Scheduling Model

Necessary Data

Description

The data is split into 2 parts:

- System-Info, contains information about the production environment
- Orders, contains orders made by customers, which need to be processed in the described production environment

System-Info

Contains information about the production environment

Tasks All steps / actions, that can be performed in the production environment. (Important: Tasks which are added as "follow up task" to a task should NOT have this task added as "preciding task"). The duration of each task can be found in the data of the workstations (section), so the processing of different tasks on different machines/workstations can be accounted for.

- id – unique ID of the task (just used for internal identification)
- name – name of the task (for easier readable/interpretable results)
- resources – (list) needed resources, to process the task (id + amonut)
- products – (list) all resources (id + amount) which are created by completing the task
- preceding_tasks – (list) tasks, which need to be scheduled before this task can be done
- follow_up_tasks – (list) tasks, which need to be scheduled after this task is completed
- independent – boolean, to flag if the task should be used in the schedule on it's own (e.g. false for tasks which only ever appear as "preciding tasks" and wouldn't make sense on their own)
- prepare_time – the time needed to prepare the machine/workstation for this task
- unpreprare_time – the time needed to unprepare a machine/workstation after it was used for this task

Recipes The recipies for the different resources, which are supposed to be produced (final products or intermediate products)

- id – unique ID of the recipe (just used for internal identification)
- name – name of the recipe (for easier readable/interpretable results)
- tasks – (list) IDs of all tasks, which need to be scheduled/processed, to finish this recipe (result resources and amounts can be looked up from the result products of the last task in the recipe)

Resources All resources, which can either be used or produced in the production environment

- id – unique ID of the resource (just used for internal identification)
- name – name of the resource (for easier readable/interpretable results)
- stock – amount of this resource currently in stock
- price – price per unit to buy this resource (0, if the resource can not be bought)
- delivery_time - expected delivery time, if the resource has to be bought
- renewable – boolean, to flag if the resource is consumed or not (e.g. a resource representing an employee is renewable, because they are usable again once a task is done)
- recipes – (list) IDs of the recipes, with which this resource can be produced (if there are any, otherwise empty list)

Workstations All machines/workstations, which are available in the production environment

- id – unique ID of the workstation (just used for internal identification)
- name – name of the workstation (for easier readable/interpretable results)
- basic_resources – (list) contains the IDs of the necessary resources + amount, which will be needed to run the workstation (independent of the resources needed to process a task)
- tasks – (list) IDs of all tasks, which can be processed on this workstation + the duration of each task on this workstation

Orders

Describes orders made by customers, which are supposed to be processed in the production environment.

- id – unique ID of the order (just used for internal identiciation)
- arrival_time – timestamp, when the order was made
- delivery_time – timestamp, when the order is supposed to be done
- latest_acceptable_time – timestampe, latest acceptable point in time to which the order needs to be completed if the given delivery time can not be met
- resources – (list) IDs + amounts of ordered resources, + the price the customer pays for each of the resources
- penalty – penalty if the order is not processed
- tardiness_fee – fee, if the order is only completed after the delivery time but before the latest acceptable time
- divisible – boolean, to flag if the order can be split and only partially fullfilled or if all of the ordered resources have to be done (or none)
- customer_id – unique ID of the customer
- optional - boolean, flags if the order has to be scheduled or if it is optional (default: true)

Jobs

Describes a specific instance of a task for an order.

- id - unique ID of the job (just used for internal identification)
- order_id - ID of the order, for which the job is being scheduled
- task_id - ID of the task which is processed
- to_id - specific assignment ID between the tasks and the orders to make it easier to link specific tasks to orders

