Lab -5

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CS-302, Modeling and Simulation

In this lab we numerically and analytically analyze two most prevailing and studied epidemic models of disease spreads namely SIR model and SARS model. The primary focus is to study and analyze the effect of change in various parameters involved in models. We have slightly modified the existing models to observe the effect of various events such as vaccination and lockdown.

I. INTRODUCTION

In order to understand the SARS model, we have to study SIR model first. SIR is very basic model which deals with the disease that is spread in closed environment i.e. no births, deaths, immigration and migration is possible in such environments. Initially, everybody is assumed to be prone to disease. In addition to that, we further assume that a recovered patient has immunity towards the disease and recovered patient can not be infected again in future. In such situations we can classify the population into three categories.

- **Susceptible (S):** Population that has no immunity
- **Infected (I):** Population that has disease and can spread it to others.
- **Recovered (R):** Population that has recovered and now immune towards disease.

This assumptions will helps us in developing complete SIR model in section II. Many models of the spread of disease including the SARS model are extensions of SIR in some ways. At last, we provide some theory backed suggestions to policy makers that may help to reduce infection.

SARS is extension of the SEIR model. The SEIR model is derivative of the SIR model which additionally consider a category called **Exposed(E)** that represents the population that has disease but they are not yet infectious. The SARS model extends the SEIR model by taking quarantine, isolation, and death into account. The SARS model has the following assumptions.

1. *There are no births.*
2. *The only deaths are because of SARS.*
3. T*he number of contacts of an infected individual with a susceptible person is constant and does not depend on the population density.*
4. *For susceptible individuals with exposure to the disease, the quarantine proportion (q) is the same for non-infected as for infected people.*
5. *Quarantine and isolation are completely effective. Someone who has the disease and is in quarantine or isolation cannot spread the disease.*

The classification of population here is also bit different as compared to the SIR model.

- **Susceptible (S):** uninfected but prone to SARS.
- **Susceptible_quarantined (S_Q):** Susceptible but quarantined due to exposure.
- **Exposed (E):** have SARS but no symptoms, not yet infectious.
- **Exposed_quarantined (E_Q):** Expose but quarantined due to exposure.
- **Infectious_undetected (I_U):** Undetected SARS and infectious.
- **Infectious_quarantined (I_Q):** have SARS, infectious but quarantined.
- **Infectious_isolated (I_D):** have SARS, infectious, isolated.
- **SARS_death (D):** dead due to SARS.
- **Recovered_immune:** recovered from SARS, immune to SARS.

The SARS model is discussed in detail in section II.

II. MODEL

A. SIR model

SIR is three compartment model as shown in figure 1.We assume that rate of change in susceptible population is proportional to product of current susceptible and infectious population. i.e.

$$\frac{dS}{dt} = -\beta S \cdot I \tag{1}$$

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here β is proportionality constant which is also known as *transmission constant*. It is measure of infectiousness of disease. The negative sign is due to the fact that population of susceptible will decrease with the time.

The rate of change of recovered population is proportional to current infected population. Proportionality constant for this equation is known as recovery rate. So, we can write

$$\frac{dR}{dt} = \alpha I \tag{2}$$

Recall the assumption of closed environment. It means the total population must remain constant. This implies that the rate of change of infected population must be difference of the rate of change of susceptible population and the rate of change of recovered population. In other words, I gains what S loses and I loses what R gains.

$$\frac{dI}{dt} = \beta S \cdot I - \alpha I \tag{3}$$

Let us define an important value to measure growth of spread of disease. **Reproductive number(R)** represents expected number of secondary infectious cases resulting from an average infectious case. When $R < 1$, there is no epidemic. When $R > 1$, there is an epidemic. For the SIR model, we use initial reproductive number R_0 which is given by following equation

$$R_0 = \frac{\beta \cdot S_0}{\alpha} \tag{4}$$

Here, S_0 is initial susceptible population.

