A Demo of HINTS

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2025-03-03

Outline

1. Basic Principles

1.1 Smoothing Principles

1. A basic residual-correction method to solve $Au = f$ for a given initial guess u_0.
   1. residual $r = f - Au_k$.
   2. correction $e = Br$ with $B \approx A^{-1}$.
   3. $u_{k+1} = u_k + e$.
2. Amplification matrix.

$$
\begin{aligned}
e_{k+1} &:= u - u_{k+1} = (I - BA)(u - u_k) = (I - BA)e_k \\
E &:= I - BA
\end{aligned}
$$

Theorem 1.1.1 The residual-correction shceme converges iff

1.1 Smoothing Principles

$$\rho(I - BA) < 1.$$

3. Jacobi iteration $B = \operatorname{diag}(A)^{-1}$.
4. 1D Poisson equation $-u'' = 0, x \in (0, 1)$ with $u(0) = u(1) = 0$.

$$\int_0^1 \varphi_i'(x)\varphi_j'(x)\,\mathrm{d}x = \frac{1}{h}\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix} =: \frac{1}{h}A$$

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where $\varphi_i(x)(i = 1, 2, ..., n)$ are hat functions and $h = \frac{1}{n+1}$. The eigenpairs (λ_k, v_k) of A are

$$\lambda_k(A) = 2(1 - \cos(hk\pi)), v_{k,j}(A) = \sqrt{2h}\sin(jkh\pi).$$

Note that $B = \frac{1}{2}I$,

$$\lambda_k(E) = \lambda_k(I - BA) = \cos(hk\pi) = \cos\left(\frac{k\pi}{n+1}\right).$$

5. Numerical experiments.

```julia
function f()
```

Julia

1.1 Smoothing Principles


```
end
```