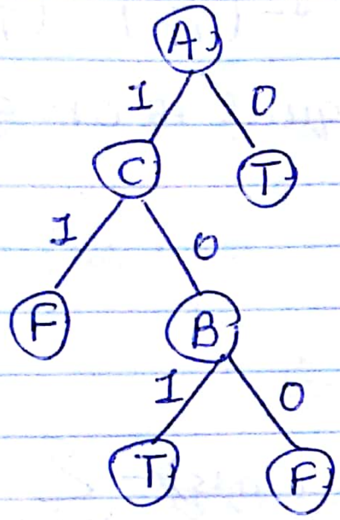
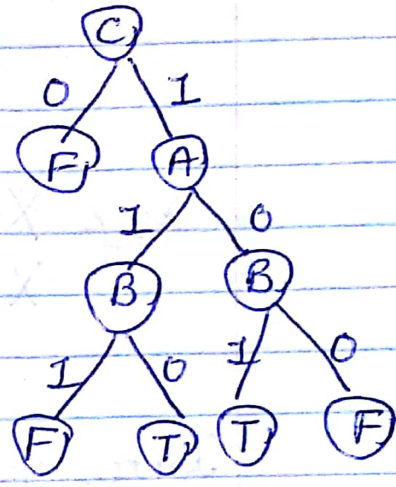
Part-I

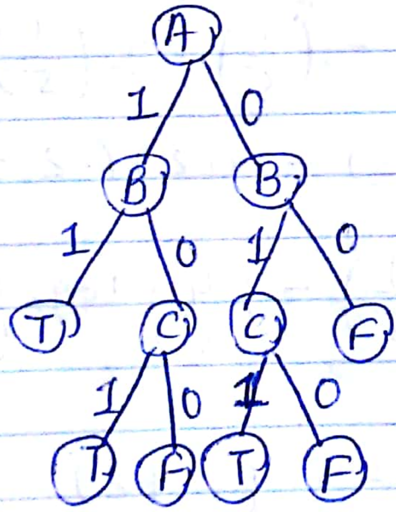
(a) $Y = (\neg A \vee B) \wedge \neg(C \wedge A)$



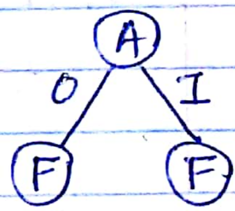
(b) $Y = (A \oplus B) \wedge C$



(c) $Y = (A \vee B) \wedge (B \vee C) \wedge (C \vee A)$



(d) $Y = (A \vee B) \wedge \neg A \wedge \neg B$



(2) Entropy (class) $= -P_{\oplus}\log_2 P_{\oplus} - P_{\ominus}\log_2 P_{\ominus}$

$$= -\left(\frac{5}{10}\right)\log_2\left(\frac{5}{10}\right) - \left(\frac{5}{10}\right)\log_2\left(\frac{5}{10}\right)$$

$$= 1 \quad \{\text{equal \# of } \oplus \text{ \& } \ominus \text{ ex}\}$$

X_1 : $X_1(1)$: [4+, 1-]
$X_1(0)$: [4-, 1+]
S : [5+, 5-]

$$\text{Info. gain}(X_1) = \text{Entropy(class)} - \sum_{v\in\{0,1\}} \frac{|S_v|}{|S|}\,\text{Ent}(S_v)$$

$$= 1 - \left(\frac{5}{10}\right)H(x_1, 1) - \left(\frac{5}{10}\right)H(X_1, 0)$$

$$H(X_1, 1) = -\frac{4}{5}\log_2\left(\frac{4}{5}\right) - \left(\frac{1}{5}\right)\log_2\left(\frac{1}{5}\right)$$

$$= -(0.8)(-0.32) - (0.2)(-2.32)$$

$$= 0.72$$

$$H(X_1, 0) = -\left(\frac{1}{5}\right)\log_2\left(\frac{1}{5}\right) - \left(\frac{4}{5}\right)\log_2\left(\frac{4}{5}\right)$$

$$= 0.72$$

$$Ig(X_1) = 1 - \frac{5}{10}(0.72) - \frac{5}{10}(0.72)$$

$$= \underline{\underline{0.28}}$$

X_2 : $X_2(1)$: [2+, 1-]
$X_2(0)$: [3+, 4-]
S : [5+, 5-]

$$H(X_2, 1) = -\left(\frac{2}{3}\right)\log_2\left(\frac{2}{3}\right) - \left(\frac{1}{3}\right)\log_2\left(\frac{1}{3}\right)$$

$$= -(0.66)(-0.599) - (0.33)(-1.599)$$

$$= 0.395 + 0.527$$

$$= 1.05$$

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$$H(X_2, 0) = -\left(\frac{3}{7}\right)\log_2\left(\frac{3}{7}\right) - \left(\frac{4}{7}\right)\log_2\left(\frac{4}{7}\right)$$

$$= -(0.428)(-1.22) - (0.57)(-0.81)$$

$$= 0.522 - 0.4617$$

$$= 0.98$$

$$Ig(X_2) = 1 - \left(\frac{3}{10}\right)(1.05) - \left(\frac{7}{10}\right)(0.98)$$

$$= \underline{\underline{0.037}}$$

X_3 : $X_3(1)$: [0+, 2-]

$X_3(0)$: [5+, 3-]

