A. G. Volosniev Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

D. H. Smith Department of Physics, The Ohio State University, Columbus, OH 43210, USA

Impenetrability in Floquet scattering in one dimension

A. G. Volosniev D. H. Smith

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FIG. 1. Panel **a**): The reflection probability $|B_0|^2$ as a function of $p/\sqrt{\omega}$ for $g_1/\sqrt{\omega} = 0.2$ – black solid curve, $g_1/\sqrt{\omega} = 0.4$ – red dashed curve, $g_1/\sqrt{\omega} = 0.8$ – green dot-dashed curve, and $g_1/\sqrt{\omega} = 1.0$ – blue dotted curve. For all curves $g_0/\sqrt{\omega} = -1$. Panel **b**): $|B_0|^2$ for $g_1/\sqrt{\omega} = 1.5$ – black solid curve, $g_1/\sqrt{\omega} = 2.5$ – red dashed curve, $g_1/\sqrt{\omega} = 3.5$ – green dot-dashed curve, and $g_1/\sqrt{\omega} = 4.5$ – blue dotted curve. For all curves $g_0 = 0$. Panel **c**): $|B_0|^2$ for $g_0/\sqrt{\omega} = 0.1$ – black solid curve, $g_0/\sqrt{\omega} = 0.5$ – red dashed curve, $g_0/\sqrt{\omega} = 1$ – green dot-dashed curve, and $g_0/\sqrt{\omega} = 1.5$ – blue dotted curve. For all curves $g_1/\sqrt{\omega} = 1.5$. Panel **d**): $|B_0|^2$ for $g_0/\sqrt{\omega} = -0.1$ – black solid curve, $g_0/\sqrt{\omega} = -0.8$ – red dashed curve, $g_0/\sqrt{\omega} = -1.4$ – green dot-dashed curve, and $g_0/\sqrt{\omega} = -2$ – blue dotted curve. For all curves $g_1/\sqrt{\omega} = 1.5$.

I. INTRODUCTION

II. SCATTERING OFF A TIME-PERIODIC ZERO-RANGE POTENTIAL IN ONE DIMENSION

III. CONCLUSIONS

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Appendix A: Appendix

1. Floquet Formalism

Scattering in time-dependent picture. Here, for convenience of the reader, we discuss the time-dependent Schrödinger equation with a general short-range time-periodic potential

$$i\frac{\partial}{\partial t}\Psi(x,t) = H(t)\Psi(x,t), \qquad H(t) = H_0 + W(x,t), \tag{A1}$$

where the operators in the coordinate representation are $H_0 = -\frac{\partial^2}{\partial x^2}$, and $W(x,t) = \sum_n e^{-\frac{2i\pi nt}{T}} W_n(x)$. We assume that the incoming square-integrable wave packet is $\Psi_0(x,t)$, i.e., $\Psi(x, t \to -\infty) = \Psi_0(x,t)$. Obviously, if $g_n = 0, \forall n$ then the time propagation is

$\Psi_0(x,t) = e^{-iH_0 t}\Psi_0$, where $\Psi_0 \equiv \Psi_0(x,0)$. Therefore, the effect of scattering is deduced by comparing $\Psi(x,t)$ with $\Psi_0(x,t)$.

To proceed we note that there is a unitary operator $U(t,s)$ that determines the time evolution

$$\Psi(t) = U(t,s)\Psi(s). \tag{A2}$$

The formal properties of $U(t,s)$ are discussed in Ref. [?]. From now on we reserve the letters t and s for time, also when it does not cause confusion we omit the coordinate variables. For convenience, we set $t = 0$ to be a reference time and introduce $\Psi \equiv \Psi(x,0)$ such that $\Psi(t) = U(t)\Psi$, where $U(t) \equiv U(t,0)$. To compare $\Psi(t)$ and $\Psi_0(t)$ one can introduce the wave operator $\Omega(s)$, such that $\Psi = \Omega\Psi_0$, $\Omega = \Omega(0)$. This operator is defined as the limit

