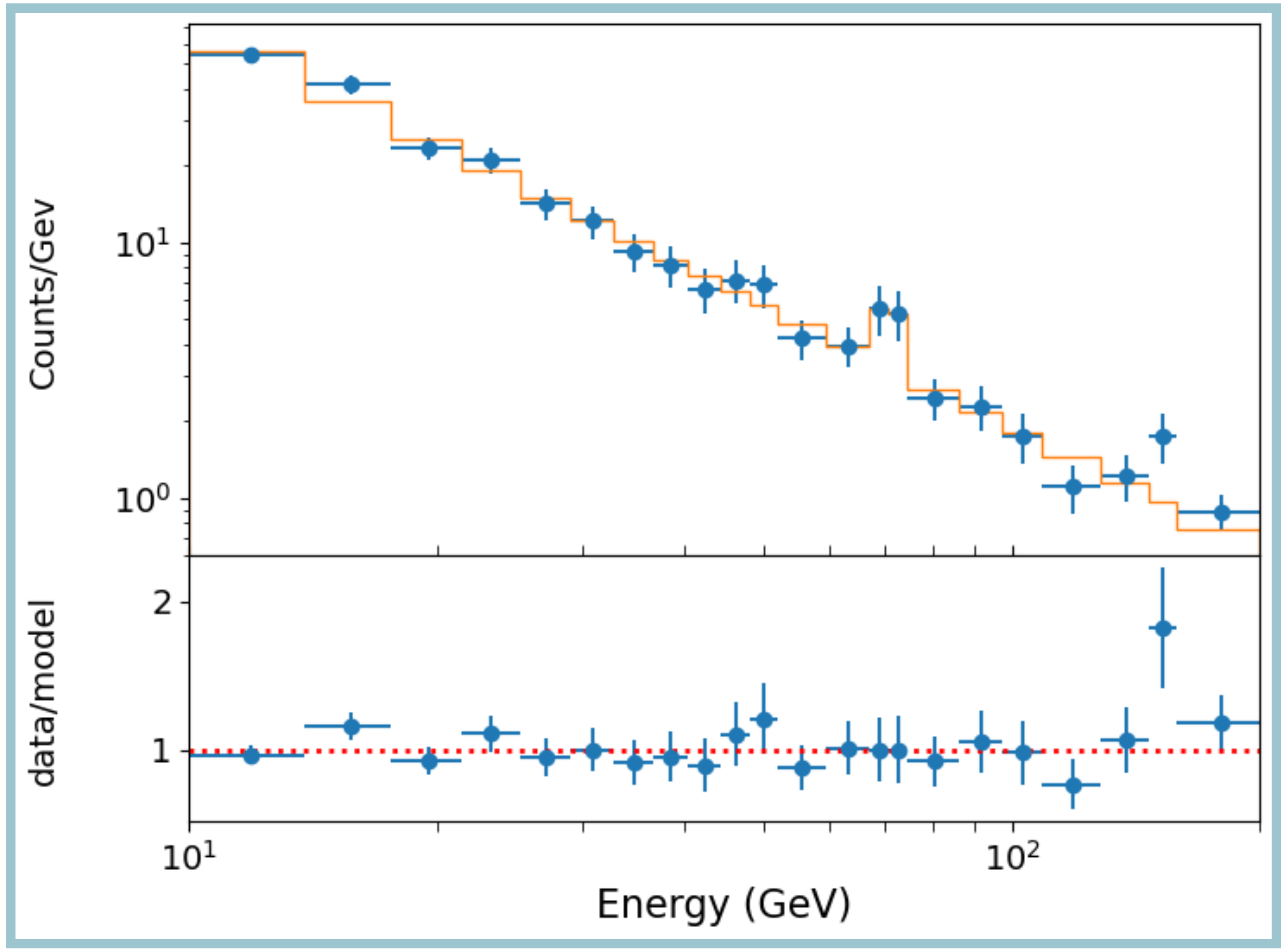
Priors

Universiteit van Amsterdam

d.huppenkothen@uva.nl

A Physics Problem



Data: γ-ray spectrum

Model: Power law + Gaussian line

Likelihood: Poisson

Parameters: A_{PL}, Γ, A_{Line}, E_{Line}, σ_{Line} A_{Line}

Priors: ???

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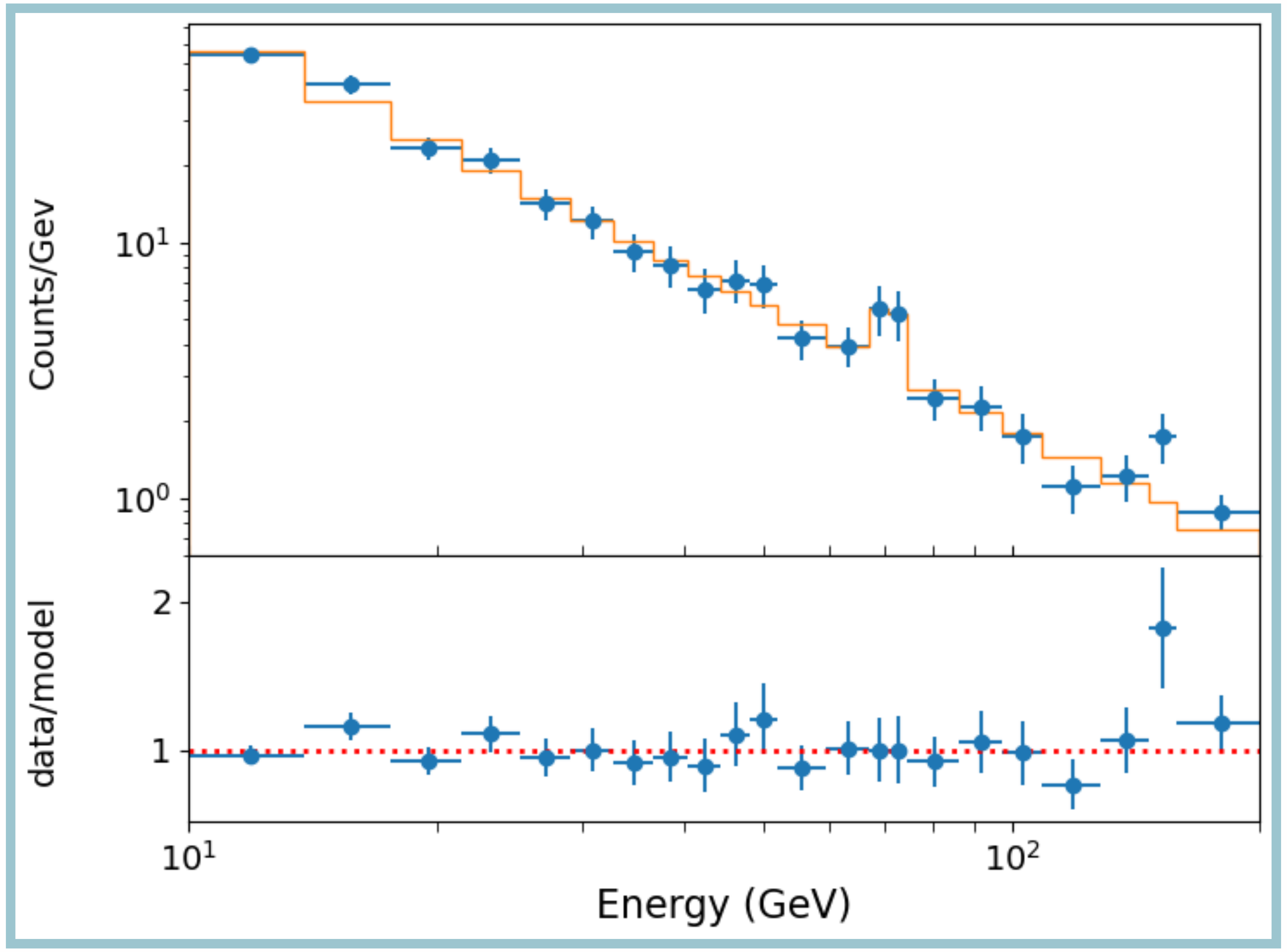