S : [5+, 5-]

$$H(class / X_3 = 0) = -\left(\frac{5}{8}\right)\log_2\left(\frac{5}{8}\right) - \left(\frac{3}{8}\right)\log_2\left(\frac{3}{8}\right)$$

$$= -(0.625)(-0.678) - (0.375)(-1.415)$$

$$= 0.423 + 0.530$$

$$= 0.953$$

$$H(class / X_3 = 1) = 0$$

$$Ig(X_3) = 1 - 0 - 0.762$$

$$= \underline{\underline{0.238}}$$

↳ X_1 has the highest gain among other attributes. So, we take X_1 as a root node.

$$H(class) \underset{X_1 = 1}{=} -\left(\frac{4}{5}\right)\log_2\left(\frac{4}{5}\right) - \left(\frac{1}{5}\right)\log_2\left(\frac{1}{5}\right)$$

$$= 0.72$$

where X_1 is 1 : [4+, 1-]

$X_2(1)$: [1+, 0-]

$X_2(0)$: [3+, 1-]

$H(\text{class}\ /\ X_2=0) = -\left(\frac{3}{4}\right)\log_2\left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right)$
$X_1 = I$

$= -(0.75)(-0.415) - (0.25)(-2)$
$= 0.311 + 0.5$
$= 0.811$

$H(X_1=1\ /\ X_2=I) = 0$

$Ig(X_2) = 0.72 - \left(\frac{1}{5}\right)(0) - \left(\frac{4}{5}\right)(0.811)$

$= 0.72 - 0.6488$
$= \underline{\underline{0.0712}}$

$X_1 = I$, $\left.\begin{array}{l} X_3(1) : [1-, 0+] \\ X_3(0) : [0-, 4+] \end{array}\right\}$ pure class

entropy $H(X_1=1, X_3(1)) = H(X_1=1, X_3(0)) = 0$

$Ig(X_3) = \underline{\underline{0.72}}$

↳ X_3 has higher ~~entropy~~ gain than X_2, we choose X_3.

⟹ Other branch, when $X_1 = 0$,

$X_1 = 0 : [1+, 4-]$

$H(X_1=0) = -\left(\frac{1}{5}\right)\log_2\left(\frac{1}{5}\right) - \left(\frac{4}{5}\right)\log_2\left(\frac{4}{5}\right)$

$= 0.72$

$X_1 = 0$, $X_2(1) : [1+, 1-]$
$X_2(0) : [0+, 3-]$

$H(X_1=0, X_2=1) = -\left(\frac{1}{2}\right)\log_2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\log_2\left(\frac{1}{2}\right)$
$= I$ {equal +ves & -ves}

$H(X_1=0 \mid X_2=0) = -\left(\frac{3}{3}\right)\log_2\left(\frac{3}{3}\right) = 0$

$Ig(X_2) = 0.72 - 0.4 = \underline{\underline{0.3219}}$

$X_1 = 0$, $X_3(1)$: $[0+, 1-]$

$X_3(0) = [1+, 3-]$

$H(X_1=0 / X_3=0) = -\left(\frac{3}{4}\right)\log_2\left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right)$

$= 0.3112 + 0.5$

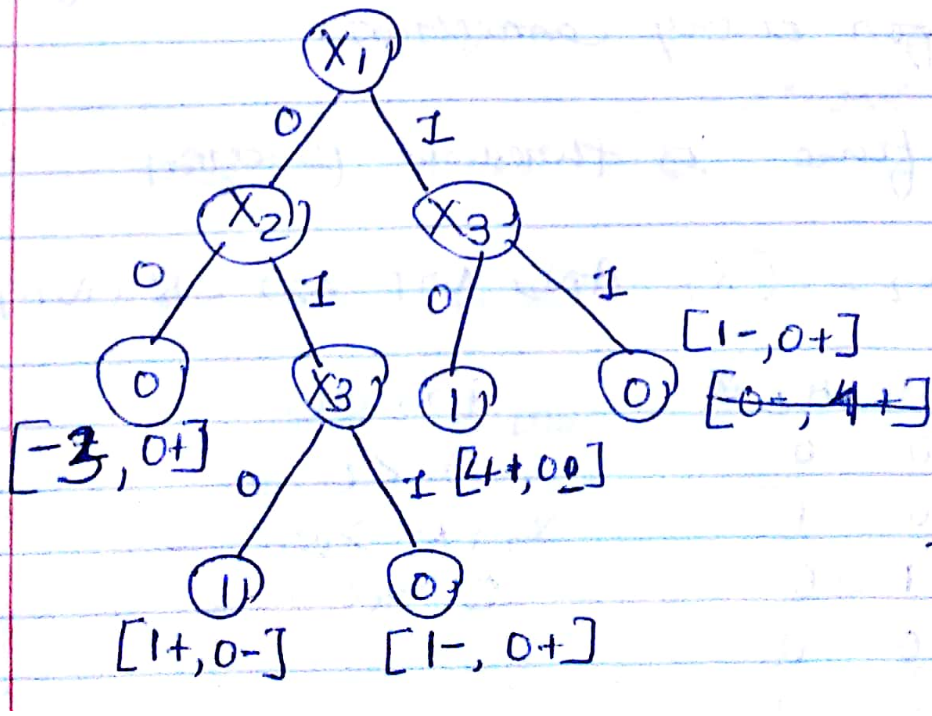
$= 0.8112$

$H(X_1=0 / X_3=1) = 0$ {pure class}

$Ig(X_3) = 0.72 - \left(\frac{4}{5}\right)(0.8112) - 0$

$= \underline{0.07104}$

↳ X_2 has higher gain than X_3, we choose X_2.



X_1	X_2	Class
0	0	0

X_1	X_3	class
1	0	1
1	1	0

X_1	X_2	X_3	Class
0	1	1	0
0	1	0	1

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