$$\Omega(s) \equiv \lim_{t\to-\infty} U^{-1}(t,s)e^{-i(t-s)H_0}. \tag{A3}$$

In scattering theory for time-independent potentials Ω is often called the Møller operator [?]. This operator exists also for time-periodic short-range potentials as discussed in Refs. [? ?]. One important property of Ω is called the intertwining relation:

$$U(t)\Omega = \Omega(t)e^{-iH_0 t}. \tag{A4}$$

We use this relation upon decomposing Ψ_0 in the eigenbasis of H_0. For convenience we refrain from using coordinate representation and use Dirac's notation for vectors instead: $|\Psi_0\rangle = \int_{-\infty}^{\infty} \mathrm{d}p\phi(p)|p\rangle$. Now if we apply the intertwining relation to this decomposition we obtain

$$\int \mathrm{d}p\phi(p)\left(U(t)\Omega|p\rangle - e^{-ip^2 t}\Omega(t)|p\rangle\right) = 0. \tag{A5}$$

Since it should be valid for all possible initial wave packets described by $\phi(p)$ we conclude that the integrand should be zero. By differentiating both sides with respect to time, we see that the only way to satisfy this condition is to assume that $|f_p\rangle(t) \equiv \Omega(t)|p\rangle$ obeys

$$\left(H(t) - i\frac{\partial}{\partial t}\right)|f_p\rangle(t) = p^2|f_p\rangle(t). \tag{A6}$$

Let us take a closer look at Eq. (??). According to the Floquet theorem $U(t+T, s+T) = U(t,s)$; see also Ref. [?]. Therefore, $\Omega(T) = \Omega$, hence $|f_p\rangle(t+T) = |f_p\rangle(t)$ and $|f_p\rangle(t) = \sum_n e^{-\frac{2i\pi nt}{T}}|\tilde{f}_{pn}\rangle$. By inserting this ansatz function into Eq. (??) and projecting onto a particular mode we obtain

$$(p^2 + n\omega - H_0)|\tilde{f}_{pn}\rangle = \sum_m W_{n-m}|\tilde{f}_{pm}\rangle, \tag{A7}$$

where $\omega = 2\pi/T$. This is a (infinite-dimensional) matrix equation, and therefore, for each p^2 there is an infinite number of solutions, which can be formally written as

$$|\tilde{f}_{pn}\rangle = |p_n\rangle\alpha_n + \frac{1}{p^2 + n\omega - H_0 + i\epsilon}\sum_m W_{n-m}|\tilde{f}_{pm}\rangle, \tag{A8}$$

where α_n and $\epsilon \in \mathbb{R}$ are coefficients, and $p_n^2 = p^2 + n\omega$. We show below that the solution that satisfies the initial condition $\Psi(x, t \to -\infty) \to \Psi_0(x, t)$ has $\alpha_n = \delta_{n,0}$ and $\epsilon > 0$. To this end we write the formal solution to Eq. (**??**)

$$|\Psi\rangle(t) = e^{-iH_0 t}\int_{-\infty}^{\infty} \mathrm{d}p\phi(p)\left(1 - i\int_{-\infty}^{t} \mathrm{d}t' e^{iH_0 t'} W(t')\Omega(t')e^{-iH_0 t'}\right)|p\rangle. \tag{A9}$$

