Modeling and Simulation, CS302

Lab-4

1. Fig. 1 shows the diffusion of innovations. Initially, the market share of a new product is zero. However, due to certain factors there are a group of people called early innovators who initially adopt the product (technology). As time progresses the product increases its market share due to many factors such as advertising, distribution of prices and contact between users and finally saturates to a maximum value. Behavior observed in Fig. 1 or some variant of it is commonly observed in a multitude of problem (e.g number of twitter users, people using smart phones, market share of apple phones etc.). In this problem we take a look at some of the models specifically for such problems.

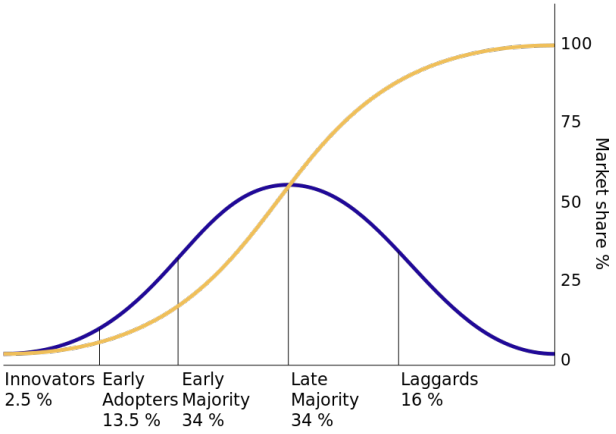

FIG. 1: The diffusion of innovations according to Rogers. With successive groups of consumers adopting the new technology (shown in blue), its market share (yellow) will eventually reach the saturation level. The yellow curve is known as the logistic function. (figure and caption taken from wikipedia)

Typically the mathematical model for such problems have the form:

$$\dot{N} = \alpha(t)\,(N_A - N(t))$$

where, $\alpha(t)$ is the coefficient of diffusion, N_A is the maximum number of potential users of the product and $N(t)$ is the total number who have adopted the product till time t. The term $\alpha(t)$ can be understood in the following way:

- external influence model in which $\alpha(t) = p$, where p is a constant and captures the

innovators or people who adopt the product on their own without being influenced by others.

- internal influence model $\alpha(t) = qN(t)/N_A$, in which the rate $\alpha(t)$ now captures the adoption due to the effect of the other users.
- mixed influence model (Bass): $\alpha(t) = p + qN(t)/N_A$, which captures both the effects.

Let us assume that the market share can be approximated by the percentage (percentage) of users. The full (mixed influence) Bass Model has two parameters p and q. The speed at which the product is being adopted or its longevity would therefore depend on the interplay between these parameters. We attempt to develop a qualitative understanding of these through numerical and quantitative analysis. Provide a systematic analysis highlighting the significance and role of the parameters. For reasonable values of the parameters you may refer to the original paper by Bass.

2. The diffusion model by Bass can be argued to have the following limitations:

   (a) Word of mouth or the adoption due to interaction remains constant over the entire time frame. This may not be true necessarily. In some products the adoption might go down in time since people lose interest over time (think of new cars for instance). In other cases it may increase over time (COVID vaccine adoption).

   (b) It can be shown that for a realistic range of the parameter values the maximum of the innovation diffusion occurs when around 50% of the potential adopters have already adopted the product.

   As a way around it there are models that allow for q to be a function of time. Specifically, $q(t) = q\left(\frac{N}{N_A}\right)^{\beta}$ is considered. Note that $\beta = 1$ gives the original Bass model. This model however, does not permit any analytical solution and hence can only be studied numerically. By obtaining the relevant equations analyze this problem. Provide a systematic analysis of this problem, highlighting what different values of β would capture in the innovation diffusion model and how this model behavior differs from the original Bass model.

Important Points

- While you analyze the two models separately your report should treat them as a single unit.
- You should look at the parameter values in the original paper by Bass.