j];
            B12[i][j] = B[i][j + k];
            B21[i][j] = B[i + k][j];
            B22[i][j] = B[i + k][j + k];
        }
    }


    mat_mul(k, A11, B11, C11);
    mat_mul(k, A12, B21, C11);

    mat_mul(k, A11, B12, C12);
    mat_mul(k, A12, B22, C12);

    mat_mul(k, A21, B11, C21);
    mat_mul(k, A22, B21, C21);

    mat_mul(k, A21, B12, C22);
    mat_mul(k, A22, B22, C22);

    for (int i = 0; i < k; i++) {
        for (int j = 0; j < k; j++) {
            C[i][j] = C11[i][j];
            C[i][j + k] = C12[i][j];
            C[i + k][j] = C21[i][j];
            C[i + k][j + k] = C22[i][j];
        }
    }

    free(A11); free(A12); free(A21); free(A22);
    free(B11); free(B12); free(B21); free(B22);
    free(C11); free(C12); free(C21); free(C22);
}

```

```c

void fill_mat(int n, int Mat[n][n])
{
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            Mat[i][j] = rand() % 10;
        }
    }
}
int main()
{
    srand(time(NULL));
    int size[] = {100, 200, 300, 400,500};
    int num = sizeof(size) / sizeof(size[0]);
    for (int i = 0; i < num; i++)
    {
        int n = size[i];
        int (*A)[n] = malloc(sizeof(int[n][n]));
        int (*B)[n] = malloc(sizeof(int[n][n]));
        int (*C)[n] = malloc(sizeof(int[n][n]));

        if (A == NULL || B == NULL || C == NULL)
        {
            printf("Memory allocation failed for size %d\n", n);
            return(1);
        }
        fill_mat(n, A);
        fill_mat(n, B);

        clock_t start = clock();
        mat_mul(n, A, B, C);
        clock_t end = clock();

        double time_taken = (double)(end - start) / CLOCKS_PER_SEC;

        printf("Matrix size: %d x %d | Execution time: %f seconds\n", n, n,

        free(A);
        free(B);
        free(C);
    }

    return 0;
}
```

Graph:



Analysis

$$T(n) = O(n^3), \quad S(n) = O(n^2)$$

Problem 3. *3(c) Given two square matrices A and B of size n × n (n is a power of 2), write a C code to multiply them using , which reduces the number of recursive multiplications from 8 to 7 by introducing additional addition/subtraction operations. Compare the execution time for different matrix sizes*

3. Strassen's Algorithm

Strassen reduces the number of recursive multiplications to 7 using clever combinations.

$$T(n) = 7T\left(\frac{n}{2}\right) + O(n^2)$$

By Master Theorem:

$$T(n) = O(n^{\log_2 7}) \approx O(n^{2.81})$$

Algorithm 3 Strassen's Algorithm

Require: Matrices A, B of size $n \times n$, n is power of 2
Ensure: $C = A \times B$
 if $n = 1$ **then**
  Multiply directly
 else
  Split A and B into quadrants
  Compute $M_1, M_2, \ldots, M_7$
  Form $C_{11}, C_{12}, C_{21}, C_{22}$
 end if

C Code:

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

void fill_matrix(int n, int **M) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            M[i][j] = rand() % 10;
}

void print_matrix(int n, int **M) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++)
            printf("%4d ", M[i][j]);
        printf("\n");
    }
    printf("\n");
}

int **alloc_matrix(int n) {
    int **M = malloc(n * sizeof(int *));
    for (int i = 0; i < n; i++)
        M[i] = calloc(n, sizeof(int));    // initialise to 0
```

