

PCFGs: Parsing & Evaluation

LING 571 — Deep Processing Techniques for NLP

October 12, 2022

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Announcements

- HW2 due **Friday** at 11:59pm
 - readme.{txt|pdf}
 - Separate upload to Canvas
 - NOT in hw2.tar.gz
 - Run check_hw2.sh before submitting!
- Start symbol: either “%start S” *or* first nonterminal
 - NB: needs to be readable by nltk’s grammar loading methods
- nltk.data.load: “**file**:path/to/grammar.cfg”
- Condor job: runs conversion, but *not* parsing before/after; that can be done separately

Roadmap

- CKY + back-pointers
- PCFGs
- PCFG Parsing (PCKY)
- Inducing a PCFG
- Evaluation
- [Earley parsing]
- HW3 + collaboration

CKY Parsing: Backpointers

Backpointers

- Instead of list of possible nonterminals for that node, each cell should have:
 - Nonterminal for the node
 - Pointer to left and right children cells
 - Either direct pointer to cell, or indices

For example:

```
bp_2 = BackPointer()  
bp_2.l_child = [X2, (1,4)]  
bp_2.r_child = [PP, (4,6)]
```

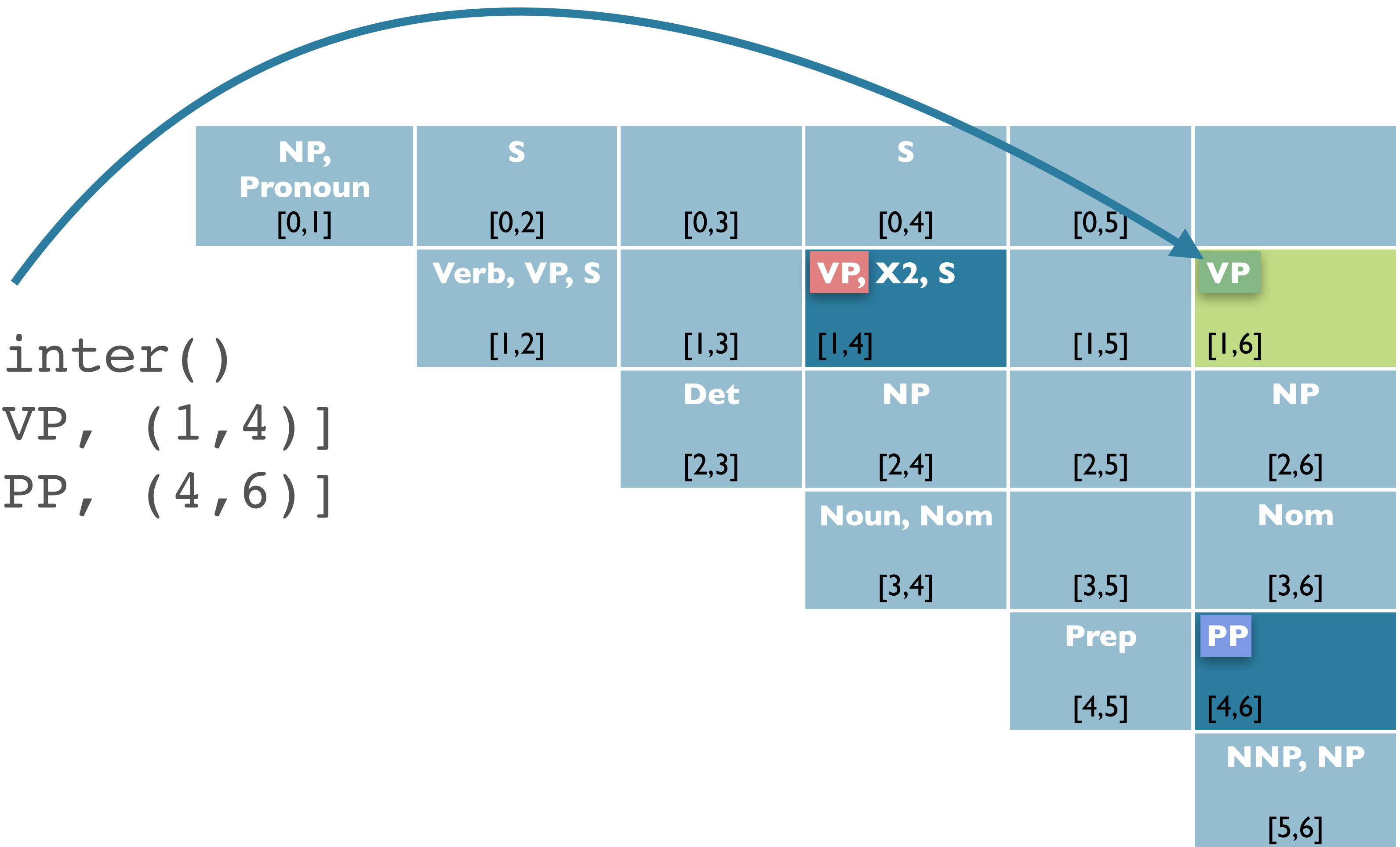
CKY *Parser*

- Pair each nonterminal with back-pointer to cells from which it was derived
- Last step:
 - construct trees from back-pointers in $[0, n]$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

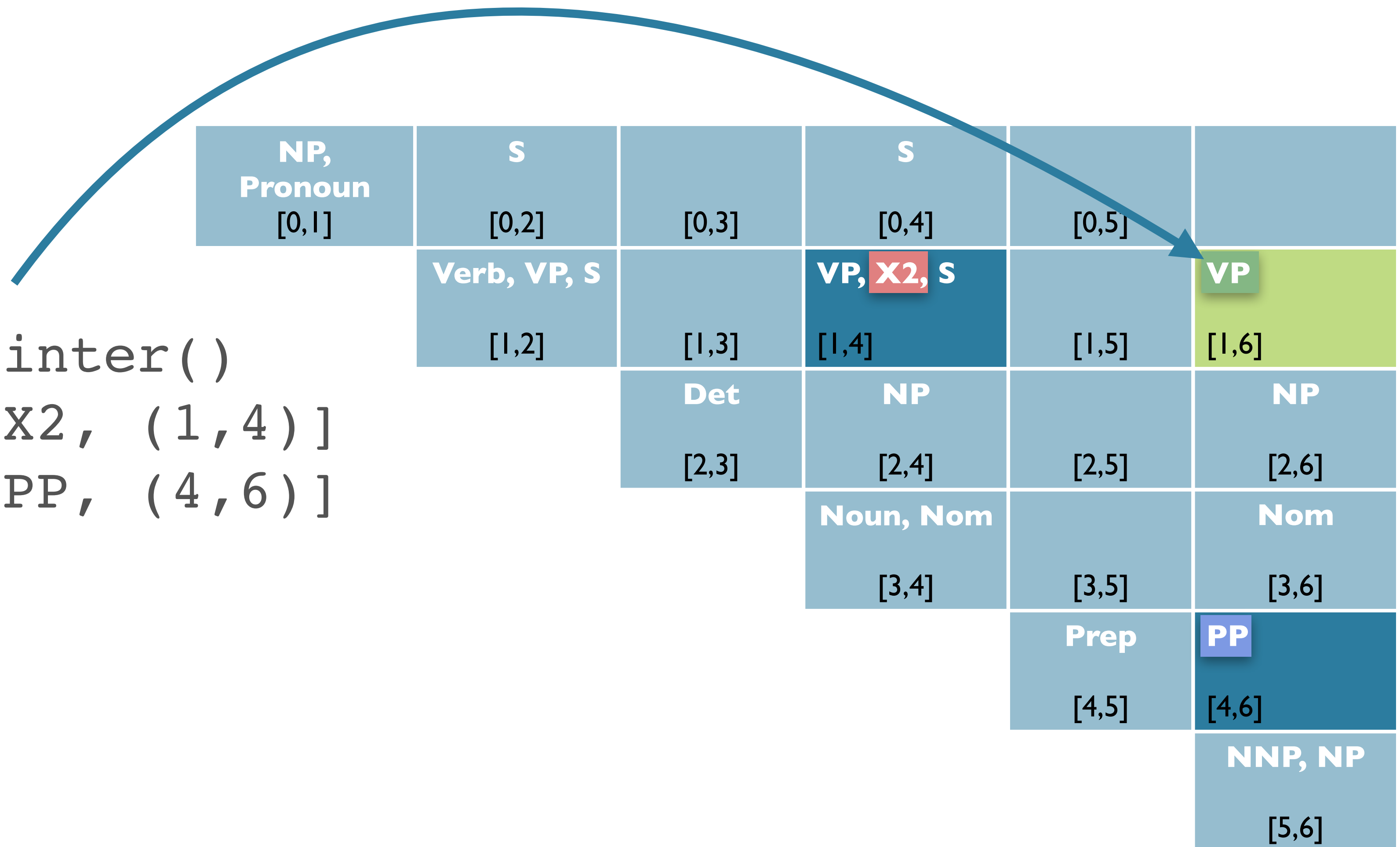
VP → *VP PP*

```
bp_1 = BackPointer()
bp_1.l_child = [VP, (1,4)]
bp_1.r_child = [PP, (4,6)]
```

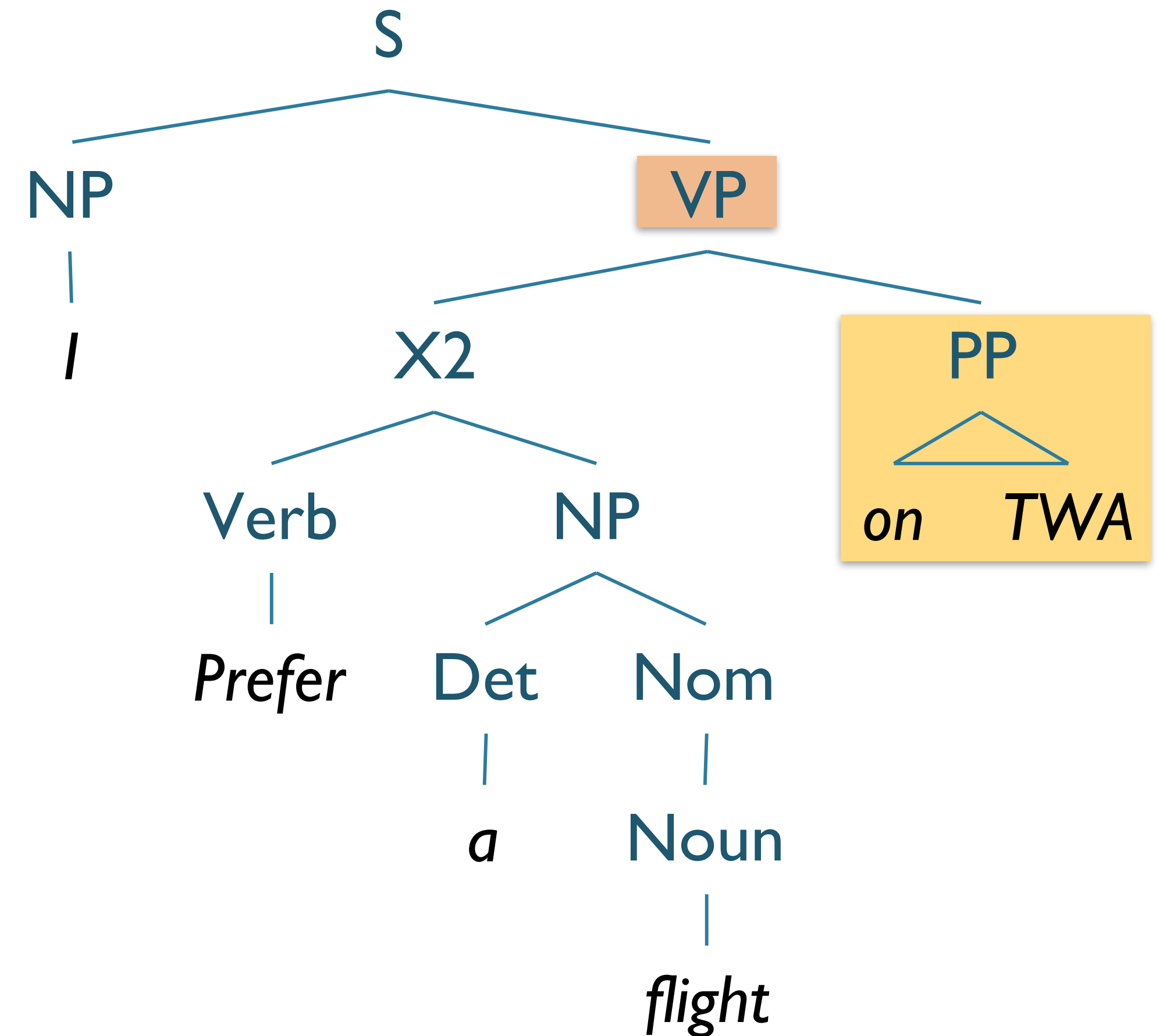
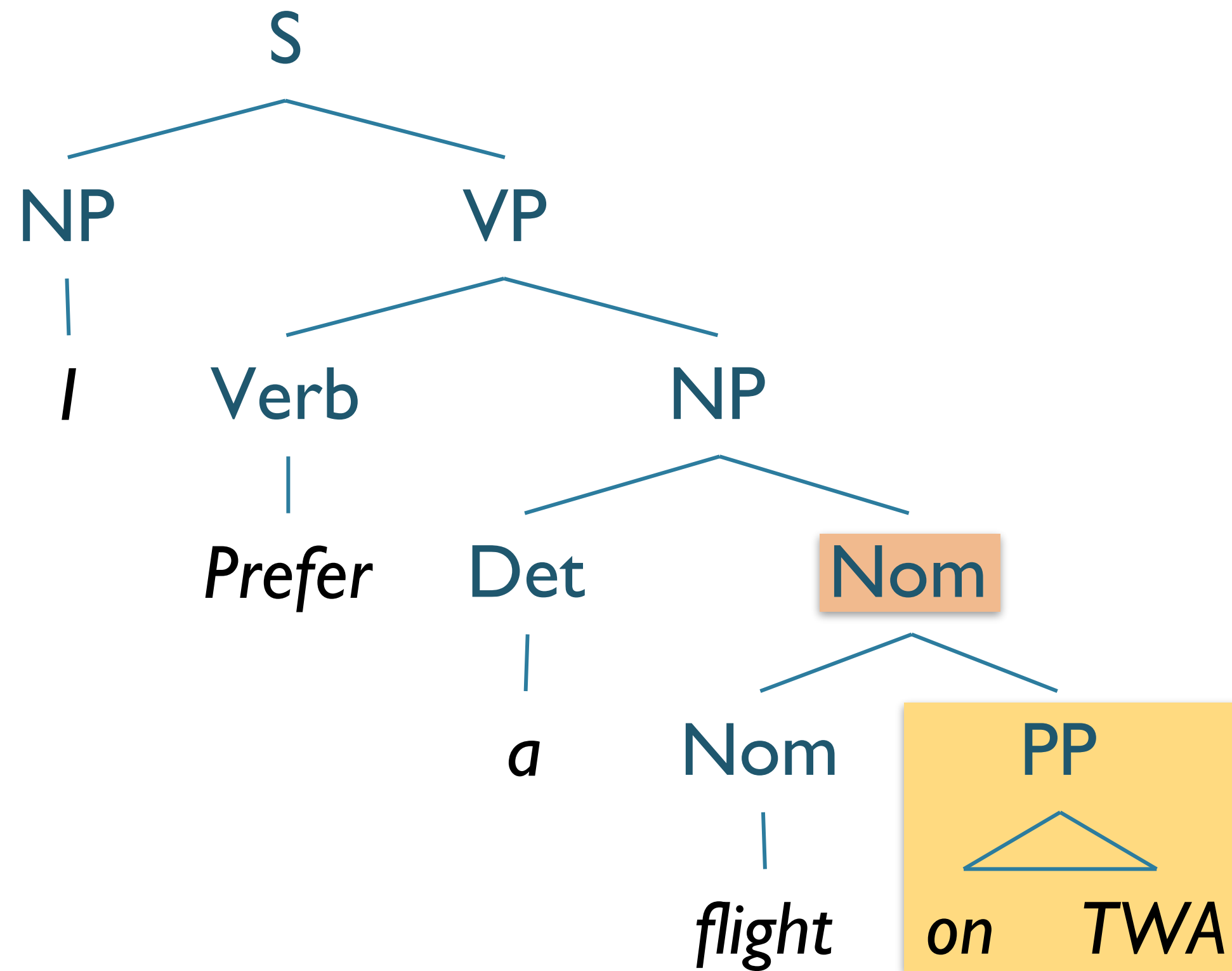


VP → *X2* *PP*

```
bp_2 = BackPointer()
bp_2.l_child = [X2, (1,4)]
bp_2.r_child = [PP, (4,6)]
```



Resulting Parses



CKY Discussion

- Running time:
 - $O(n^3)$ where n is the length of the input string
 - Inner loop grows as square of # of non-terminals
- Expressiveness:
 - As implemented, requires CNF
 - Weak equivalence to original grammar
 - Doesn't capture full original structure
 - Back-conversion?
 - Can do binarization, terminal conversion
 - Unit productions requires change in CKY