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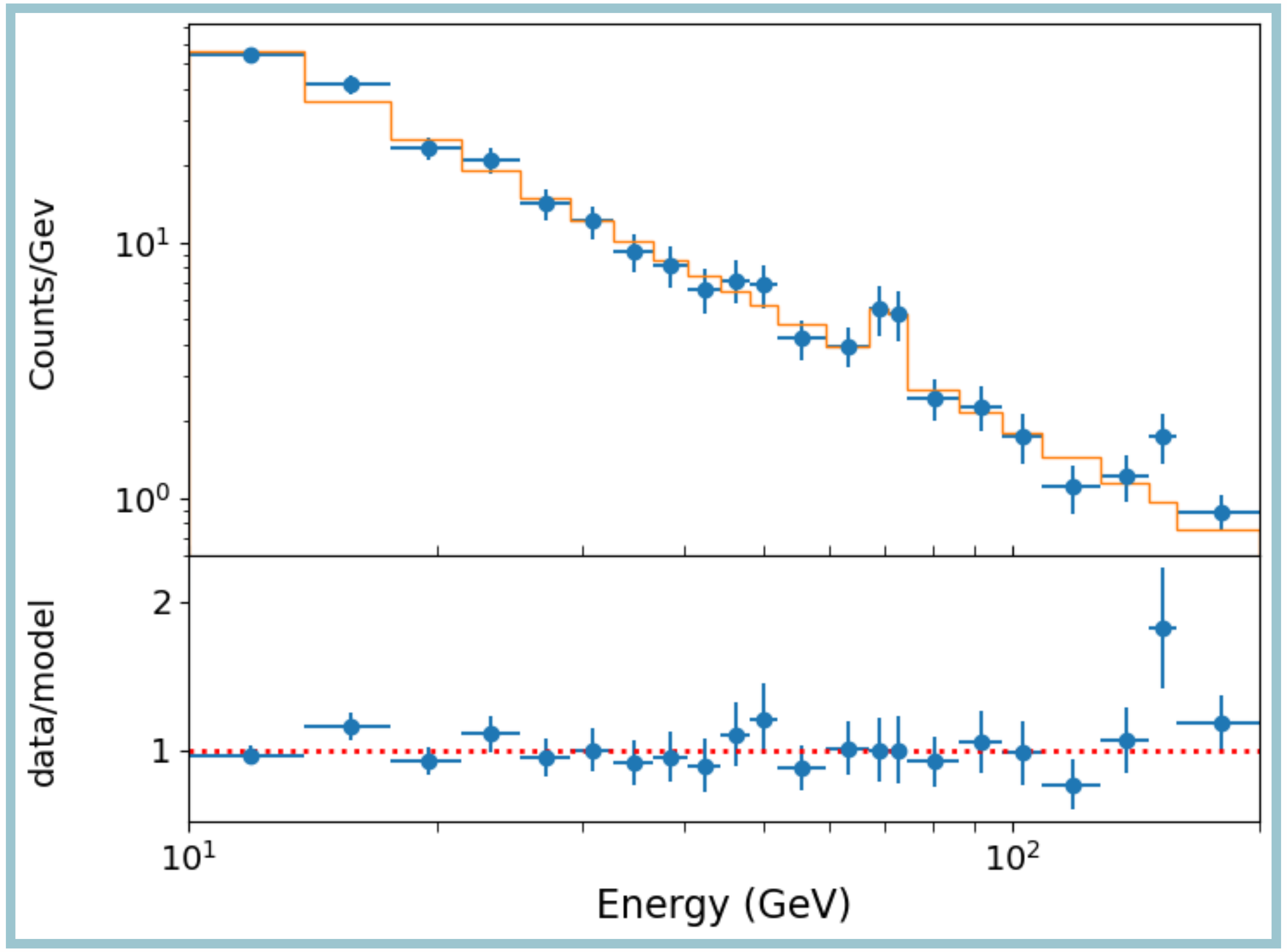

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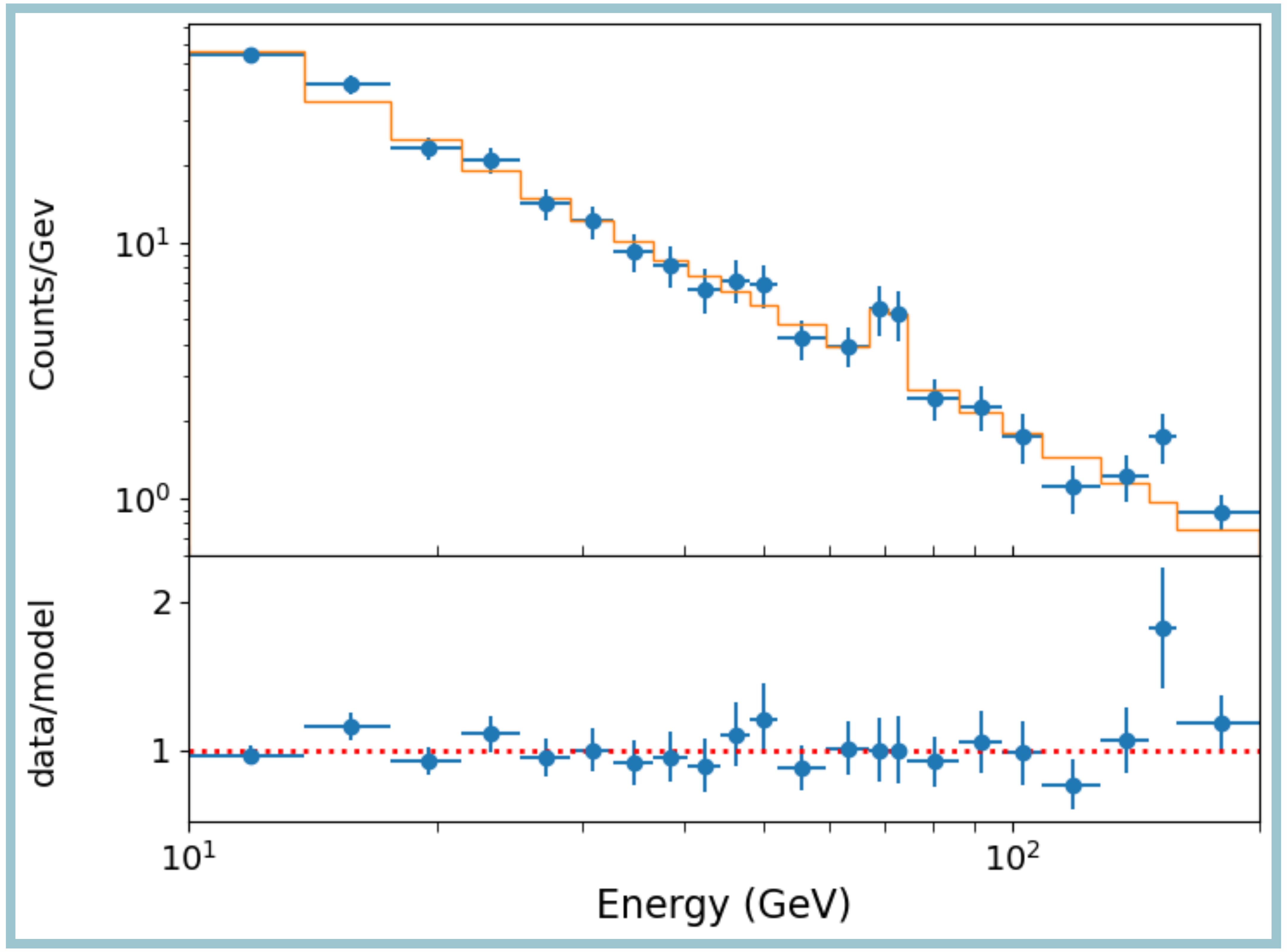

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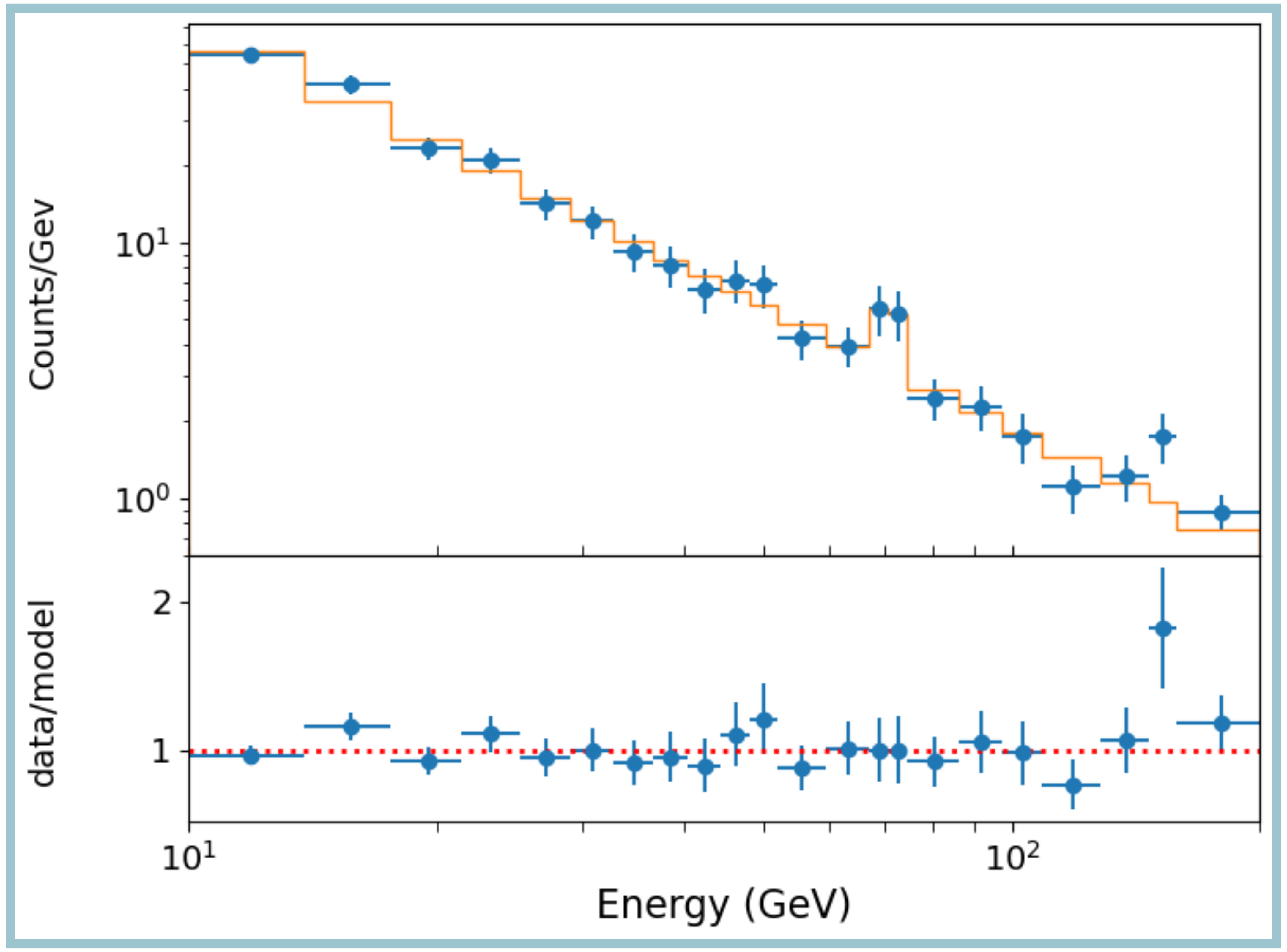

