



A practical introduction to Bayesian Inference

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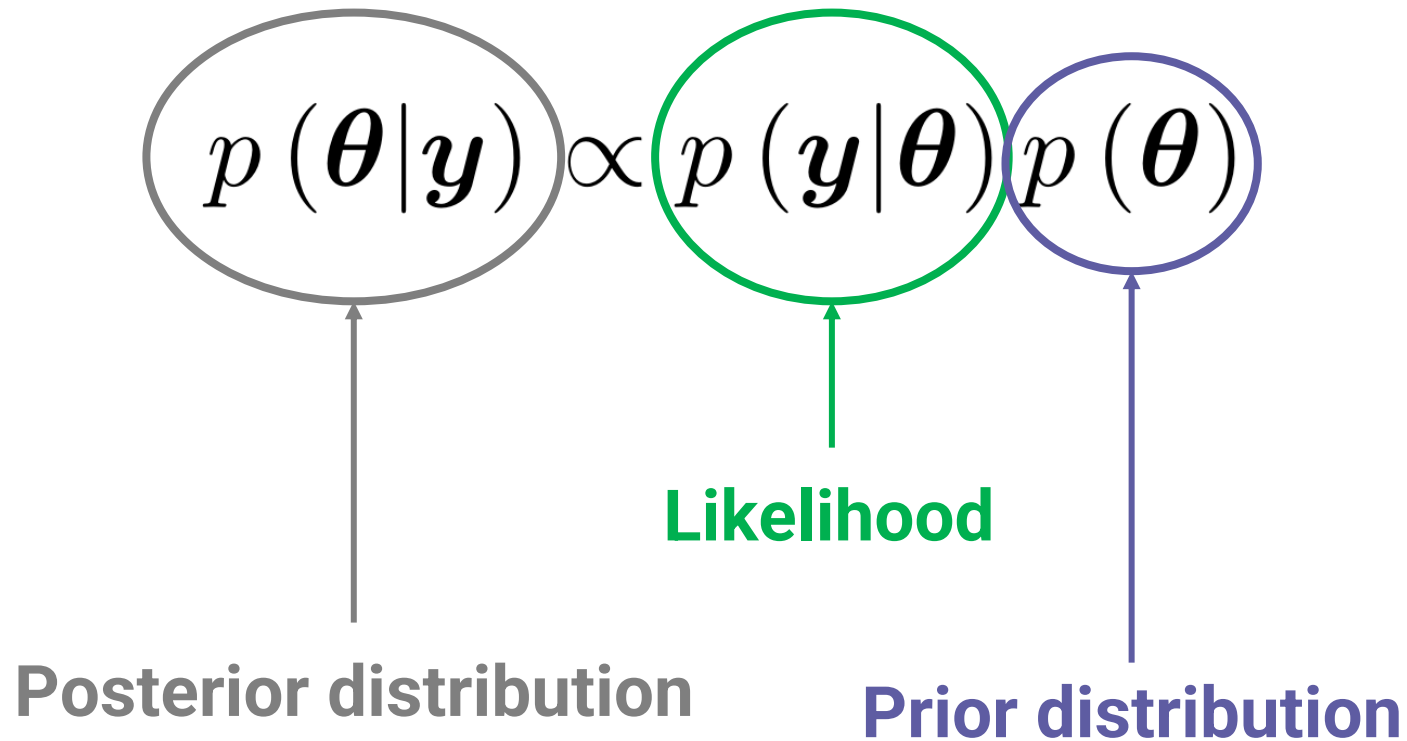
SO-IAA School on Bayesian Statistics



Bayes' Theorem

$$p(\boldsymbol{\theta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{y})}$$

- $\boldsymbol{\theta} \rightarrow$ vector of model parameters
- $\mathbf{y} \rightarrow$ data



Bayesian Inference

- Parameters are not fixed
- The quantity of interest is the distribution of the parameters
- We want to calculate exactly or, if we can't, estimate the *posterior distribution*

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

- Computational methods for estimating the posterior
 - Grid of θ 's \rightarrow evaluate the posterior at every point, given the data (\$\$\$)
 - Drawing samples from the posterior
 - E.g., Markov Chain Monte Carlo, Hamilton Monte Carlo, Gibbs Sampling, or some other sampling method
 - Variational Bayes
 - Other methods