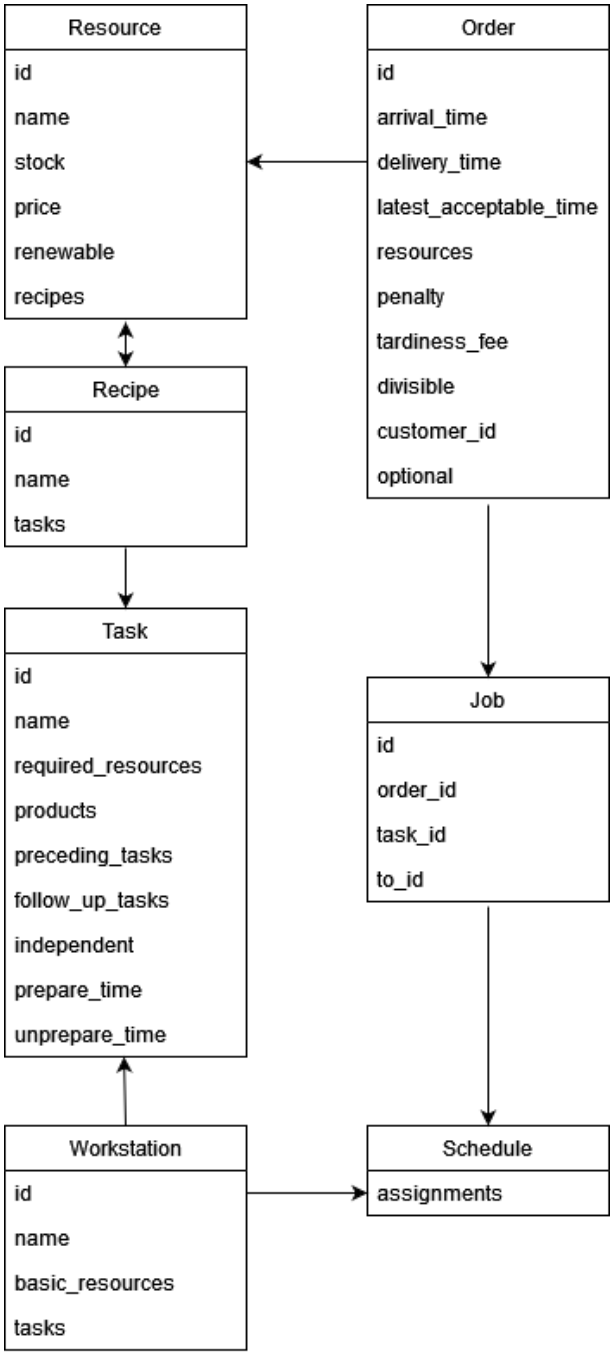

Figure 1: Dependencies

Input / Output

Exact structure of the data for with example data can be seen at "JSON-Example" (section).

Input:
Orders:
$OrderA$
$Task_1 - Task_n$
$price_1 - price_n$
$penalty$
$tardiness\_fee$
$delivery\_date$
$latest\_date$
$optional$

Mapping of tasks to jobs example::
$TaskA.1$
$TaskID$
$JobID$

Intermediate:
<Workstation ID, Task (Operation) ID, Start Time Slot> $j_1 - (w, t, s)$
$j_2 - (w, t, s)$
$j_2 - (w, t, s)$
.
.
.
$j_n - (w, t, s)$

Output:

$$\begin{bmatrix} (j_{11}, s_{11}) & (j_{12}, s_{12}) & (j_{13}, s_{13}) & \dots & (j_{1n}, s_{1n}) \\ (j_{21}, s_{21}) & (j_{22}, s_{22}) & (j_{23}, s_{23}) & \dots & (j_{2n}, s_{2n}) \\ \dots & \dots & \dots & \dots & \dots \\ (j_{m1}, s_{m1}) & (j_{m2}, s_{m2}) & (j_{m3}, s_{m3}) & \dots & (j_{mn}, s_{mn}) \end{bmatrix}$$

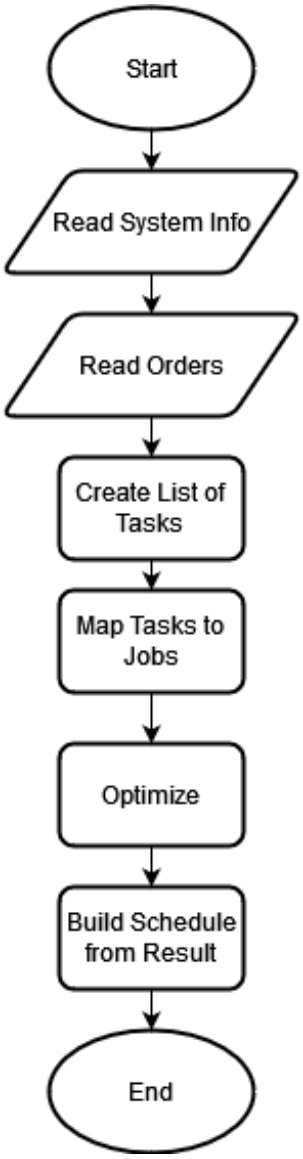

Figure 2: Ablauf

Model

Variable Definition

OLD:
O - Set of all Orders
o - Specific Order, $o \in O$
W - Set of all Workstations

