

Introduction to AMC

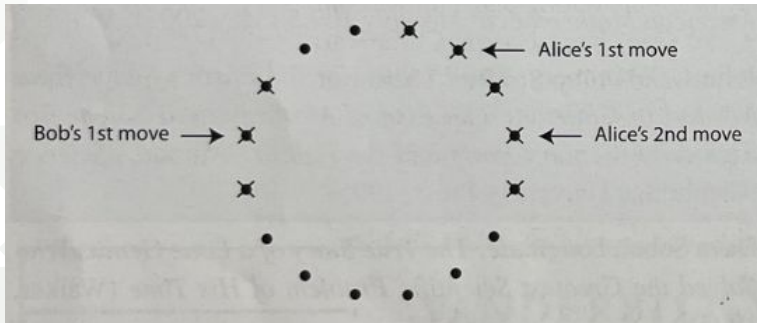
Topics

Housekeeping

- ⇒ Sign in is on paper
- ⇒ AMC Signups Closed
- ⇒ Eldrick's Calculus Website
 - <https://chaddypratt.org/calculus/>

Problem of the Week

- ⇒ Alice and Bob play a game on a circle of n dots
- ⇒ On each person's turn, they cross out an uncrossed dot and the two dots next to it (if they are uncrossed)
- ⇒ Winner is the one who crosses out the last dot
- ⇒ Winning strategy for $n = 19$? $n = 20$?
- ⇒ Example: $n = 16$





1.

Algebra

Topics

- ⇒ Conic Sections
- ⇒ Polynomials
 - Factorization
- ⇒ Complex Numbers
 - Roots of Unity
- ⇒ Inequalities
- ⇒ Logarithms

2010 AMC 12A Problem 10

The first four terms of an arithmetic sequence are p , 9 , $3p - q$, and $3p + q$. What is the 2010th term of this sequence?

(A) 8041 (B) 8043 (C) 8045 (D) 8047 (E) 8049

- ⇒ Arithmetic sequence: the difference between each two consecutive terms is constant
- ⇒ $9 - p = (3p - q) - 9 = (3p + q) - (3p - q)$
- ⇒ Solve for the first term and the common difference

The background of the slide is a dark teal color with a repeating pattern of overlapping triangles in various shades of teal and green, creating a textured, geometric effect.

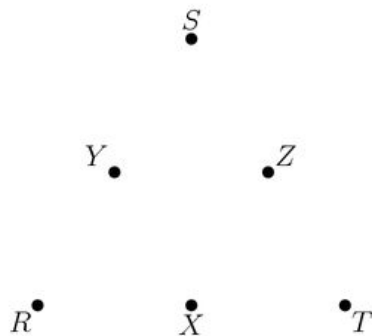
2. Counting

Topics

- ⇒ Distinguishability
- ⇒ Casework Counting
- ⇒ Sticks and Stones/Stars and Bars
- ⇒ Choose, Permutation, Factorial
- ⇒ Complementary Counting
- ⇒ Constructive Counting

2001 AMC 8 Problem 23

Points R , S and T are vertices of an equilateral triangle, and points X , Y and Z are midpoints of its sides. How many noncongruent triangles can be drawn using any three of these six points as vertices?



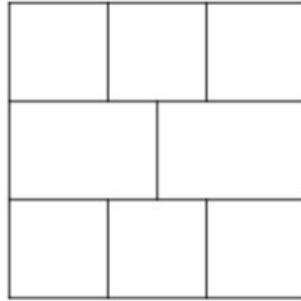
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 20

2001 AMC 8 Problem 23

- ⇒ Casework counting: subtracting the unwanted number of possibilities from the total number of possibilities.
- ⇒ Casework counting: brute force the answer
- ⇒ In this case, brute forcing the answer is faster.

2023 Cowconuts Competition - #2

2. Alice wants to paint the tiles in the diagram below. She has three different colors of paint: red, blue, and green. Find the number of ways she can paint the tiles such that no two tiles that share a border are painted with the same color.





3. Probability

Topics

- ⇒ Probability is (number desired)/(number total)
- ⇒ Independent vs. Dependent probabilities
- ⇒ Multiplying together vs. Adding together
- ⇒ Complementary probability
- ⇒ Expected Value

2001 AMC 8 Problem 23

Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?

(A) all 4 are boys (B) all 4 are girls (C) 2 are girls and 2 are boys (D) 3 are of one gender and 1 is of the other gender (E) all of these outcomes are equally likely

⇒ A) Probability = $(\frac{1}{2})^4$

⇒ B) Probability = $(\frac{1}{2})^4$

⇒ C) Probability = $(\frac{1}{2})^2 * (\frac{1}{2})^2 = (\frac{1}{2})^4$

⇒ D) Probability = 3 boys 1 girl: $(\frac{1}{2})^3 * (\frac{1}{2}) + 3 \text{ girls 1 boy: } (\frac{1}{2})^3 * (\frac{1}{2})$

Number Theory

Topics

- ⇒ Divisibility
- ⇒ Prime Factorization
- ⇒ Number bases
- ⇒ Modular Arithmetic/Linear Congruence

2023 AIME II Problem 2

Recall that a palindrome is a number that reads the same forward and backward. Find the greatest integer less than 1000 that is a palindrome both when written in base ten and when written in base eight, such as $292 = 444$ (base 8)

Geometry

Topics

- ⇒ Length, Angle or Area
- ⇒ Similar Triangles
- ⇒ Power of a Point
- ⇒ Angle Chasing

2022 AMC 10A #10

Daniel finds a rectangular index card and measures its diagonal to be 8 centimeters. Daniel then cuts out equal squares of side 1 cm at two opposite corners of the index card and measures the distance between the two closest vertices of these squares to be $4\sqrt{2}$ centimeters, as shown below. What is the area of the original index card?

