HOMEWORK - 4

Group - 17

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Problem - 1

Using multinomial distribution

$$f(x_1, x_2, \ldots, x_k; p_1, p_2 \ldots p_k, n)$$

$$= \left(\frac{n}{x_1, x_2 \ldots x_k}\right) p_1^{x_1} \, p_2^{x_2} \ldots p_k^{x_k}.$$

with

$$\sum_{i=1}^{k} x_i = n \quad \& \quad \sum_{i=1}^{k} p_i = 1$$

$$f(3, 3, 1, 2; 0.4, 0.2, 0.3, 0.1, 9)$$

$$= \binom{9}{3, 3, 1, 2} (0.4)^3 (0.2)^3 (0.3)^1 (0.2)^2$$

$$= \left(\frac{9!}{3!\,3!\,1!\,2!}\right) (0.4)^3 (0.2)^3 (0.3)^1 (0.2)^2$$

$$= 0.0077$$

Problem 2

$n = 20 \qquad p = 0.3 \qquad q = 0.7$

(a) $P(X \geq 8) = 1 - P(X < 8)$

$$= 1 - \left[\binom{20}{x} (0.3)^x (0.7)^{20-x} \right]$$

$$= 1 - (0.00079 + 0.0068 + 0.027 + 0.071 + 0.13 + 0.178 + 0.191 + 0.164)$$

$$= 1 - (0.772)$$

$$= 0.228.$$

$$\boxed{P(X \geq) = 0.228}$$

(b) $P(X \leq 3) = \sum_{x=0}^{3} b(x; 20, 0.3)$

$$P(X \leq 3) = 0.00079 + 0.0068 + 0.027 + 0.0716$$

$$\boxed{P(X \leq 3) = 0.107}$$

(c) $P(X=5) = b(5; 20, 0.3)$

$\boxed{P(X=5) = 0.1789}$

Prob of five failing is 17.89%

$\therefore$ 30% applies to this plant.

Problem 3

$N = 10 \qquad n = 4$

$K = (10-2) = 8$

(a) All 4 will fire.

$= h(x; N, n, k)$

$= h(4, 10, 4, 8)$

$$h(4, 10, 4, 8) = \frac{({}^{8}C_4)({}^{2}C_0)}{{}^{10}C_4}$$

$$= \boxed{1/3}$$

(b) At most 2 will fire

$= h(x; N, n, k)$

$x = 0, 1, 2)$

$$= \sum_{0}^{2} h(x, 10, 4, 2)$$

for $(x=0) \Rightarrow h(0, 10, 4, 2)$

$$= \frac{({}^{2}C_0)({}^{8}C_4)}{{}^{10}C_4} = 1/3$$

$x = 1$

$$= \frac{({}^{2}C_1)\ ({}^{8}C_3)}{{}^{10}C_4} = \frac{8}{15}$$

$x = 2$

$$= \frac{({}^{2}C_2)\ ({}^{8}C_2)}{{}^{10}C_4} = \frac{2}{15}$$

P(at most 2 will not fire) $= \frac{1}{3} + \frac{8}{15} + \frac{2}{15}$

$$= \underline{\underline{1}}$$

Problem 4

a) Probability that exactly 4 small aircrafts arrive during a 1-hour period.

$\mu = 6$

$$P(X=4) = \frac{e^{-\mu}(\mu)^4}{4!}$$

$$= \frac{e^{-6}(6)^4}{4!}$$

$$\boxed{P(X=4) = 0.13385}$$

b) Probability that atleast 4 arrive during a 1-hour period.