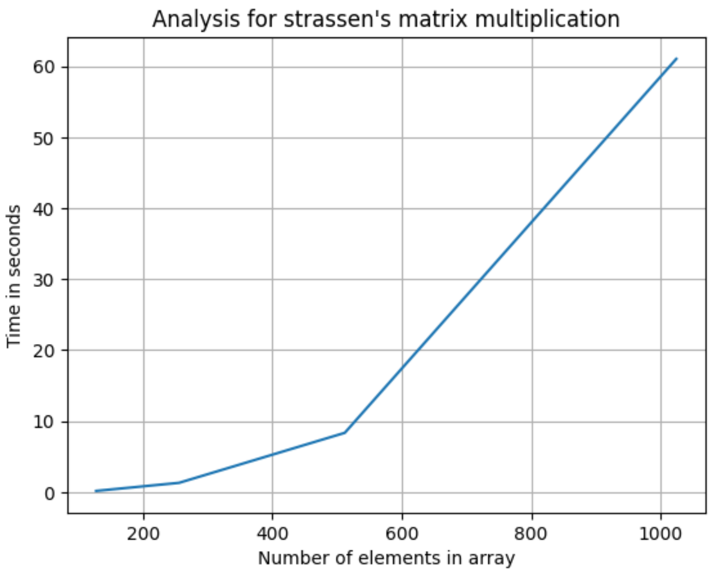
```c
    return M;
}

void free_matrix(int n, int **M) {
    for (int i = 0; i < n; i++) free(M[i]);
    free(M);
}

void add_matrix(int n, int **A, int **B, int **C) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            C[i][j] = A[i][j] + B[i][j];
}

void sub_matrix(int n, int **A, int **B, int **C) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            C[i][j] = A[i][j] - B[i][j];
}

void naive_mul(int n, int **A, int **B, int **C) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            C[i][j] = 0;
            for (int k = 0; k < n; k++)
                C[i][j] += A[i][k] * B[k][j];
        }
}


void strassen(int n, int **A, int **B, int **C) {
    if (n <= 2) {  // base case: use naive multiplication
        naive_mul(n, A, B, C);
        return;
    }

    int k = n / 2;


    int **A11 = alloc_matrix(k), **A12 = alloc_matrix(k);
    int **A21 = alloc_matrix(k), **A22 = alloc_matrix(k);
    int **B11 = alloc_matrix(k), **B12 = alloc_matrix(k);
    int **B21 = alloc_matrix(k), **B22 = alloc_matrix(k);
    int **C11 = alloc_matrix(k), **C12 = alloc_matrix(k);
    int **C21 = alloc_matrix(k), **C22 = alloc_matrix(k);

    int **M1 = alloc_matrix(k), **M2 = alloc_matrix(k), **M3 = alloc_matrix(
    int **M4 = alloc_matrix(k), **M5 = alloc_matrix(k), **M6 = alloc_matrix(
    int **T1 = alloc_matrix(k), **T2 = alloc_matrix(k);

    // split A and B into 4 parts
```

```
    for (int i = 0; i < k; i++) {
        for (int j = 0; j < k; j++) {
            A11[i][j] = A[i][j];
            A12[i][j] = A[i][j + k];
            A21[i][j] = A[i + k][j];
            A22[i][j] = A[i + k][j + k];

            B11[i][j] = B[i][j];
            B12[i][j] = B[i][j + k];
            B21[i][j] = B[i + k][j];
            B22[i][j] = B[i + k][j + k];
        }
    }

    // M1 = (A11 + A22) * (B11 + B22)
    add_matrix(k, A11, A22, T1);
    add_matrix(k, B11, B22, T2);
    strassen(k, T1, T2, M1);

    // M2 = (A21 + A22) * B11
    add_matrix(k, A21, A22, T1);
    strassen(k, T1, B11, M2);

    // M3 = A11 * (B12 - B22)
    sub_matrix(k, B12, B22, T2);
    strassen(k, A11, T2, M3);

    // M4 = A22 * (B21 - B11)
    sub_matrix(k, B21, B11, T2);
    strassen(k, A22, T2, M4);

    // M5 = (A11 + A12) * B22
    add_matrix(k, A11, A12, T1);
    strassen(k, T1, B22, M5);

    // M6 = (A21 - A11) * (B11 + B12)
    sub_matrix(k, A21, A11, T1);
    add_matrix(k, B11, B12, T2);
    strassen(k, T1, T2, M6);

    // M7 = (A12 - A22) * (B21 + B22)
    sub_matrix(k, A12, A22, T1);
    add_matrix(k, B21, B22, T2);
    strassen(k, T1, T2, M7);

    // C11 = M1 + M4 - M5 + M7
    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++)
            C11[i][j] = M1[i][j] + M4[i][j] - M5[i][j] + M7[i][j];

    // C12 = M3 + M5
```

```
    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++)
