3. Markov Chains

Computational Music Creativity

Universitat Pompeu Fabra Barcelona

MTG Music Technology Group

to recap the theory

What’s a Markov chain (MC)?

What’s a Markov chain (MC)?

- Mathematical system that undergoes transitions from one state to another
- Model sequence of events probabilistically

Key assumption

The next state depends only on the current state and not on the sequence of events that preceded it (memoryless)

Markov chain formalisation

- *States*: The possible conditions (e.g., weather condition, coin side, note)

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Markov chain formalisation

- *States*: The possible conditions (e.g., weather condition, coin side, note)
- *Initial probabilities*: Likelihood of starting the sequence in a state
- *Transition probabilities*: Likelihood of moving from one state to another

Melody generation with MCs

$$S = \{C, D, E, G, A\}$$

$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

Melody generation with MCs

- First pitch

Melody generation with MCs

- First pitch
  a. Use Ip vector

$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice

$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip

$$I_p = \begin{pmatrix} p_C \\ p_D \\ \boxed{p_E} \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use lp vector
  b. Roll dice
  c. Get pitch from lp
- Subsequent pitches

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice
  d. Get new pitch

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E -> G

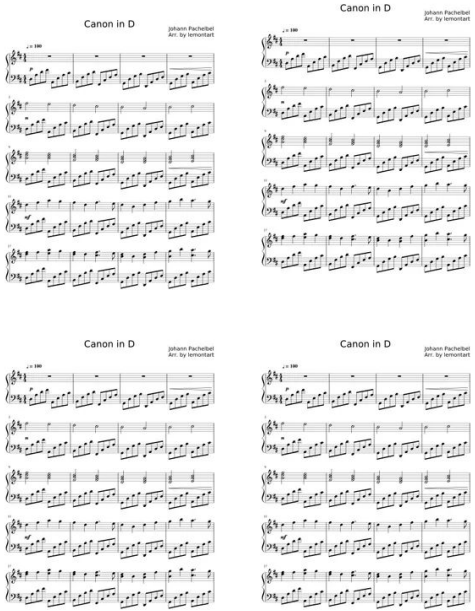
Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice
  d. Get new pitch

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E -> G -> A

Training a MC melody generator



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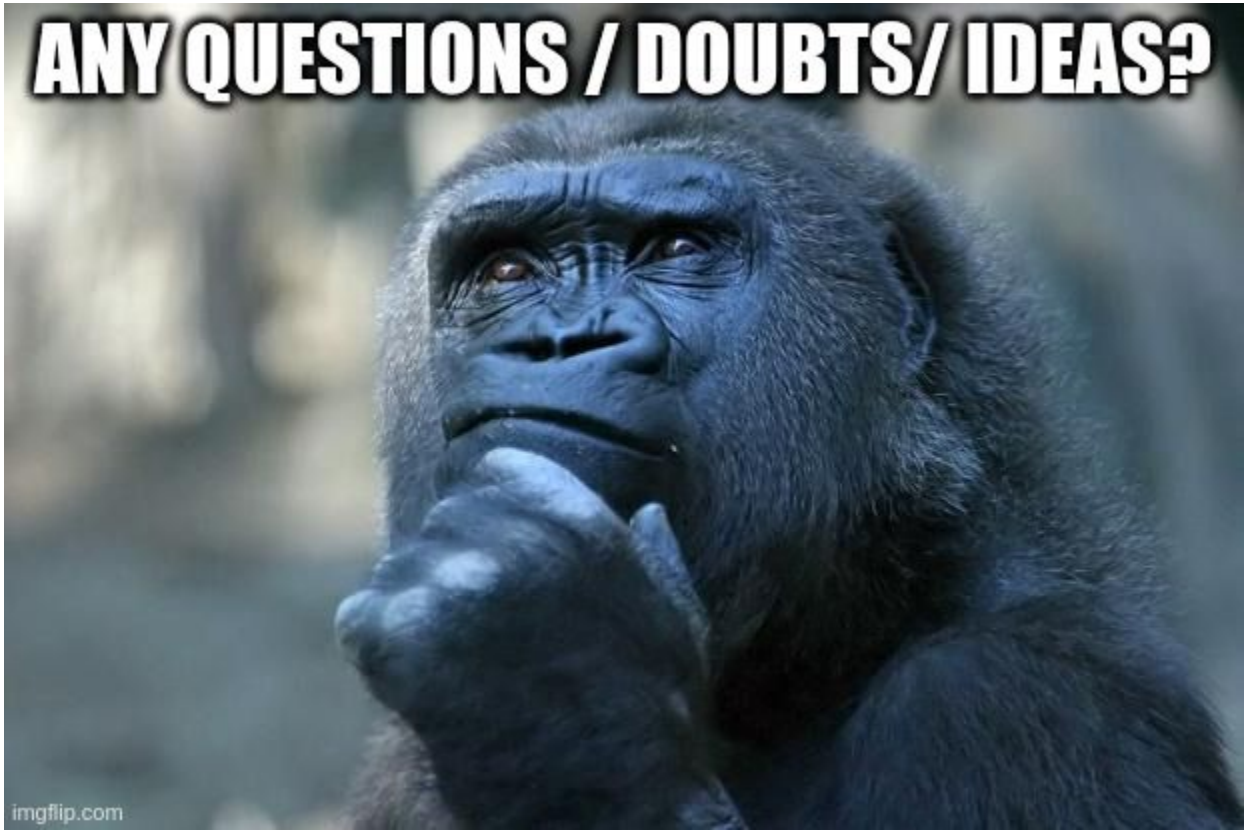
How do you “train” a Markov Chain?