This equation yields $|\Psi\rangle$

$$|\Psi\rangle = \int_{-\infty}^{\infty} \mathrm{d}p\phi(p)\left(|p\rangle + \sum_{m,n}\frac{1}{p^2 + (n+m)\omega - H_0 + i\delta} W_n|\tilde{f}_{pm}\rangle\right). \tag{A10}$$

where δ is a small positive quantity. To obtain $|\Psi\rangle$ we used the prescription $W(t) \to e^{\delta t}W(t)$, which is justified by noticing that for $t \to -\infty$ the square-integrable wave packet cannot be affected by the finite-range potential. Now if we look at Eqs. (**??**) and (**??**) and notice that $|\Psi\rangle = \Omega|\Psi_0\rangle = \int \mathrm{d}p\phi(p)\sum_n |\tilde{f}_{pn}\rangle$ we deduce that $\alpha_n = \delta_{n,0}$ and $\epsilon > 0$. At t we have $|\Psi\rangle(t) = \int \mathrm{d}p\phi(p)e^{-ip^2 t}\sum_n e^{-i\omega n t}|\tilde{f}_{pn}\rangle$. This provides us with the wave function $\Psi(x, t)$ for $x \to \infty$, which determines characteristics of transmission

$$\Psi(x, t) = \Psi_0(x, t) - \frac{i}{2}\sum_{m,n}\int \mathrm{d}p\phi(p)\frac{e^{ip_n x - ip_n^2 t}}{p_n}\int \mathrm{d}x' e^{-ip_n x'} W_{n-m}(x')\langle x'|\tilde{f}_{pm}\rangle, \tag{A11}$$

a similar expression can be derived for $x \to -\infty$. Here we use the coordinate representation of the Green's function

$$G(x, x'; k^2) \equiv \langle x|\frac{1}{H_0 - k^2 - i\epsilon}|x'\rangle = \frac{i}{2k}e^{ik|x-x'|}. \tag{A12}$$

Scattering in time-independent picture. In this subsection we consider Eq. (**??**) that determines the properties of the scattering in time-independent picture in more detail. In the coordinate representation it reads

$$\tilde{f}_{pn}(x) = \delta_{n,0}e^{ip_n x} - \sum_{m=-\infty}^{\infty}\int \mathrm{d}x' G(x, x'; p_n^2)W_{n-m}(x')\tilde{f}_{pm}(x'), \tag{A13}$$

where $\delta_{m,n}$ is Kronecker's delta. This function at $x \to \infty$ has the form $\delta_{n,0}e^{ip_nx} + B_ne^{ip_nx}$ where

$$B_n = -\frac{i}{2p_n}\sum_{m=-\infty}^{\infty}\int \mathrm{d}x' e^{-ip_nx'}W_{n-m}(x')\tilde{f}_{pm}(x') \tag{A14}$$

whereas at $x \to -\infty$ it has the form $\delta_{n,0}e^{ip_nx} + \tilde{B}_ne^{-ip_nx}$ with

$$\tilde{B}_n = -\frac{i}{2p_n}\sum_{m=-\infty}^{\infty}\int \mathrm{d}x' e^{ip_nx'}W_{n-m}(x')\tilde{f}_{pm}(x'), \tag{A15}$$

Apparently, Eqs. (**??**), (**??**) and (**??**) contain all information about the scattering process and can be used to derive Eq. (**??**) of the main text. It is worthwhile to notice that for a plane wave with a given p^2 the total probability to find a particle with the energies $p_n^2, n = 0, \pm 1, ...$ is conserved in the scattering process (see the next subsection). This can be seen as the conservation of the quasi-energy. At the same time the total energy is not conserved and can become larger or smaller, depending on the problem.

2. Conservation of Flux.

The Floquet modes, f_p, fully describe the scattering process. Since they always contain scattering states, there should be no probability to find a particle close to the potential at $t \to \infty$ (assuming a square-integrable wave function at $t \to -\infty$): the total outgoing flux should be equal to the total incoming flux. Note that since the states with $p^2 - \omega(m+n) < 0$ do not give any contribution to the fluxes, the particle will leave these modes after some time. Physically it is easily understood, since a particle in these modes can undergo a transition to a scattering state and leave the range of the potential. This appendix shows the conservation of flux explicitly in a time-independent picture. Let us start with the scattering off zero-range potential described by Eq. (**??**), for which the flux is

$$\vec{j} = i(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi). \tag{A16}$$