CKY + Back-pointers Example

cky_table[0,6][S] = {(NP, (0,1)),
VP, (1,6))}

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

S

NP

VP

I prefer a flight on TWA

$S[S] = \{ (NP, ($
VP, (

S

NP VP

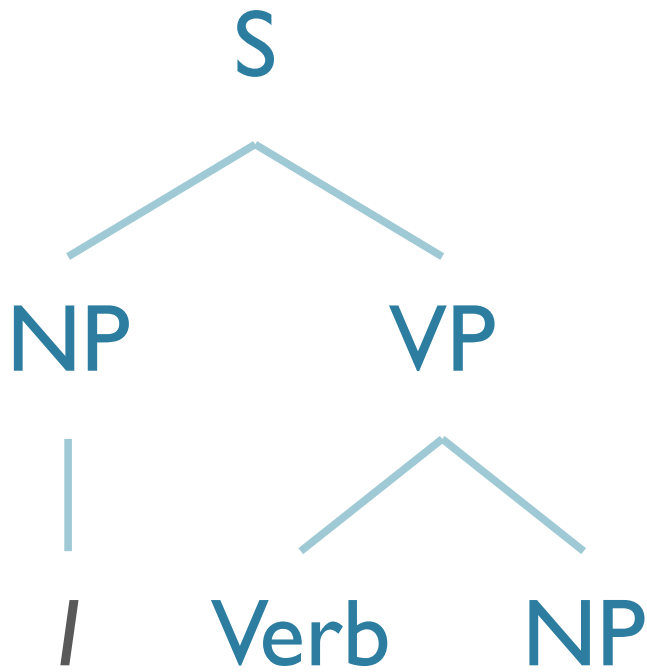
|

—

I prefer a flight on TWA

cky_table[0,6][S] = {(NP, (0,1),
VP, (1,6))}
cky_table[0,1][NP] = {'I'}
cky_table[1,6][VP] = {(Verb, (1,2),
NP, (2,6)),
(X2, (1,4),
PP, (4,6))}

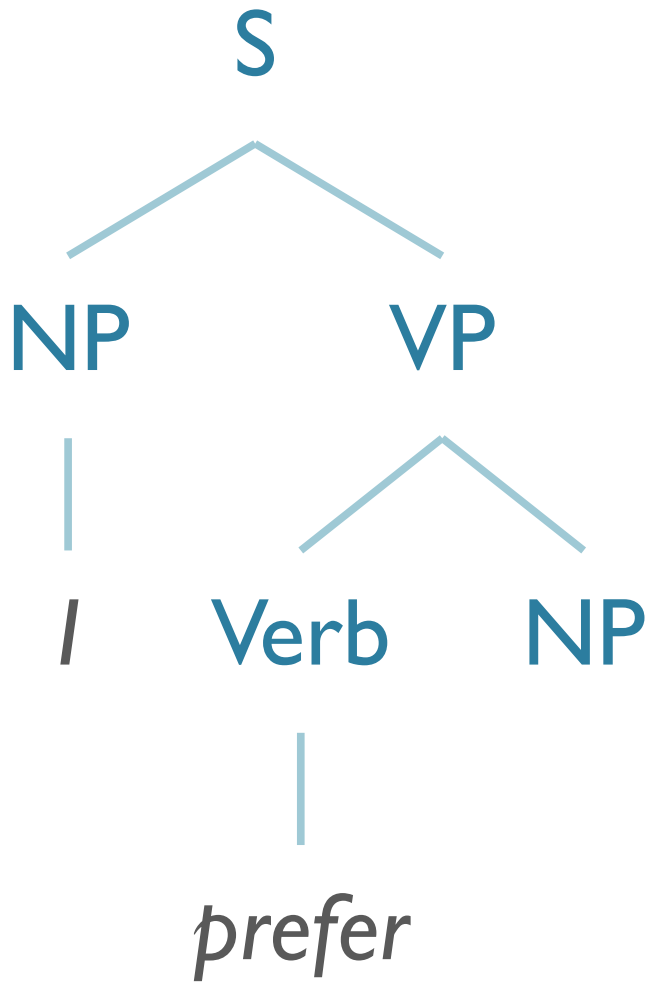
NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
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cky_table[1,6][VP] = {(Verb, (1,2),
NP, (2,6)),
(X2, (1,4),
PP, (4,6))}
cky_table[1,2][Verb] = {'prefer'}

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
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				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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```
cky_table[0,6][S] = {(NP, (0,1),  
                      VP, (1,6))}
```

```
cky_table[0,1][NP] = {('I')}
```

```
cky_table[1,6][VP] = {(Verb, (1,2)),  
                        (NP, (2,6)),  
                        (X2, (1,4)),  
                        (PP, (4,6))}
```

```
cky_table[1,2][Verb] = {('prefer')}
```

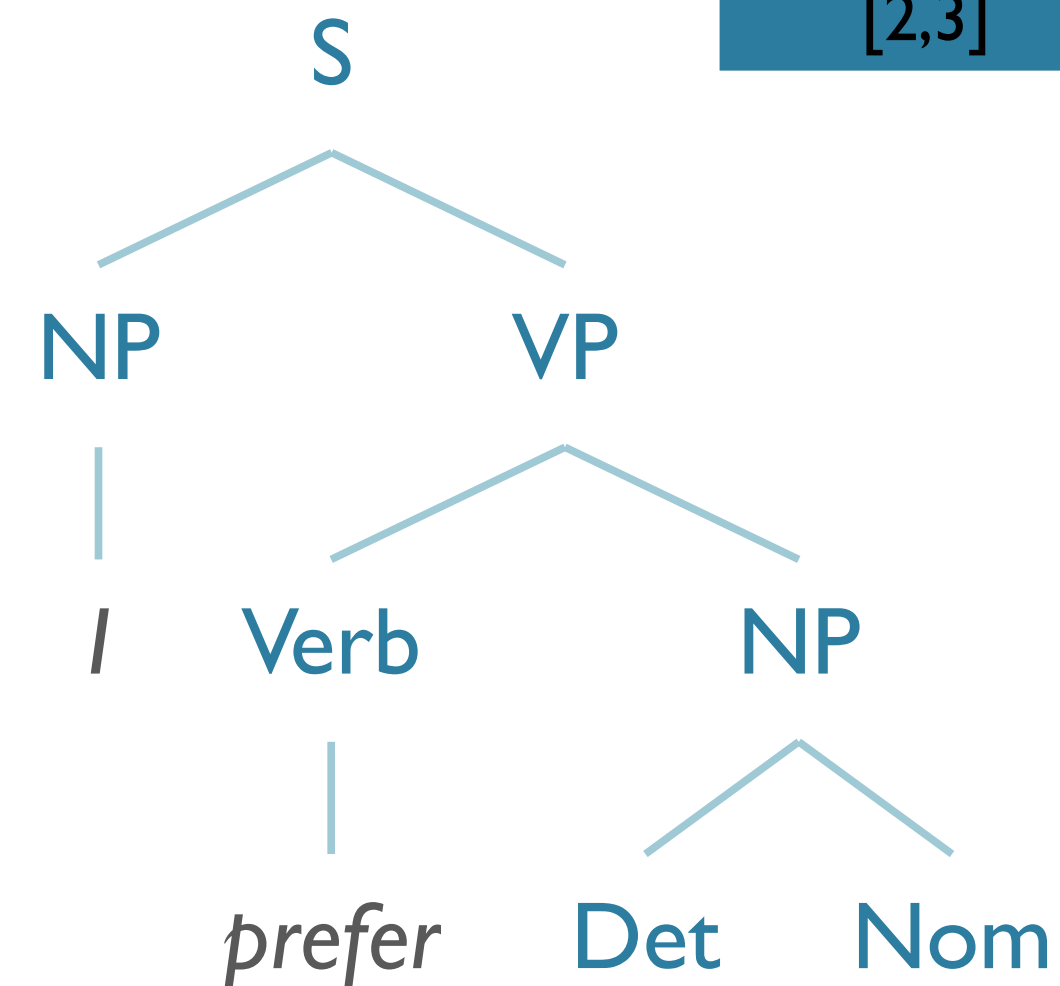
```
cky_table[2,6][NP] = { (Det, (2,3)),  
                        (Nom, (3,6)) }
```

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
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Diagram illustrating the hierarchical structure of a sentence (S) and its components:

```

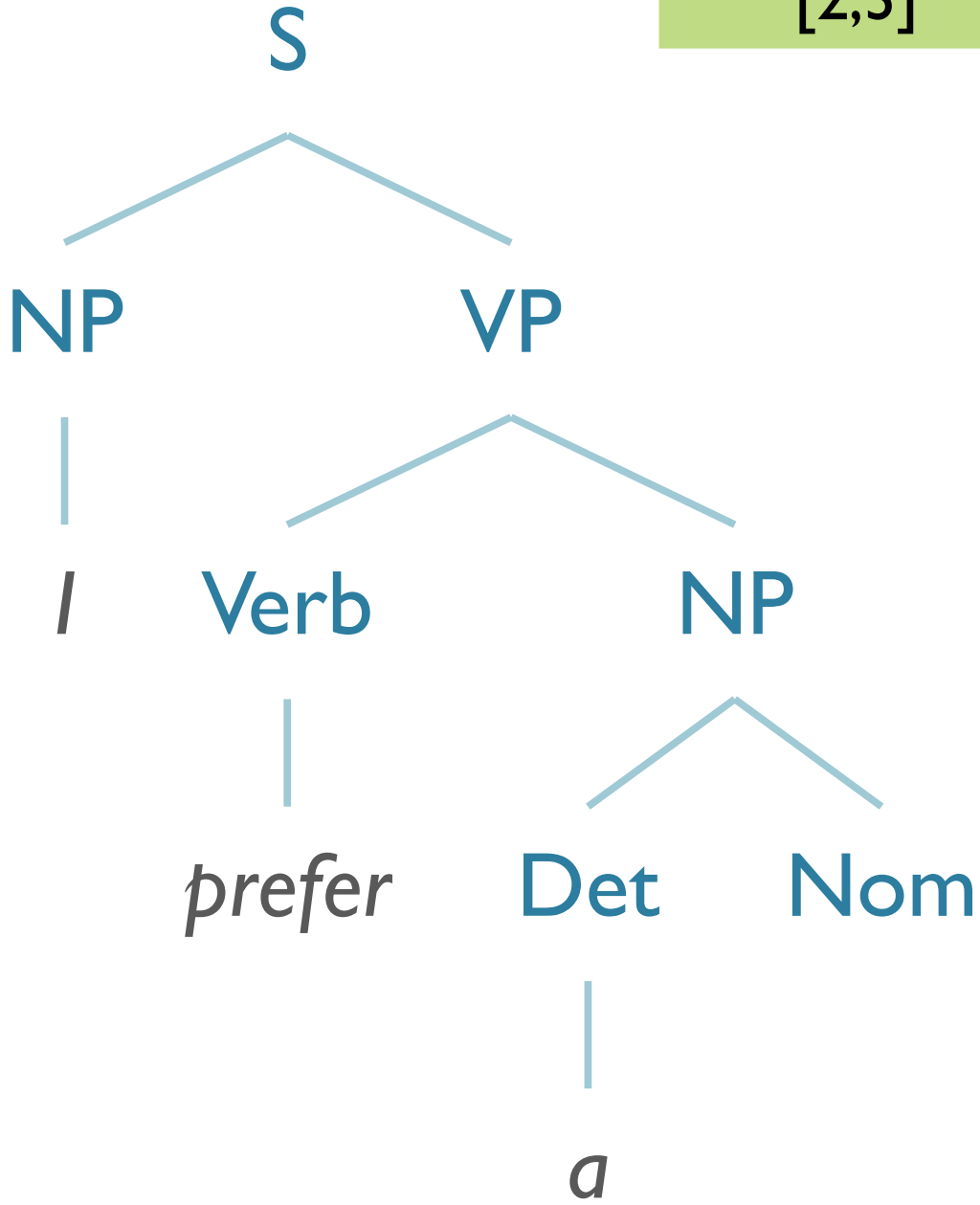
graph TD
    S --> NP1[NP]
    S --> VP1[VP]
    NP1 --> I[/ /]
    VP1 --> Verb[Verb]
    VP1 --> NP2[NP]
    Verb --> V[ ]
    NP2 --> N1[N]
    NP2 --> N2[N]
    
```



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```
cky_table[0,6][S] = {(NP, (0,1)),
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cky_table[1,6][VP] = {(Verb, (1,2)),
                    (NP, (2,6)),
                    (X2, (1,4)),
                    (PP, (4,6))}
cky_table[1,2][Verb] = {'prefer'}
cky_table[2,6][NP] = {(Det, (2,3)),
                    (Nom, (3,6))}
cky_table[2,3][Det] = {'a'}
```

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
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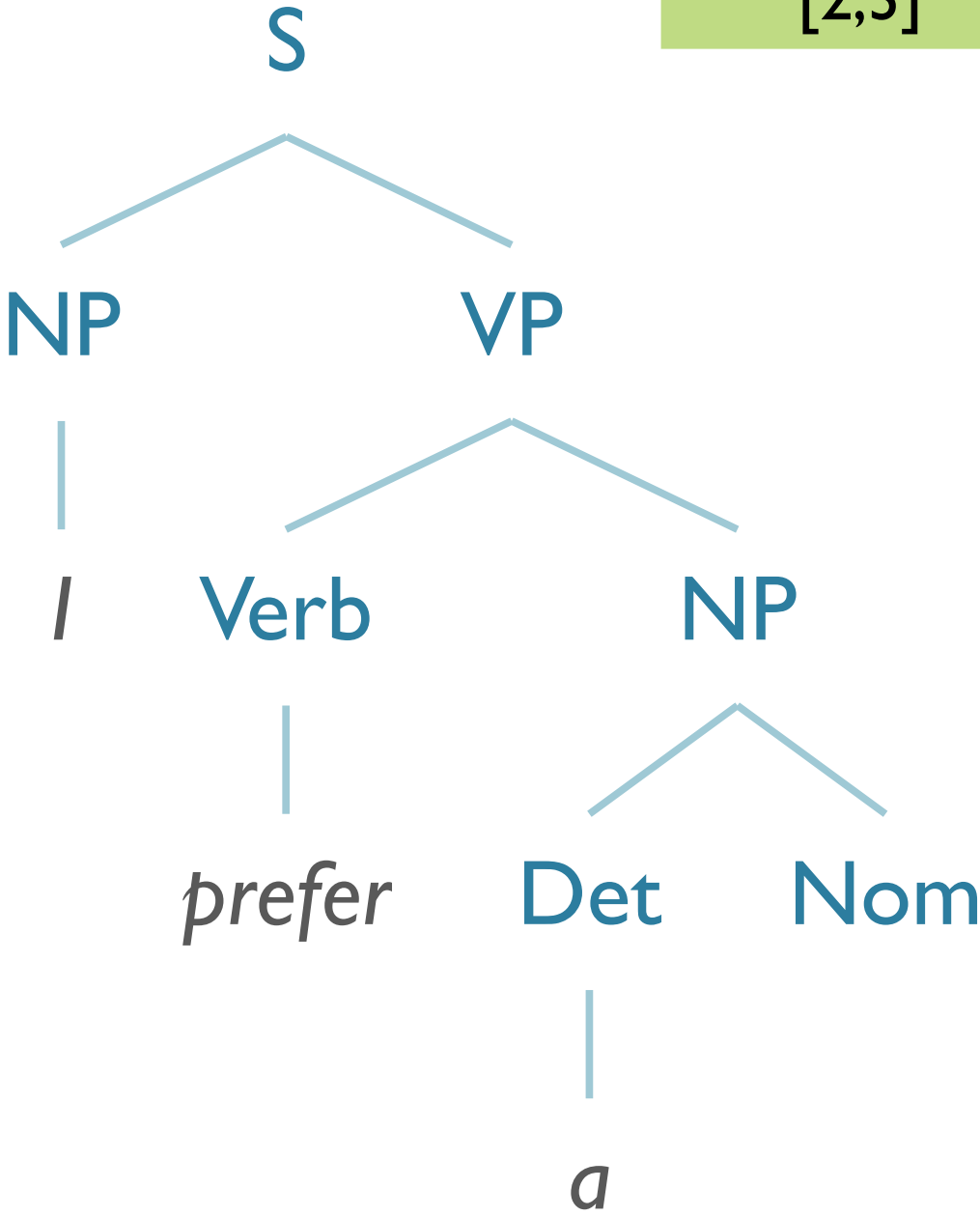
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```