Today's activity: Bayesian inference with m&m's

- Find the **posterior distribution** for the percentage of **blue** m&m's[®] made at the factory — *using Bayes' theorem and 15 candies from a bag of m&m's.*

$$p(\theta|y) \propto p(y|\theta) p(\theta)$$

*No googling for answers!
That spoils the fun.*



Initial exploratory questions

- What kind of data are the colours of m&m's?
 - Numerical
 - Categorical
 - Continuous
- How will you record the data?
- Will you sample with replacement or without?
- How might we model the probability of drawing a blue m&m?

What's (y)our prior information?

- How many different colours of m&m's are there?
- Do you think the m&m's are well-mixed before they go into a bag at the factory?
- What percentage of blue m&m's do you think are made at the factory?
- Do you think every bag will have the same percentage of blue m&m's?
- Sketch (or at least think about) what your prior distribution for θ looks like

**Sketch of a prior distribution for
the percentage of blue m&m's made at the factory**

Likelihood of drawing y
blue m&m's[®] given n trials:

$y \rightarrow$ # of successes (blue m&m's[®])

$n - y \rightarrow$ # of failures (not a blue m&m's[®])

Binomial Distribution:

$$p(\theta) \propto \theta^y (1 - \theta)^{n-y}$$

Prior knowledge

- Outlined/sketch previously
- For convenience, use the conjugate prior
- find parameter values that make the conjugate prior look like our prior distribution sketch.

Beta Distribution:

$$p(\theta) \propto \theta^{\alpha-1} (1 - \theta)^{\beta-1}$$

Binomial Distribution:

$$p(y|\theta) \propto \theta^y (1 - \theta)^{n-y}$$

Beta distribution:

$$p(\theta) \propto \theta^{\alpha-1} (1 - \theta)^{\beta-1}$$

$$***p(\theta|y) \propto p(y|\theta)p(\theta)***$$

In-Class Activity

- Go to the R markdown or Jupyter notebook on Github
 1. write a function for the prior that takes α and β as inputs
 2. Write code to plot the prior distribution
 3. Choose parameters to define the prior distribution
 - a) adjust the α and β hyperparameters that best match your sketched prior distribution
 4. Calculate the mean and variance of the prior you chose

Binomial Distribution:

$$p(y|\theta) \propto \theta^y (1 - \theta)^{n-y}$$

Beta distribution:

$$p(\theta) \propto \theta^{\alpha-1} (1 - \theta)^{\beta-1}$$

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

$$p(\theta|y) \propto \theta^{y+\alpha-1} (1 - \theta)^{n-y+\beta-1}$$

What distribution is this?

Exercise

- Gather some data!
- Open your bag of m&m's
- Take out the first 10 , and record how many colours you have of each
- Perform Bayesian inference to predict the percentage of blue m&m's made at the factory



Think-Pair-Share

- Is the posterior distribution what you expected?
- Compare the posterior distribution to the prior distribution
- Is this the result you expected, given six different colours? Does this result tell you about the percentages of the other colours?
- How sensitive is the posterior to the prior distribution?

Pool the data from the entire class

- How would you expect the posterior to change given more data?
- **Let's pool the data and find out!**
 - (What assumptions are we making here?)
- Plot the posterior again
- Compare to prior

Surprise Twist!

Different factories make different colour distributions of m&m's!