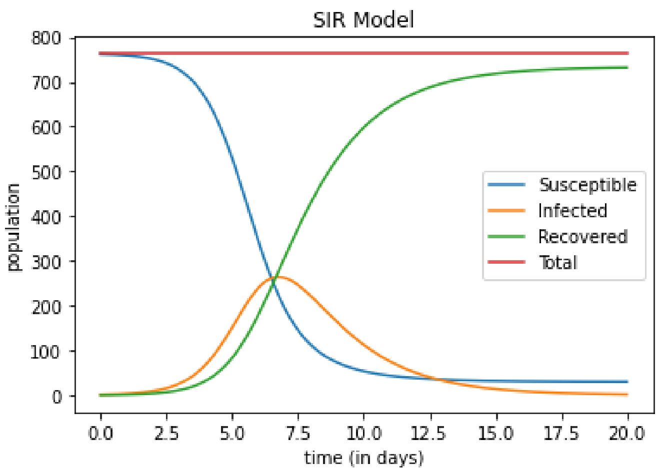

FIG. 1: SIR model: time(days) vs S, I, R population

B. SIR model with Vaccination

We will modify the basic SIR model to analyze role vaccination. We will consider three vaccination process namely vaccination with immediate immunization, vaccination with delayed immunization and partial vaccination.

1. **Vaccination with immediate immunization:** In this case, we assume that immunization starts immediately after vaccination. So, the vaccinated population should be considered equivalent to recovered population. For this case we rename R as **Recovered_vaccinated** population. Suppose, p fraction of susceptible population is being vaccinated everyday. Rewriting equations,

$$\frac{dS}{dt} = -\beta S \cdot I - pS \tag{5}$$

$$\frac{dR}{dt} = pS + \alpha I \tag{6}$$

$$\frac{dI}{dt} = \beta S \cdot I - \alpha I \tag{7}$$

2. **Vaccination with delayed immunization:** In this case, we assume that it takes d days to get immunized after being vaccinated. We can analyze this case in two phase. The first phase last from day 1 to day d. During this period, we can apply the basic SIR model. The second phase starts from $(d+1)^{th}$ day. We can use the SIR model with vaccination here. Of course the values of S, R and I on d^{th} day will be used as initial values while analysing second phase using vaccination model.

3. **Partial vaccination:** In this case, we assume that vaccine is partial effective and vaccinated population becomes susceptible at rate of μ^{-1}. In order to incorporate partial vaccination, we have to add a compartment that represents vaccinated population. Suppose V denotes the total vaccinated population at any point of time. We write the new equations for each compartments as following.

$$\frac{dS}{dt} = -\beta S \cdot I - pS + \mu V \tag{8}$$

$$\frac{dR}{dt} = pS + \alpha I \tag{9}$$

$$\frac{dI}{dt} = \beta S \cdot I - \alpha I \tag{10}$$

$$\frac{dV}{dt} = pS - \mu V \tag{11}$$

C. SIR model with Lockdown

We consider two different scenarios of lockdown.

1. **Ideal lockdown:** Lock down effects the rate at which contacts happen. Reduction in the number of contacts helps in reducing the spread rate. In ideal lockdown, there is sudden decrease in value of transmission rate. This in turn leads to sudden decrease in infection spread. The transmission constant remains constant during lockdown. When lockdown is called off, the transmission constant suddenly increases.

2. **Lockdown with behaviour of people:** Ideal lockdown does not represents real scenarios. Due to behaviour of people, the high value of transmission constant decreases slowly after lockdown is imposed. It remains at lower value for some time. Then it increases gradually when people become relax and they start venturing out.

D. Lipsitch's SARS model

The parameters used in modelling the spread of SARS are S for susceptible, S_q for susceptible quarantined, E for exposed, E_q for exposed quarantined, I_u for infectious undetected, I_q for infectious quarantined, I_d for infectious isolated, D for deaths due to SARS and R for the recovered population. The mathematical model for SARS can be represented as follows,