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cky_table[1,2][Verb] = {'prefer'}
cky_table[2,6][NP] = {(Det, (2,3),
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cky_table[2,3][Det] = {'a'}

```

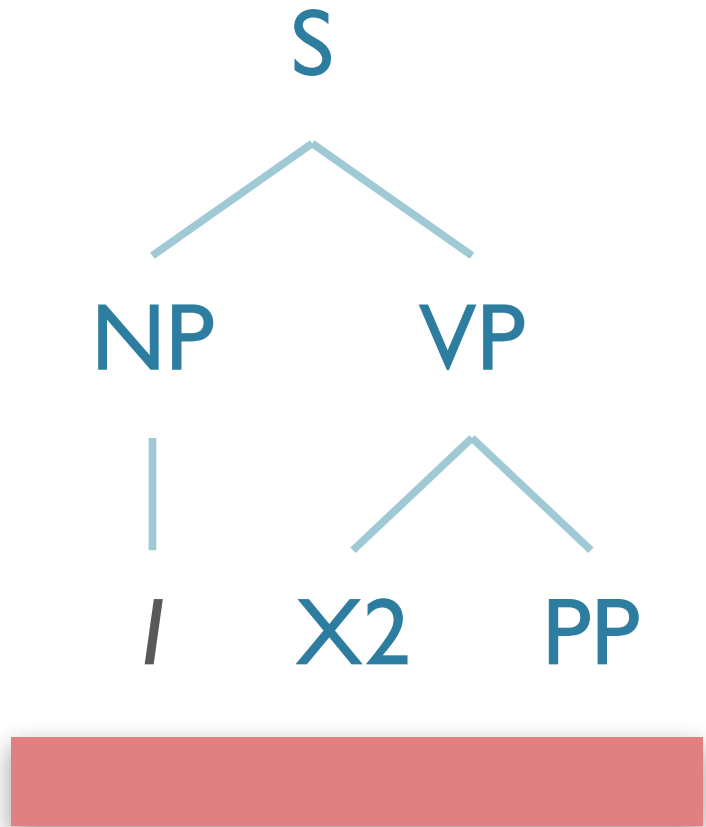
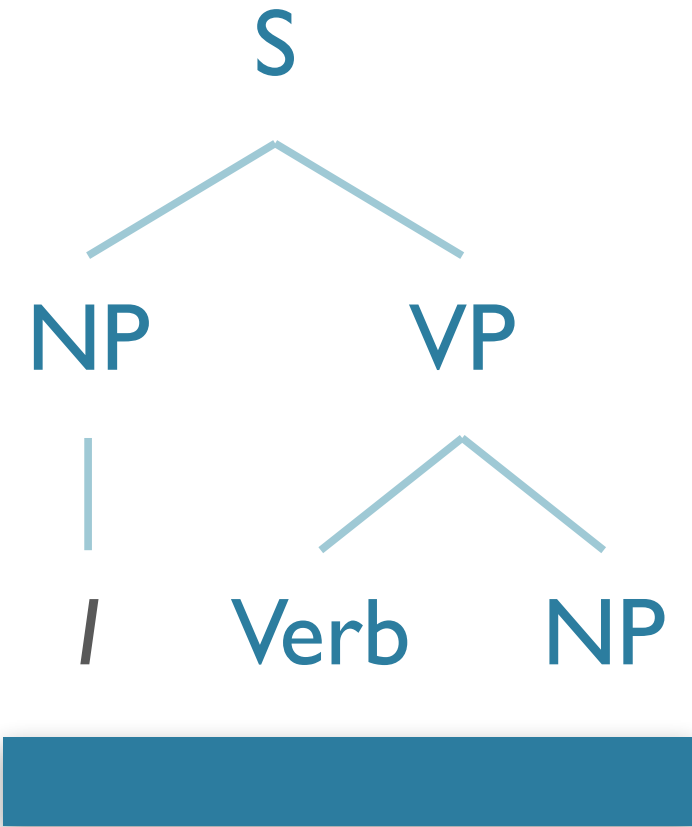
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
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I prefer a flight on TWA

Probabilistic Context-Free Grammars

Probabilistic Context-free Grammars: Roadmap

Motivation: Ambiguity

Approach:

Definition

Disambiguation

Parsing

Evaluation

Enhancements

Motivation

What about ambiguity?

Current algorithm can *represent* it

...can't resolve it.

Probabilistic Parsing

- Provides strategy for solving disambiguation problem
 - Compute the probability of all analyses
 - Select the most probable
- Employed in language modeling for speech recognition
 - N-gram grammars predict words, constrain search
 - Also, constrain generation, translation

PCFGs: Formal Definition

N

a set of **non-terminal symbols** (or **variables**)

PCFGs: Formal Definition

N	a set of non-terminal symbols (or variables)
-----	---

Σ	a set of terminal symbols (disjoint from N)
----------	---

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Σ	a set of terminal symbols (disjoint from N)
R	a set of rules of productions, each of the form $A \rightarrow \beta[p]$, where A is a non-terminal where A is a non-terminal, β is a string of symbols from the infinite set of strings $(\Sigma \cup N)^*$ and p is a number between 0 and 1 expressing $P(\beta A)$

PCFGs: Formal Definition

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Σ	a set of terminal symbols (disjoint from N)
R	a set of rules of productions, each of the form $A \rightarrow \beta[p]$, where A is a non-terminal where A is a non-terminal, β is a string of symbols from the infinite set of strings $(\Sigma \cup N)^*$ and p is a number between 0 and 1 expressing $P(\beta A)$
S	a designated start symbol

PCFGs

- Augment each production with probability that LHS will be expanded as RHS
 - $P(A \rightarrow \beta)$
 - $P(A \rightarrow \beta | A)$
 - $P(\beta | A)$
 - $P(RHS \mid LHS)$
- NB: the first is often used; but the latter are what's really meant.

PCFGs

- Sum over all possible expansions is 1

$$\sum_{\beta} P(A \rightarrow \beta) = \underline{1}$$

- A PCFG is **consistent** if sum of probabilities of all sentences in language is 1
- Recursive rules often yield inconsistent grammars (*Booth & Thompson, 1973*)

Example PCFG: Augmented \mathcal{L}_1

Grammar		Lexicon
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book [.10] \mid flight [.30] \mid meal [.15] \mid money [0.95]$
$S \rightarrow VP$	[.05]	$\mid flights [0.30] \mid dinner [.10]$
$NP \rightarrow Pronoun$	[.35]	$Verb \rightarrow book [.30] \mid include [.30] \mid prefer [.40]$
$NP \rightarrow Proper-Noun$	[.30]	$Pronoun \rightarrow I [.40] \mid she [.05] \mid me [.15] \mid you [.40]$
$NP \rightarrow Det Nominal$	[.20]	$Proper-Noun \rightarrow Houston [.60] \mid NWA [.40]$
$NP \rightarrow Nominal$	[.15]	$Aux \rightarrow does [.60] \mid can [.40]$
$Nominal \rightarrow Noun$	[.75]	$Preposition \rightarrow from [.30] \mid to [.30] \mid on [.20] \mid near [.15]$
$Nominal \rightarrow Nominal Noun$	[.20]	$\mid through [.05]$
$Nominal \rightarrow Nominal PP$	[.05]	
$VP \rightarrow Verb$	[.35]	
$VP \rightarrow Verb NP$	[.20]	
$VP \rightarrow Verb NP PP$	[.10]	
$VP \rightarrow Verb PP$	[.15]	
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$PP \rightarrow Preposition NP$	[1.0]	

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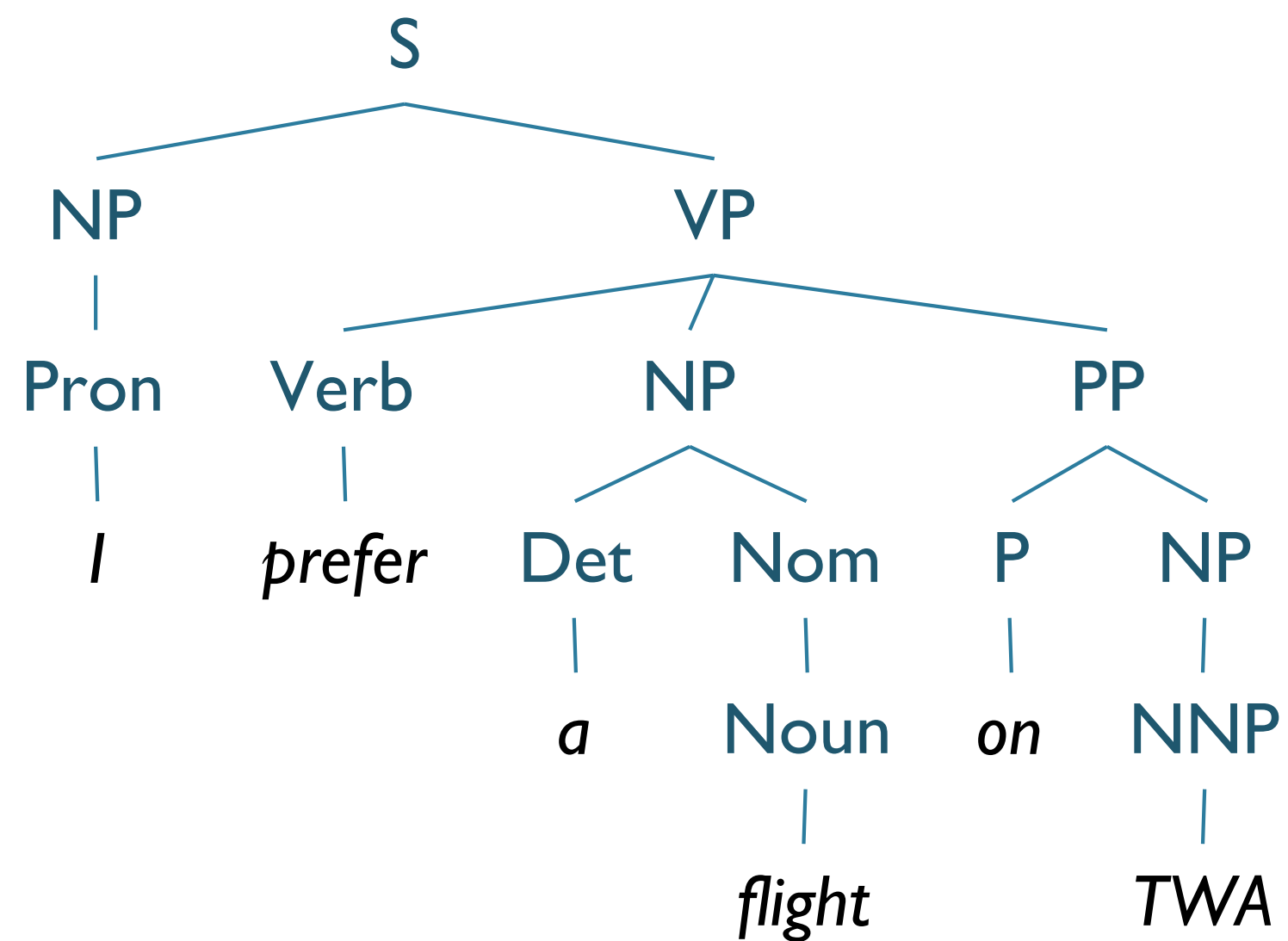
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Disambiguation

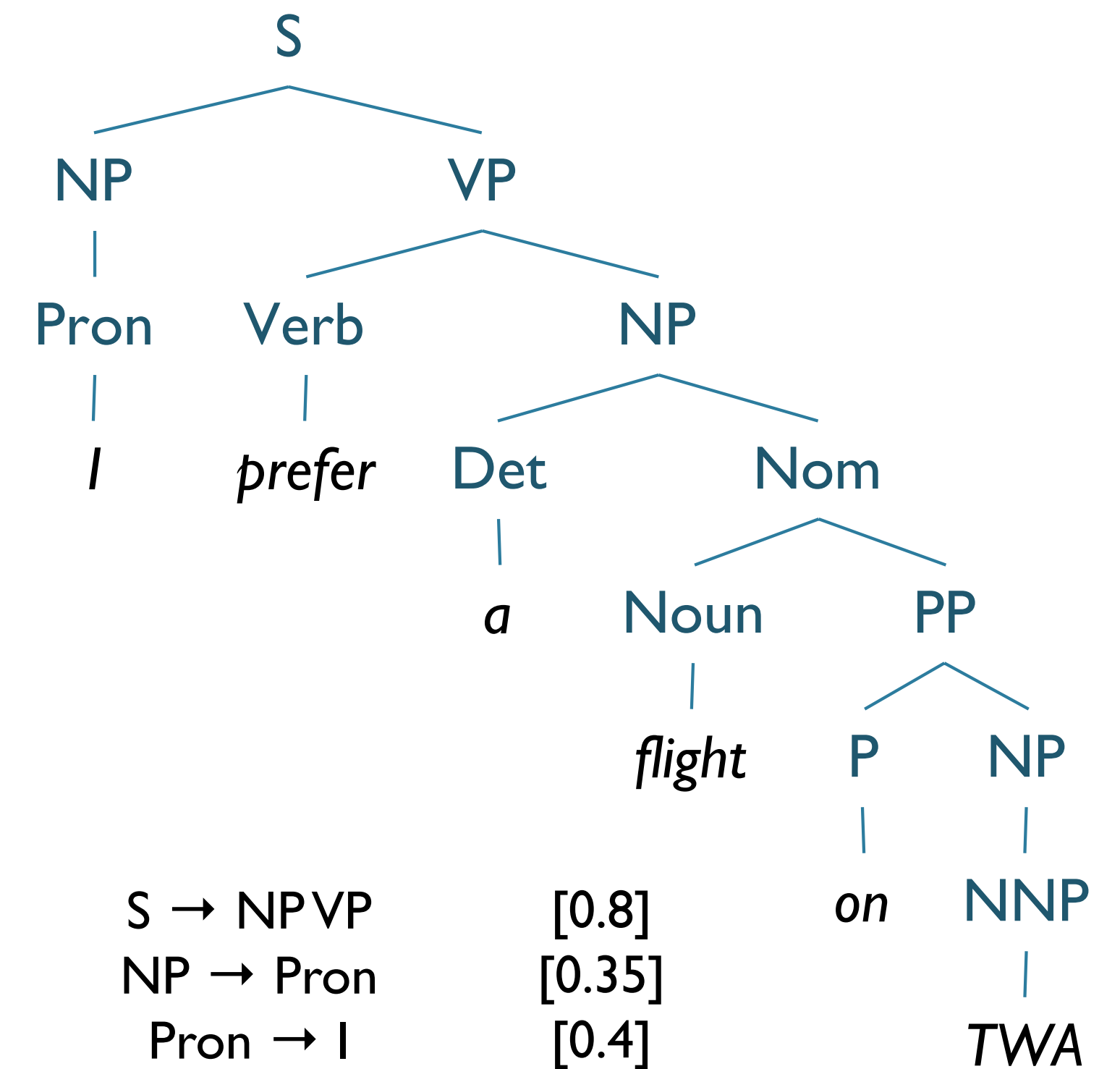
- A PCFG assigns probability to each parse tree T for input S
- Probability of T : product of all rules used to derive T

$$P(T, S) = \prod_{i=1}^n P(RHS_i | LHS_i)$$

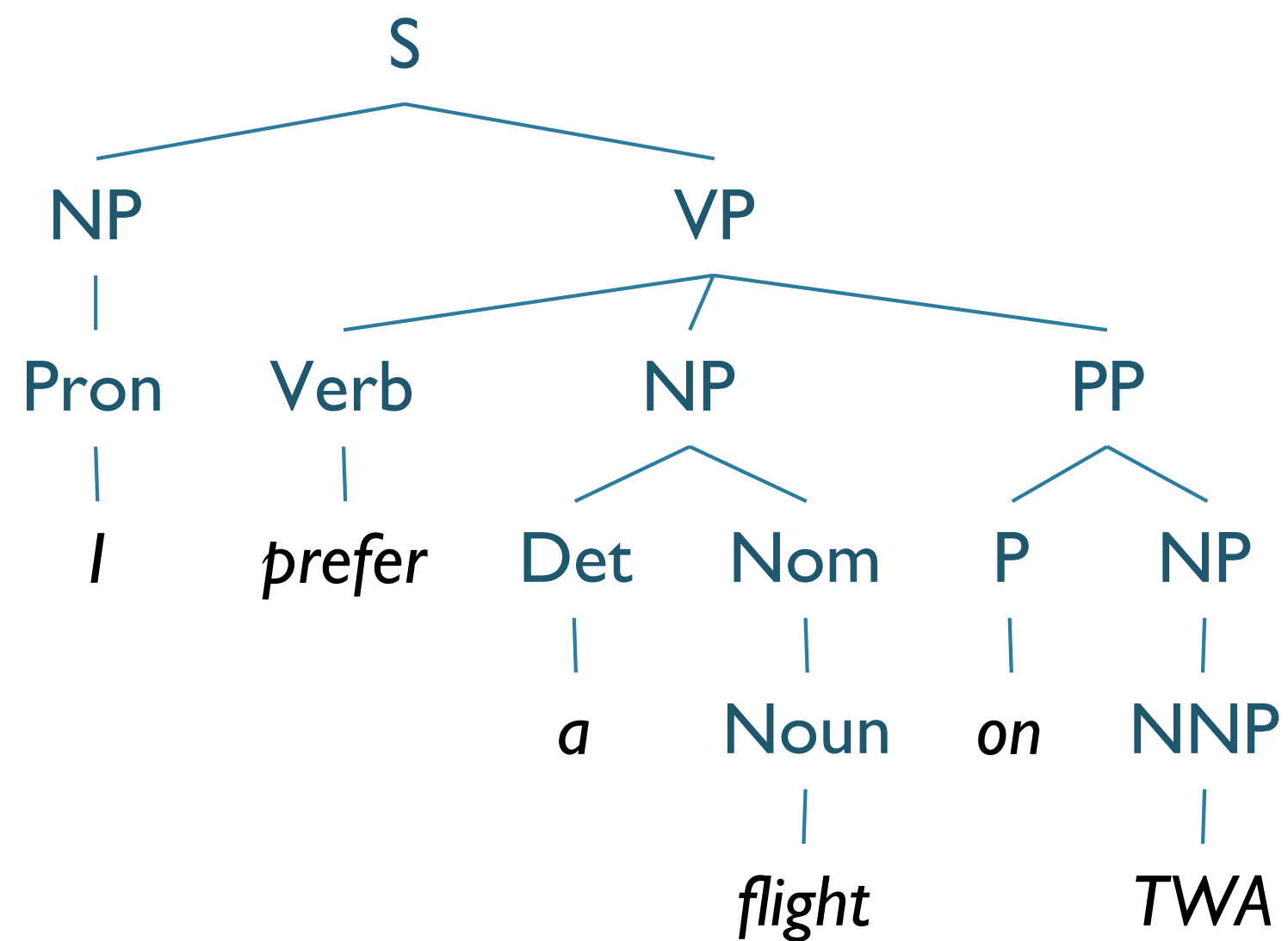
$$P(T, S) = P(T)P(S | T) = P(T)$$



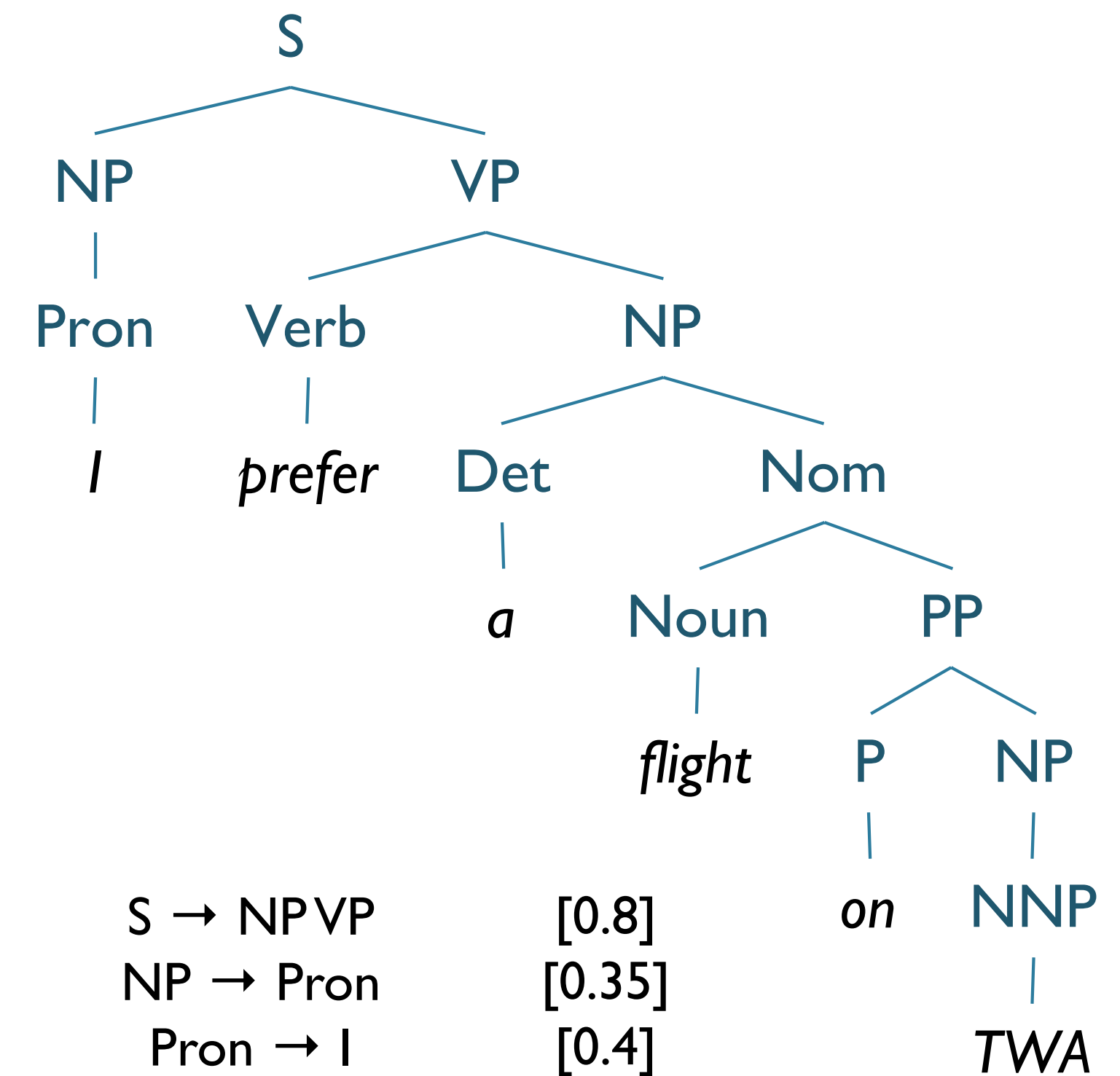
$S \rightarrow NP VP$	[0.8]
$NP \rightarrow Pron$	[0.35]
$Pron \rightarrow I$	[0.4]
$VP \rightarrow V NP PP$	[0.1]
$V \rightarrow prefer$	[0.4]
$NP \rightarrow Det Nom$	[0.2]
$Det \rightarrow a$	[0.3]
$Nom \rightarrow N$	[0.75]
$N \rightarrow flight$	[0.3]
$PP \rightarrow P NP$	[1.0]
$P \rightarrow on$	[0.2]
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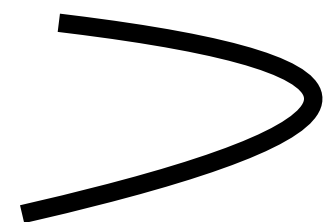
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$NP \rightarrow NNP$	[0.3]
$NNP \rightarrow NWA$	[0.4]

prefer

$\sim 1.452 \times 10^{-6}$



$\sim 1.452 \times 10^{-7}$

Parsing Problem for PCFGs

- Select T such that (*s.t.*)

$$\hat{T}(S) = \operatorname{argmax}_{T \text{ s.t. } S=\text{yield}(T)} P(T)$$

- String of words S is *yield* of parse tree
- Select the tree \hat{T} that maximizes the probability of the parse

Application: Language Modeling

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 - Approximate using conditioning on limited context:

$$P(w_i | w_{i-1}) = \frac{P(w_{i-1}, w_i)}{P(w_{i-1})}$$

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$$P(w_i | w_{i-1}) = \frac{P(w_{i-1}, w_i)}{P(w_{i-1})}$$
- PCFGs are able to give probability of entire string without as bad sparsity
- Model probability of *syntactically valid* sentences
 - Not just probability of sequence of words

PCFGs: Parsing

Probabilistic CKY (PCKY)

- Like regular CKY
 - Assumes grammar in Chomsky Normal Form (CNF)
 - $A \rightarrow B C$
 - $A \rightarrow w$
 - Represent input with indices b/t words:
 - $_0 \text{ Book } _1 \text{ that } _2 \text{ flight } _3 \text{ through } _4 \text{ Houston } _5$

Probabilistic CKY (PCKY)

- For input string length n and non-terminals V
 - Cell $[i, j, A]$ in $(n+1) \times (n+1) \times V$ matrix
 - Contains probability that A spans $[i, j]$

$$C_{ij} = [A, p]$$

PCKY Algorithm