The incoming flux is along the x axis and amounts to $2p$. The outgoing flux consists of the two parts: the first is along the x direction and equals to $\sum_{p_n\geq 0} 2p_n|C_n|^2$. The second piece is along the $(-x)$ direction and amounts to $\sum_{p_n\geq 0} 2p_n|C_n - \delta_{n,0}|^2$. Let us show that from Eq. (**??**) it follows that

$$\sum_{p_n\geq 0} p_n|C_n|^2 = p\mathrm{Re}C_0, \tag{A17}$$

which means that the total flux is conserved. To this end, we multiply Eq. (??) with C_n^*,

$$2p_n|C_n|^2 = 2p\delta_{n,0}C_n^* - ig_0|C_n|^2 - \frac{ig_1}{2}(C_{n+1} + C_{n-1})C_n^*. \tag{A18}$$

Next we conjugate Eq. (??) and then multiply with C_n,

$$2p_n^*|C_n|^2 = 2p\delta_{n,0}C_n + ig_0|C_n|^2 + \frac{ig_1}{2}(C_{n+1}^* + C_{n-1}^*)C_n. \tag{A19}$$

Now we add these two equations and sum over all states

$$\sum(p_n + p_n^*)|C_n|^2 = 2p\mathrm{Re}C_0. \tag{A20}$$

Since $p_n + p_n^*$ is non-zero only for the scattering states we obtain Eq. (??). Similar steps can be taken to show that the total flux is conserved for any short-range potential.

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[1] Cheng Chin, Rudolf Grimm, Paul Julienne, and Eite Tiesinga. Feshbach resonances in ultracold gases. *Rev. Mod. Phys.*, 82:1225–1286, Apr 2010.

[2] M. Girardeau. Relationship between systems of impenetrable bosons and fermions in one dimension. *Journal of Mathematical Physics*, 1(6):516–523, 1960.

[3] Belen Paredes, Artur Widera, Valentin Murg, Olaf Mandel, Simon Folling, Ignacio Cirac, Gora V. Shlyapnikov, Theodor W. Hansch, and Immanuel Bloch. Tonks-girardeau gas of ultracold atoms in an optical lattice. *Nature*, 429:277–281, 2004.

[4] Toshiya Kinoshita, Trevor Wenger, and David S. Weiss. Observation of a one-dimensional tonks-girardeau gas. *Science*, 305(5687):1125–1128, 2004.

[5] M. D. Girardeau, E. M. Wright, and J. M. Triscari. Ground-state properties of a one-dimensional system of hard-core bosons in a harmonic trap. *Phys. Rev. A*, 63:033601, Feb 2001.

[6] F. Deuretzbacher, K. Fredenhagen, D. Becker, K. Bongs, K. Sengstock, and D. Pfannkuche. Exact solution of strongly interacting quasi-one-dimensional spinor bose gases. *Phys. Rev. Lett.*, 100:160405, Apr 2008.

[7] A. G. Volosniev, D. V. Fedorov, M. Jensen, A. S. Valiente, and N. T. Zinner. Strongly interacting confined quantum systems in one dimension. *Nat. Commun.*, 5:5300, 2014.

[8] M. Olshanii. Atomic scattering in the presence of an external confinement and a gas of impenetrable bosons. *Phys. Rev. Lett.*, 81:938–941, Aug 1998.

[9] Philip F. Bagwell and Roger K. Lake. Resonances in transmission through an oscillating barrier. *Phys. Rev. B*, 46:15329–15336, Dec 1992.

[10] S. T. Thompson, E. Hodby, and C. E. Wieman. Ultracold molecule production via a resonant oscillating magnetic field. *Phys. Rev. Lett.*, 95:190404, Nov 2005.

[11] M. Büttiker and R. Landauer. Traversal time for tunneling. *Phys. Rev. Lett.*, 49:1739–1742, Dec 1982.

