

Use the PRF in some secure mode of operation to obtain a CPA-secure encryption scheme

The basic idea of pseudorandom functions F_k is $c = (r, F_k(r) \oplus m)$. These are easy to compute and computationally indistinguishable from a random function and there are $2^{n(2^n)}$ possible functions.

Definition of PRF :

Let $f : \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$ be an efficient, length-preserving, keyed function. We say that F is a pseudorandom function if for all probabilistic polynomial-time distinguishers D , there exists a negligible function negl such that:

$$|\Pr[D_k^F(\cdot)(1^n) = 1] - \Pr[D^f(\cdot)(1^n) = 1]| \leq \text{negl}(n),$$

Where $k \leftarrow \{0,1\}^n$ is chosen uniformly at random and f is chosen uniformly at random from the set of functions mapping n -bit strings to n -bit strings.

There are three modes of operation as discussed in class, namely, Cipher block Chaining (CBC), Output Feedback Mode (OFB), and Randomized Counter Mode.

To obtain a CPA-secure encryption scheme using PRF, Let F be a PRF. First, one needs to define a private-key encryption scheme for messages of length n .

The encryption scheme is a collection of three algorithms, namely, key generation algorithm (Gen), encryption algorithm (Enc) and decryption algorithm (Dec). The Gen gives key to both the sender and receiver, the Enc with key, plain text and local randomness gives the ciphertext as the output, and Dec takes the ciphertext and key to output the message.

For the CPA-secure encryption :

1. Gen : on input 1^n , choose $k \leftarrow \{0,1\}^n$ uniformly at random and output it as the key
2. Enc : on input a key where k is belonging to $\{0,1\}^n$ and a message ' m ' belonging to $\{0,1\}^n$, choose $r \leftarrow \{0,1\}^n$ uniformly at random and output the ciphertext
$$c := \langle r, F_k(r) \oplus m \rangle$$
3. Dec : on input a key ' k ' belonging to $\{0,1\}^n$ and a ciphertext $c = \langle r, s \rangle$, output the plaintext message
$$m := F_k(r) \oplus s$$

By following the above mentioned instructions, a CPA-secure encryption scheme from any pseudorandom function can be obtained.