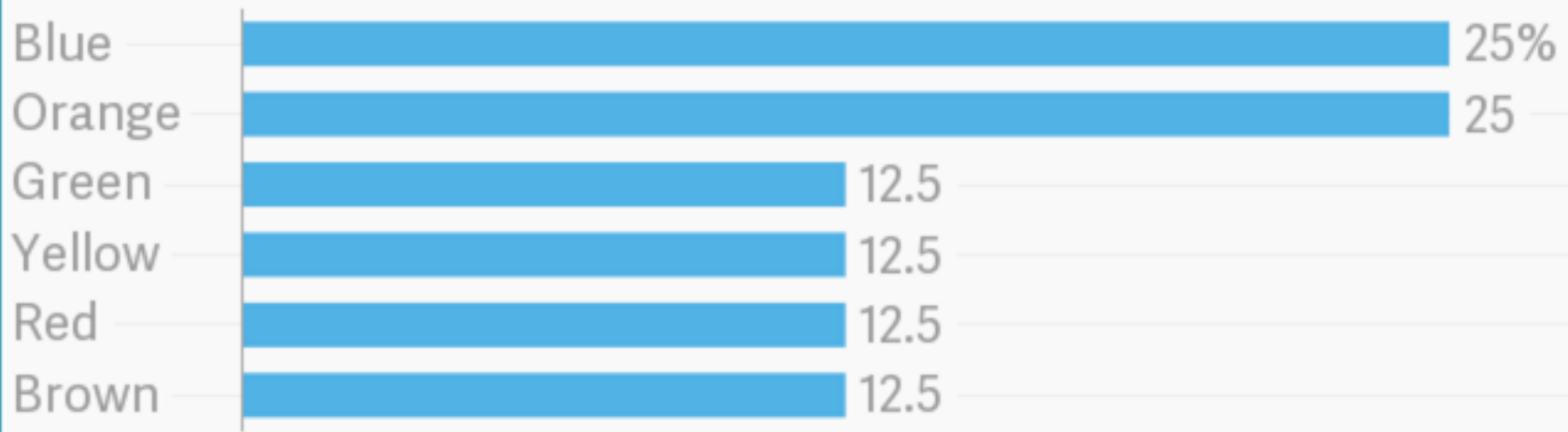
Based on your posterior distribution, which factory did your m&m's come from?

HKP = New Jersey
CLV = Tennessee

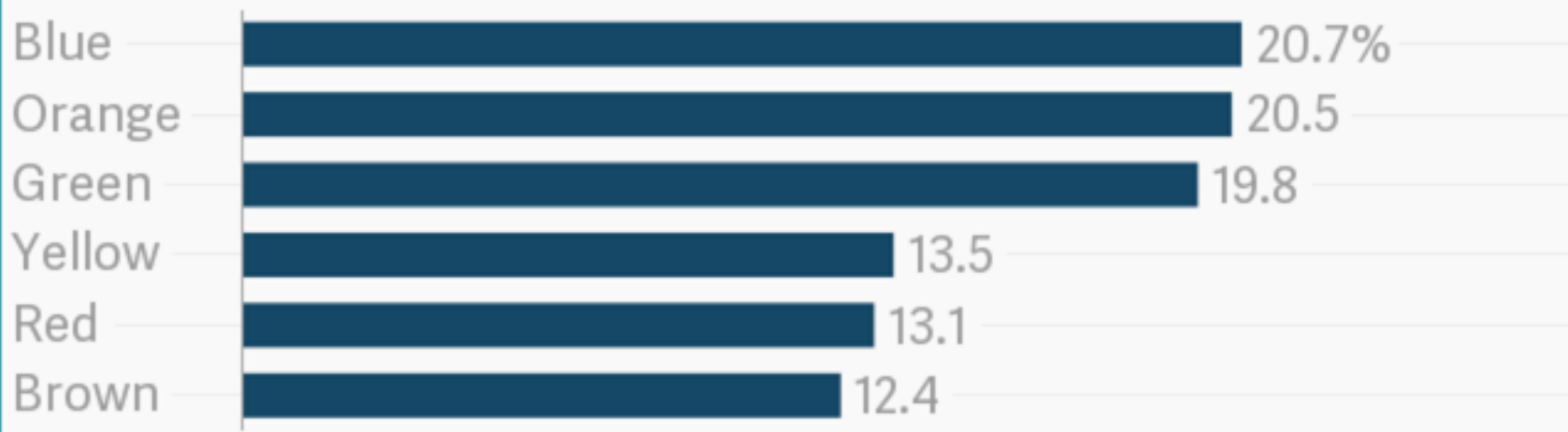


M&Ms color distribution, c. 2017

New Jersey factory



Tennessee factory





Thomas Bayes (1701-1761)



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