Lab - 4: Diffusion of Innovations

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In this lab we numerically and analytically analyze the diffusion of innovation models. We starts with external influence model, followed by internal influence model and mixed influence model (full Bass model). Al last, we attempts to develop a model with minor modification in the Bass model to incorporate real-life scenarios.

I. INTRODUCTION

Diffusion of Innovations is a framework that studies the behaviour of new products or new ideas in terms of their spread in social system. The people usually adopt any new idea or product through mutual exchange and communication in social systems. This process of adoption of new ideas and products is knows as diffusion of innovations. The adoption of of innovation occurs in phases (simply because it is a process). Hence, the adopters of innovations can be classified based on the phase they belong. Rogers[1] came up with categories of the adopters based on their timing of adoptions. He proposed five categories as following

1. **Innovators:** *Adopt innovations independent of decisions of other individuals in the social system, the first adopters of innovations*

2. **Early Adopters:** *People with the highest degree of opinion leadership in the social system.*

3. **Early Majority:** *People having above average status in social system, take significantly longer time than innovators and early adaptors to adopt a innovation.*

4. **Late Majority:** *People who are skeptical about innovations, wait to see outcomes of adoption made by early majority.*

5. **Laggards:** *People who are the last one to adopt innovations, probably the hardest social group to influence. Almost no opinion leadership.*

Majority of the population falls in the middle categories of the classifications.This classification helps innovator and producers to identified the potential adopters their ideas and products and make strategies accordingly. Brass merged category (2) and (5) and called them **imitators**. we shall use Bass classifications as we develop our model in next sections.

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II. MODEL

The diffusion of innovations is mathematically represented by differential equation,

$$\frac{dN}{dt} = \alpha(t) \cdot (N_A - N(t)) \tag{1}$$

Here, $N(t)$ represents the maximum number of adopters of innovation at time t. N_A is the total number of potential adopters. And $\alpha(t)$ is coefficient of diffusion.

Normalized form the Eq. (1) can be given by,

$$\frac{dn}{dt} = \alpha(t) \cdot (1 - n(t)) \tag{2}$$

where $n(t) = N(t)/N_A$.

It is quit clear that, the solution of Eq. (2) depends on definition of $\alpha(t)$. Each model discussed here differs only by the definition of $\alpha(t)$. We solve the differential equation to find the fractional adopters of innovations of total population as function of time. We would analyze how the fraction changes as the definition of $\alpha(t)$ changes.

A. External Influence Model

In this model, we define $\alpha(t) = p$, where p is a constant which depicts the probability of initial purchase. The mathematical modelling for the external influence model can done as follows,

$$\frac{dn}{dt} = p \cdot (1 - n(t)) \tag{3}$$

Here diffusion coefficient is constant. It implies that the diffusion is independent of other adopters of the innovations. Hence, it captures the spread in innovators. Fig. (1) justifies our thinking.

B. Internal Influence Model

For the internal influence model the value of $\alpha(t) = q \cdot n(t)$, for which the mathematical modelling can be done as follows,

$$\frac{dn}{dt} = q \cdot n(t) \cdot (1 - n(t)) \tag{4}$$

Diffusion coefficient in this model is function of the current fractional population of adopters. This means that the spread is influenced by the adopters of innovation. See fig. (2).

C. Mixed Influence Model: Bass Model

This model was proposed by Bass. Here, $\alpha(t) = p + q \cdot n(t)$. So, the differential equation is given by,

$$\frac{dn}{dt} = (p + q \cdot n(t))(1 - n(t)) \tag{5}$$

As the name suggests, mixed influence model incorporates both external and internal influence in calculations. This is the usual case observed in the social system. We have just added the coefficient of both models to capture mixed behaviour. Refer to fig. (3).

Further implications of these models along with graphical analysis are detailed in section IV.

D. Modified Bass Model

In this section we would analyze modified version of Bass model where q is also a function of time. Specifically, we define $q(t) = q \cdot (\frac{N(t)}{N_A})^{\beta}$. Hence, differential equation is given by

$$\frac{dn}{dt} = (p + q \cdot n(t)^{\beta+1}) \cdot (1 - n(t)) \tag{6}$$

Here, β is quit important parameter. When $\beta = 0$, we get mixed influence model. When β is positive, the internal influence is dominating the diffusion. So, the diffusion is quite similar to that of internal influence model. On the other hand, if β is negative, external influence dominates the diffusion process. we can expect the diffusion to be similar to the external influence model. See fig. (4).

The detail behaviour of diffusion in terms of diffusion coefficient is discussed in section IV.

III. LIMITATIONS OF BASS MODEL AND ITS DERIVATIVES

- This models fails to capture the situation of loss of interest in idea or product. This is possible in many products and ideas such as car, technology, electronics items etc.

- In real life, the parameter p and q must depends on the geometry. These model does not consider this simple fact.

- External and internal influence are not only two factors that contribute to the diffusion process. There is several other factors that are responsible to diffusion[2].

IV. RESULTS

In this section we visualize the models defined in section II by varying different parameters used in the models.

1. External Influence model

In this case we consider 5 different values of p starting from 0.01 to 0.05 and we found that as the value of p increases the amount of time it takes for the entire population to accept the product or idea decreases i.e. a larger fraction of the population adopts the product faster.

