Finite state transducer (FST)

LING 570

Fei Xia

Applications of FSTs

- ASR
- Tokenization
- Stemmer
- Text normalization
- Parsing
- …

Outline

- Regular relation
- Finite-state transducer (FST)

Regular relation

Definition of regular relation

- The set of regular relations is defined as follows:
  - For all $(x, y) \in \Sigma_1 \times \Sigma_2$, $\{(x, y)\}$ is a regular relation
  - The empty set is a regular relation
  - If R_1, R_2 are regular relations, so are
    $R_1 \bullet R_2 = \{(x_1\, x_2, y_1\, y_2) \mid (x_1, y_1) \in R_1, (x_2, y_2) \in R_2\}$,
    $R_1 \bigcup R_2$, and R^*.
  - Nothing else is a regular relation.

Defining R*:

- $R^1 = R$
- $R^n = R^{n-1} \bullet R$
- $R^* = \{(\varepsilon, \varepsilon)\} \cup R^1 \cup R^2 \cup \dots$

Examples:

- $R = \{(a, b), (c, d)\}$
- $R^2 = \{(aa, bb), (ac, bd), (ca, db), (cc, dd)\}$
- $R^* = \{(\varepsilon, \varepsilon), (a, b), (c, d), (aa, bb), (ac, bd), (ca, db), (cc, dd), \dots\}$

Closure properties

- Like regular languages, regular relations are closed under
  - union
  - concatenation
  - Kleene closure

- Unlike regular languages, regular relations are <u>NOT</u> closed under
  - Intersection: R1={(a^nb^*, c^n)}, R2={(a^*b^n, c^n)}, the intersection is {(a^nb^n, c^n)} and it is not regular
  - difference:
  - complementation

Closure properties (cont)

- New operations for regular relations:
  - Composition: $\{ (x,z) \mid \exists\, y, (x,y) \in R_1 \text{ and } (y,z) \in R_2 \}$
  - Projection: $\{ x \mid \exists\, y, (x,y) \in R \}$
  - Inversion: $\{ (y,x) \mid (x,y) \in R \}$
  - Take a regular language and create the identity regular relation: $\{ (x,x) \mid x \in L \}$
  - Take two regular languages and create the cross product relation: $\{ (x,y) \mid x \in L_1, y \in L_2 \}$

Finite state transducer

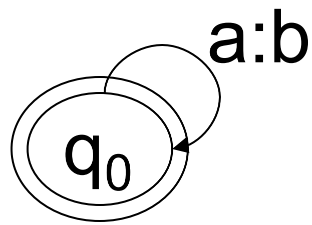
Finite-state transducers

- x:y is a notation for a mapping between two alphabets: $x \in \Sigma_1, y \in \Sigma_2$

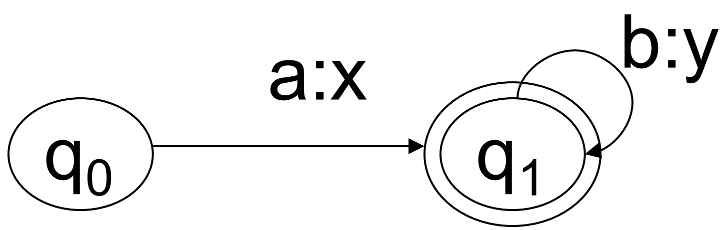
- An FST processes an input string, and outputs another string as the output.

- Finite-state automata equate to regular languages, and FSTs equate to regular relations.
  – Ex: R = { (a^n, b^n) | n >= 0} is a regular relation.
    It maps a string of a's into an equal length string of b's.

FST examples



R(T) = { (ϵ, ϵ), (a, b), (aa, bb), …}



R(T) = { (a, x), (ab, xy), (abb, xyy), …}

Definition of FST

A FST is $(Q, \Sigma, \Gamma, I, F, \delta)$

- Q: a finite set of states
- Σ: a finite set of input symbols
- **Γ: a finite set of output symbols**
- I: the set of initial states
- F: the set of final states
- $\delta \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \times Q$: the transition relation between states.

➔ FSA can be seen as a special case of FST.

Definition of transduction

- The extended transition relation δ^* is the smallest set such that

$$\delta \subseteq \delta^*$$

$$(q,x,y,r) \in \delta^* \wedge (r,a,b,s) \in \delta \Rightarrow (q,xa,yb,s) \in \delta^*$$

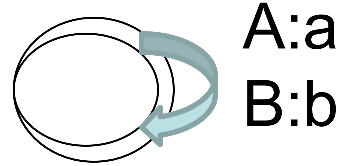
- T transduces a string x into a string y if there exists a path from an initial state to a final state whose input is x and whose output is y:

$$x[T]y \quad (a.k.a. \quad (x,y) \in R(T))$$

$$iff \quad \exists q \in I \, \exists f \in F \; s.t. \, (q,x,y,f) \in \delta^*$$

More FST examples

- Case folding:
  “Go away” ➔ “go away”



- Tokenization:
  he said: ”Go away.” ➔ he said : “ Go away . ”