$$\frac{dS}{dt} = uS_q - \frac{kSI_u}{N}\left(q + 2b(1-q)\right) \tag{12}$$

$$\frac{dS_q}{dt} = -uS_q + \frac{qk(1-b)SI_u}{N} \tag{13}$$

$$\frac{dE}{dt} = -pE + \frac{bk(1-q)SI_u}{N} \tag{14}$$

$$\frac{dE_q}{dt} = -pE_q + \frac{bkqSI_u}{N} \tag{15}$$

$$\frac{dI_u}{dt} = pE - (m+v+w)I_u \tag{16}$$

$$\frac{dI_q}{dt} = pE_q - (m+v+w)I_q \tag{17}$$

$$\frac{dI_d}{dt} = w(I_u + I_q) - (m+v)I_d \tag{18}$$

$$\frac{dR}{dt} = v(I_q + I_d + I_u) \tag{19}$$

$$\frac{dD}{dt} = m(I_q + I_d + I_u) \tag{20}$$

where b is the probability of contact between person in I_u and S, k is the mean number of contacts per day someone from I_u has with someone in S, q is fraction per day of people in S who had exposure to SARS , u is the fraction of S_q who are allowed to leave quarantine and go to S, p is fraction per day of exposed people who become infectious, m is the per capita death rate, v is the per capita recovery rate and w is fraction per day of people who are transferred from I_u to I_d.

III. RESULTS

A. SIR Model

1. **Role of R_0:** When R_0 is less than 1, infected population decreases rapidly. When R_0 is greater than 1, infected population decreases slowly. We get peak only in the case when $R_0 > 1$. Otherwise, infected population decreases. Also, Peak infection is greater when R_0 is greater.

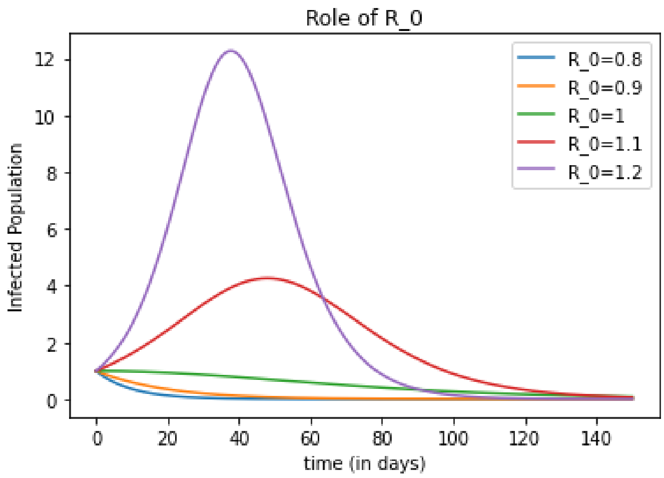

FIG. 2: Infected population for different R_0

2. **Role of S_0:** When S_0 is large, there is slow increase in infected population initially. Therefore, peak is achieved later and epidemic last longer. On other hand, When S_0 is smaller, peak is achieved rapidly, and hence, epidemic last shorter than former case.

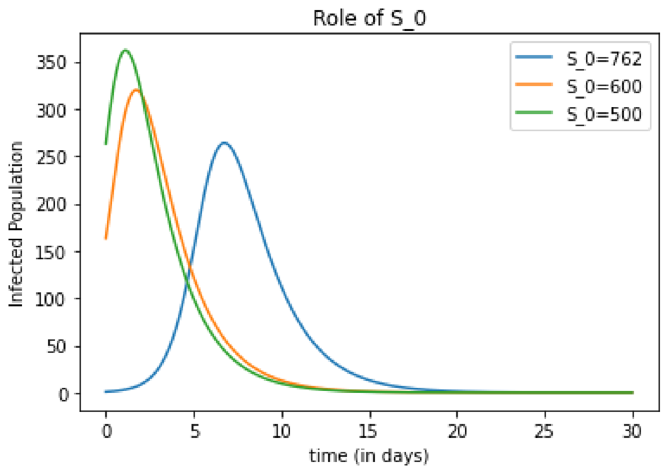

FIG. 3: Infected population for different S_0

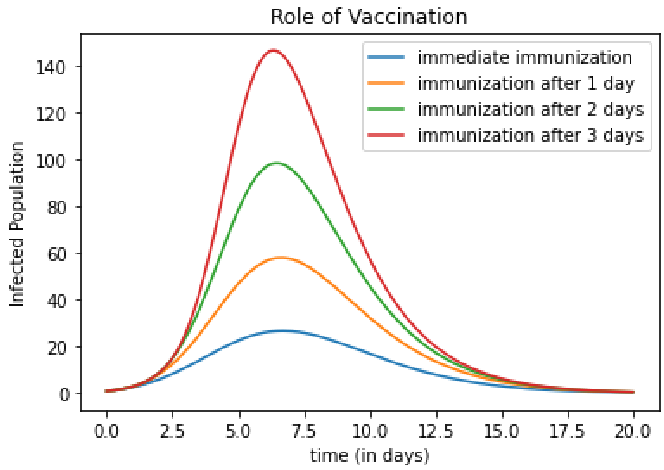

FIG. 5: time vs Infected population. Delayed immunization.