```
function PROBABILISTIC-CKY-PARSE(words, grammar) returns most probable parse and its probability
for j ← from 1 to LENGTH(words) do
  for all { A |  $A \rightarrow words[j] \in grammar$  }
     $table[j-1, j, A] \leftarrow P(A \rightarrow words[j])$ 
  for i ← from j-2 downto 0 do
    for k ← i + 1 to j-1 do
      for all { A |  $A \rightarrow B C \in grammar$ ,
        and  $table[i, k, B] > 0$  and  $table[k, j, C] > 0$  }
        if ( $table[i, j, A]$  <  $P(A \rightarrow BC) \times table[i, k, B] \times table[k, j, C]$ ) then
           $table[i, j, A]$  ←  $P(A \rightarrow BC) \times table[i, k, B] \times table[k, j, C]$  choose
           $back[i, j, A]$  ← { k, B, C }
return BUILD_TREE( $back[1, LENGTH(words), S]$ ,  $table[1, LENGTH(words), S]$ )
```

best way

PCKY Algorithm

```
function PROBABILISTIC-CKY-PARSE(words, grammar) returns most probable parse and its probability
for j  $\leftarrow$  from 1 to LENGTH(words) do
  for all { A |  $A \rightarrow words[j] \in grammar$  }
     $table[j-1, j, A] \leftarrow P(A \rightarrow words[j])$ 
  for i  $\leftarrow$  from j-2 downto 0 do
    for k  $\leftarrow$  i + 1 to j-1 do
      for all { A |  $A \rightarrow B C \in grammar,$ 
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```

PCKY Grammar Segment

$S \rightarrow NP VP$ [0.80]

$NP \rightarrow Det N$ [0.30]

$VP \rightarrow V NP$ [0.20]

$Det \rightarrow \text{the}$ [0.40]

$Det \rightarrow \text{a}$ [0.40]

$V \rightarrow \text{includes}$ [0.05]

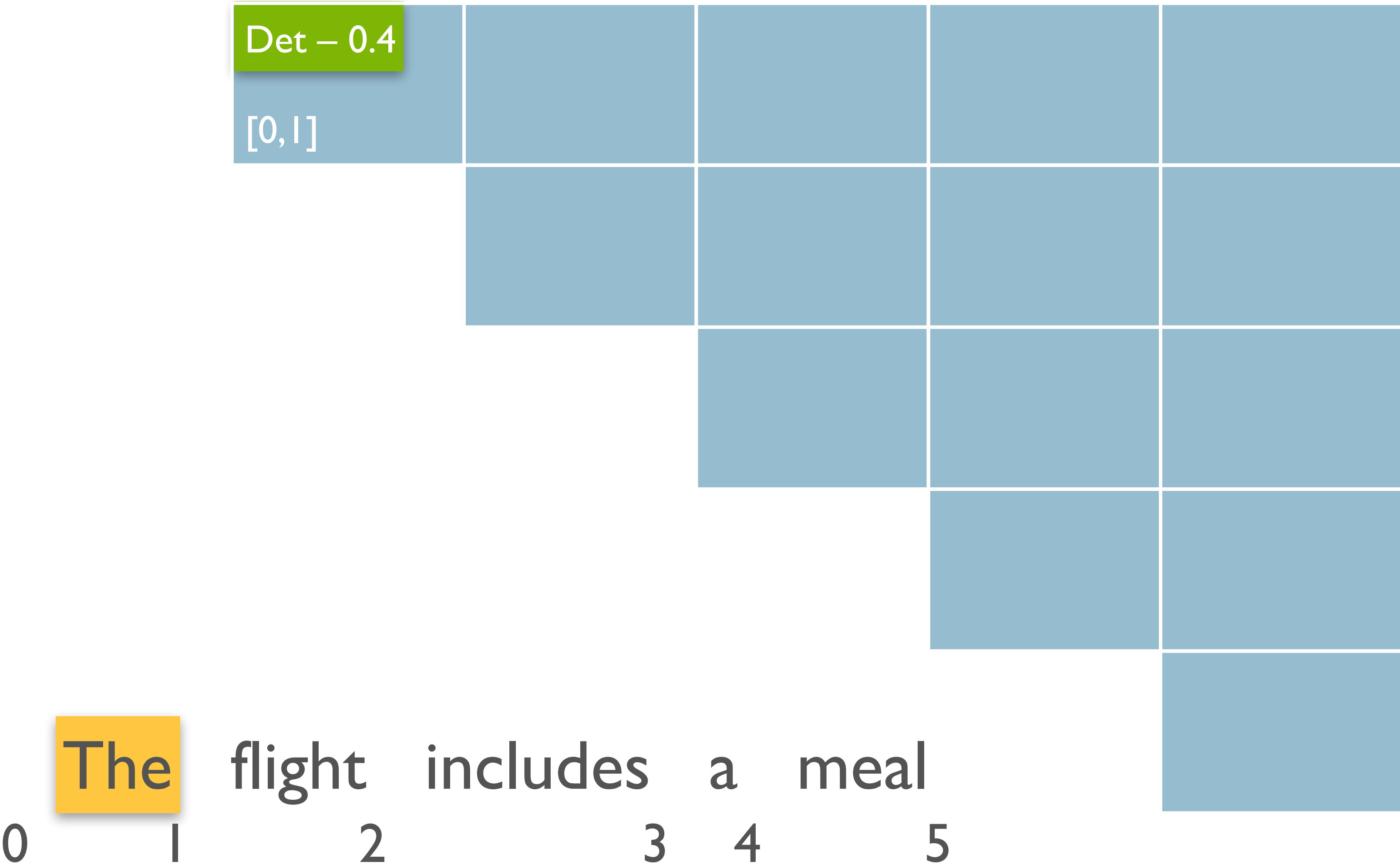
$N \rightarrow \text{meal}$ [0.01]

$N \rightarrow \text{flight}$ [0.02]

