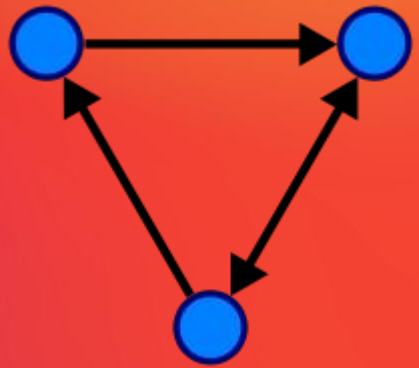
Graph Theory

Graph Theory Review

- Graphs have edges and vertices
- problems such as Travelling Salesman's Problem
- Dijkstra's algorithm, etc. can solve search problems

- directed/undirected edges
- weighted edges



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Friendship Theorem: In a party of n persons, if every pair of persons has exactly one mutual friend, then there is someone in the party who is everyone else's friend. (A “friendship” is a symmetric relation).

Friendship Paradox: Sociologist Scott Feld found in 1991 that most people have fewer friends than their friends have, on average.

Proof

- **P** = set of persons
- arbitrary select person **p** from **P**
- **E** = set of friendships
- **deg(p)** = # of friends person **p** has

- **μ** = average expected number of friends

- person **p** has **deg(p)** friends
- **μ** = Sum **deg(p)** / **|P|**
  = 2 |E| / |P|

- friend **f** is friends with **p** with probability **def(f)** / 2 **|E|**
- **average # of friends of F**
  = Sum [**def(f)**]^2 / 2 **|E|**
  >= **|P|** * **μ**^2 / 2 **|E|**
  = **μ**