3. **Vaccination with immediate immunization:** In the case of immediate immunization, peak infected population decrease significantly. As we consider vaccinated population as recovered population, we observe increase in recovered population.

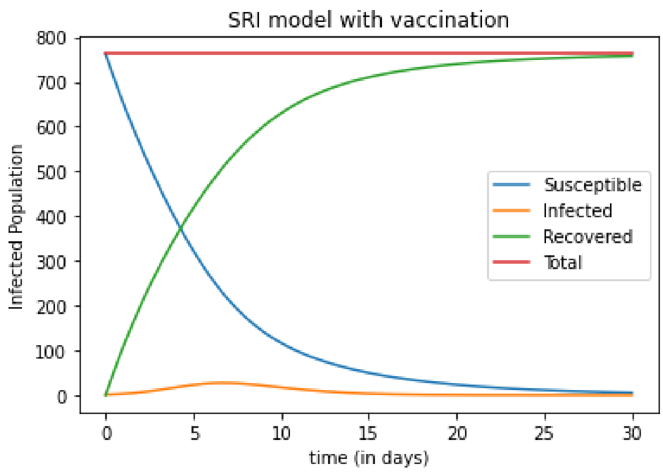

FIG. 4: time vs S, I, R population. Fraction of population getting vaccinated everyday=0.15

4. **Vaccination with delayed immunization:** In the case of delayed immunization, peak infected population increases as delay increases(See fig. 5). This is because susceptible keep getting infected before immunization starts. So, more delay leads to more infections in susceptible.

5. **Partial vaccination:** For partial vaccination model, we have added *Vaccinated* compartment. Fig. (6) depicts the distribution of net population into S, I, R and V. As we increase μ, we observe that peak infected population increases. This because, more population from vaccinated compartment would shift into susceptible compartment (see fig. (7).

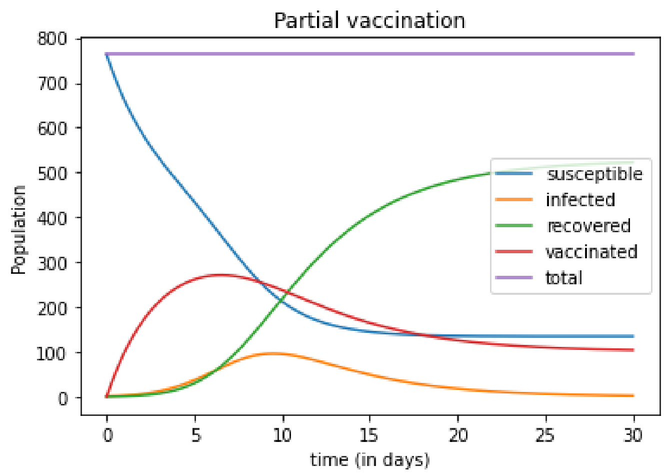

FIG. 6: time vs S, I, R, V. Fraction of population getting vaccinated everyday=0.15

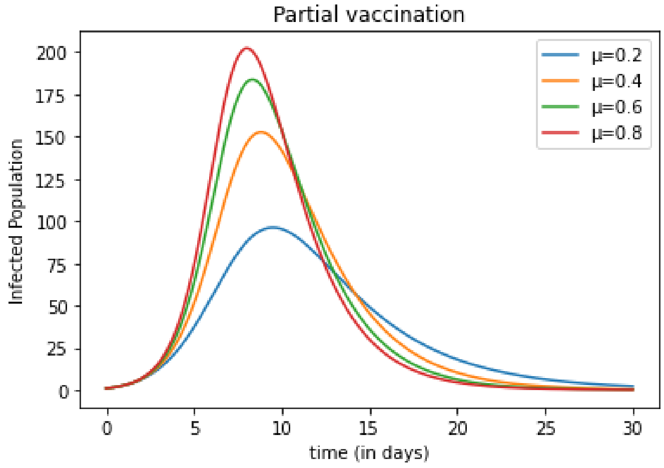

FIG. 7: time vs Infected population. Fraction of population getting vaccinated everyday=0.15

