Math Club 10/13

Sign in is on paper :)

Simon’s Favorite Factoring Trick

- Simon’s Favorite Factoring Trick (SFFT) is often used in a Diophantine equation where factoring is needed

Consider the following:

$$xy + 66x - 88y = 23333$$

Simon Rubinstein-Salzedo
PhD in mathematics, teacher at Euler Circle · Author has **105** answers and **348.4K** answer views · 4y

I didn't discover it; it was already well-known when I was in high-school, although perhaps not exactly stated like that. You can see from the name that I do not claim any sort of priority over it, only that it's a trick that I happen to be fond of.

It got named after me simply because I was at the right place at the right time. The Art of Problem Solving website went live in late May of 2003, less than a month before I graduated high school. I was a very frequent poster from the very beginning, probably one of the two or three most prolific posters for quite some time. After just a few months, Richard Rusczyk invited me to become an assistant instructor for their classes, so I was very well-known on AoPS both from my frequent forum posts and from assisting in the classes. A few months later, I believe in October of 2003, I solved a problem that someone posted on the forum, saying that I would solve it using Simon's Favorite Factoring Trick. I don't know why I referred to myself in the third person at that particular moment, but I did.

Apparently people thought that that was a good name, and Richard and Mathew and the other instructors started calling it that in their classes and books. I didn't do anything at all after that initial forum post to popularize it under that name; everyone else did it for me.

It is very strange to me that my greatest claim to fame in life is a single forum post I made when I was 18 years old. I think I've done much better things in my life, especially running Euler Circle and trying to revolutionize gifted mathematics education, giving strong students an opportunity to get a dignified and challenging education that no one else is willing to offer them. However, it does seem that people find my factoring trick memorable with my name on it, so it seems to have done a small part in improving students' problem-solving abilities; I'm

Example #1

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

(A) 22 **(B)** 60 **(C)** 119 **(D)** 180 **(E)** 231

2000 AMC 12 #6

One way to solve this problem is through SFFT.
We can rearrange the equation to look like this:

$$(p-1)(q-1)-1$$

This way, it is easier to notice that 11 and 13 will make 119, or C

Example #2

Find $3x^2y^2$ if x and y are integers such that $y^2 + 3x^2y^2 = 30x^2 + 517$.

1987 AIME #5

If we move the x squared term to left side, we can use SFFT to factor the equation

$$(3x^2 + 1)(y^2 - 10) = 517 - 10$$

We see that 3 * 13^2. Since x and y must be integers, we see that (3x^2 + 1) can’t be the multiple of 3. This means that (3x^2 + 1) either equals 169 or 13.

3 x 4 x 49 = **588**

Example #3

A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2008 AMC 12B #16

$$A_{outer} = ab$$

$$A_{inner} = (a-2)(b-2)$$

$$A_{outer} = 2A_{inner}$$

$$ab = 2(a-2)(b-2) = 2ab - 4a - 4b + 8$$

$$0 = ab - 4a - 4b + 8$$

$$8 = ab - 4a - 4b + 16 = (a-4)(b-4)$$

Since $8 = 1 \times 8$ and $8 = 2 \times 4$ are the only positive factorings of 8.

$(a, b) = (5, 12)$ or $(a, b) = (6, 8)$ yielding $\Rightarrow$ $\textbf{(B)}$ 2 solutions. Notice that because $b > a$, the reversed pairs are invalid.

Thank you for coming!