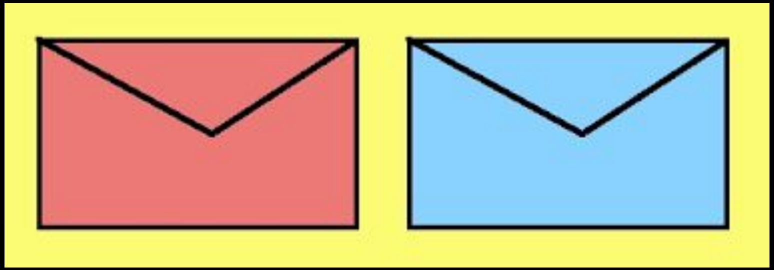
The Two Envelopes Problem

Introduction

The Two Envelope Problem is a paradox stemming from the usage of expected value. The problem statement goes like this:

> You are presented with two identical envelopes, each containing money. One envelope contains twice the amount of money as the other, and you are only allowed to choose one. After choosing an envelope, before checking the amount of money you have, you are given the opportunity to switch envelopes. Should you do so?



01

Expected Value

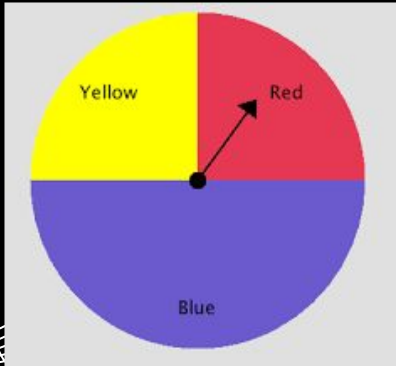
A probabilistic tool to find the expected gain from an action

What is Expected Value?

In simple terms, the expected value is how much you expect to get in return for doing something. It is the weighted average of all the possible outcomes. (Note that it is NOT the most likely outcome)

Example

Bill is playing a game. Every turn, he spins the spinner. If the spinner lands on Yellow, Bill gets 3 points. If the spinner lands on Blue, Bill gets 1 point. If the spinner lands on red, he loses 4 points. What is the expected points Bill will earn in one turn?



Example

Suppose you are betting on a roulette table. You can bet on either red or black. If you guess correctly, you get $1. If you don’t, you pay $1. What is the expected value for playing once?

37 total numbers

18 red number

18 black numbers

1 green number

02

Back to the Problem

Applying Expected Value to the Two Envelopes Problem

Problem Statement

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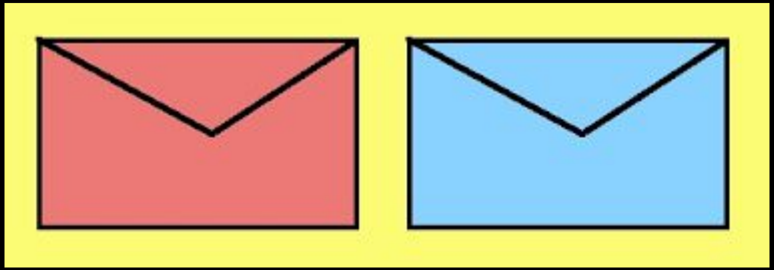

...Whaaat?

There have been multiple proposed solutions that “solve” this problem and explain how the approach is wrong. However, someone else then comes along and does a slight alteration to the problem statement that invalidates the solution, and once again creates the paradox.

Fixed Total Amount

Assume that the total amount of money is 3x, with x in one envelope and 2x in the other. With this, the expected gain from switching envelopes becomes $\frac{1}{2}(x-2x) + \frac{1}{2}(2x-x) = 0$, so there is no anticipated change in money gained from switching envelopes, and the paradox is resolved.

The confusion from the problem comes from the fact that it is assumed that the amount of money in the other envelope is always relative to the one you have. If you instead calculate off a fixed total, it is resolved.

Revising the Problem Statement

You are presented with two identical envelopes. You are told that one was filled with a certain amount of money, and then a coin was flipped. If the coin landed on heads, the other envelope is filled with twice the amount of the first one. If the coin landed on tails, the other envelope is filled with half the amount of money as the first envelope. Do you switch envelopes?