6. **Ideal lockdown:** In ideal lockdown scenario, we observe sudden decrease in infected population when lockdown is imposed. Depending upon when the lockdown is imposed and the duration of the lockdown, we may or may not get peak. See orange colored curve in fig. (9), we have even after imposing lockdown. But for green colored curve, we don't get peak. So, Timing of imposing lockdown plays important role in minimizing peak infected population.

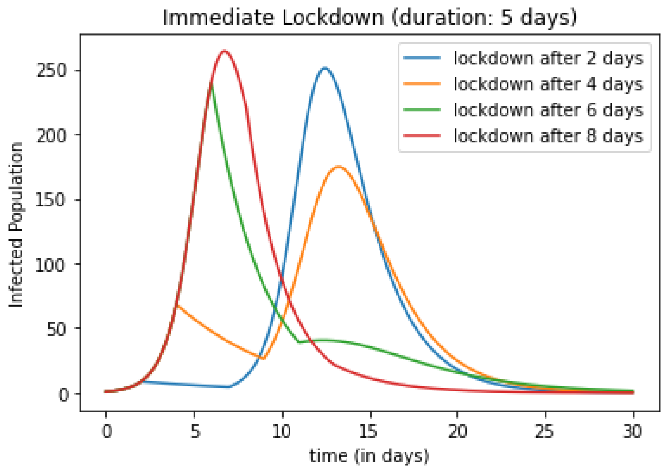

FIG. 8: time vs Infected population, ideal lockdown

7. **Lockdown with people's behaviour:** The ideal lockdown is not quite practical. Instead of sudden decrease in β, we consider that β slowly decrease for some time and then increase same way. Fig. (9) shows such scenario. We don't observe rapid decrease in infected population, but we get smooth graph.In this case, We can get peak after imposing lockdown similar to ideal lockdown.

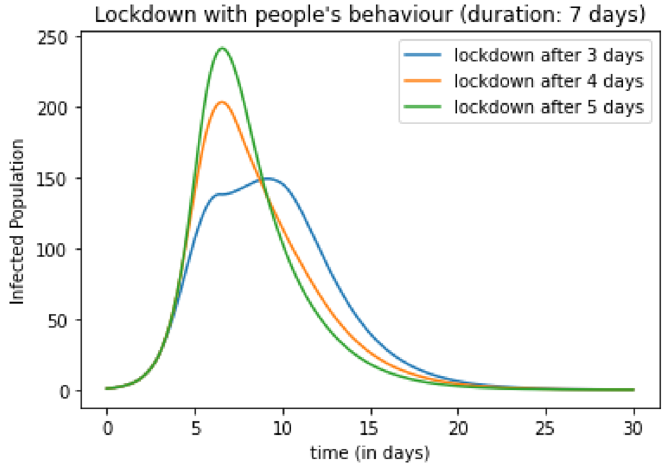

FIG. 9: time vs Infected population. Lockdown with people's behaviour.

B. Lipsitch's SARS model

1. **Varying q**: In this section we consider the effect of changing the value of 'q' which represents the fraction per day of individuals in susceptible (S) who go into quarantine (either susceptible quarantined or exposed quarantined).