T - Set of all Tasks
T_o - Set of all Tasks needed for Order o, $T_o \subset T$
x - Specific Task, $x \in T$
T_{xp} - Set of all Tasks preceding Task x, $T_{xp} \subset T$
T_{xf} - Set of all Tasks following Task x, $T_{xf} \subset T$
J - Set of all Jobs
J_w - Set of all Jobs assigned to Workstation w, $w \in W$
j_x - Job linked to Task j, $x \in T$, $j \in J$
x_j - Task linked to Job j, $x \in T$, $j \in J$
W_j - Set of Workstations eligible for Job j, $W_j \subset W$
R - Set of all Resources
w - Specific Workstation $w \in W$
R_j - Set of Resources needed for Job j, $R_j \subset R$
r - Specific Resource, $r \in R$
d_{xw} - Duration of Task x on Workstation w
j_w - Selected Workstation for a Job j, $w \in W_j$
j_s - Start time slot of Job j
j_{dt} - Deliver time slot of Job j
j_{lt} - Latest allowed time slot of Job j
i_{rt} - Amount of Resource r in the Inventory at time slot t
l_j - Binary Variable, 1 if Job j is late (Tardiness Fee applies)
l_o - Binary Variable, 1 if Order o is late (Tardiness Fee applies)
u_o - Binary Variable, 1 if Order o can only be fulfilled partially
p_o - Price the customer pays for Order o
f_o - Price reduction for tardy orders
O_c - Set of all Orders which could be completed in the derived schedule, $O_c \subset O$

NEW SUGGESTION:
O - Set of all Orders
P - Set of all Tasks
W - Set of all Workstations
R - Set of all Resources
X - Set of all Jobs
o - Specific Order, $o \in O$
p - Specific Task, $p \in P$
w - Specific Workstation $w \in W$
r - Specific Resource, $r \in R$
x - Specific Job, $x \in X$
P_o - Set of all Tasks needed to complete Order o, $P_o \subset P$

$P_{p_{pre}}$ - Set of all Tasks preceding Task p, $P_{p_{pre}} \subset P$
P_{p_f} - Set of all Tasks following Task p, $P_{p_f} \subset p$
X_w - Set of all Jobs assigned to Workstation w, $w \in W$, $X_w \subset X$
W_x - Set of all Workstations eligible for Job x, $x \in X$, $W_x \subset W$
R_x - Set of all Resources required for Job x, $x \in X$, $R_x \subset R$
d_{pw} - Duration of Task p on Workstation w, $p \in P$, $w \in W$, $d_{pw} \geq 0$
x_w - Selected Workstation w for Job x, $x \in X$, $w \in W$, $x_w \in W$
x_s - Start time slot for Job x, $x \in X$, $x_s \geq 0$
x_p - Task p corresponding to Job x, $x \in X$, $p \in P$, $x_p \in X$
x_{dt} - Delivery time slot for Job x, $x \in X$, $x_{dt} \geq 0$
x_{lt} - Latest acceptable time slot for Job x, $x \in X$, $x_{lt} \geq 0$, $x_{lt} \geq x_{dt}$
$a_{r,t}$ - Amount of Resource r in stock at time slot t, $r \in R$, $t \geq 0$, $a_{r,t} \geq 0$
l_x - Binary Variable, 1 if Job x is late (Tardiness Fee applies), $x \in X$
l_o - Binary Variable, 1 if Order o is late (Tardiness Fee applies), $o \in O$
u_o - Binary Variable, 1 if Order o can only be fulfilled partially, $o \in O$,
z_o - Price the customer pay for Order o, $o \in O$, $z_o \geq 0$
f_o - Price reduction for tardy orders , $o \in O$, $f_o \geq 0$
o_{opt} - Binary Variable, 1 if Order o is optional, $o \in O$
o_{lt} - Latest acceptable time for Order o, $o \in O$, $lt \geq 0$
b_o - Penalty, if Order o is not scheduled to meet o_{lt}, $o \in O$, $b \geq 0$
O_c - Set of all Orders which could be completed in the schedule, $O_c \subset O$