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What assumption have we made here?

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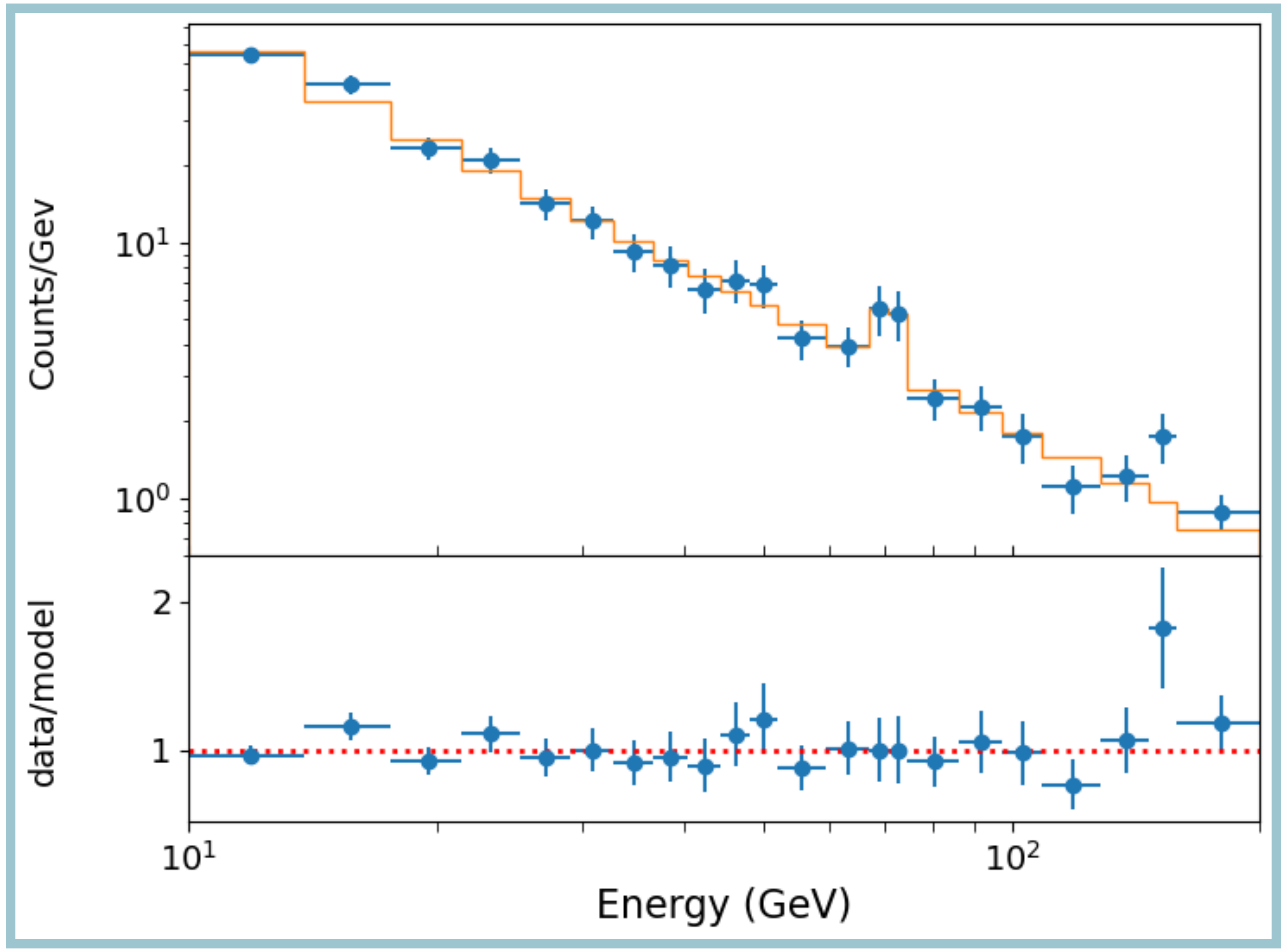

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What assumption have we made here?

Independence of priors!

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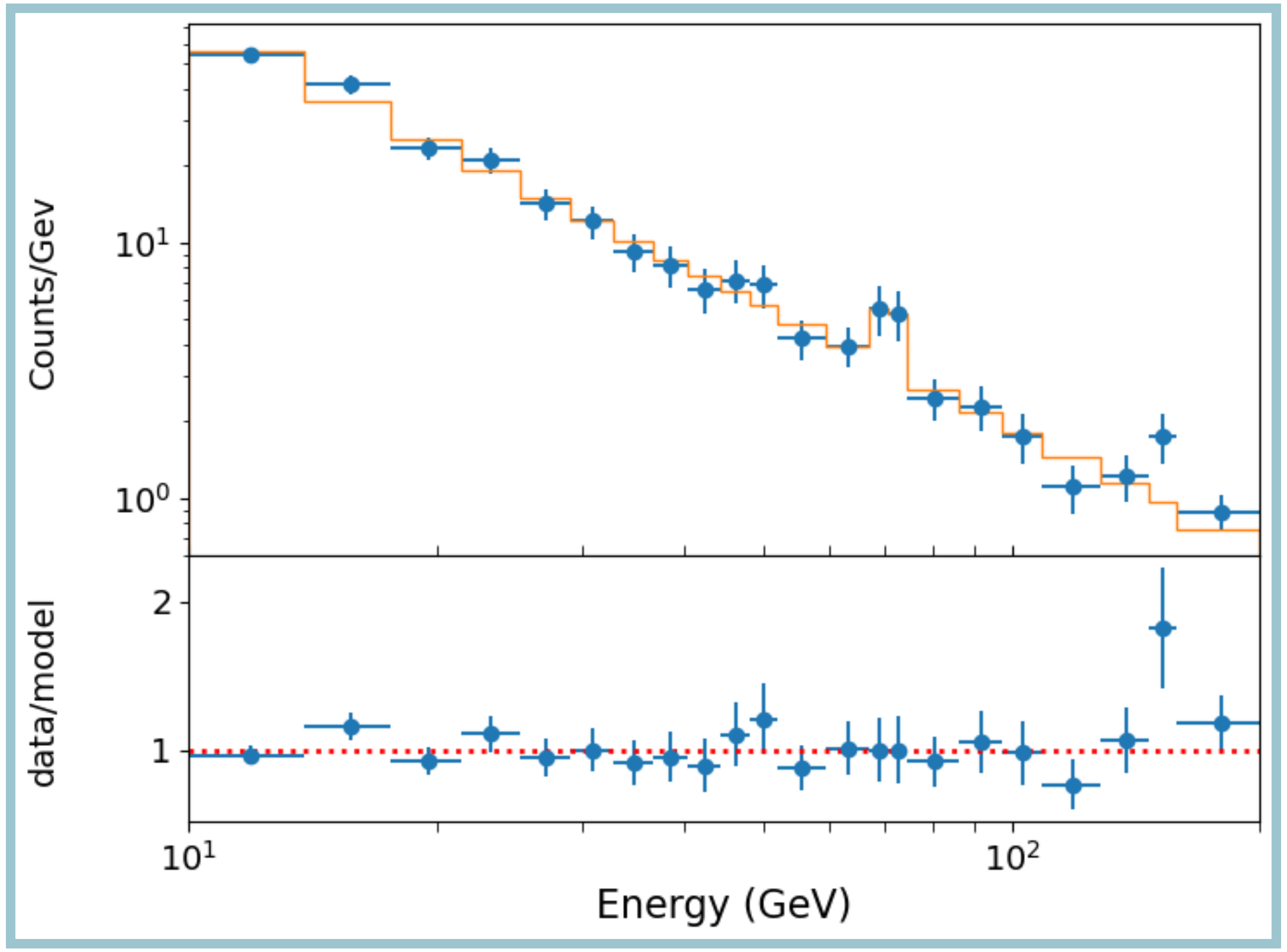

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What priors should we choose?