[12] M Büttiker and R Landauer. Traversal time for tunneling. *Physica Scripta*, 32(4):429, 1985.

[13] Mathias Wagner. Photon-assisted transmission through an oscillating quantum well: A transfer-matrix approach to coherent destruction of tunneling. *Phys. Rev. A*, 51:798–808, Jan 1995.

[14] G. Burmeister and K. Maschke. Scattering by time-periodic potentials in one dimension and its influence on electronic transport. *Phys. Rev. B*, 57:13050–13060, May 1998.

[15] Wenjun Li and L. E. Reichl. Floquet scattering through a time-periodic potential. *Phys. Rev. B*, 60:15732–15741, Dec 1999.

[16] Michael Henseler, Thomas Dittrich, and Klaus Richter. Classical and quantum periodically driven scattering in one dimension. *Phys. Rev. E*, 64:046218, Sep 2001.

[17] Thomas M. Hanna, Thorsten Köhler, and Keith Burnett. Association of molecules using a resonantly modulated magnetic field. *Phys. Rev. A*, 75:013606, Jan 2007.

[18] Christian Langmack, D. Hudson Smith, and Eric Braaten. Association of atoms into universal dimers using an oscillating magnetic field. *Phys. Rev. Lett.*, 114:103002, Mar 2015.

[19] D. Hudson Smith. Inducing resonant interactions in ultracold atoms with a modulated magnetic field. *Phys. Rev. Lett.*, 115:193002, Nov 2015.

[20] Smith, D. Hudson. Induced two-body scattering resonances from a square-well potential with oscillating depth. *EPJ Web of Conferences*, 113:02005, 2016.

[21] D. H. Smith. *Resonant Floquet Scattering of Ultracold Atoms, PhD thesis.* arXiv:1611.05412, 2016.

[22] A. G. Sykes, H. Landa, and D. S. Petrov. Two- and three-body problem with floquet-driven zero-range interactions. *Phys. Rev. A*, 95:062705, Jun 2017.

[23] P. Hänggi. *Quantum Transport and Dissipation, chapter 5.* Wiley-VCH, Weinheim, edited by T. Dittrich, 1998.

[24] Ranjan K. Mallik. The inverse of a tridiagonal matrix. *Linear Algebra and its Applications*, 325(13):109 – 139, 2001.

[25] Moawwad E.A. El-Mikkawy. On the inverse of a general tridiagonal matrix. *Applied Mathematics and Computation*, 150(3):669 – 679, 2004.

[26] G. Burmeister and K. Maschke. Bound states revealed by time-periodic perturbing scattering potentials. *Phys. Rev. B*, 59:4612–4614, Feb 1999.

[27] U. Fano. Effects of configuration interaction on intensities and phase shifts. *Phys. Rev.*, 124:1866–1878, Dec 1961.

[28] A. G. Volosniev, H.-W. Hammer, and N. T. Zinner. Real-time dynamics of an impurity in an ideal bose gas in a trap. *Phys. Rev. A*, 92:023623, Aug 2015.

[29] Martin Ebert, Artem Volosniev, and Hans-Werner Hammer. Two cold atoms in a time-dependent harmonic trap in one dimension. *Annalen der Physik*, 528(9-10):693–704, 2016.

[30] X. Y. Yin, Yangqian Yan, and D. Hudson Smith. Dynamics of small trapped one-dimensional fermi gas under oscillating magnetic fields. *Phys. Rev. A*, 94:043639, Oct 2016.

[31] Kenji Yajima. Scattering theory for schrödinger equations with potentials periodic in time. *J. Math. Soc. Japan*, 29:729–743, 1977.

[32] Jonh R. Taylor. *Scattering Theory. The Quantum Theory of Nonrelativistic Collisions*. Dover Publications, Inc., 2006.

[33] J. Howland. Scattering theory for hamiltonians periodic in time. *Indiana J. Math*, 28:471–494, 1979.