$$P(X \geq 4) = 1 - P(X \leq 3)$$

$$P(X \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!}\right]$$

$$= 1 - [0.00247 + 0.0148 + 0.0446 + 0.0892]$$

$$= 1 - [0.15307)$$

$$= 0.84$$

c) $P(X \geq 75) = 1 - P(X \leq 74)$

$= \cancel{1 - \sum_{x=0}^{4} e^{-6}}$

$\mu = 6t = 6(12) = \underline{\underline{72}}$

$$P(X \geq 75) = 1 - \sum_{0}^{74} \frac{e^{-6}(72)^x}{x!}$$

$= 1 - 0.6226$

$= \underline{\underline{0.3773}}$

$\boxed{P(X \geq 75) = 0.3773}$

Problem 5

(a) $P(X > 31.7) = P\left(\frac{31.7 - 30}{2}\right) = 0.85$

$P(Z > 0.85) = 1 - P(Z \leq 0.85)$

$= 1 - 0.80234$

$\boxed{P(Z > 0.85) = 0.1977}$

Problem 5

(b) $29.3 < x < 33.5$

$$\left(\frac{29.3-30}{2}\right) < x < \left(\frac{33.5-30}{2}\right)$$

$$\frac{-0.7}{2} < x < \frac{3.5}{2}$$

$$-0.35 < x < 1.75$$

Using std normal table

$P(29.3 < X < 33.5) = P(-0.35 < Z < 1.75)$

$= P(Z < 1.75) - P(Z < -0.35)$

$= 0.95994 - 0.36317$

$= 0.59677$

$= \underline{59.677\%}$

c) Shorter than 25.5 cm

$$P(X<25.5) = P\left(Z<\frac{25.5-30}{2}\right)$$

$$= P\left(Z<\frac{-4.5}{2}\right)$$

$$= P(Z<-2.25)$$

Using std normal table.

$P(Z<-2.25) = 0.0122$

$\boxed{= 1.22\% \text{ leaves are shorter than } 25.5\text{ cm.}}$

Problem 6

(a) $\Rightarrow P(X \le 1)$

$\alpha = 2 \qquad \beta = 1/2$

$$G(x) = \frac{\beta^{-\alpha} e^{-x/\beta} x^{\alpha - 1}}{\Gamma \alpha} = 4xe^{-2x}$$

$$P(X \le 1) = \int_{-\infty}^{1} G(x)\,dx = \int_{0}^{1} 4xe^{-2x}\,dx$$

$$= 4\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right]_0^1$$

$$\boxed{P(X \le 1) = 1 - \frac{3}{e^{x}} = \underline{\underline{0.594}}}$$

(b) $$P(X \ge 2) = \int_{2}^{\infty} G(x)\,dx.$$

$$= 4\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right]_0$$

$$= 4\left[0 - \left(-e^{-4} - \frac{e^{4}}{4}\right)\right]$$

$$= \cancel{4} \times \frac{5}{\cancel{4}} e^{-4} = 5e^{-4}$$

$$\boxed{P(X \ge 2) = 0.0916}$$

Problem 7

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$\beta > 0$

(a) $\beta = 3$

$$P(X > 5) = \int_5^\infty \frac{1}{3} e^{-x/3}\, dx$$

$$= \frac{1}{3}\left[-3e^{-x/3}\right]_5^\infty$$

$$= e^{-5/3}$$

$$\boxed{P(X>5) = 0.1889}$$

(b) $$P(X>10) = \int_{10}^\infty \frac{1}{3} e^{-x/3}\, dx$$

$$= \frac{1}{3}\left[-3e^{-x/3}\right]_{10}^\infty$$

$$= e^{-10/3}$$

$$\boxed{P(X>10) = 0.0357}$$

Problem 8

$\mu = 5 \qquad \sigma = 2$

$$P(X \geq 50{,}000) = 1 - \phi\left(\frac{\ln 50{,}000 - 5}{2}\right)$$

$$= 1 - \phi(2.9099)$$

Using normal std table

$$\phi(2.9099) = 0.9982$$

$\therefore$

$$P(X \geq 50{,}000) = 1 - 0.9982$$

$$\boxed{P(X \geq 50{,}000) = 0.0018}$$