# A practical introduction to Bayesian Inference

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# Bayes' Theorem

$$\boldsymbol{p}(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{\boldsymbol{p}(\boldsymbol{y}|\boldsymbol{\theta})\boldsymbol{p}(\boldsymbol{\theta})}{\boldsymbol{p}(\boldsymbol{y})}$$

- $\boldsymbol{\theta}$ → vector of model parameters
- $\boldsymbol{y}$ → data

$$\underbrace{p\left(\boldsymbol{\theta}|\boldsymbol{y}\right)}_{\text{Posterior distribution}} \propto \underbrace{p\left(\boldsymbol{y}|\boldsymbol{\theta}\right)}_{\text{Likelihood}}\underbrace{p\left(\boldsymbol{\theta}\right)}_{\text{Prior distribution}}$$

# Bayesian Inference

- Parameters are not fixed
- The quantity of interest is the distribution of the parameters
- We want to calculate exactly or, if we can't, estimate the ***posterior distribution***

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

- Computational methods for estimating the posterior
  - Grid of $\theta$'s → evaluate the posterior at every point, given the data ($$$)
  - Drawing samples from the posterior
    - E.g., Markov Chain Monte Carlo, Hamilton Monte Carlo, Gibbs Sampling, or some other sampling method
  - Variational Bayes
  - Other methods

# Today's activity: Bayesian inference with m&m's

- Find the posterior distribution for the percentage of blue m&m's® made at the factory — *using Bayes' theorem and 15 candies from a bag of m&m's.*

$$p\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \propto p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right)$$

*No googling for answers!*

*That spoils the fun.*

# Initial exploratory questions

- What kind of data are the colours of m&m’s?
  - Numerical
  - Categorical
  - Continuous

- How will you record the data?

- Will you sample with replacement or without?

- How might we model the probability of drawing a blue m&m?

# What’s (y)our prior information?

- How many different colours of m&m’s are there?

- Do you think the m&m’s are well-mixed before they go into a bag at the factory?

- What percentage of blue m&m’s do you think are made at the factory?

- Do you think every bag will have the same percentage of blue m&m’s?

- Sketch (or at least think about) what your prior distribution for $\theta$ looks like

# Sketch of a prior distribution for the percentage of blue m&m's made at the factory

# Likelihood of drawing $y$ blue m&m's® given $n$ trials:

$y \rightarrow$ # of successes (blue m&m®)

$n - y \rightarrow$ # of failures (not a blue m&m®)

## Binomial Distribution:

$$p(\theta) \propto \theta^{y}(1-\theta)^{n-y}$$

# Prior knowledge

- Outlined/sketch previously
- For convenience, use the conjugate prior
- find parameter values that make the conjugate prior look like our prior distribution sketch.

## Beta Distribution:

$$p(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

Binomial Distribution:

$$p(y|\theta) \propto \theta^{y}(1-\theta)^{n-y}$$

Beta distribution:

$$p(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$\boldsymbol{p(\theta|y) \propto p(y|\theta)p(\theta)}$$

# In-Class Activity

- Go to the R markdown or Jupyter notebook on Github

1. write a function for the prior that takes $\alpha$ and $\beta$ as inputs
2. Write code to plot the prior distribution
3. Choose parameters to define the prior distribution
   a) adjust the $\alpha$ and $\beta$ hyperparameters that best match your sketched prior distribution
4. Calculate the mean and variance of the prior you chose

Binomial Distribution:

$$p(y|\theta) \propto \theta^{y}(1-\theta)^{n-y}$$

Beta distribution:

$$p(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

$$\boldsymbol{p(\theta|y) \propto \theta^{y+\alpha-1}(1-\theta)^{n-y+\beta-1}}$$

What distribution is this?

# In-class activity

- Gather some data!
- Open your bag of m&m's
- Take out the first $10$ , and record how many colours you have of each
- Perform Bayesian inference to predict the percentage of blue m&m's made at the factory

# Think-Pair-Share

- Is the posterior distribution what you expected?
- Compare the posterior distribution to the prior distribution
- Is this the result you expected, given six different colours? Does this result tell you about the percentages of the other colours?
- How sensitive is the posterior to the prior distribution?

# Posterior predictive distribution

- We can check our model in much the same way as we did in regression
  → with predictive checks
- *Posterior predictive distribution*

$$p(y_{future}|y_{observed}) = \int p(y_{future}|\theta)p(\theta|y_{observed})d\theta$$

# In-class Activity (cont'd)

- Perform a *posterior predictive check*
  - Draw random $\theta$ value from your posterior
  - Given $\theta$, draw random $y$ from the binomial
  - Repeat many times, to get a predictive distribution of $y$'s
  - Predictive distribution of $y$'s → $p(y_{future}|y_{oberved})$
- Plot your predictive distribution of $y$'s, compare to your neighbour's

# Think-Pair-Share

- Comparing the predictive distribution to your own data?
  - In this example, basically one data point
- Better to compare the predictive distribution of $y$'s to the distribution of $y_{observed}$'s from the entire class
- Let's get everyone's data to do this comparison!