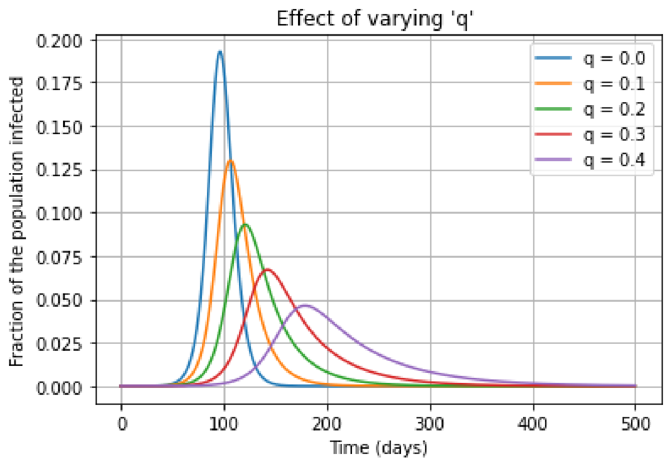

FIG. 10: Fraction of the population infected as the value of q increases from 0.1 to 0.5

From Fig. 10 we can see that as the value of q increases i.e. more number of people go into quarantine, the peak value of the number of infection decreases significantly and the occurrence of peak gets delayed, which should happen practically and hence quarantine is a good measure to reduce the rate of infection.

2. **Varying k**: In this section we consider the effect of changing the value of 'k' which represents the mean number of contacts per day someone from

infectious undetected (I_q) has with someone from susceptible (S).

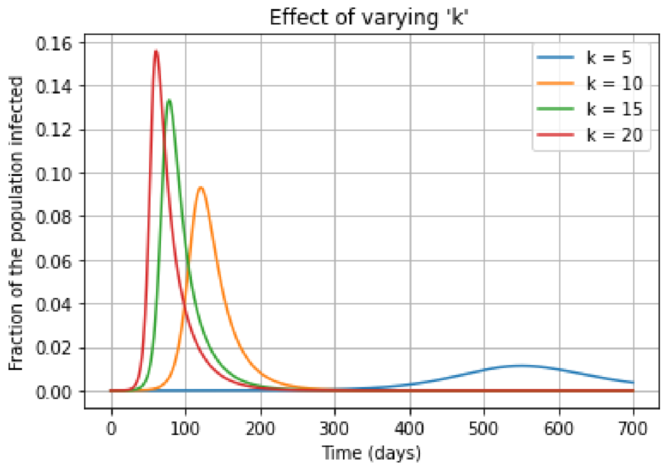

FIG. 11: Fraction of the population infected as the value of k increases from 5 to 20

From Fig. 11 we can see that as the value of k increases from 5 to 20 i.e. more number of susceptibles get into contact with those in infectious undetected, the peak value of the number of infection increases significantly and the peak of the infections occur much earlier. Hence from this we can conclude that interventions and quarantine are very effective since they help in reducing the number of contacts made by susceptibles with the infectious undetected population.

3. **Varying** $\frac{1}{v+m+w}$: In this section we consider the effect of changing the value of $\frac{1}{v+m+w}$ which represents the number of days a person remains infectious i.e. can spread the virus to other people. Here m represents the per capita death rate, v represents the per capita recovery rate and w represents the fraction per day of those in I_q which are transfered to I_d.

   From Fig. 12 we can see that as the value of $\frac{1}{v+m+w}$ increases from 1 to 5 i.e. a person remains infectious for a longer period of time, the peak of the fraction of population infected increases because a single person can infect many more since the duration of the person being infected increases.

4. **Effect of delayed quarantine**: In this section we consider the effect of delaying the quarantine or imposing lockdown.

   From Fig. 13 we can see that as the time taken to quarantine people increases, the peak of the fraction of people infected increases significantly after a delay of about 70 days. Since there is a delay in quarantine the virus spreads much faster leading to a the peak arriving much faster along with many

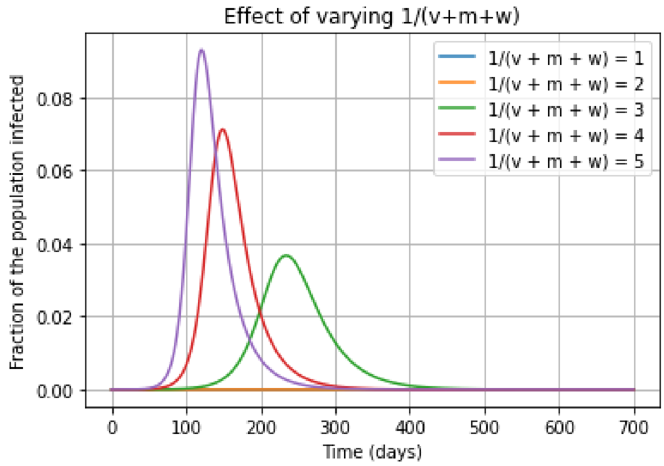

FIG. 12: Fraction of the population infected as the value of $\frac{1}{v+m+w}$ increases from 1 to 5 days