PCKY Matrix

$S \rightarrow NP VP$ [0.80]
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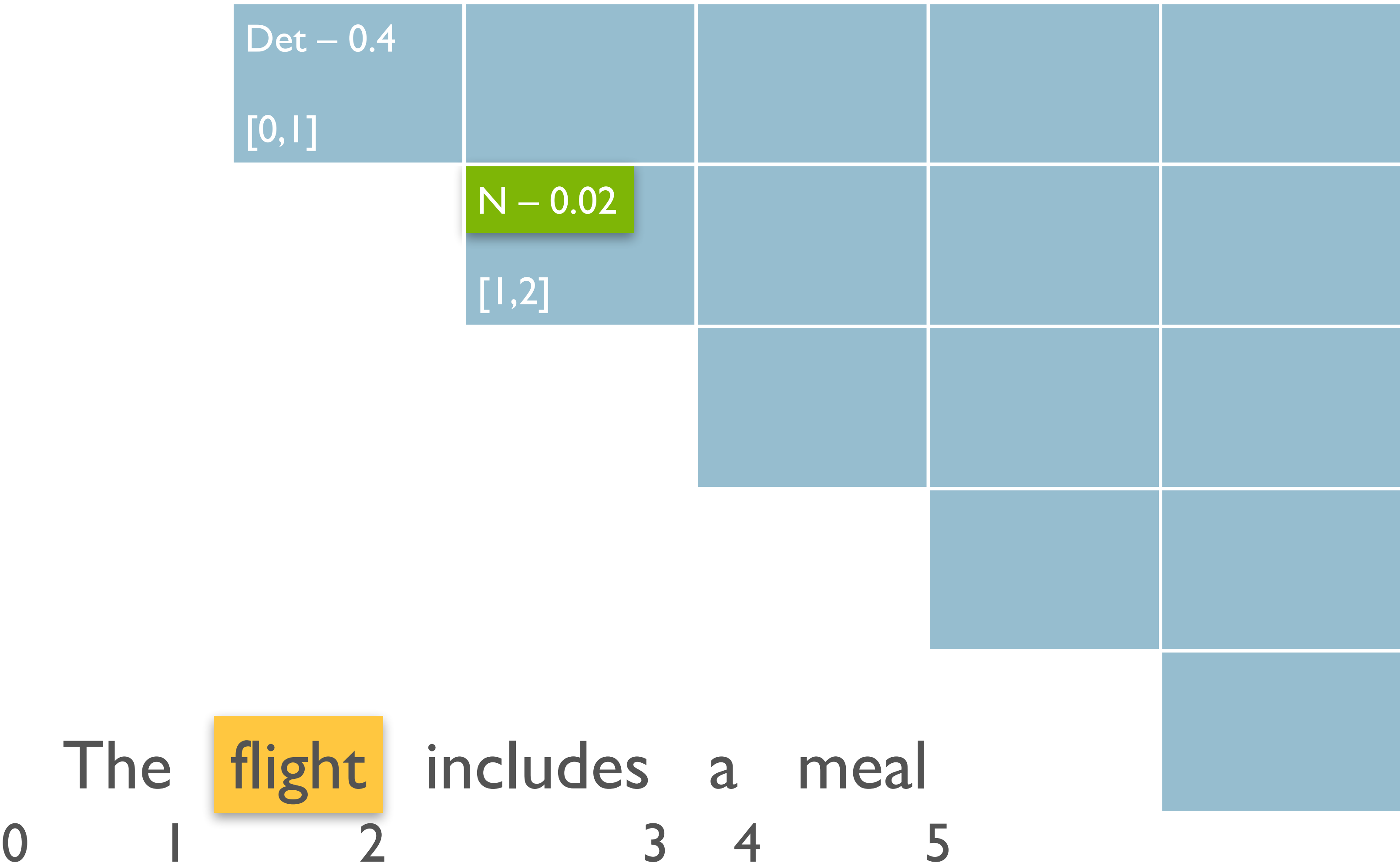
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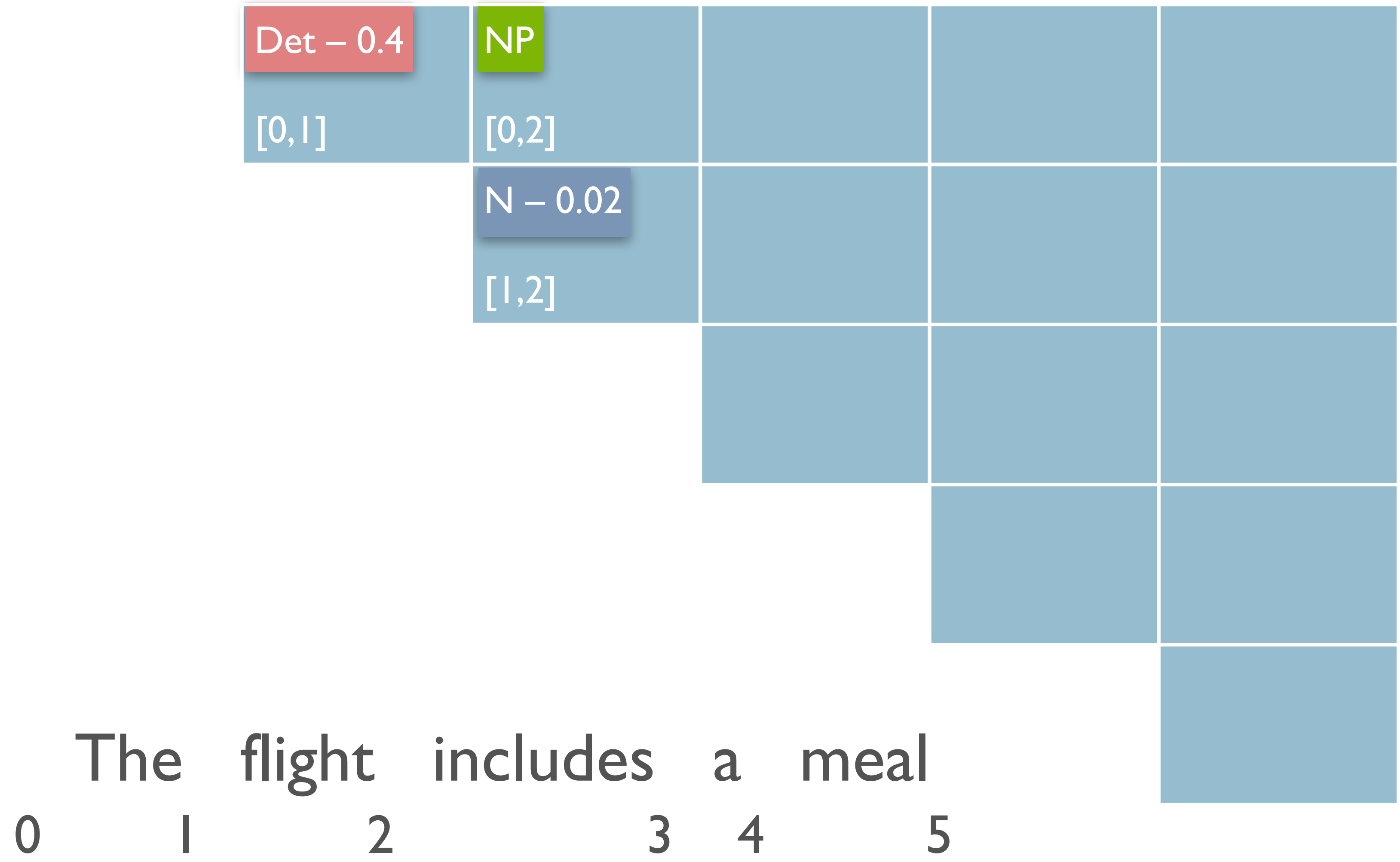
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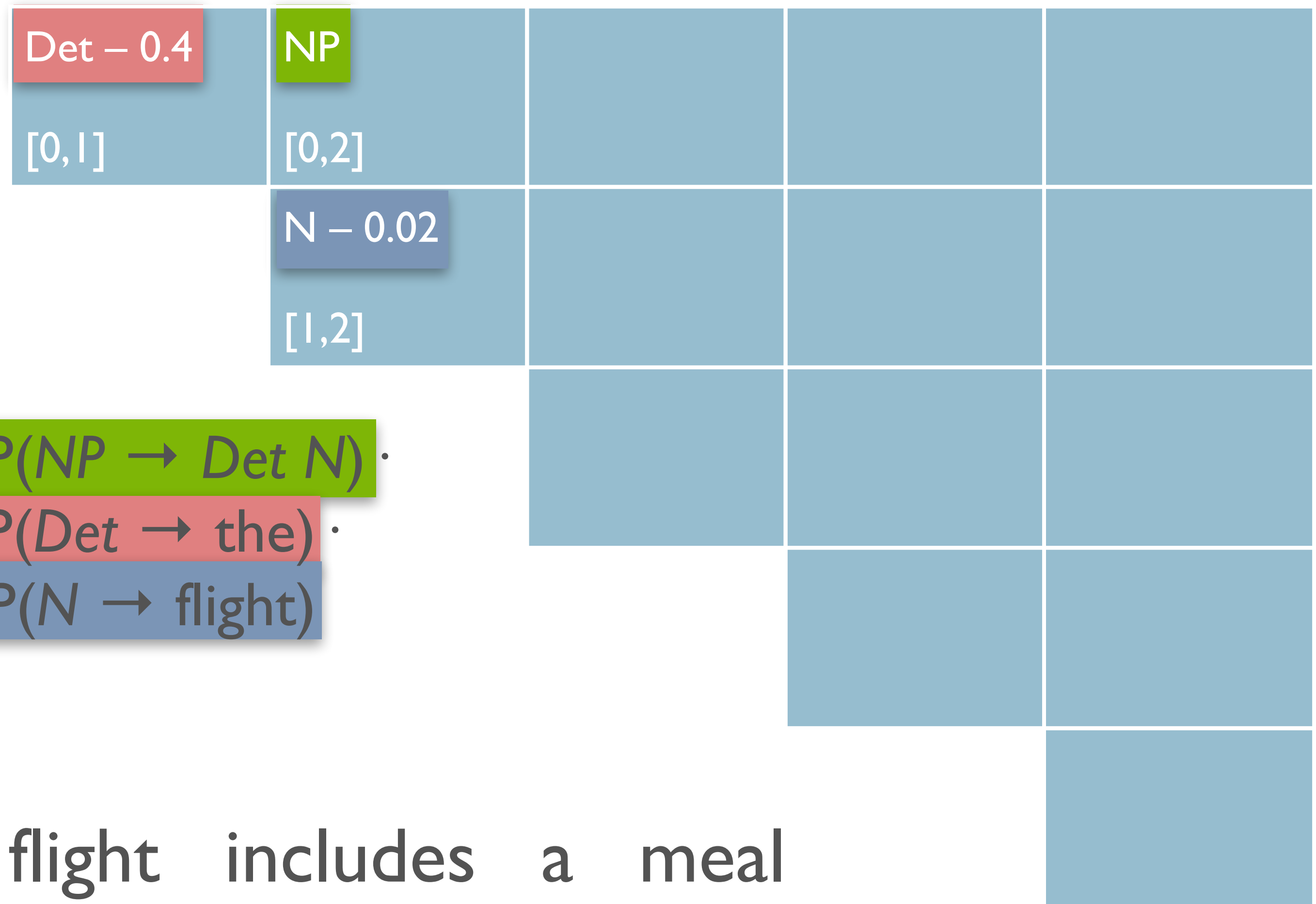
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$$P = P(NP \rightarrow Det N) \cdot P(Det \rightarrow the) \cdot P(N \rightarrow flight)$$

0 1 2 3 4 5
 The flight includes a meal



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Det – 0.4	NP				
[0,1]	[0,2]				
	N – 0.02				
	[1,2]				

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$$P = 0.3 \cdot 0.4 \cdot 0.02 = 0.00024$$

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PCKY Matrix

$S \rightarrow NP VP$ [0.80]
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Det – 0.4	NP – 0.0024				
[0,1]	[0,2]				
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Det – 0.4 [0,1]	NP – 0.0024 [0,2]			S – 2.304×10 ⁻⁸ [0,5]
	N – 0.02 [1,2]			
		V – 0.05 [2,3]		VP – 1.2×10 ⁻⁵ [2,5]
			Det – 0.4 [3,4]	NP – 0.0012 [3,5]
				N – 0.01 [4,5]

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Inducing a PCFG

Learning Probabilities

- Simplest way:
 - Use treebank of parsed sentences

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- Alternative: Learn probabilities by re-estimating
 - (Later)

Probabilistic Parser Development Paradigm

	Train	Dev	Test
Size	Large (eg. WSJ 2–21, 39,830 sentences)	Small (e.g. WSJ 22)	Small/Med (e.g. WSJ, 23, 2,416 sentences)
Usage	Estimate rule probabilities	Tuning/Verification, Check for Overfit	Held Out, Final Evaluation

Parser Evaluation

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- Assume a ‘gold standard’ set of parses for test set

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 - Constituents in output match those in reference

Parser Evaluation

- Assume a ‘gold standard’ set of parses for test set
- How can we tell how good the parser is?
- How can we tell how good a parse is?
 - Maximally strict: identical to ‘gold standard’
 - Partial credit:
 - Constituents in output match those in reference
 - Same start point, end point, non-terminal symbol

Parseval

- How can we compute parse score from constituents?
- Multiple Measures:

$$\text{Labeled Recall (LR)} = \frac{\# \text{ of } \mathbf{correct} \text{ constituents in } \mathbf{hypothetical} \text{ parse}}{\# \text{ of } \mathbf{total} \text{ constituents in } \mathbf{reference} \text{ parse}}$$

$$\text{Labeled Precision (LP)} = \frac{\# \text{ of } \mathbf{correct} \text{ constituents in } \mathbf{hypothetical} \text{ parse}}{\# \text{ of } \mathbf{total} \text{ constituents in } \mathbf{hypothetical} \text{ parse}}$$

Parseval

- **F-measure:**

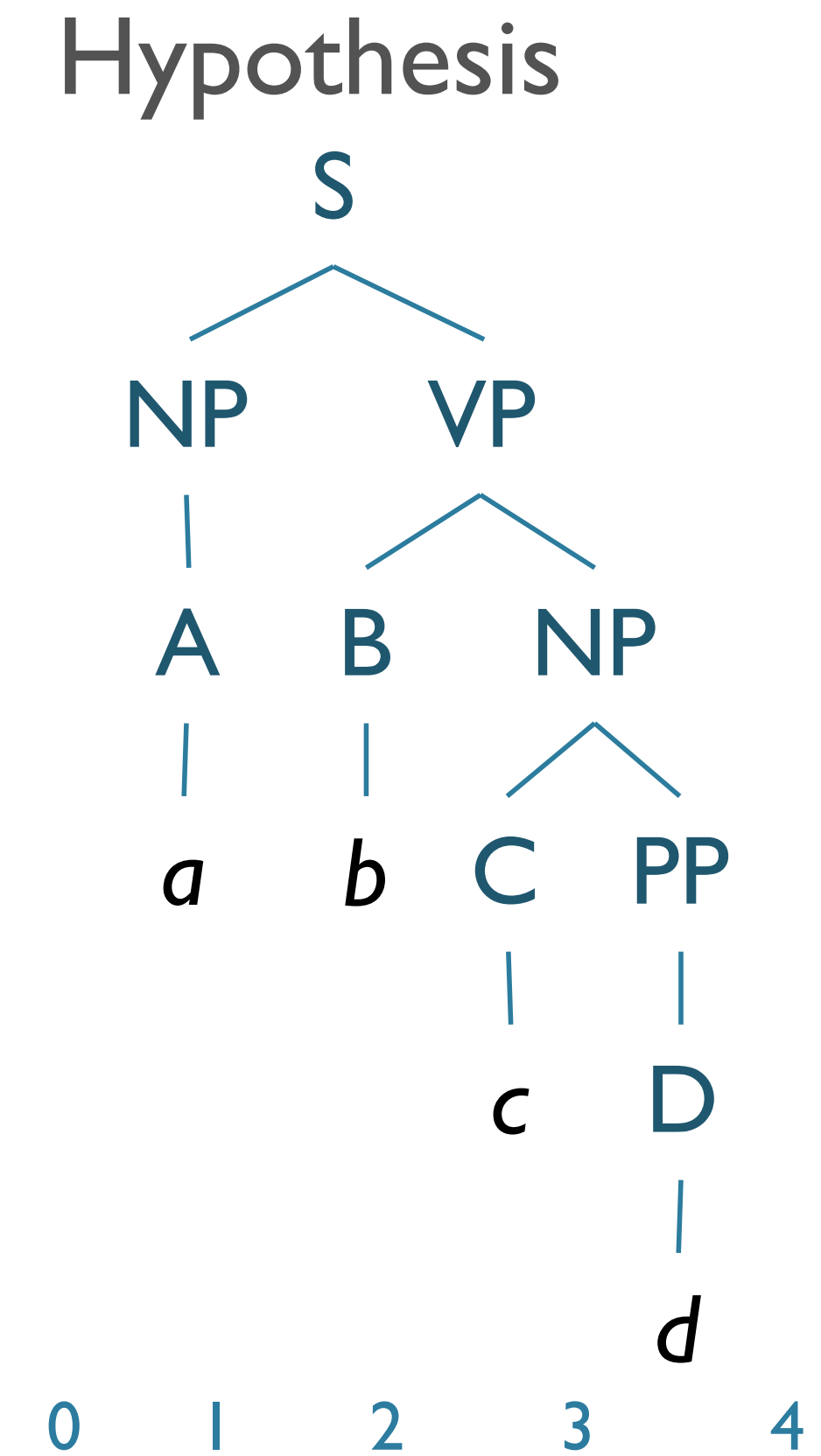
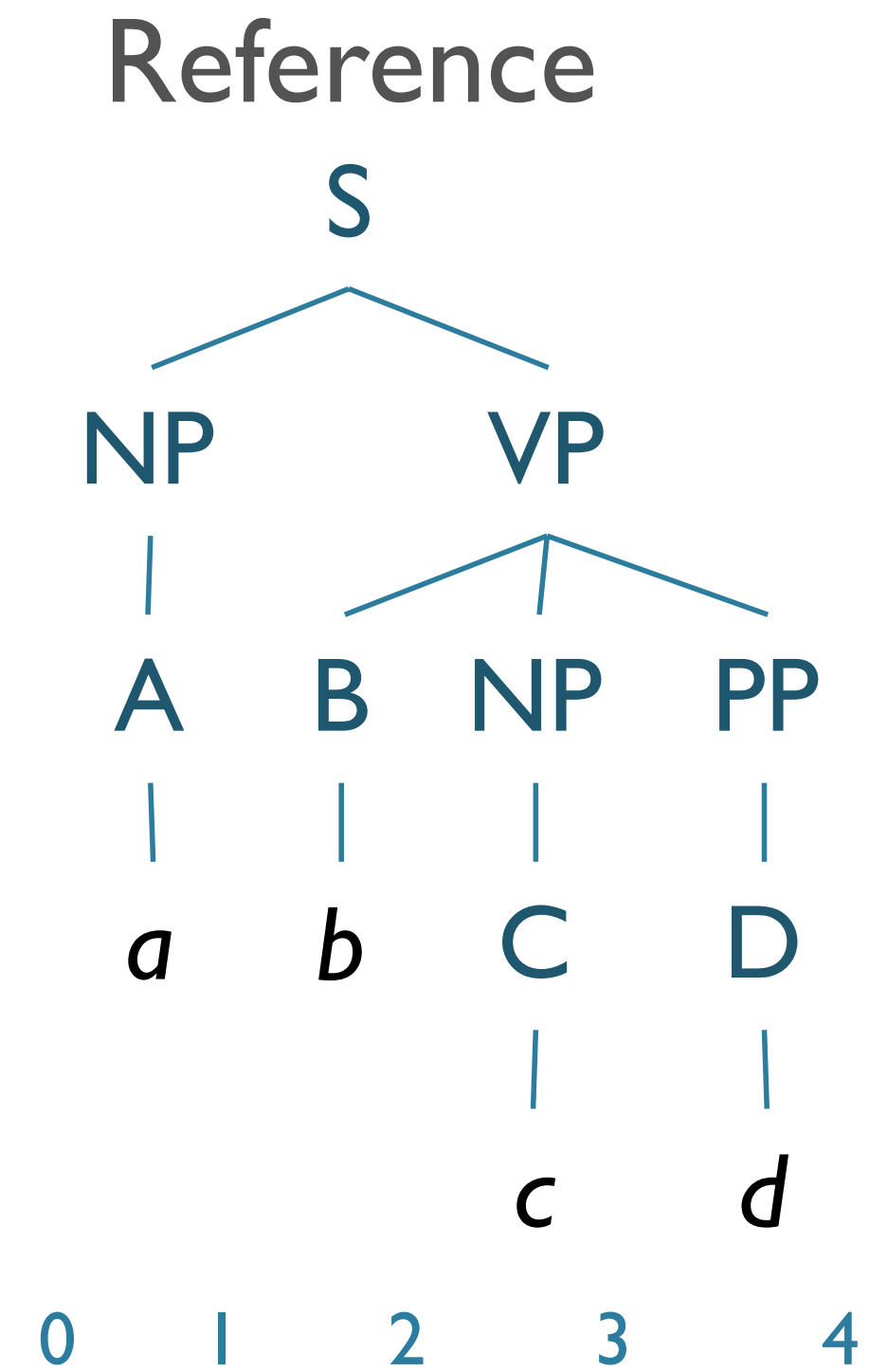
- Combines precision and recall

- Let $\beta \in \mathbb{R}$, $\beta > 0$ that adjusts P vs. R s.t. $\beta \propto \frac{R}{P}$