# Pool the data from the entire class

- How would you expect the posterior to change given more data?
- **Let's pool the data and find out!**
  - (What assumptions are we making here?)
- Plot the posterior again
- Compare to prior

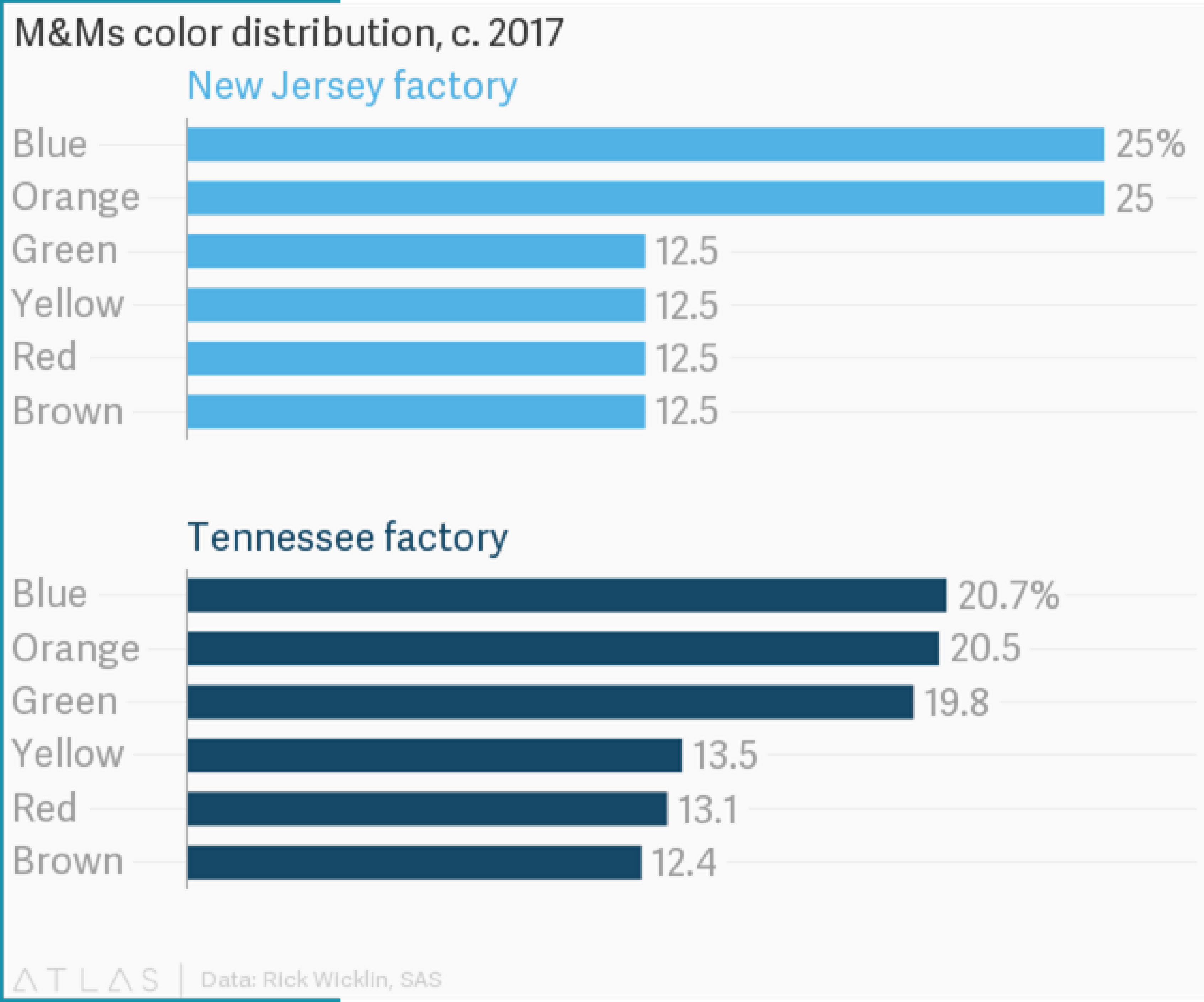
# Surprise Twist!

Different factories make different colour distributions of m&m’s!

***Based on your posterior distribution,* which factory did your m&m's come from?**

HKP = New Jersey

CLV = Tennessee

Thomas Bayes (1701-1761)

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# Introducing Bayesian Analysis With m&m's®: An Active-Learning Exercise for Undergraduates

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