Objective Function + Constraints

Objective Functions

Equation 1 Minimize tardy jobs
Equation 2 Maximize earning
Equation 3 Minimize deviation from the expected delivery date
Equation 4 Maximize Set Size of scheduled jobs
Equation 5 Maximize Set Size of scheduled jobs with least tardy jobs
Equation 6 Maximize Set Size of scheduled jobs with least deviations

$$\begin{aligned} X &:= X_i \\ minimize &\sum_{i=0}^{X} l_x \end{aligned} \tag{1}$$

$$
\begin{aligned}
o &:= O_{c,i} \\
maximize \sum_{i=0}^{O_c} p_o - (l_o * f_o)
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
x &:= X_i \\
y &:= p_j \\
minimize \sum_{i=0}^{X} |(x_s + d_{y,w}) - x_{dt}|
\end{aligned}
\tag{3}
$$

$$
maximize|O_c| \tag{4}
$$

$$
\begin{aligned}
x &:= X_i \\
maximize|O_c| - \sum_{i=0}^{X} l_x
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
x &:= X_i \\
y &:= p_x \\
z ... threshold for acceptable deviation \\
d &:= \sum_{i=0}^{X} (|(x_s + d_{y,w}) - x_{dt}| > z) \\
maximize|O_c| - d
\end{aligned}
\tag{6}
$$

Constraints

Equation 7 to make sure jobs finish before last possible time slot
Equation 8 checks if tardiness fee applies
Equation 9 starting times need to be after timeslot 0
Equation 10 to make sure jobs have sufficient resources to start
Equation 11 jobs can't start before preceding tasks finish
Equation 12 follow up jobs can't start before job finished
Equation 13 only one active job at one time slot for each Workstation

UPDATED:

$$
x_s \leq x_{lt} - d_{x_p,x_w} \tag{7}
$$

$$
l_x := x_{dt} \leq x_s \leq x_{lt} \tag{8}
$$

$$
\begin{aligned}
x &:= X_i \\
\sum_{i=0}^{X} (x_s < 0) &< 1
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
r &:= R_{xi} \\
\sum_{i=0}^{R_x} (a_{r,t} - r \leq 0) &< 1
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
y &:= P_{x_p pre,i} \\
y' &:= P_{x_p pre,i-1} \\
z &:= \sum_{i=1}^{P_{x_p pre}} (x_{ys} + d_{yw}) - (x_{y'} + d_{y'w}) < 0 \\
z &< 1
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
y &:= P_{x_p f,i} \\
y' &:= P_{x_p f,i-1} \\
z &:= \sum_{i=1}^{P_{x_p f}} (x_{y's} + d_{y'w}) < x_{ys} \\
z &< 1
\end{aligned}
\tag{12}
$$

NOT YET UPDATED:

$$
\begin{aligned}
\forall w &\in W \\
i &= J_{wm} \\
j &= J_{wn} \\
x &= x_i \\
x' &= x_j \\
\sum_{m=0}^{J_w} (\sum_{n=0}^{J_w - 1} &((1 - (j_s + d_{x',w} < i_s)) + (1 - (j_s > i_s + d_{x,w})))) < 1
\end{aligned}
\tag{13}
$$

0.1 Tests & Experiments

0.1.1 Used Objective Functions

0.1.1.1 Makespan

0.1.1.2 Min Deadline Deviation

$$\begin{aligned} x &:= X_i \\ y &:= p_j \\ minimize \sum_{i=0}^{X} &|(x_s + d_{y,w}) - x_{dt}| \end{aligned} \tag{14}$$

0.1.1.3 Min Idle Time

0.1.2 Genetic Algorithm Approach

Tested own implementation and PyGAD library - currently using PyGAD library (same results, but faster runtime)

0.1.2.1 Encoding

0.1.2.1.1 1-Stage Optimization For every Job which needs to be scheduled, on pair of <workstation, start time> is used - the duration for each job can be looked up using the workstation and the type of Job for each gene.

0.1.2.1.2 2-Stage Optimization In Stage 1, every Job is represented by a single integer <workstation> In Stage 2, every Job is represented by a single integer <start time> - the workstation assignments needed for the fitness function are taken of the result of Stage 1

0.1.2.2 Selection

The selection method used for all current experiments / tests was the "Roulette Wheel Selection", both with the PyGAD library and the self implemented GA version.