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What assumption have we made here?

Independence of priors!

Conjugate priors

$$p(q \mid k) \propto p(k \mid q)p(q)$$

Conjugate priors

likelihood

$$\boxed{p(k \mid q, N) \propto q^k (1-q)^{N-k}}$$

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likelihood

$$\boxed{p(k \mid q, N) \propto q^k (1-q)^{N-k}}$$

prior

$$\boxed{p(q) \propto q^{\alpha-1} (1-q)^{\beta-1}}$$

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prior

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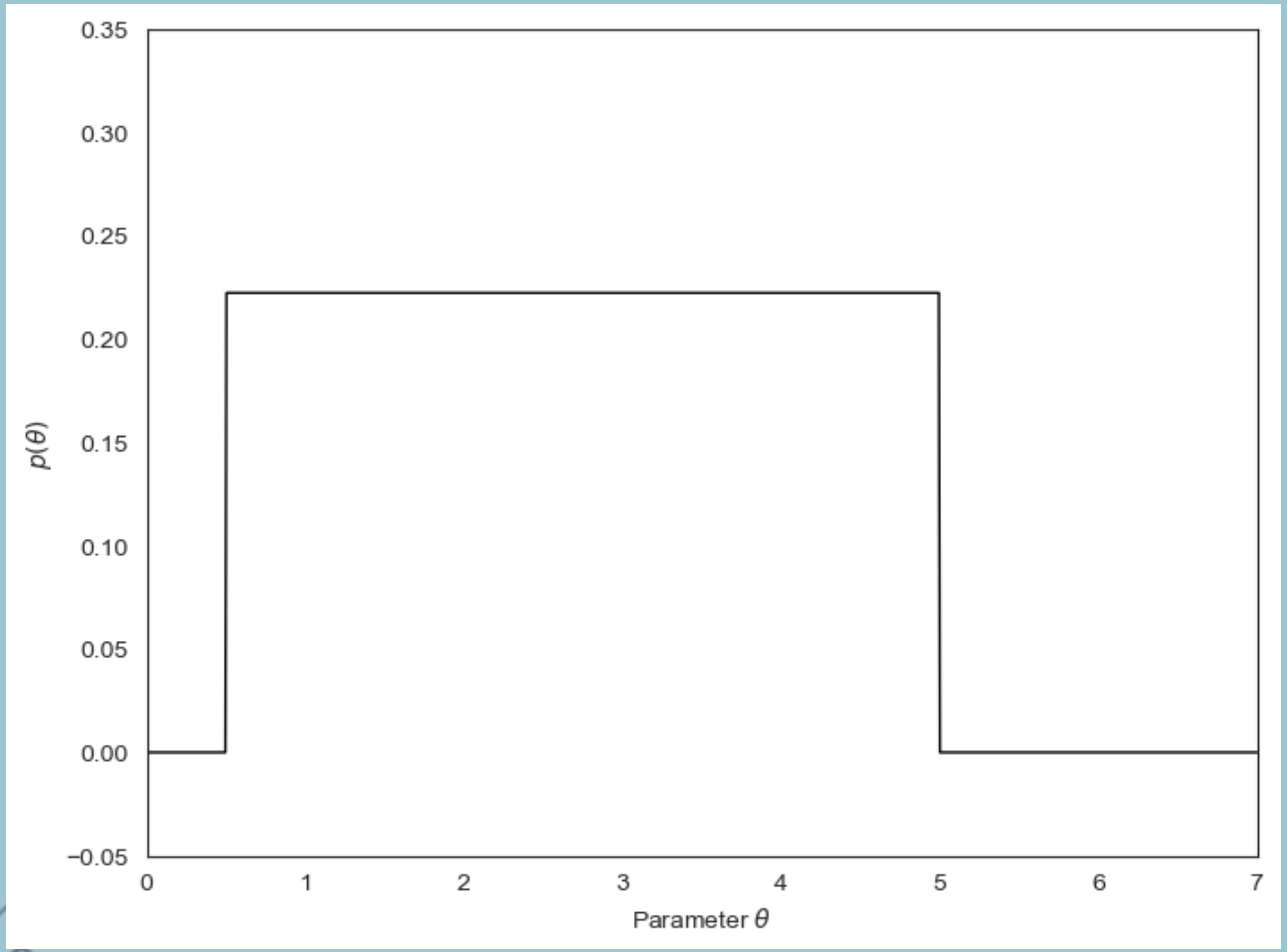
$$p(q\,|\,k) \propto p(k\,|\,q)\,p(q)$$

$$\boxed{p(q\,|\,k,N) \propto q^{k+\alpha-1}(1-q)^{N-k+\beta-1}}$$

Flat Priors:

Γ

"I know nothing but I'm 100% sure that my parameter is between $\theta_{\min}$ and $\theta_{\max}$



Scale-free priors:

A_{PL}, A_{Line}

"I believe with equal prior probability that my parameter could be between 0.1 and 1 or between 1 and 10"

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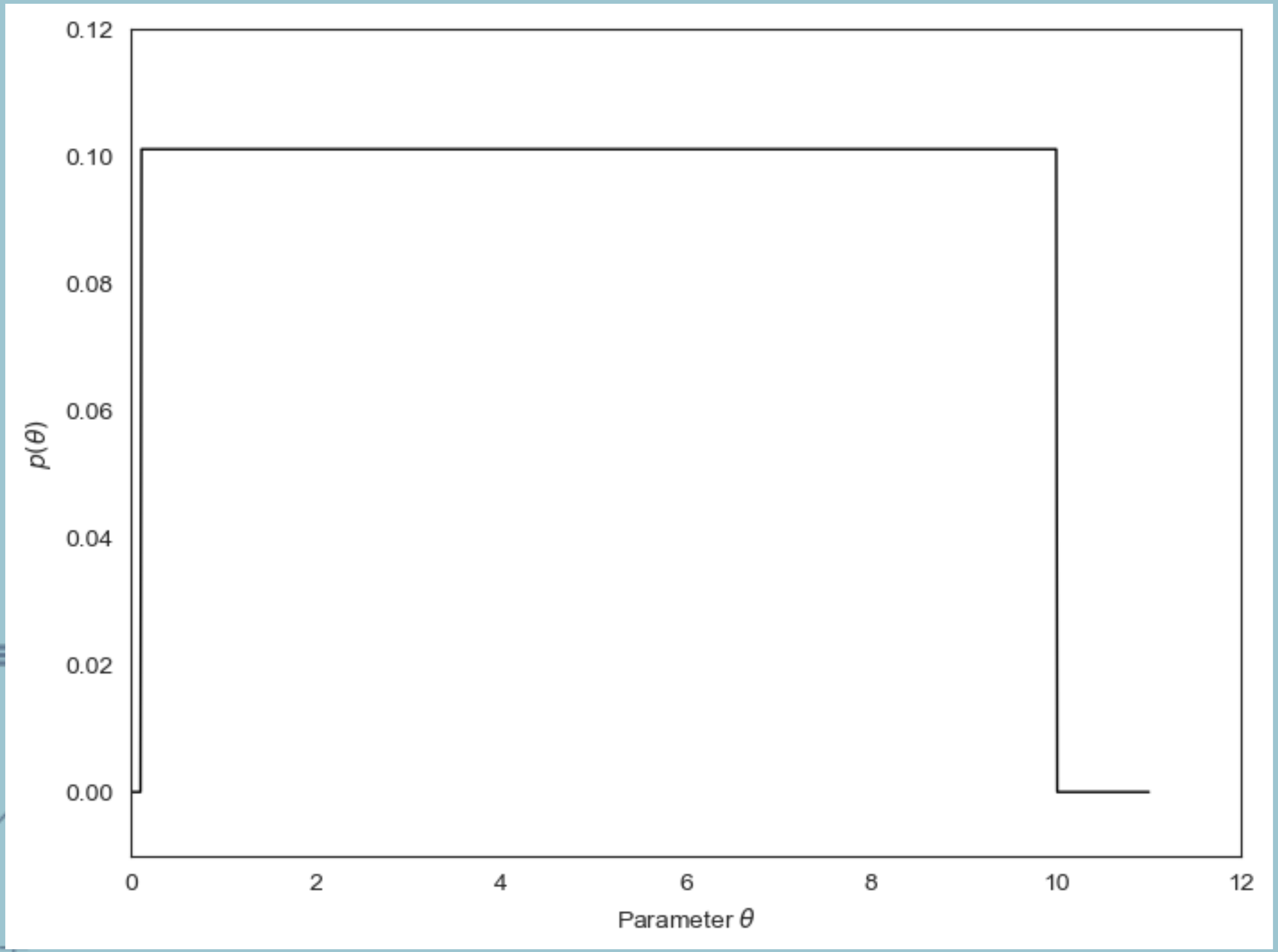