            C12[i][j] = M3[i][j] + M5[i][j];

    // C21 = M2 + M4
    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++)
            C21[i][j] = M2[i][j] + M4[i][j];

    // C22 = M1 - M2 + M3 + M6
    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++)
            C22[i][j] = M1[i][j] - M2[i][j] + M3[i][j] + M6[i][j];

    // join C
    for (int i = 0; i < k; i++) {
        for (int j = 0; j < k; j++) {
            C[i][j]         = C11[i][j];
            C[i][j + k]     = C12[i][j];
            C[i + k][j]     = C21[i][j];
            C[i + k][j+k]   = C22[i][j];
        }
    }

    // free memory
    free_matrix(k, A11); free_matrix(k, A12); free_matrix(k, A21); free_matr
    free_matrix(k, B11); free_matrix(k, B12); free_matrix(k, B21); free_matr
    free_matrix(k, C11); free_matrix(k, C12); free_matrix(k, C21); free_matr
    free_matrix(k, M1);  free_matrix(k, M2);  free_matrix(k, M3);
free_matrix(k, M4);
    free_matrix(k, M5);  free_matrix(k, M6);  free_matrix(k, M7);
    free_matrix(k, T1);  free_matrix(k, T2);
}


int main() {
    int size[] = {100, 200, 300, 400,500};
    int num = sizeof(size) / sizeof(size[0]);
    for (int i = 0; i < num; i++)
    {
    srand(time(NULL));
    int n= size[i];

    int **A = alloc_matrix(n);
    int **B = alloc_matrix(n);
    int **C = alloc_matrix(n);

    fill_matrix(n, A);
    fill_matrix(n, B);

    clock_t start = clock();
```

```
        strassen(n, A, B, C);
        clock_t end = clock();

        double secs = (double)(end - start) / CLOCKS_PER_SEC;
        printf("Matrix size %d x %d | Time taken: %f seconds\n", n, n, secs);

        // print_matrix(n, C);

        free_matrix(n, A);
        free_matrix(n, B);
        free_matrix(n, C);
    }
        return 0;
    }
```

Graph:



Analysis

$$T(n) = O(n^{2.81}), \quad S(n) = O(n^2)$$

Practical Notes on Strassen's Algorithm

While Strassen's algorithm has a better theoretical complexity ($O(n^{2.81})$ compared to $O(n^3)$), in practice we observed that it can be slower than the naïve and recursive methods for smaller input sizes. This is due to:

- *Overhead of recursive function calls.*
- *Extra memory allocations for temporary submatrices.*
- *Increased number of additions/subtractions, which outweigh the savings in multiplications for small n.*
- *Poorer cache locality compared to the simple iterative method.*

*Therefore, Strassen's algorithm usually only becomes faster for **very large** n (in the order of thousands). On typical laptops, the cross-over point where Strassen starts to outperform classical methods is usually between $n = 512$ and $n = 2048$.*

Conclusion

- *The classical iterative and recursive approaches both run in $O(n^3)$.*
- *Strassen's algorithm asymptotically improves runtime to $O(n^{2.81})$.*
- *For small n, iterative may outperform Strassen due to overhead.*