ANY QUESTIONS / DOUBTS/ IDEAS?
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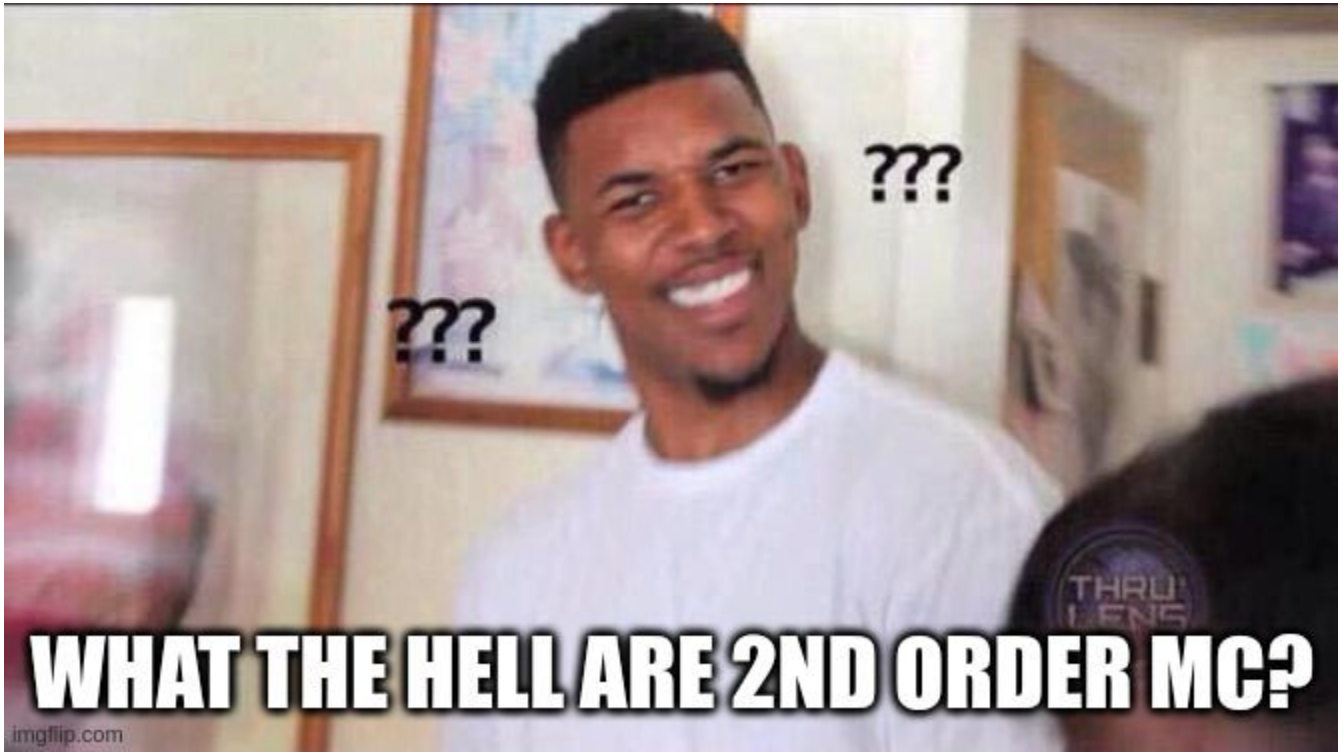
My experience with MCs