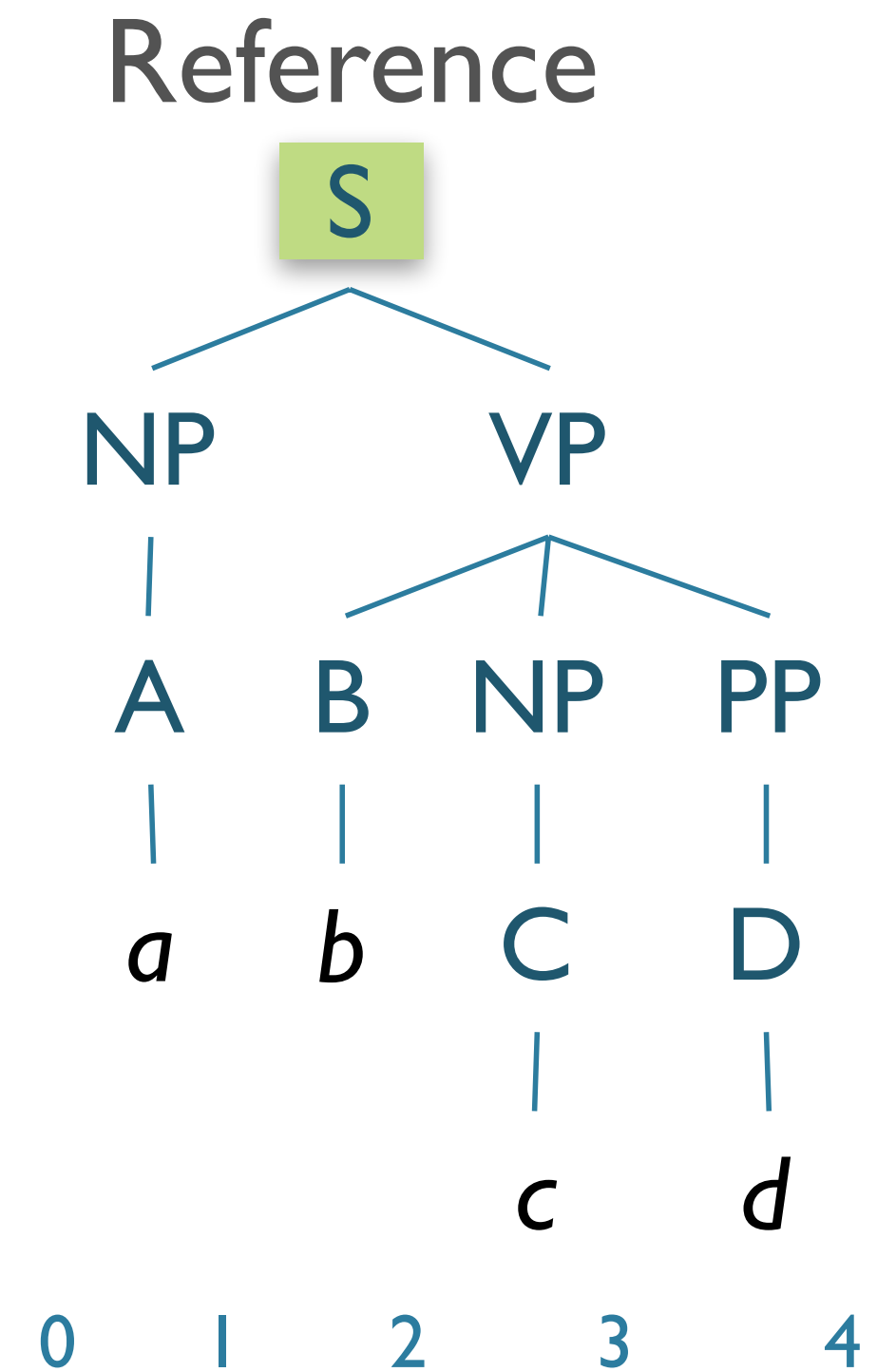
- F_β -measure is then:
$$F_\beta = (1 + \beta^2) \cdot \frac{P \cdot R}{\beta^2 \cdot P + R}$$

- With F1-measure as
$$F_1 = \frac{2PR}{P + R}$$

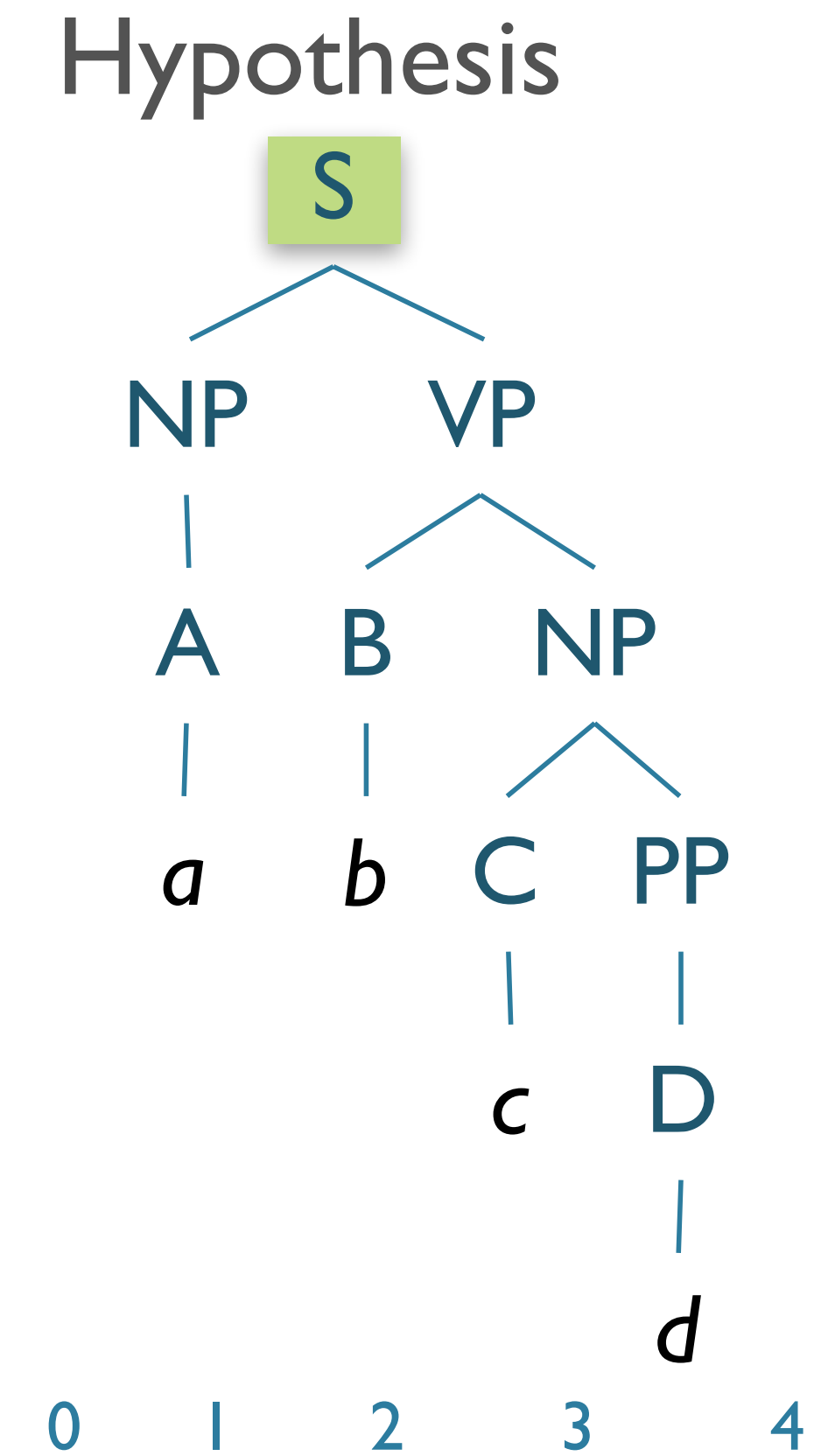
Evaluation: Example



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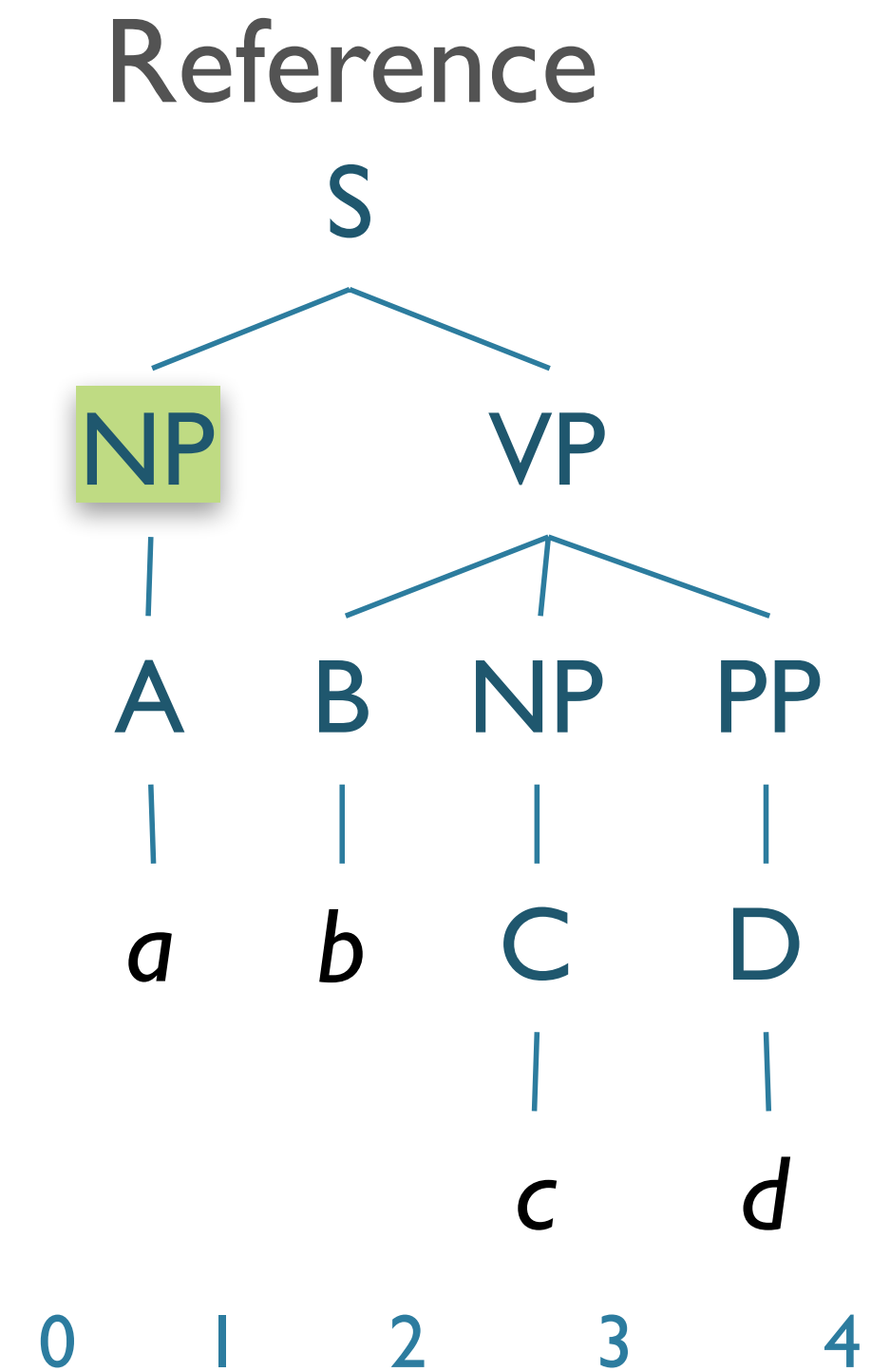


S(0,4)

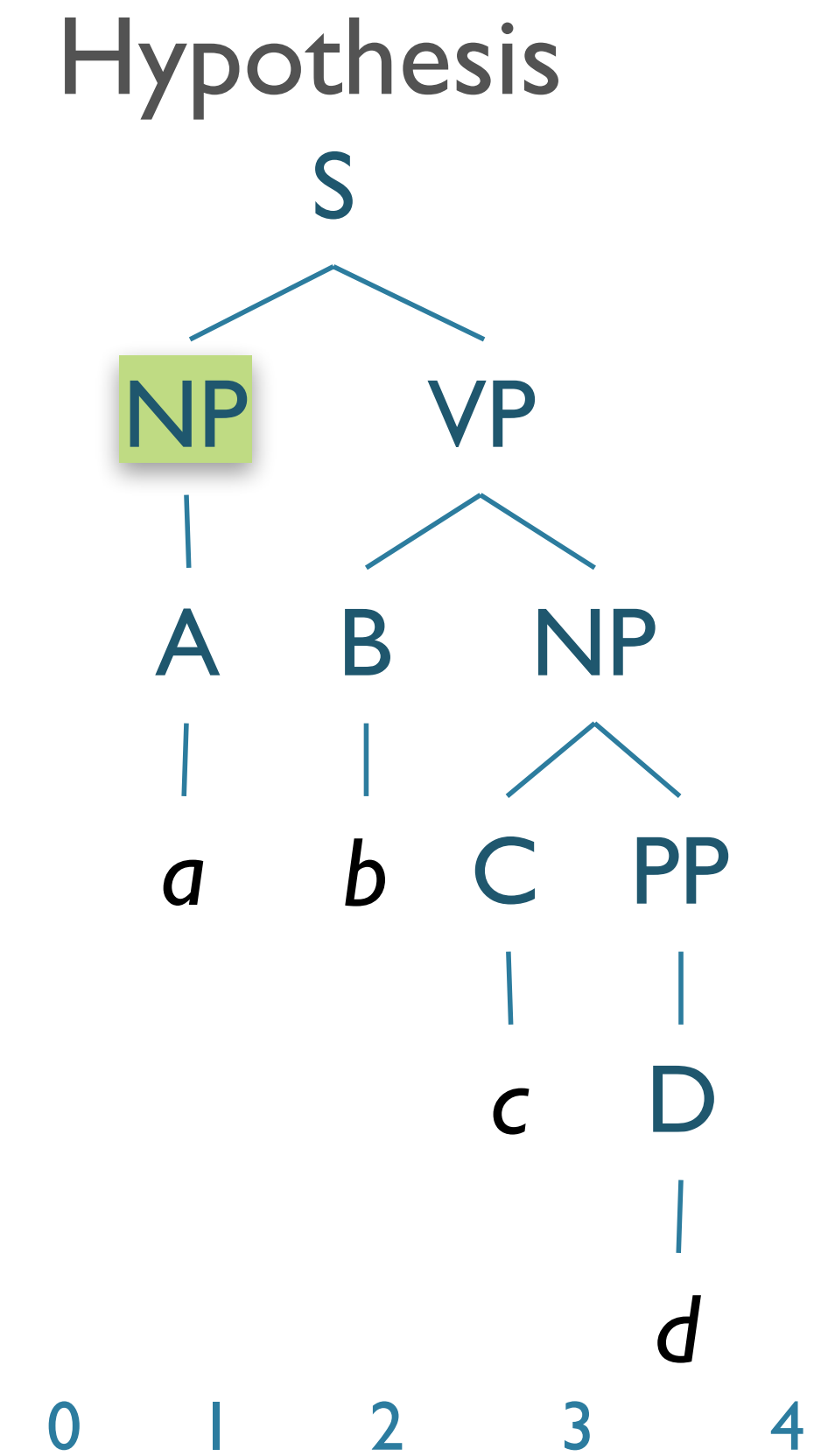


S(0,4)

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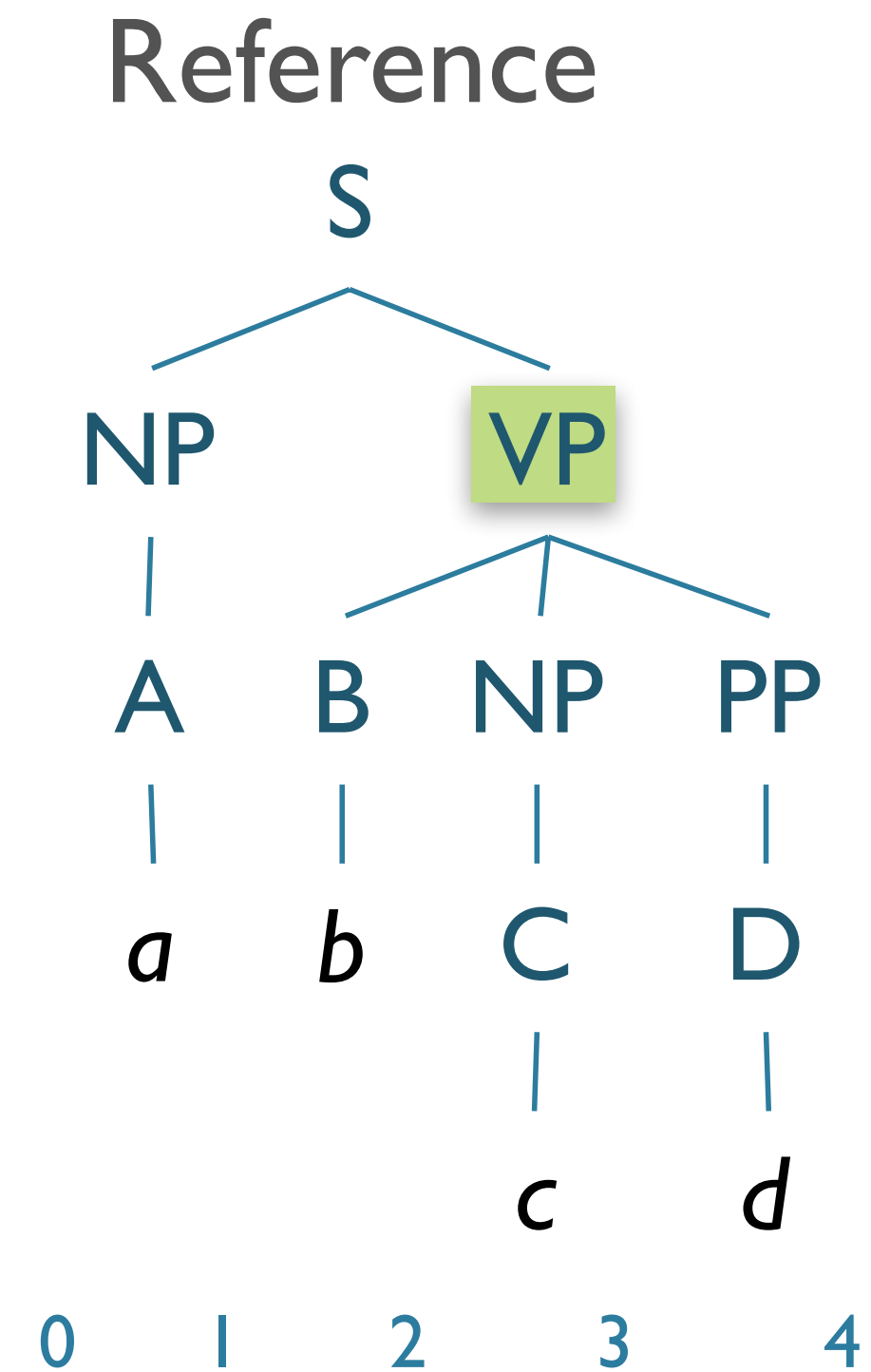