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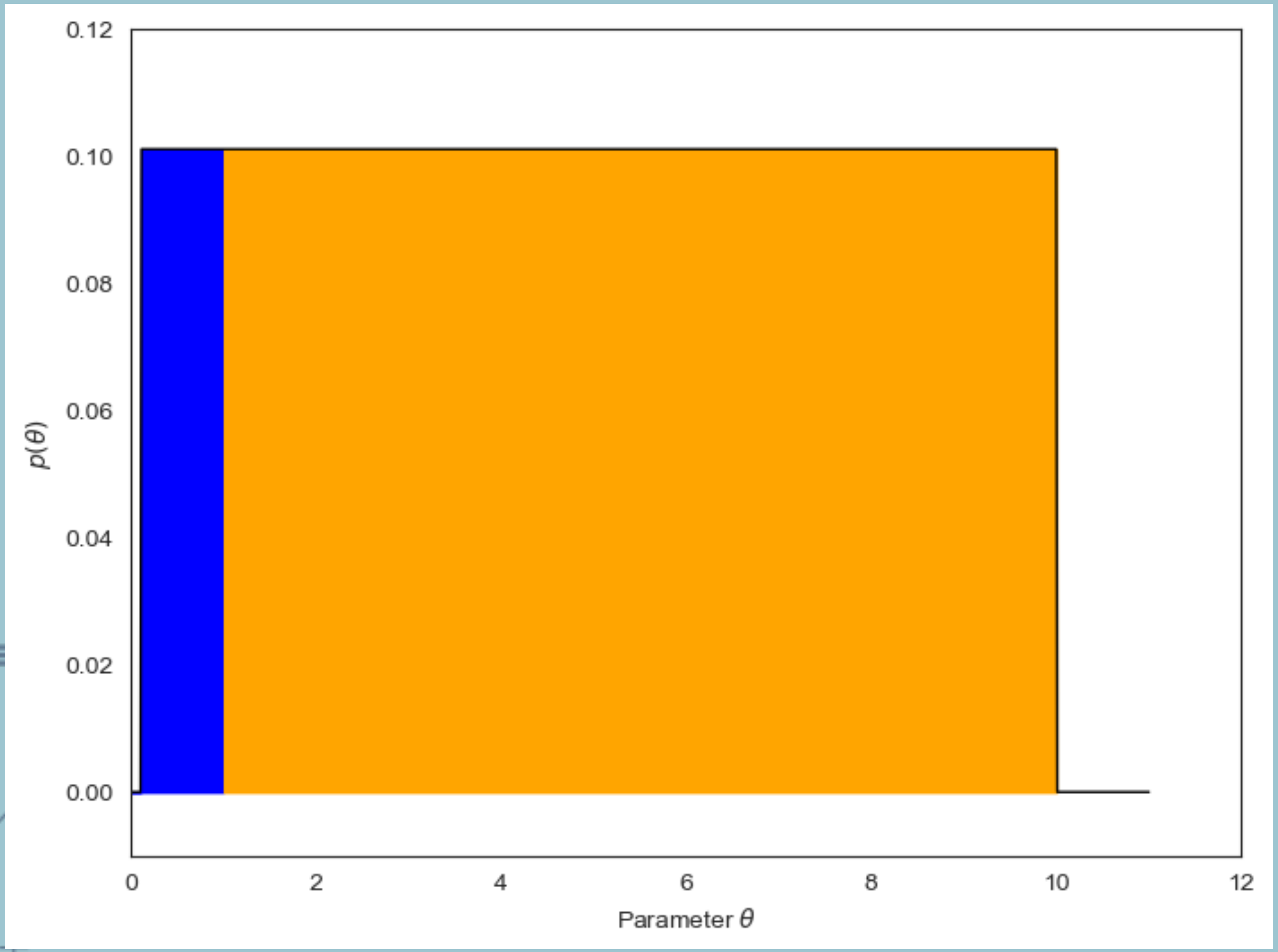

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$\downarrow$

$\log(A_{\mathrm{PL}})$, $\log(A_{\mathrm{Line}})$

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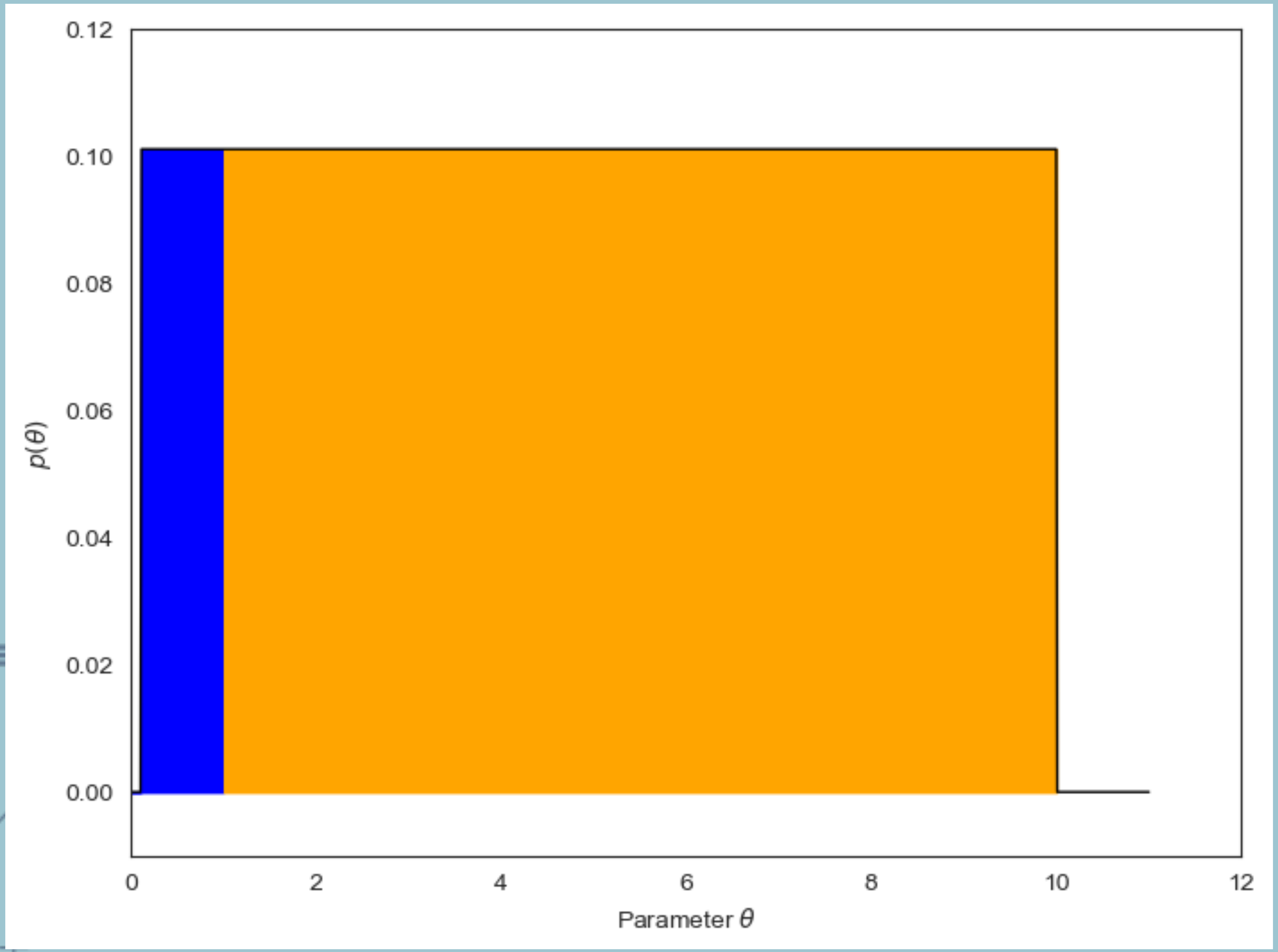

