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Part – I

1-A) $P(wn) = 0.85$

$P(ga) = 0.95$

$P(ga|wn) = 0.99$

$P(wn|ga) = (?)$

$$P(wn|ga) = \frac{P(ga|wa) \cdot P(wn)}{P(ga)}$$

$$= \frac{0.99 \times 0.85}{0.95} = 0.8857$$

$$\boxed{\therefore P(wn|ga) = 0.8857}$$

b) $P(ga|\neg wn) = (?)$

$P(\neg wn) = 1 - P(wn)$

$= 1 - 0.85$

$\boxed{P(\neg wn) = 0.15}$

$P(\neg wn|ga) = 1 - P(wn|ga)$

$= 1 - 0.8857$

$= 0.1143$

$$\therefore P(ga|\neg wn) = \frac{P(\neg wn|ga) \cdot P(ga)}{P(\neg wn)}$$

$$= \frac{0.1143 \times 0.95}{0.15}$$

$$\boxed{P(ga|\neg wn) = 0.722}$$

2) $P(T_{seen Blue} | T_{Blue}) = 0.75$ ($\because$ 75% reliability given)

$P(T_{seen green} | T_{Blue}) = 0.25$

$P(T_{green}) = 0.9$	$P(T_{seen Blue} \mid T_{green}) = 0.25$
$P(T_{Blue}) = 0.1$	$P(T_{seen green} \mid T_{green}) = 0.75$

we need to find,

$$P(T\,Blue \mid T\,SeenBlue) = \frac{P(T\,SeenBlue \mid T\,Blue) \cdot P(T\,Blue)}{P(T\,SeenBlue)}$$

$$= \frac{0.75 \times 0.1}{P(T\,SeenBlue \mid T\,Blue) \cdot P(T\,Blue) + P(T\,SeenBlue \mid T\,green) \cdot P(T\,green)}$$

$$= \frac{0.75 \times 0.1}{(0.75)(0.1) + (0.25 \times 0.9)}$$

$$= \frac{0.75 \times 0.1}{0.3}$$

$$= 0.25$$

Now, $P(T\,green \mid T\,SeenBlue) = 1 - 0.25$
$= 0.75$

⟹ here, The probability of taxi being green is more likely than the probability of taxi being blue, Thus, Most likely color of taxi is green.

3) a) given, ω = good, S = pass, N = out

$P(\omega/yes) = 1/3$ $\quad$ $P(\omega/No) = 3/5$

$P(S/yes) = 3/3$ $\quad$ $P(S/No) = 1/5$

$P(N/yes) = 1/3$ $\quad$ $P(N/No) = 3/5$

$P(yes) = 3/8$ $\quad$ $P(No) = 5/8$

→ $P(yes/S, N, \omega) = P(S/yes) \cdot P(N/yes) \cdot P(\omega/yes) \cdot P(yes)$
$= (3/3) \cdot (1/3) \cdot (1/3) \cdot (3/8)$
$= 0.04167$

→ $P(No/S, N, \omega) = P(S/No) \cdot P(N/No) \cdot P(\omega/No) \cdot P(No)$
$= (1/5) \cdot (3/5) \cdot (3/5) \cdot (5/8) = 0.045$

⇒ here probability of No is more than Yes, so,
Jim is not happy.

b) Using Bayes: X = (Weather = Good, Study = pass, Neighbor = out)

$$P(\text{Happy} = \text{yes} / w = \text{Good}, S = \text{pass}, N = \text{out})$$

$$= \frac{P(w = \text{Good}, S = \text{pass}, N = \text{out} / \text{Happy}/\text{yes})\, P(\text{yes})}{P(w = \text{Good}, S = \text{pass}, N = \text{out})}$$

$$= \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{8}} = 1$$

$$P(\text{No}/X) = \frac{P(X/\text{No}) \cdot P(\text{No})}{P(X)}$$

$$= \frac{0 \cdot 5/8}{1/8} = 0$$

∴ Jim is happy.

A4) $P(C++) = 0.5$

$P(\text{Java}) = 0.4$

$P(\text{Macrosoft worker}) = 0.01 = P(MS)$

$P(C++/MS) = 0.99$

$P(\text{Java}/MS) = 0.98$

$$P\left(MS/C++ \wedge \text{Java}\right) = (?)$$

$$= \frac{P(C++/MS) \cdot P(\text{Java}/MS) \cdot P(MS)}{P(C++ \wedge \text{Java}) \leftarrow \left[\{P(C++) \cdot P(\text{Java})\}\right]}$$

$$= \frac{(0.99)(0.98)(0.01)}{0.5 \times 0.4}$$

$$= 0.04851$$

$$P(\neg MS / C++ \wedge \text{Java}) = 1 - P(MS / C++ \wedge \text{Java})$$

$$= 1 - 0.04851$$

$$= 0.951$$

⇒ hence, we can find that more probable to be not be an employer of macrosoft, who knows C++ & Java.

⇒ so, Ans is <u>No</u>.