S(0,4)
NP(0,1)

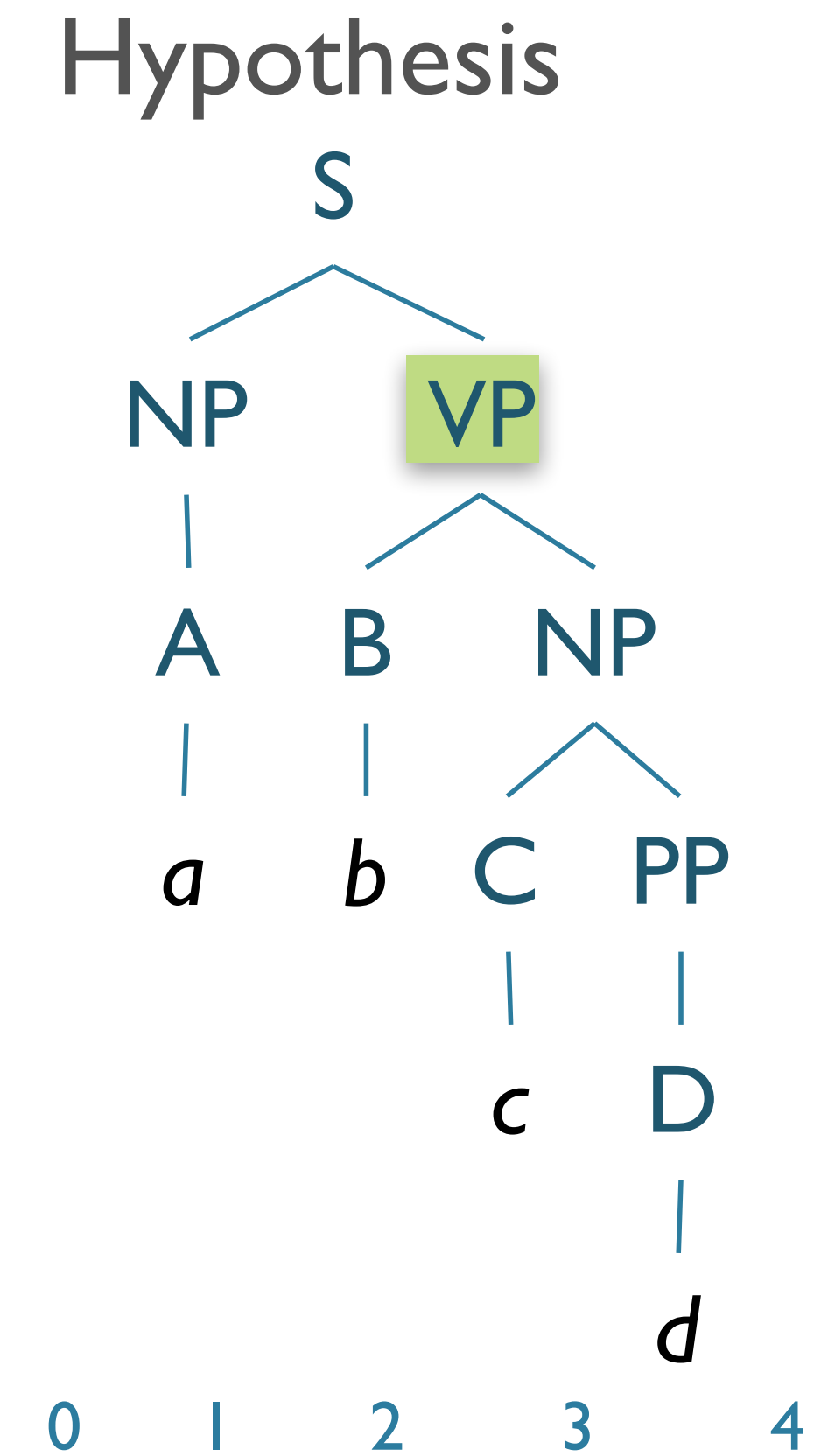


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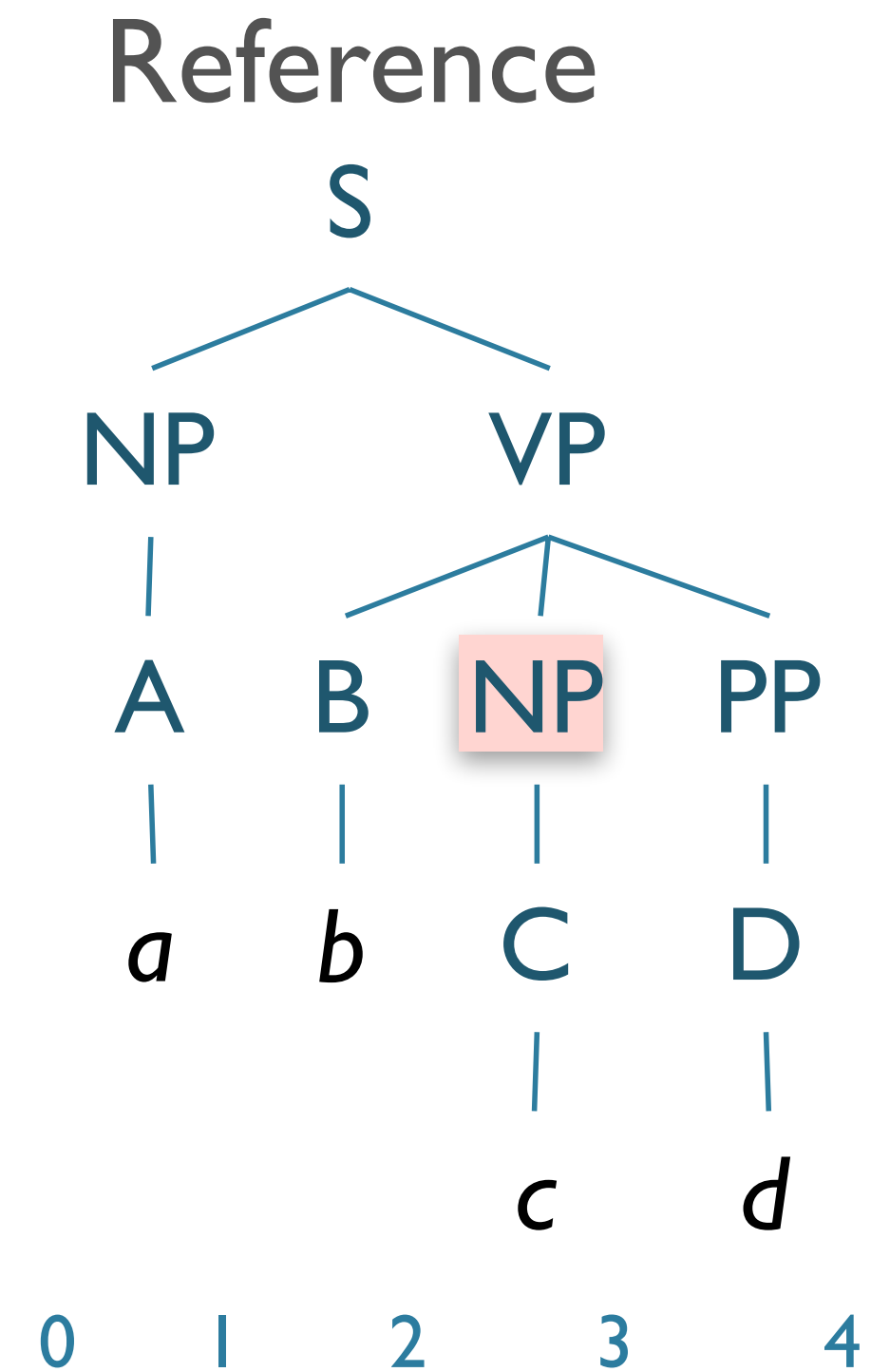


S(0,4)
NP(0,1)
VP(1,4)

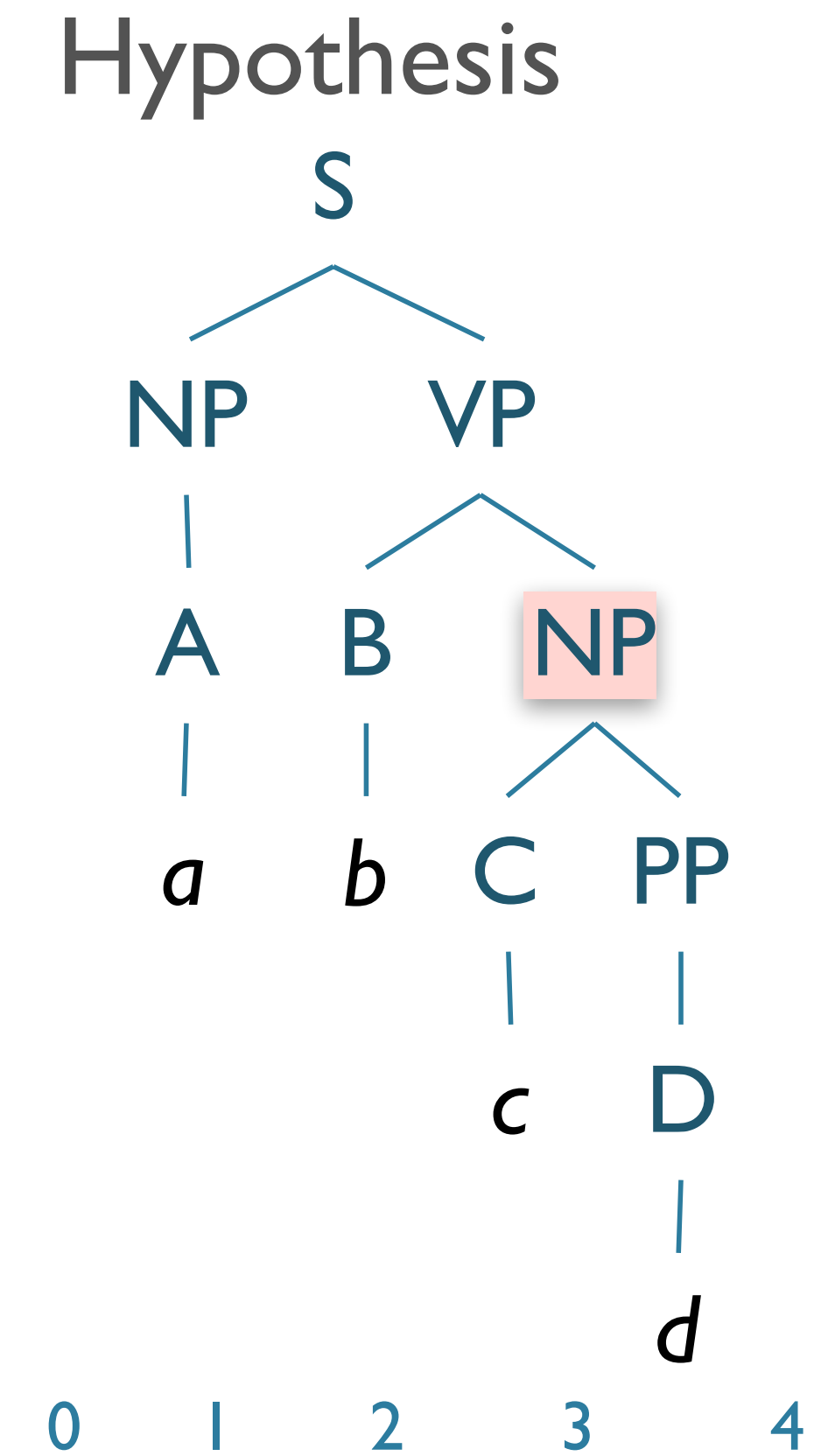


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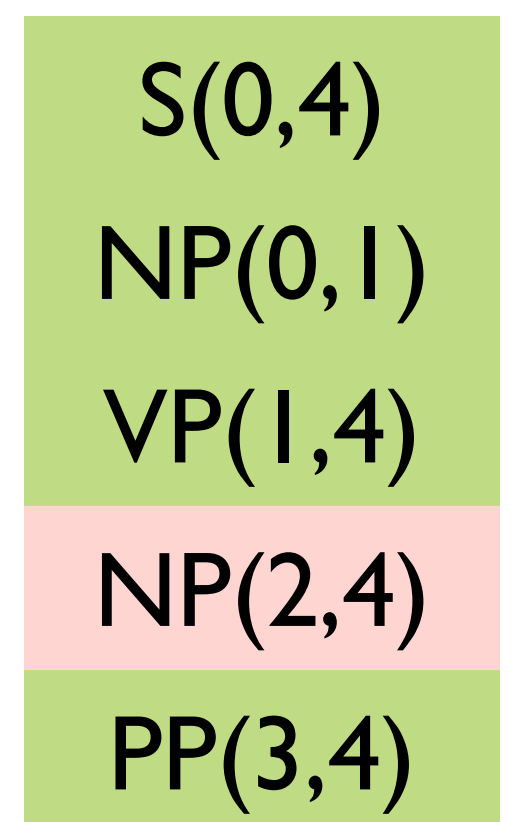
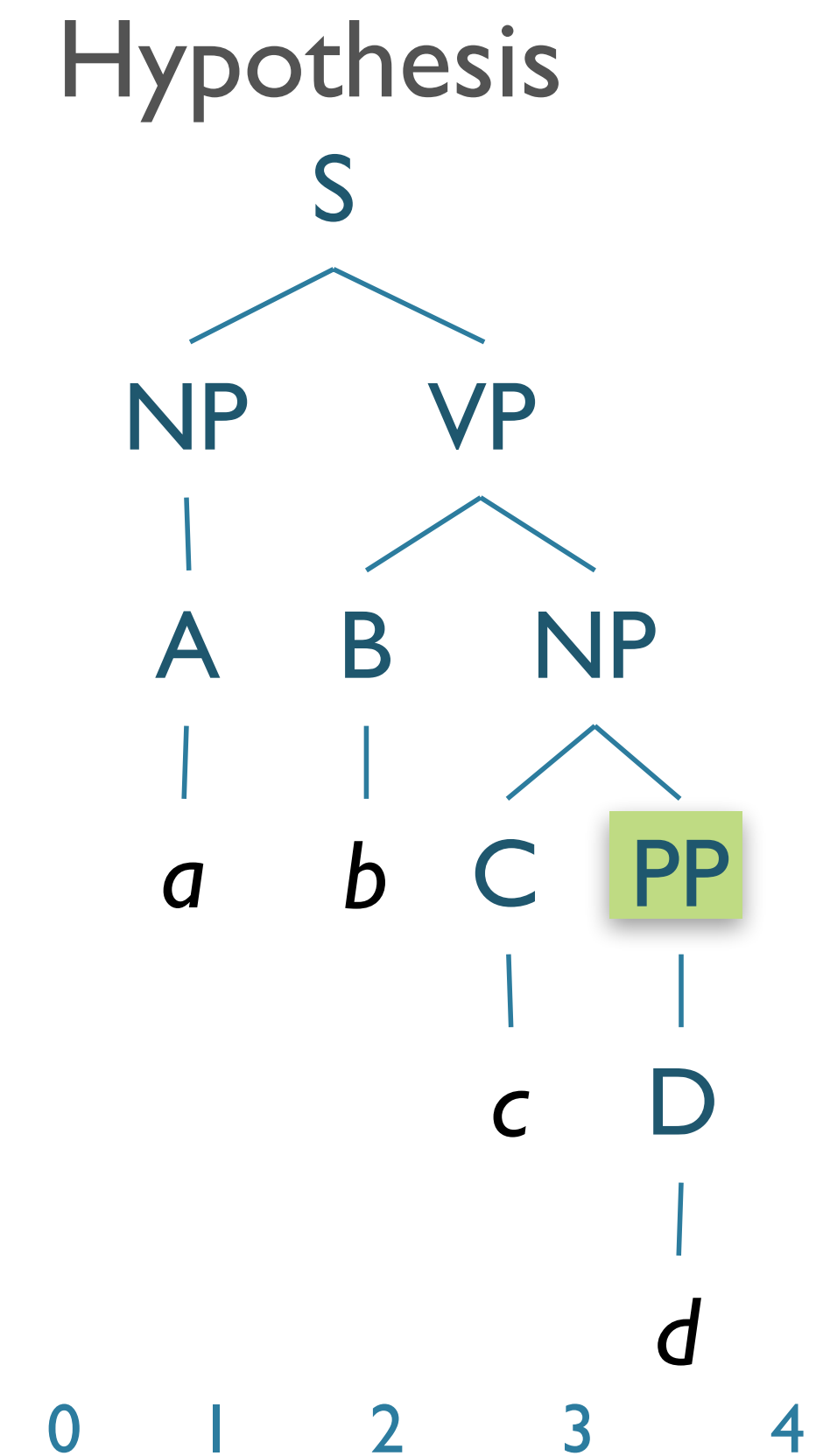
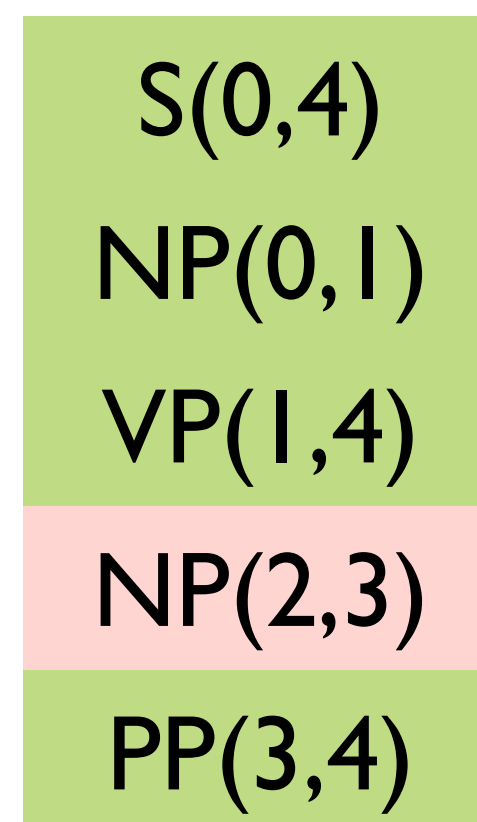
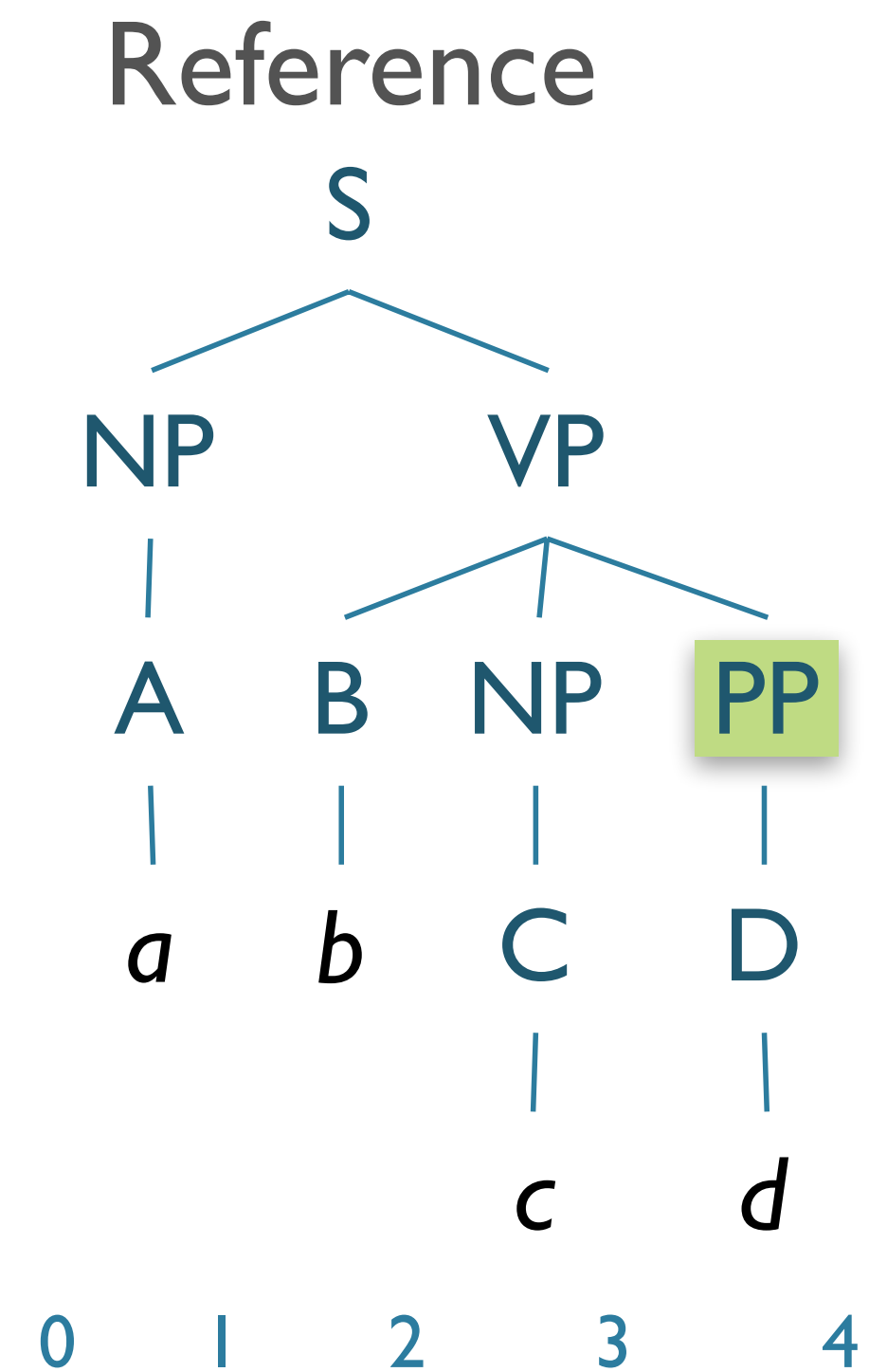