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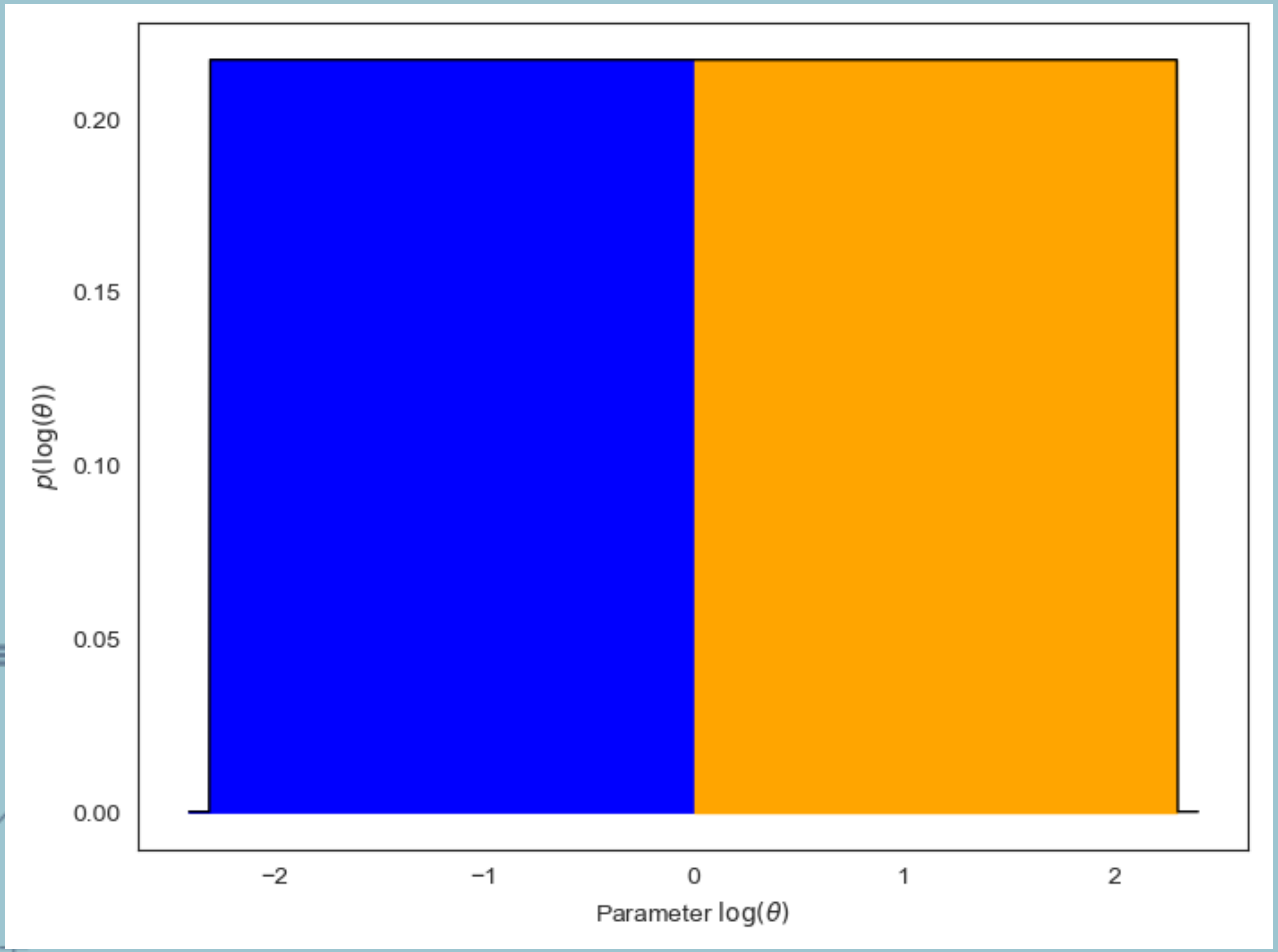

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Also use this for parameters that cannot be negative!

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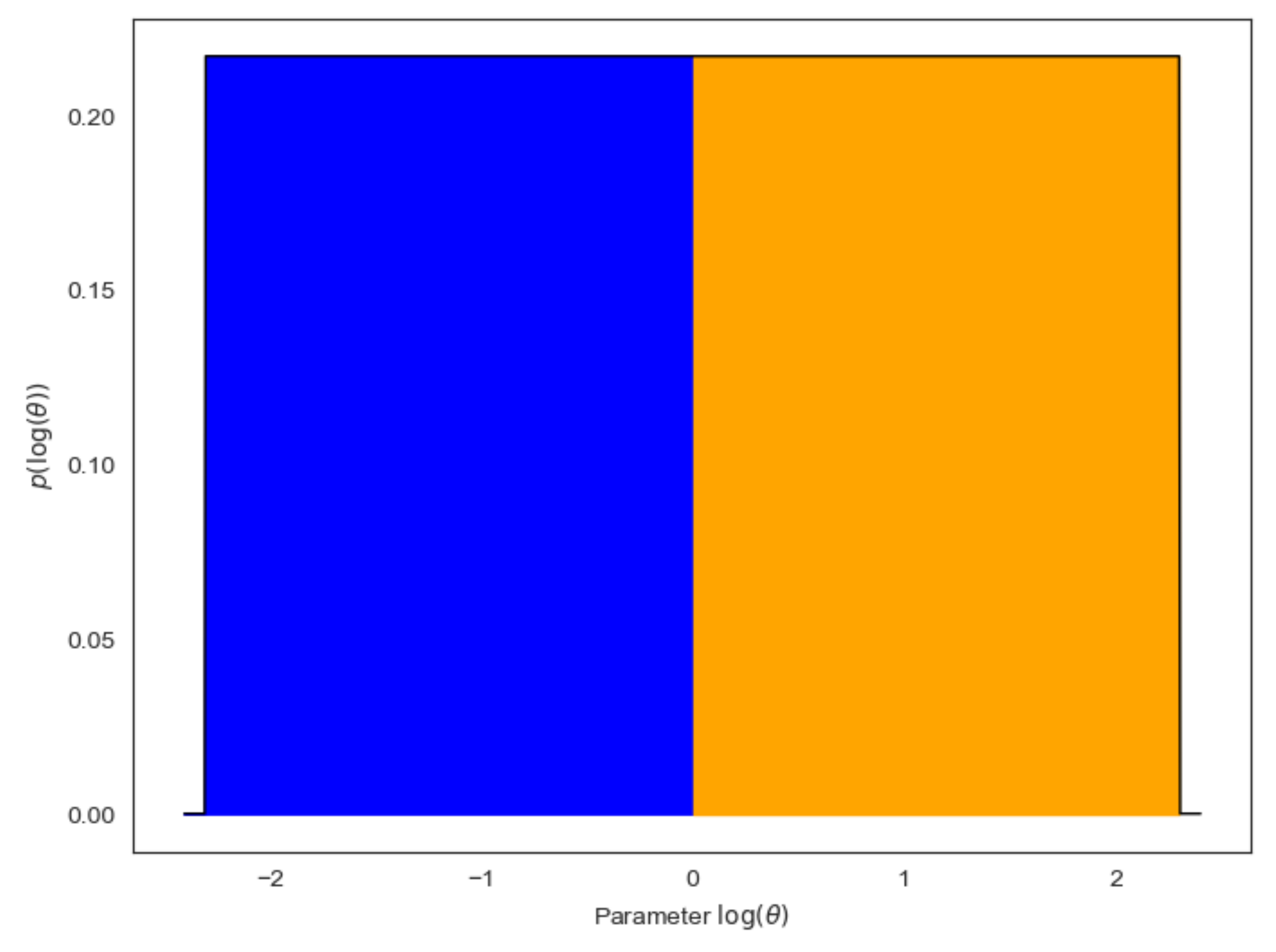
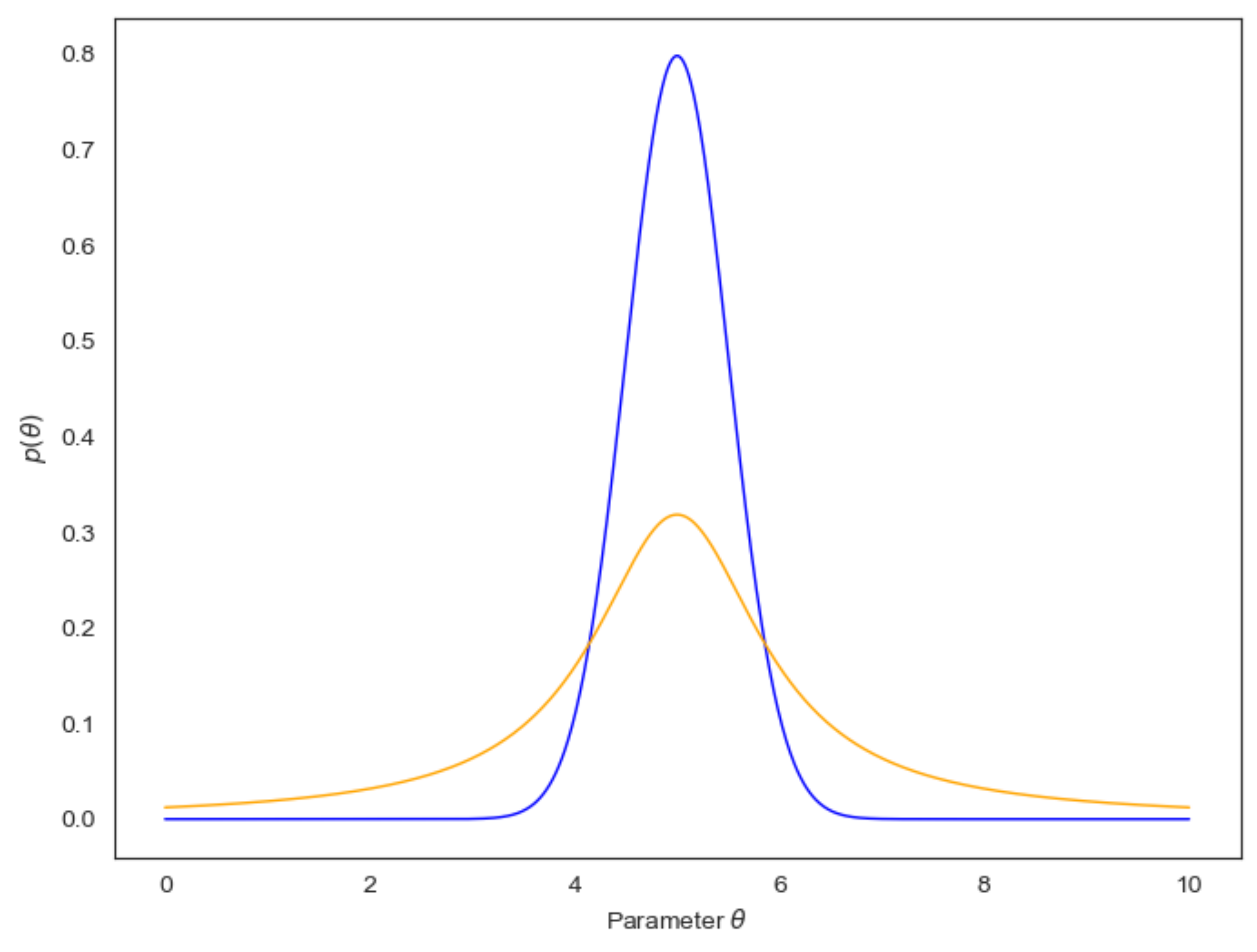


