Bayesian Inference

Universiteit van Amsterdam

d.huppenkothen@uva.nl

Everything you learned about statistics so far is wrong*!

*just kidding. Mostly.

Estimating Probabilities for Rare Events

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What would you do?

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Result: expect 19 "conspicuous weapon accidents" per year. Led to implementation of major safety features.

A related Astronomy problem:

estimating event rates in astronomical surveys

The LVK Collaboration (2021)

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LIGO has found a single GW event from an intermediate-mass black hole.

What is the underlying rate of of IMBH mergers?

What is the predicted number of IMBH GW events in Advanced LIGO?

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Is it a random variable?

https://app.wooclap.com/STATPHYSASTRO25

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IS A RANDOM VARIABLE!

Statistics Battle of the Century

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Karl Pearson

Ronald A Fisher

Jerzy Neyman

Egon Pearson

+ most of 20th century academic statisticians

Frequentist

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Thomas Bayes

Bruno de Finetti

Alan Turing

Pierre Simon
Maquis de Laplace

Harold Jeffreys

Bayesian

Frequentist

*except the mathematical assumptions of your statistical tests

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- Our belief in an outcome is a combination of our prior beliefs in that outcome, and the information supplied by the data.
- Enables direct computation of probabilities of hypotheses

Conditional Probabilities

DANIELA HUPPENKOTHEN

UNIVERSITEIT VAN AMSTERDAM

d.huppenkothen@uva.nl

Example: Monty Hall Problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

What will we eat at the dinner tomorrow?

$$F = (\text{tapas}, \text{pizza})$$

$$P(F=\text{tapas}) \qquad P(F=\text{pizza})$$

Basic rule of probability

(First and second axioms of probability)

$$0 \leq \mathbf{P(F)} \leq 1$$

If p(tapas)=0.7,
what is p(pizza)?

a) 0.3
b) 0.5
c) 0.01
d) 0.7

If P(tapas)=0.7,
what is P(pizza)?

P(tapas) + P(pizza) = 1

Second axiom of probability

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$$\sum_i P_i(x) = 1$$

Second axiom of probability

$$P(\text{tapas}) + P(\text{pizza}) + P(\text{curry}) = 1$$

“complement”

$$P(\text{tapas}) + P(\text{pizza}) + P(\text{curry}) = 1$$

$$P(\text{pizza}) + P(\text{curry}) = P(\text{not tapas})$$

“complement”

$$P(\text{tapas}) + P(\text{pizza}) + P(\text{curry}) = 1$$

$$P(\text{pizza}) + P(\text{curry}) = P(\text{not tapas})$$

$$P(\text{tapas}) = 1 - P(\text{not tapas})$$

Let’s add another category

Where’s the school?

P(L=Granada) = the school is in Granada
P(L=Rome) = the school is in Rome

The joint distribution

P(tapas ∩ Granada) = “P(tapas and Granada)”
= P(tapas, Granada)

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F, L are independent:

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$$\boxed{P(\text{tapas}, \text{Granada}) = P(\text{tapas})P(\text{Granada})}$$

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P(tapas ∪ Granada) = “P(tapas or Granada)”

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F , L are independent:

$$\boxed{P(\text{tapas or Granada}) = P(\text{tapas}) + P(\text{Granada})}$$

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$$\boxed{\begin{aligned}P(\text{tapas or Granada}) = P(\text{tapas}) + P(\text{Granada}) \\ - P(\text{tapas, Granada})\end{aligned}}$$

Marginalisation:
What is p(F)?

$$P(F) = \sum_{L=\{Granada,\ Rome\}} P(F,L)$$

Conditional probabilities: What is the probability of having tapas at dinner, given that we're in Granada?

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Which mode of transport will I use given the weather?

	Tapas	Pizza	Curry
Granada	0.2	0.15	0.08
Rome	0.5	0.05	0.02

- P(Curry) = ?
- P(Granada | tapas) = ?
- P(curry | Granada) = ?
- Is p(tapas | Granada) = p(Granada | tapas)?
- $\sum_{L} p(\text{tapas}, L) = ?$

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- $\sum_{L} p(\text{tapas}, L) = ?$ **0.7**

Important: p(rain | clouds) $\neq$ p(clouds | rain)

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$$P = 1/3 \,/\, (1/3 + 1/6) = 2/3$$

Joint Probability Distributions

Questions

$$\int p(x \mid y)dx = ?$$

$$\int p(x \mid y)dy = ?$$

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$$\int p(x \mid y)dx = 1$$

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Bayes Theorem

Multiplication Rule

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What does this have to do with atomic bomb accidents?

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$p(x|y) =$ posterior probability of observing future accidents with nuclear bombs, given our data on plane accidents and our prior assumptions

$$P(\theta \mid D) = \frac{P(D \mid \theta) P(\theta)}{P(D)}$$

Bayes rule

$$P(\theta \mid D) = \frac{P(D \mid \theta) P(\theta)}{P(D)}$$

Bayes rule

$\theta =$ model parameters

$$P(\theta \mid D) = \frac{P(D \mid \theta) P(\theta)}{P(D)}$$

Bayes rule

$\theta =$ model parameters

$D =$ data