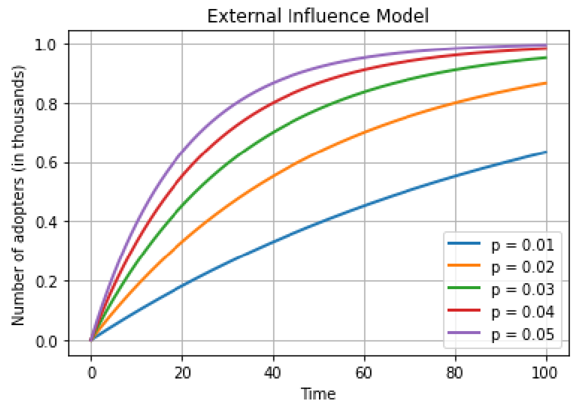

FIG. 1: Number of adopters vs Time for different values of the parameter p.

2. Internal Influence model

In this case we vary the parameter q which is the imitation coefficient and found out that as the value of q increases the amount of time taken for the number of adopters to saturate decreases. Further in this case, if the initial population of adopters is 0, then it would remain 0 for the entire duration. So for more people to accept the product there has to be some initial fraction of population which adopts the product

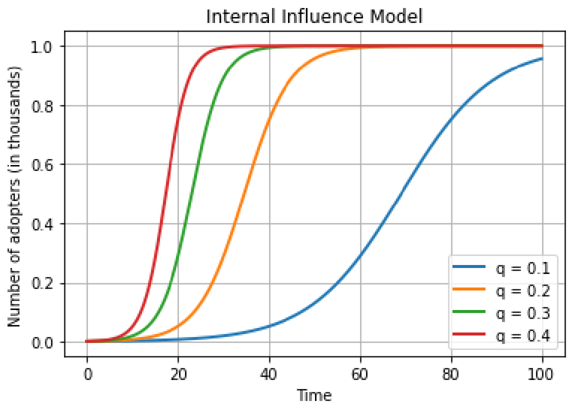

FIG. 2: Number of adopters vs Time for different values of the parameter q.

3. Bass model

In this case we consider different values of p and q, since the Bass model depends on both the parameters, where the former specifies the propensity to consume independent of the number of people currently using the product and latter is proportional to the number of people who have accepted the product at a time t.

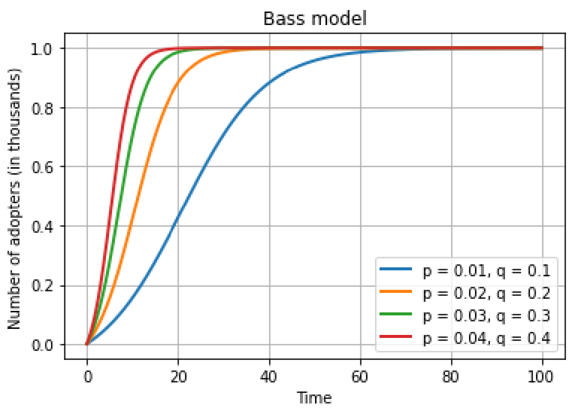

FIG. 3: Number of adopters vs Time for different values of the parameters p and q.

4. Modified Bass model

In this case we modified the value of parameter β and found that as the value β increases the more time it takes for the a significant amount of people to adopt the product i.e. the rate of adoption decreases.

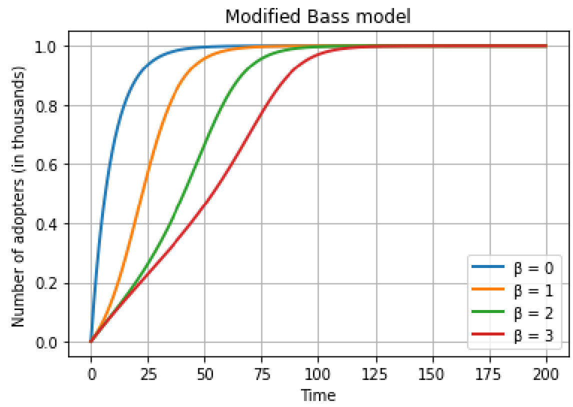

FIG. 4: Number of adopters vs Time for different values of the parameter β

5. Combining all 4 models

In this case we combined all the 4 different models which we considered in section II and found that in the external influence model it takes the longest time for a significant amount of population to adopt the product whereas in the Bass model (with $p = 0.01$ and $q = 0.1$ it takes the shortest time to reach the saturation population.

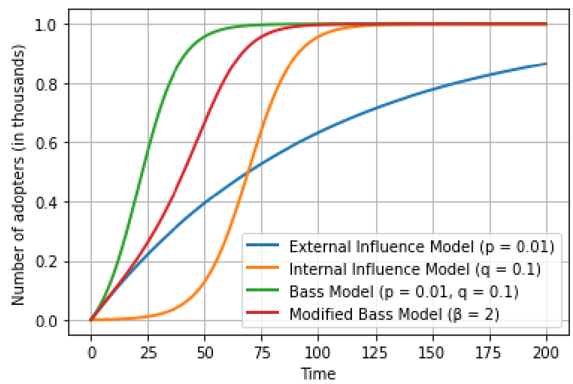

FIG. 5: Number of adopters vs Time for the 4 different models specified above

V. CONCLUSIONS

We started with two basic influence model namely external influence model and internal influence model. These two models capture two different kind of process of diffusion of innovations. External influence model incorporates the diffusion in innovators who make independently. Internal influence model capture diffusion process influenced by adopters of innovations. Then we developed the model that is mix of both these models. The Bass model is more realistic. The model strongly depends on parameters p and q. However, The Bass model fall shorts to explain several diffusion processes. At last, we analyzed the modified Bass model in which we defined q as function of time. The sign of the parameter β dominates the behaviour of diffusion process.

[1] Rogers, Everett M. Diffusion of Innovations. New York: Free Press, 1995

[2] Kumar, Ashish. “The Bass Diffusion Model does not explain diffusion.” (2015).

[3] Bass, Frank M. ”A new product growth for model consumer durables.” Management science 15, no. 5 (1969): 215-227.