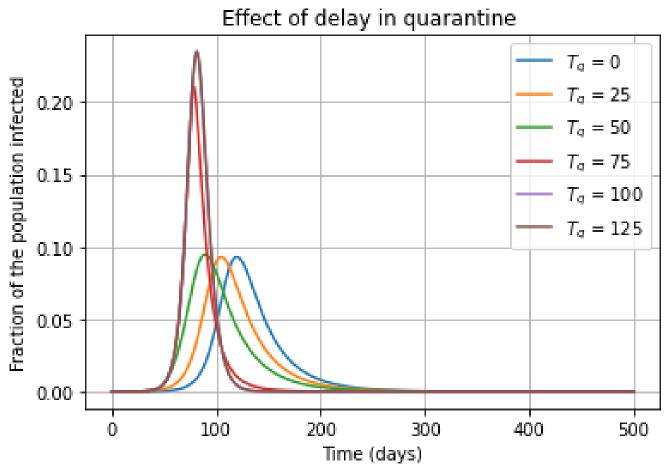

FIG. 13: Fraction of the population infected as the delay in in quarantine increases from 0 to 125 days.

   more deaths occurring than if the quarantine was more effective.

5. **Effect of intervention**: In this section we consider the effect of intervention and how it helps in the reducing the value of the reproduction number, which should be less than 1 for the pandemic to turn to an endemic.

   From Fig. 14 we can see that successful interventions lead to lower value of the reproduction number and as a result the reduction in infection is higher. However, for a particular value of the reproduction number, as the value of q increases the proportional reduction reduces. This is because as the value of q increases, the reproduction number decreases leading to a lesser effect of interventions as compared to that with higher reproduction number.

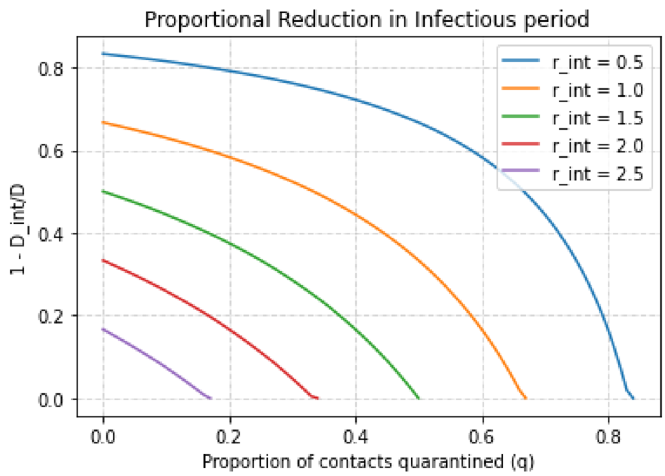

FIG. 14: Contour plot showing the values of reproduction number based on the fraction of the population quarantined.

IV. CONCLUSIONS

In this lab we modelled the SIR model and the SARS model, both computationally and analytically along with graphical visualizations for both of the models for better understanding of the model of epidemic spread. Based on our modelling we observed the following points,

1. $R_0 > 1$ implies epidemic spread, hence all the policies and intervention methods of the government should be focused on keeping the R_0 less than 1.

2. Vaccination is the key to keep the peak infected population low and hence the rate of vaccination should be scaled rapidly.

3. Quarantine of susceptible and infectious people is also a very important measure and hence a delay in quarantine would lead to an increase in the spread of the virus.

4. Lockdown is also a quite effective measure, but the timing and duration of the lockdown is critical. If the lockdown is too short, then we may get the peak after unlock and if lockdown is imposed too late, then there no effect at all.

[1] A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).
[2] Lipsitch et. al (2003). Transmission Dynamics and Control of Severe Acute Respiratory Syndrome. Science (New York, N.Y.). 300. 1966-70. 10.1126/science.1086616.