Other Distributions:

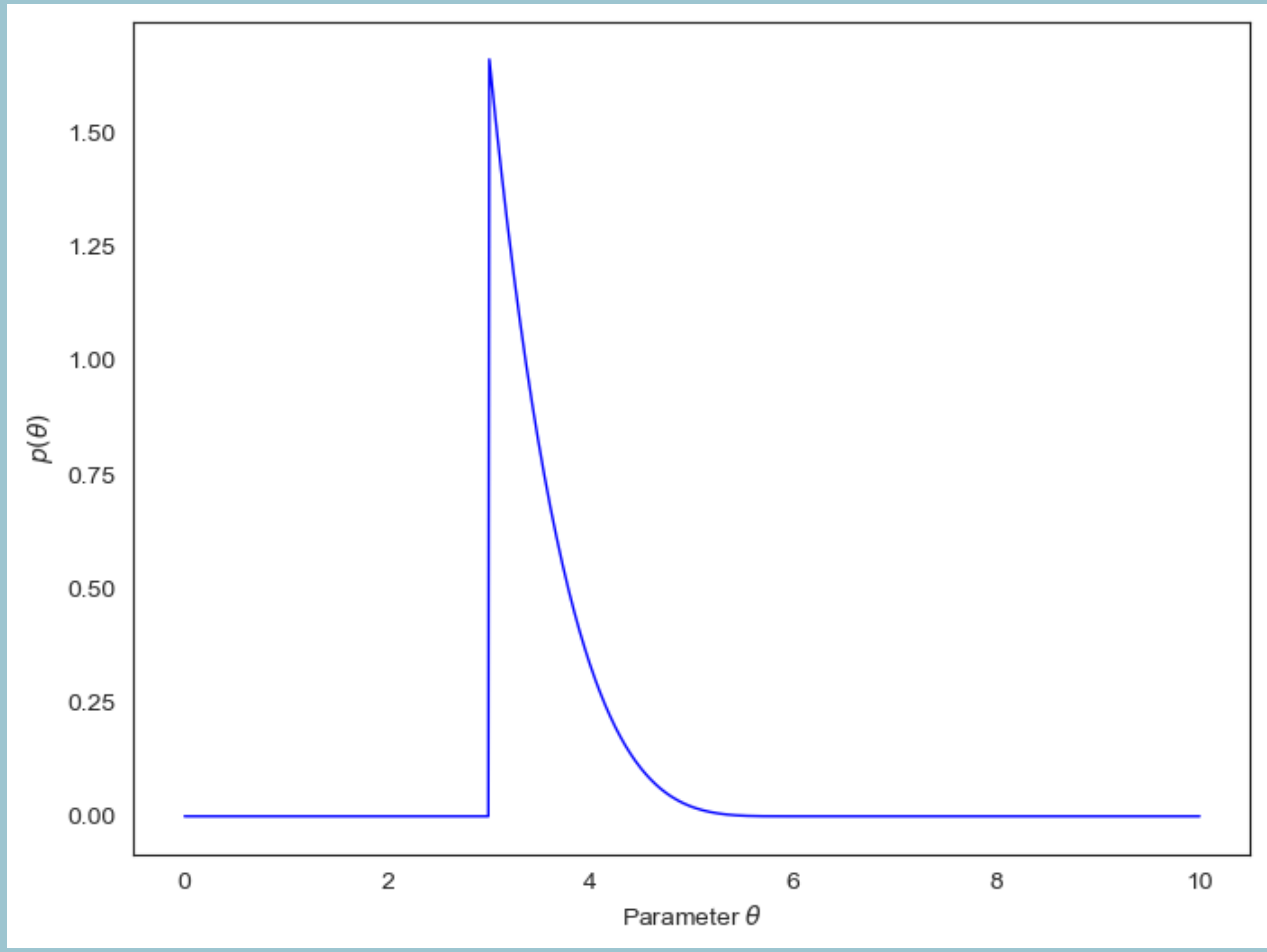
E_{line}

"I have prior information that says that my parameter θ should be around a value of $\hat{\theta}$, but I'm not sure."

Gaussian and t-distribution



Beta-distribution



Reparametrizations

σ_{line}

"The possible width of a line generally scales with the line's energy."

$$p(E_{\text{line}}, \sigma_{\text{line}}) \neq p(E_{\text{line}})p(\sigma_{\text{line}})$$

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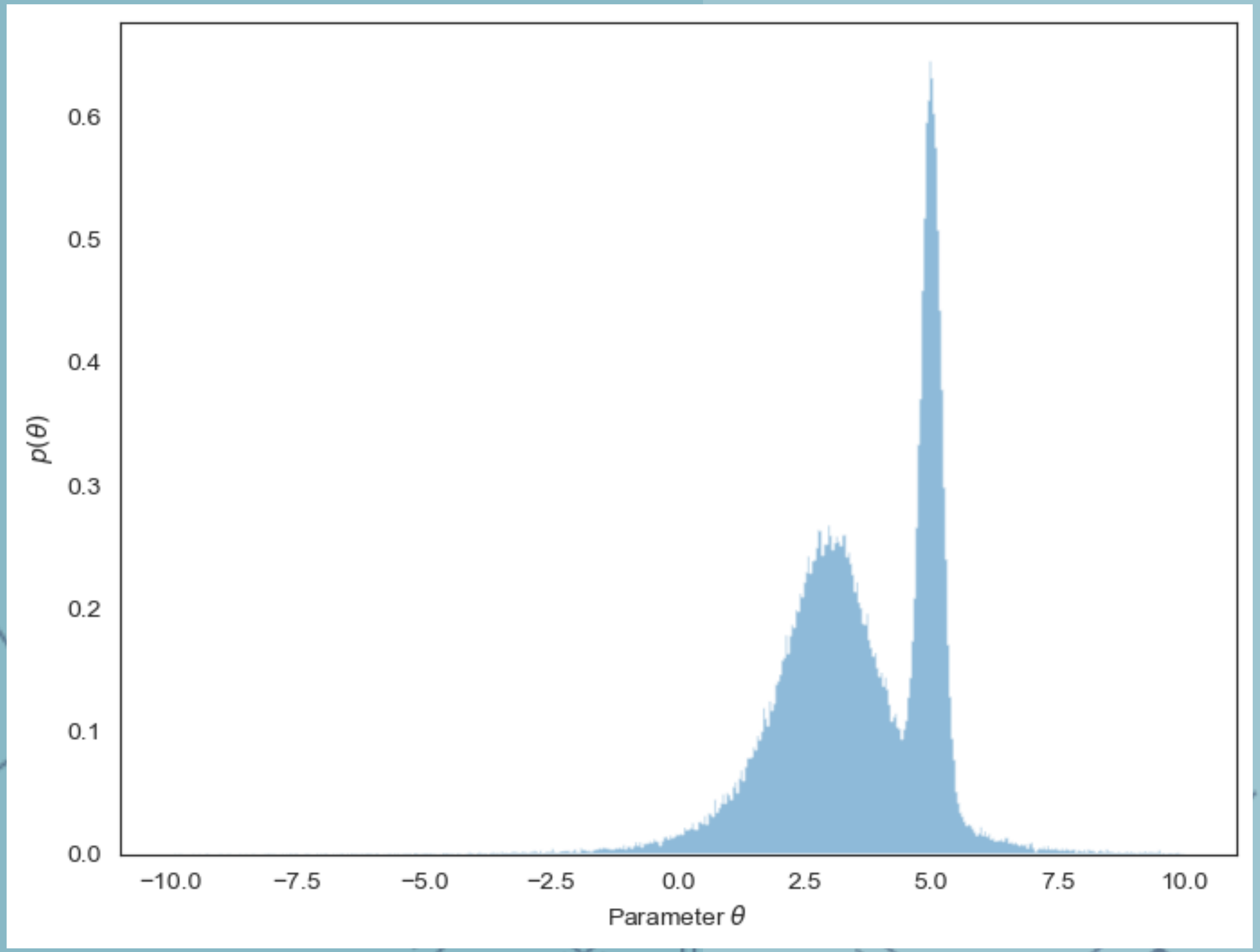
$$p(E_{\text{line}}, q) = p(E_{\text{line}})p(q)$$