- Morphological analysis:
  cats ➔ cat s

- POS tagging:
  He called Mary ➔ PN V N

- Map Arabic numbers to words
  123 ➔ one hundred and twenty three

Operations on FSTs

- Union:

$$(x, y) \in R(T_1 \cup T_2) \;\; iff \;\; (x, y) \in R(T_1) \; or \; (x, y) \in R(T_2)$$

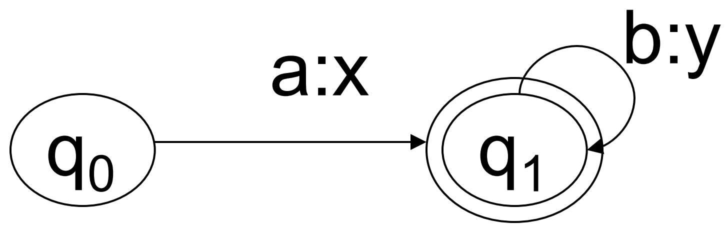
- Concatenation:

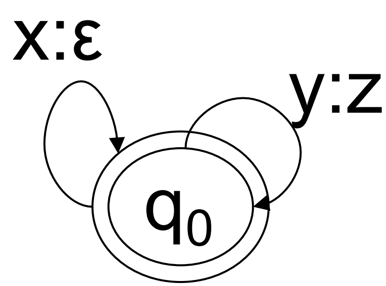
$$(wx, yz) \in R(T_1 \bullet T_2) \; iff \;\; (w, y) \in R(T_1) \; and \; (x, z) \in R(T_2)$$

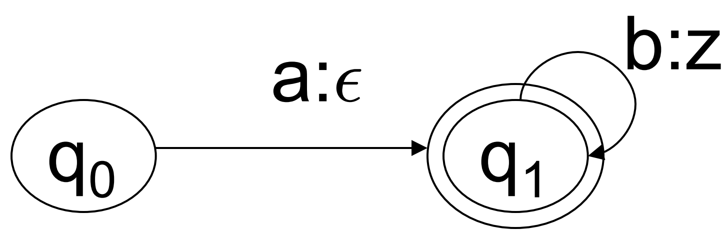
- Composition:

$$(x, z) \in R(T_1 \circ T_2) \quad iff \quad \exists y \; s.t. \, (x, y) \in R(T_1) \; and \; (y, z) \in R(T_2)$$

An example of composition operation

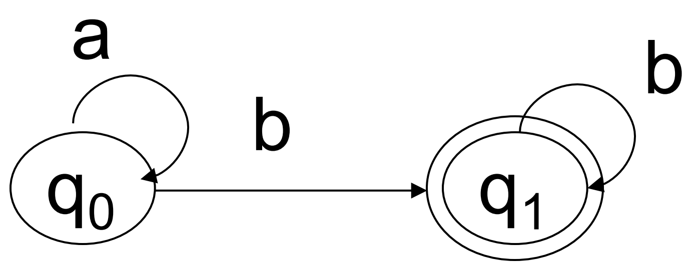




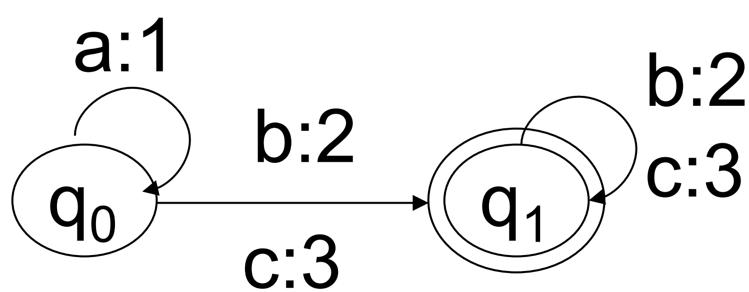


Hw1 Q7

Q7(a):



Q7(b):



FST Algorithms

- **Recognition**: Is a given pair of strings accepted by an FST?
  - (x,y) ➔ yes/no
- **Composition**: Given two FSTs T_1 and T_2 defining regular relations R_1 and R_2, create the FST that computes the composition of R_1 and R_2.
  - R1={(x,y)}, R2={(y,z)} ➔ $\{(x,z) \mid (x,y) \in R_1, (y,z) \in R_2\}$
- **Transduction**: given an input string and an FST, provide the output as defined by the regular relation?
  - x ➔ y

Weighted FSTs

A FST is $(Q, \Sigma, \Gamma, I, F, \delta, P)$

- Q: a finite set of states
- Σ: a finite set of input symbols
- Γ: a finite set of output symbols
- I: Q ➔R+ (initial-state **probabilities**)
- F: Q ➔R+ (final-state **probabilities**)
- $\delta \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \times Q$: the transition relation between states.
- **P:** $\delta \rightarrow R^{+}$ **(transition probabilities)**

Summary

- Finite state transducers specify regular relations

- FST closure properties: union, concatenation, composition

- FST special operations:
  - creating regular relations from regular languages (Id, crossproduct);
  - creating regular languages from regular relations (projection)

- FST algorithms
  - Recognition
  - Transduction
  - Composition
  - …

- Not all FSTs can be determinized.

- Weighted FSTs are used often in NLP.