0.1.2.3 Recombination

0.1.2.3.1 One-Point-Crossover

0.1.2.3.2 Two-Point-Crossover

0.1.2.4 Mutation

0.1.2.5 Manual Result Improvement

0.1.3 Agent-Based Approach

0.1.4 CMA-ES

JSON-Example

```
{
"system-info":
        {
        "tasks":
        [
                {
                        "id": 0,
                        "name": "Task 1",
                        "resources":
                        [
                                {
                                        "id": 0,
                                        "amount": 10
                                }
                        ],
                        "products":
                        [
                                {
                                        "resource_id":
                                            1,
                                        "amount": 1
                                }
                        ],
                        "preceding_tasks":
                        [],
                        "follow_up_tasks":
                        [
                                1
                        ],
                        "independent": true,
                        "prepare_time": 5,
```

```
                "unprepare_time": 5
            },
            {
                "id": 1,
                "name": "Task 2",
                "resources":
                [
                    {
                        "id": 1,
                        "amount": 1
                    }
                ],
                "products":
                [
                    {
                        "resource_id":
                            2,
                        "amount": 1
                    }
                ],
                "preceding_tasks":
                [],
                "follow_up_tasks":
                [],
                "independent": false,
                "prepare_time": 0,
                "unprepare_time": 5
            },
            {
                "id": 2,
                "name": "Task 3",
                "resources":
                [
                    {
                        "id": 3,
                        "amount": 20
                    },
                    {
                        "id": 4,
                        "amount": 10
```

```
                        }
                ],
                "products":
                [
                        {
                                "resource_id":
                                    5,
                                "amount": 2
                        },
                        {
                                "resource_id":
                                    6,
                                "amount": 1
                        }
                ],
                "preceding_tasks":
                [
                        3, 4
                ],
                "follow_up_tasks":
                [],
                "independent": true,
                "prepare_time": 10,
                "unprepare_time": 5
        },
        {
                "id": 3,
                "name": "Task 4",
                "resources":
                [
                        {
                                "id": 7,
                                "amount": 5
                        }
                ],
                "products":
                [
                        {
                                "resource_id":
                                    3,
```

```
                                        "amount": 20
                                }
                        ],
                        "preceding_tasks":
                        [],
                        "follow_up_tasks":
                        [],
                        "independent": false,
                        "prepare_time": 5,
                        "unprepare_time": 5
                },
                {
                        "id": 4,
                        "name": "Task 5",
                        "resources":
                        [
                                {
                                        "id": 8,
                                        "amount": 1
                                }
                        ],
                        "products":
                        [
                                {
                                        "resource_id":
                                            4,
                                        "amount": 5
                                }
                        ],
                        "preceding_tasks":
                        [],
                        "follow_up_tasks":
                        [],
                        "independent": true,
                        "prepare_time": 1,
                        "unprepare_time": 5
                },
                {
                        "id": 5,
                        "name": "Task 6",
```

```
                        "resources":
                        [
                                {
                                        "id": 9,
                                        "amount": 20
                                },
                                {
                                        "id": 10,
                                        "amount": 1
                                }
                        ],
                        "products":
                        [
                                {
                                        "resource_id":
                                            8,
                                        "amount": 2
                                }
                        ],
                        "independent": true,
                        "preceding_tasks":
                        [],
                        "follow_up_tasks":
                        [],
                        "prepare_time": 5,
                        "unprepare_time": 5
                }
        ],
        "recipes":
        [
                {
                        "id": 0,
                        "name": "Recipe 1",
                        "tasks":
                        [
                                5
                        ]
                },
                {
                        "id": 1,
```

```
                        "name": "Recipe 2",
                        "tasks":
                        [
                                2
                        ]
                },
                {
                        "id": 2,
                        "name": "Recipe 3",
                        "tasks":
                        [
                                0
                        ]
                },
                {
                        "id": 3,
                        "name": "Recipe 4",
                        "tasks":
                        [
                                4
                        ]
                }
                ],
        "resources":
        [
                {
                        "id": 0,
                        "name": "Resource 1",
                        "stock": 50,
                        "price": 20,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                },
                {
                        "id": 1,
                        "name": "Resource 2",
                        "stock": 0,
                        "price": 300,
```

```
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                },
                {
                        "id": 2,
                        "name": "Resource 3",
                        "stock": 10,
                        "price": 1000,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        [
                                2
                        ]
                },
                {
                        "id": 3,
                        "name": "Resource 4",
                        "stock": 100,
                        "price": 10,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                },
                {
                        "id": 4,
                        "name": "Resource 5",
                        "stock": 10,
                        "price": 100,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        [
                                3
                        ]
                },
                {
```

```
                        "id": 5,
                        "name": "Resource 6",
                        "stock": 4,
                        "price": 50,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        [