A5) $$P(y=1/X) = \frac{1}{1+\exp(\omega_0 + \sum_{i=1}^{n} \omega_i X_i)}$$

$$P(y=0/X) = \frac{\exp(\omega_0 + \sum_{i=1}^{n} \omega_i X_i)}{1+\exp(\omega_0 + \sum_{i=1}^{n} \omega_i X_i)}$$

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Y is boolean, govened by Bernoulli distribution with parameter $\pi = P(Y=1)$

$X = \{X_1 \ldots X_n\}$ where each X_i is boolean

According to Bayes rule

$$P(Y=1/X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X|Y=1)\,P(Y=1) + P(X|Y=0)\cdot P(Y=0)}$$

$$= \frac{1}{1 + \frac{P(X|Y=0)\cdot P(Y=0)}{P(X|Y=1)\cdot P(Y=1)}}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{P(X/Y=0)\cdot P(Y=0)}{P(X/Y=1)\cdot P(Y=1)}\right)}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{P(Y=0)}{P(Y=1)} + \sum_i^n \ln \frac{P(X_i/Y=0)}{P(X_i/Y=1)}\right)}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{1-\pi}{\pi} + \sum_i \ln \frac{P(X_i/Y=0)}{P(X_i/Y=1)}\right)}$$

$$\sum_i \ln \frac{P(X_i/Y=0)}{P(X_i/Y=1)} = \sum_i \ln \left(\frac{\theta_{i0}^{X_i}\,(1-\theta_{i0})^{(1-X_i)}}{\theta_{i1}^{X_i}\,(1-\theta_{i1})^{(1-X_i)}}\right)$$

[$\rightarrow$ here X_i are boolean variables

$\theta_{i1} = P(X_i=1/Y=1)$ & $\theta_{i0} = P(X_i=1/Y=0)$]

$$\sum_i \ln \frac{P(X_i/Y=0)}{P(X_i/Y=1)} = \sum_i \left[X_i \ln\left(\frac{\theta_{i0}}{\theta_{i1}}\right) + (1-X_i)\ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]$$

$$= \sum_i \left[\ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right) X_i + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]$$

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$$P(Y=1/X) = \frac{1}{1+\exp\left(\ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \left[\ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right)X_i + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]\right)}$$

$$= \frac{1}{1+\exp(w_0 + \sum_{i=1}^{n} w_i X_i)}$$

where $w_i = \ln\left[\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right]$

$$w_0 = \ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

For, $P(Y=0/X) = 1 - P(Y=1/X)$

$$= 1 - \frac{1}{1+\exp(w_0 + \sum_{i=1}^{n} w_i X_i)}$$

$$= \frac{\exp(w_0 + \sum_{i=1}^{n} w_i X_i)}{1+\exp(w_0 + \sum_{i=1}^{n} w_i X_i)}$$

$\Rightarrow$ hence, $P(Y/X)$ follows the same form, just values are different.

6) Y is a boolean with parameter $\pi = P(Y=1)$
$X = \{X_1, X_2, \dots X_n\}$ where each X_i follows exponential distribution.

According to Bayes rule.

$$P(Y=1|X) = \frac{P(X|Y=1)\cdot P(Y=1)}{P(X|Y=1)\cdot P(Y=1) + P(X|Y=0)\cdot P(Y=0)}$$

$$= \frac{1}{1 + \frac{P(X|Y=0)\cdot P(Y=0)}{P(X|Y=1)\cdot P(Y=1)}}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{P(X|Y=0)\, P(Y=0)}{P(X|Y=1)\cdot P(Y=1)}\right)}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{P(Y=0)}{P(Y=1)} + \sum_i \ln \frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

$$= \frac{1}{1 + \exp\left(\ln \frac{1-\pi}{\pi} + \sum_i \ln \frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

$$\sum_i \ln \frac{P(X_i|Y=0)}{P(X_i|Y=1)} = \sum_i \ln \frac{h(X_i,\theta_{i0}) \exp(-\theta_{i0}^T X_i + c)}{h(X_i,\theta_{i1}) \exp(-\theta_{i1}^T X_i + c)}$$

$$= \sum_i \ln \frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})} + \ln(\exp(-\theta_{i0}^T X_i + c - (-\theta_{i1}^T X_i + c))$$

$$= \sum_i \ln \frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})} + (\theta_{i1}^T - \theta_{i0}^T) X_i$$

$$P(Y=1|X) = \frac{1}{1 + \exp\left(\ln \frac{1-\pi}{\pi} + \sum_i (\theta_{i1}^T - \theta_{i0}^T) X_i + \ln\left(\frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})}\right)\right)}$$

$$= \frac{1}{1+\exp(-z)}$$

$$-z = \ln \frac{1-\pi}{\pi} + \sum_i \left((\theta_{i1}^T - \theta_{i0}^T) x_i + \ln \frac{h(x_i, \theta_{i0})}{h(x_i, \theta_{i1})} \right)$$

$$z = \ln \frac{\pi}{1-\pi} - \sum_i \left((\theta_{i1}^T - \theta_{i0}^T) x_i + \ln \frac{h(x_i, \theta_{i0})}{h(x_i, \theta_{i1})} \right)$$

$$= \omega_0 + \sum \omega_i x_i$$

$$\boxed{\therefore \omega_0 = \ln \frac{\pi}{1-\pi} - \sum_i \ln \frac{h(x_i, \theta_{i0})}{h(x_i, \theta_{i1})}}$$

$$\boxed{\omega_i = \theta_{i0}^T - \theta_{i0}^T}$$

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