Empirical Distributions

“My prior is actually someone else’s posterior”



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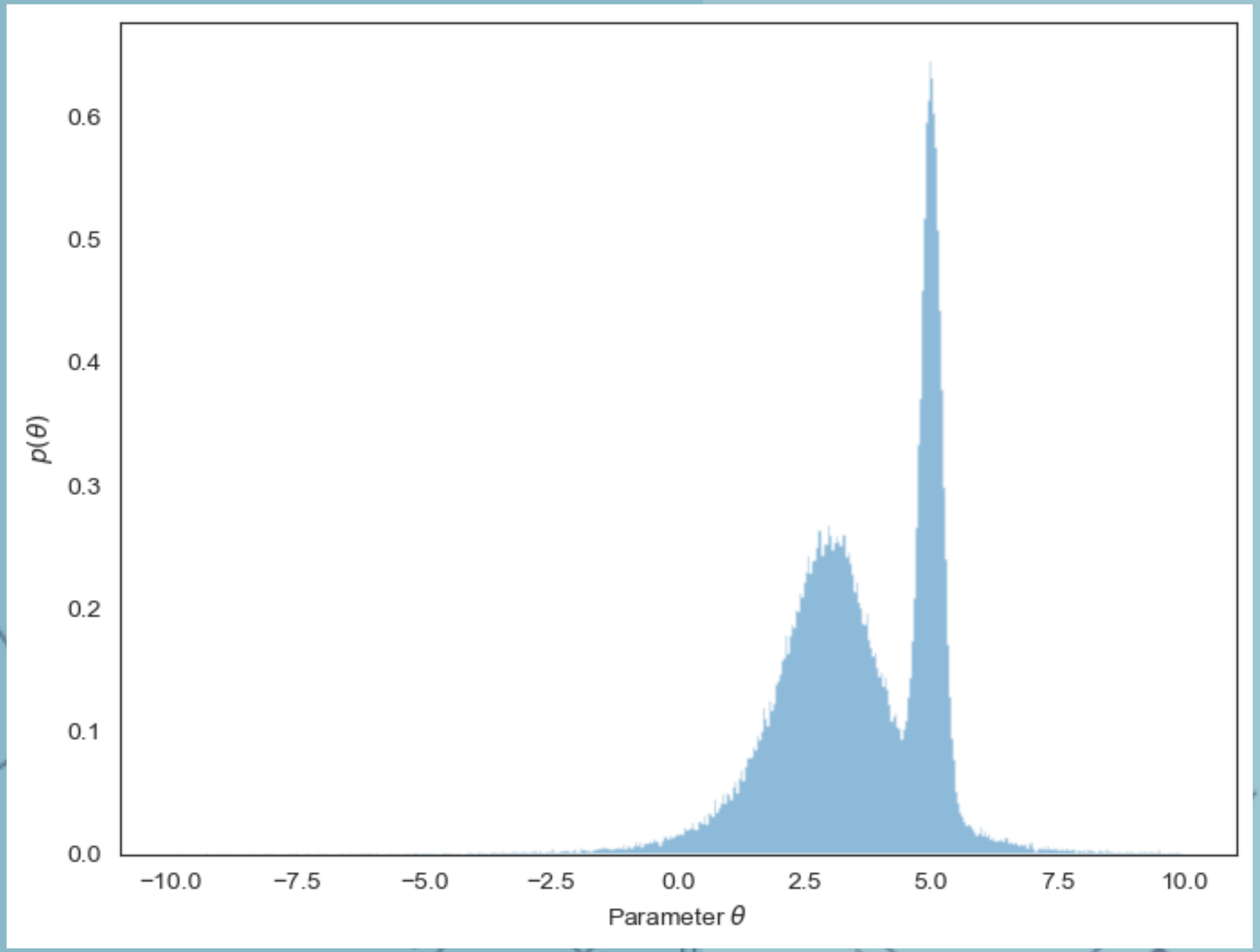
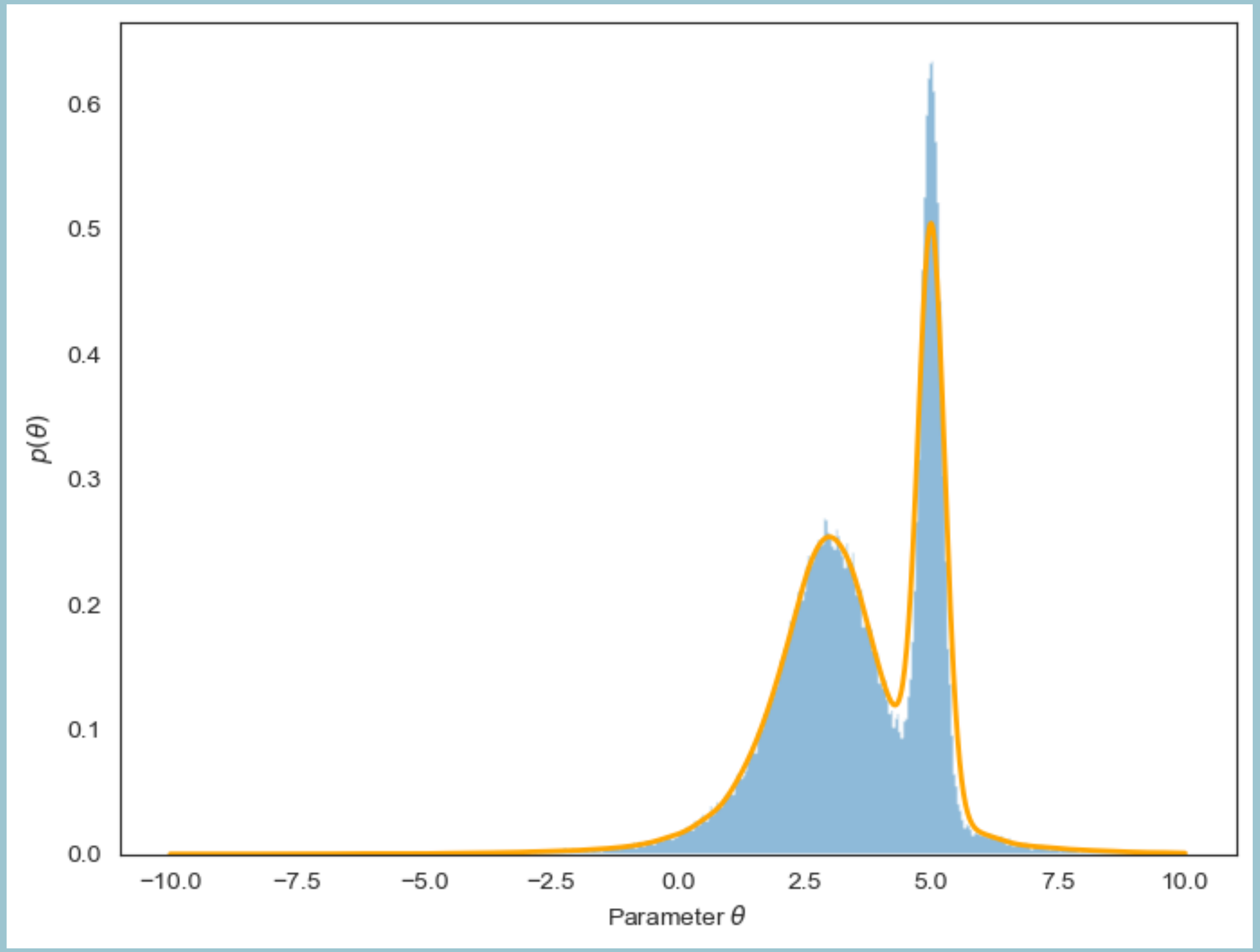


Kernel Density Estimation



What prior would you choose for the M&M exercise, if you weren’t restricted to a conjugate prior?

How can we numerically approximate the posterior pdf?

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… that's what we'll find out next!