                                1
                        ]
                },
                {
                        "id": 6,
                        "name": "Resource 7",
                        "stock": 3,
                        "price": 70,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        [
                                1
                        ]
                        },
                {
                        "id": 7,
                        "name": "Resource 8",
                        "stock": 15,
                        "price": 15,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                },
                {
                        "id": 8,
                        "name": "Resource 9",
                        "stock": 6,
                        "price": 700,
                        "delivery_delay": 0,
                        "renewable": false,
```

```
                        "recipes":
                        [
                                0
                        ]
                },
                {
                        "id": 9,
                        "name": "Resource 10",
                        "stock": 40,
                        "price": 200,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                },
                {
                        "id": 10,
                        "name": "Resource 11 (Employee
                           )",
                        "stock": 10,
                        "price": 0,
                        "delivery_delay": 0,
                        "renewable": true,
                        "recipes":
                        []
                },
                {
                        "id": 11,
                        "name": "Resource 12",
                        "stock": 1000,
                        "price": 5,
                        "delivery_delay": 0,
                        "renewable": false,
                        "recipes":
                        []
                }
        ],
        "workstations":
        [
                {
```

```
                        "id": 0,
                        "name": "Workstation 1",
                        "basic_resources":
                        [
                                {
                                        "id": 10,
                                        "amount": 1
                                }
                        ],
                        "tasks":
                        [
                                {
                                        "task_id": 0,
                                        "duration": 10
                                },
                                {
                                        "task_id": 2,
                                        "duration": 5
                                }
                        ]
                },
                {
                        "id": 1,
                        "name": "Workstation 2",
                        "basic_resources":
                        [
                                {
                                        "id": 10,
                                        "amount": 1
                                },
                                {
                                        "id": 11,
                                        "amount": 2
                                }
                        ],
                        "tasks":
                        [
                                {
                                        "task_id": 1,
                                        "duration": 20
```

```
                                        },
                                        {
                                                "task_id": 2,
                                                "duration": 7
                                        }
                                ]
                        },
                        {
                                "id": 2,
                                "name": "Workstation 3",
                                "basic_resources":
                                [],
                                "tasks":
                                [
                                        {
                                                "task_id": 3,
                                                "duration": 1
                                        },
                                        {
                                                "task_id": 4,
                                                "duration": 2
                                        },
                                        {
                                                "task_id": 5,
                                                "duration": 15
                                        }
                                ]
                        },
                        {
                                "id": 3,
                                "name": "Workstation 4",
                                "basic_resources":
                                [
                                        {
                                                "id": 10,
                                                "amount": 1
                                        }
                                ],
                                "tasks":
                                [
```

```
                                        {
                                                "task_id": 4,
                                                "duration": 1
                                        },
                                        {
                                                "task_id": 5,
                                                "duration": 10
                                        }
                                ]
                        }
                ]
},
"orders":
[
                {
                        "id": 0,
                        "arrival_time": "2022-02-07 00:00:00",
                        "delivery_time": "2022-02-20 14:30:00"
                           ,
                        "latest_acceptable_time": "2022-02-22
                           00:00:00",
                        "resources":
                        [
                                {
                                        "id": 2,
                                        "amount": 5,
                                        "price": 700
                                },
                                {
                                        "id": 4,
                                        "amount": 10,
                                        "price": 300
                                }
                        ],
                        "penalty": 300,
                        "tardiness_fee": 100,
                        "divisible": false,
                        "customer_id": 1,
                        "optional": true
                },
```

```
        {
                "id": 1,
                "arrival_time": "2022-02-17 00:00:00",
                "delivery_time": "2022-02-21 15:30:00",
                "latest_acceptable_time": "2022-02-24 00:00:00",
                "resources":
                [
                        {
                                "id": 4,
                                "amount": 20,
                                "price": 600
                        },
                        {
                                "id": 5,
                                "amount": 10,
                                "price": 2000
                        }
                ],
                "penalty": 500,
                "tardiness_fee": 100,
                "divisible": false,
                "customer_id": 1,
                "optional": true
        },
        {
                "id": 2,
                "arrival_time": "2022-02-10 00:00:00",
                "delivery_time": "2022-02-19 14:00:00",
                "latest_acceptable_time": "2022-02-26 00:00:00",
                "resources":
                [
                        {
                                "id": 2,
                                "amount": 20,
                                "price": 2700
                        },
```

```
                        {
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                "optional": true
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        {
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}
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