- MCs are s**t for melody / rhythm generation
- Work well for form / instrumentation
- Expert-crafted probabilities can work really well

Tips to use MCs

- Train different MCs for different styles
- Combine rules + MCs
- Use 2nd order chains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WHAT THE HELL ARE 2ND ORDER MC?
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Subsequent token depends on previous 2 tokens

What’s the relationship between MC and Transformer?


markov chains
2nd order markov chains
RNN
LSTM
Transformer
imgflip.com


MUSICAL CONTEXT
LIGHTYEAR
MUSICAL CONTEXT EVERYWHERE
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Activity 1: Improve Markov Chain

Come up with strategies to improve a MC that generates the lead melody for a song (e.g., rules, weighting). For each:

- Pros and cons
- Limitations

Instructions:

- Work in groups (5 people)
- 10’ to come up with solution
- 5’ to discuss together

GEDMAS

Activity 2: GEDMAS paper

Read the paper. Answer these questions:

- What's GEDMAS? What does it generate?
- What's GEDMAS' architecture?
- What do MCs have to do with GEDMAS?
- How does GEDMAS generate a full piece?
- What's GEDMAS trained on?
- What are GEDMAS' limitations? How would you improve it?

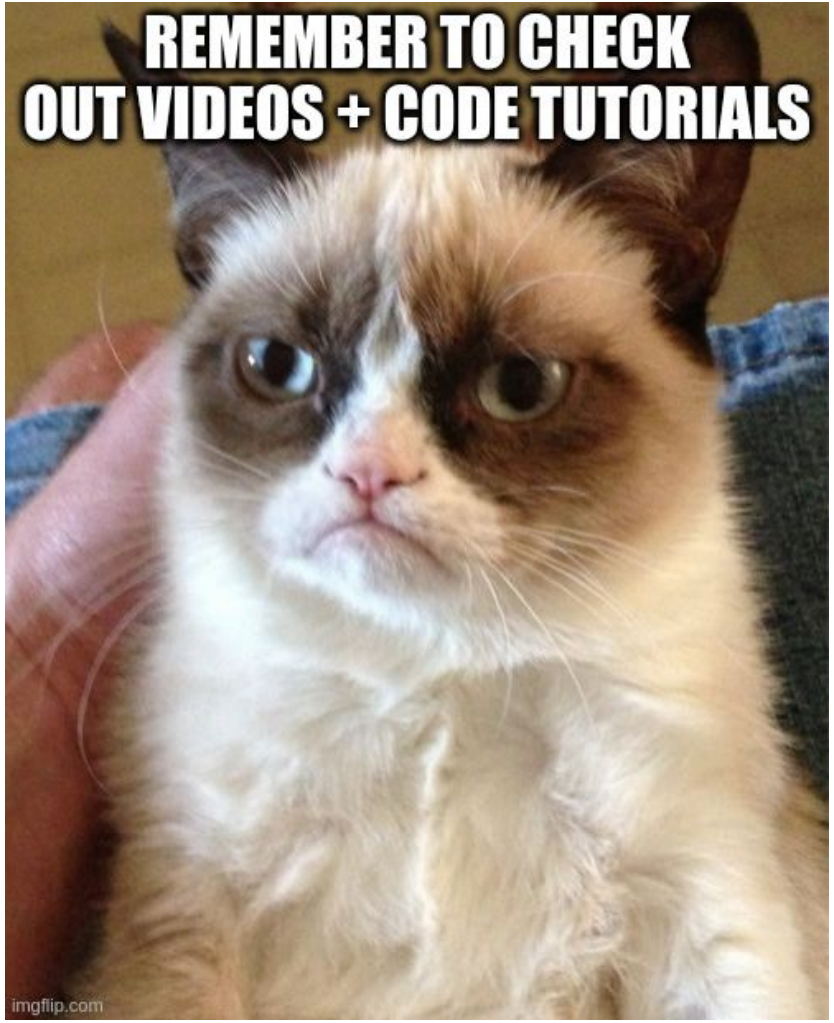
Instructions:

- Work in pairs
- 15' to read and take notes
- 8' to discuss together

Assignment 2: Second Order Markov Chain

Re-adapt the MC melody generator you've seen in the tutorial to a 2nd order Markov Chains.

Deadline: 25 January at midnight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OUT VIDEOS + CODE TUTORIALS
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