

## Homework 4

**Problem 1 (10 points):** The probabilities are 0.4, 0.2, 0.3, and 0.1, respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. What is the probability that among 9 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by automobile, and 2 arrived by train?

**Problem 2 (20 points):** According to Chemical Engineering Progress (November 1990), approximately 30% of all pipework failures in chemical plants are caused by operator error.

(a) What is the probability that out of the next 20 pipework failures at least 8 are due to operator error?

(b) What is the probability that no more than 3 out of 20 such failures are due to operator error?

(c) Suppose, for a particular plant, that out of the random sample of 20 such failures, exactly 5 are due to operator error. Do you feel that the 30% figure stated above applies to this plant? Comment.

**Problem 3 (10 points):** From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 2 defective missiles that will not fire, what is the probability that

(a) all 4 will fire?

(b) at most 2 will not fire?

**Problem 4 (20 points):** Changes in airport procedures require considerable planning.

The arrival rates of aircraft are important factors that must be taken into account.

Suppose small aircraft arrive at a certain airport, according to a Poisson process, at the rate of 6 per hour. Thus, the Poisson parameter for arrivals over a period of hours is  $\mu = 6t$ .

(a) What is the probability that exactly 4 small aircraft arrive during a 1-hour period?

(b) What is the probability that at least 4 arrive during a 1-hour period?

(c) If we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a working day?

**Problem 5 (10 points):** The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what percentage of the loaves are

- (a) longer than 31.7 centimeters?
- (b) between 29.3 and 33.5 centimeters in length?
- (c) shorter than 25.5 centimeters?

**Problem 6 (10 points):** Suppose that the time, in hours, required to repair a heat pump is a random variable  $X$  having a gamma distribution with parameters  $\alpha = 2$  and  $\beta = 1/2$ . What is the probability that on the next service call

- (a) at most 1 hour will be required to repair the heat pump?
- (b) at least 2 hours will be required to repair the heat pump?

**Problem 7 (10 points):** Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

- (a) What is the probability that response time exceeds 5 seconds?
- (b) What is the probability that response time exceeds 10 seconds?

**Problem 8 (10 points):** The average rate of water usage (thousands of gallons per hour) by a certain community is known to involve the lognormal distribution with parameters  $\mu = 5$  and  $\sigma = 2$ . It is important for planning purposes to get a sense of periods of high usage. What is the probability that, for any given hour, 50,000 gallons of water are used?

**Submission Format**

1. Submit pdf or word document of the solutions
2. All the calculation steps must be included in the solution. However, students can use R to get the numerical answers
3. No need to attach the R codes
4. The solutions can be handwritten and scanned or typed
5. Submit the solutions by 13th November
6. This homework is a group effort and only one member needs to submit
7. Mention the group member contribution when submitting via Canvas