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NP(2,3)

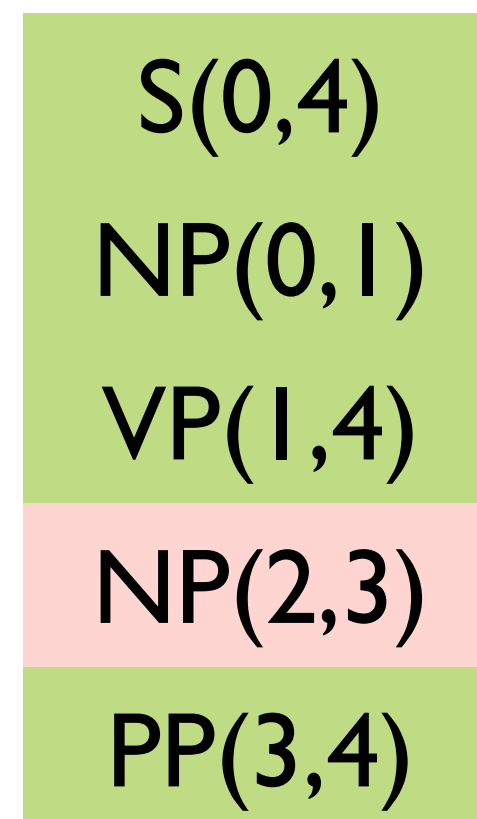
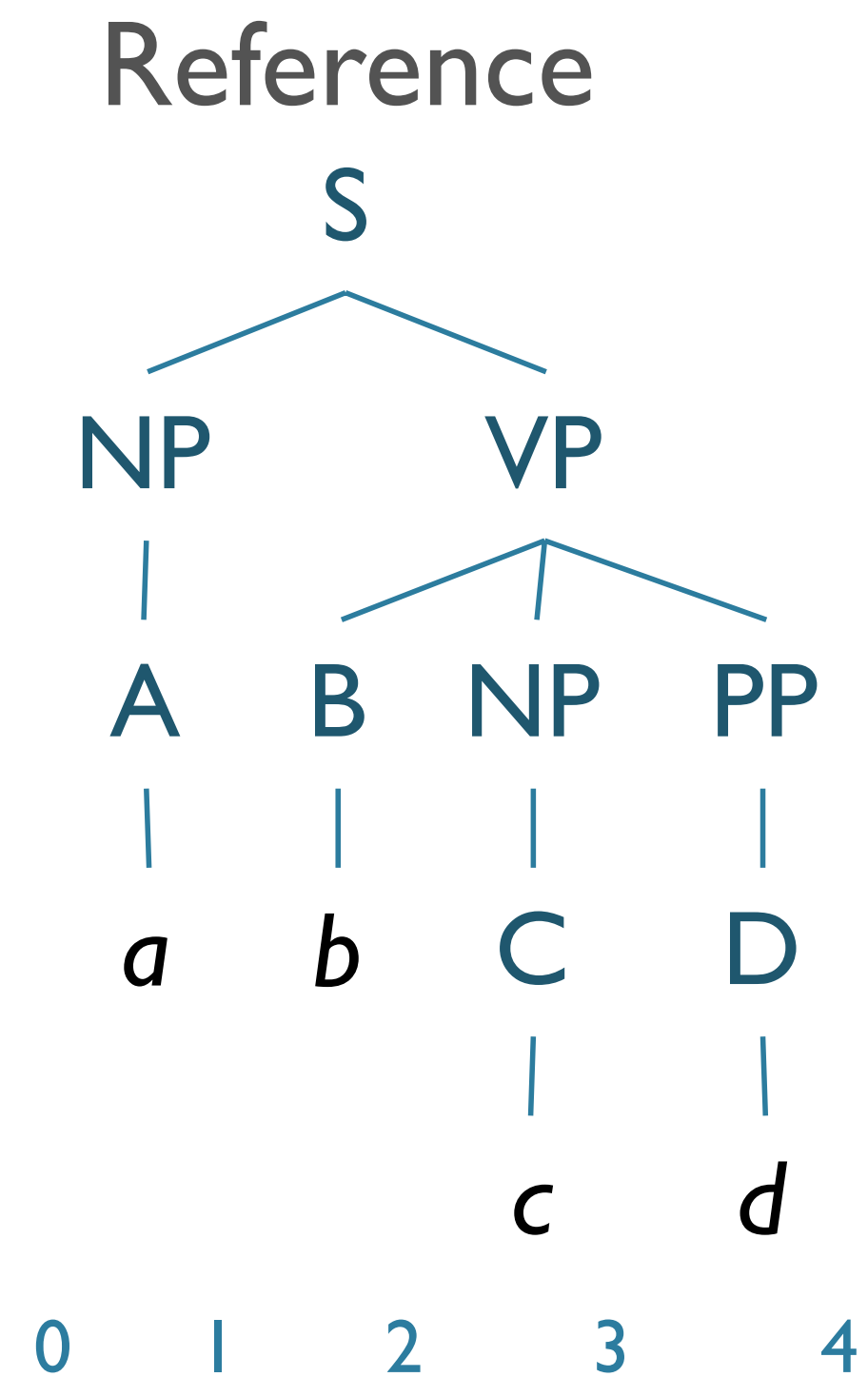


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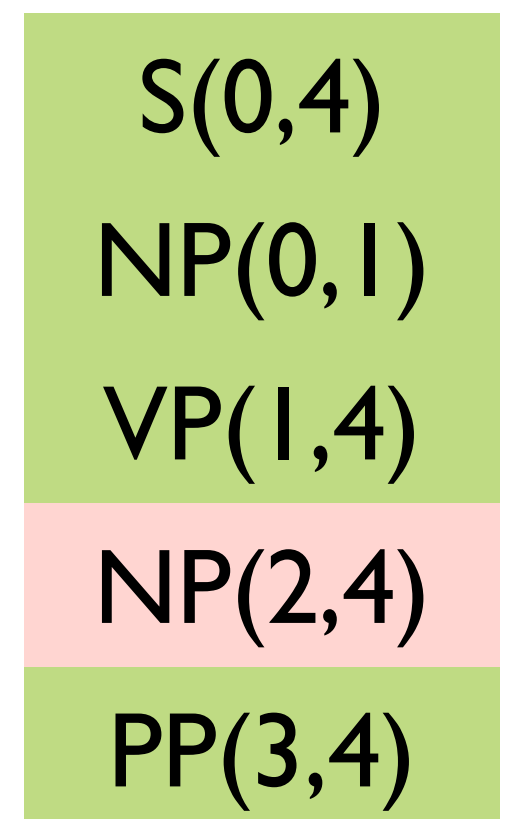
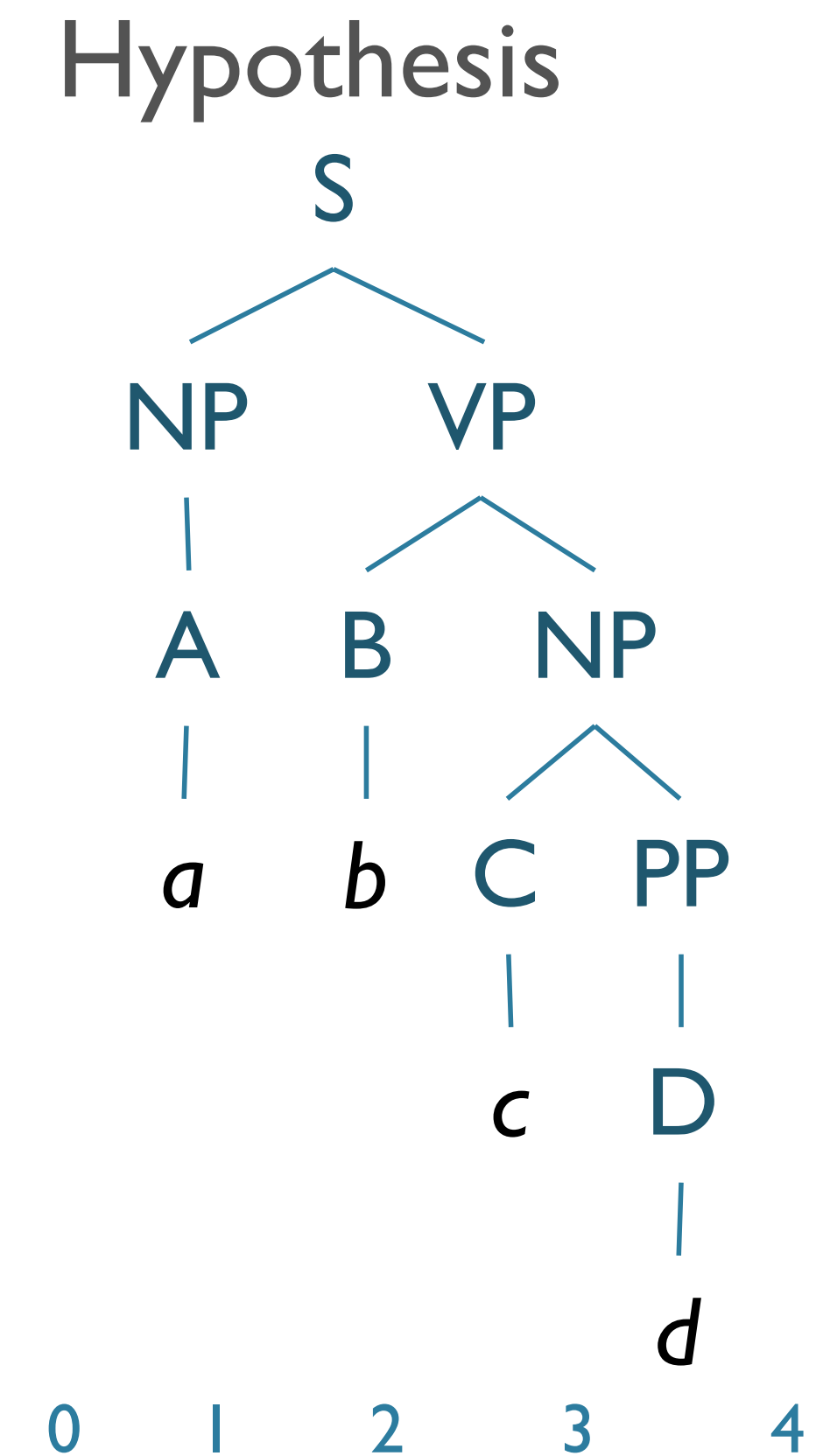
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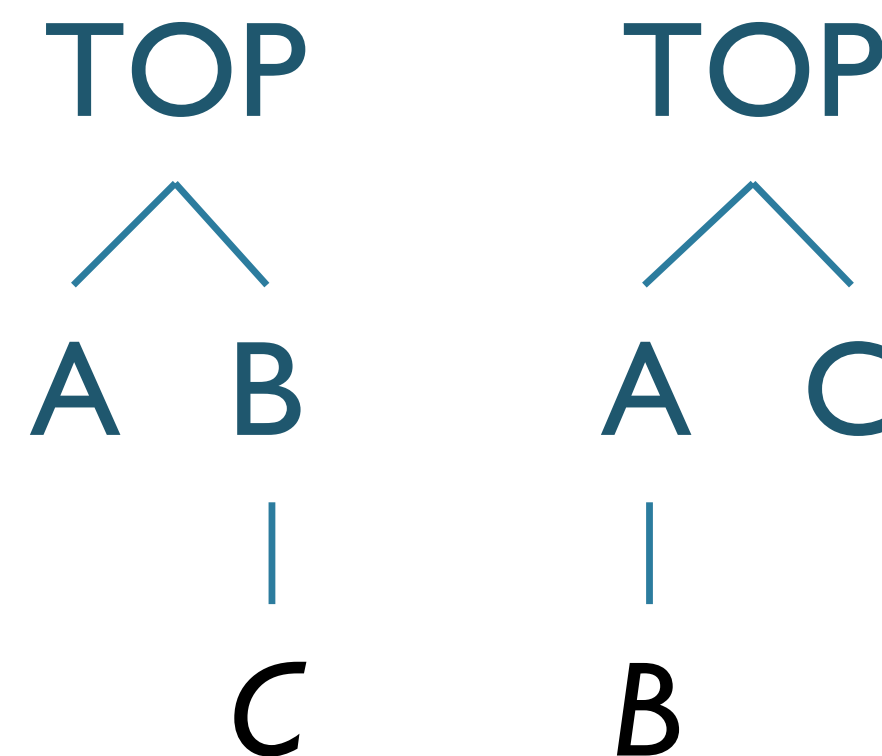


LP:	4/5
LR:	4/5
F ₁ :	4/5



Parser Evaluation

- Crossing Brackets:
 - # of constituents where produced parse has bracketings that overlap for the siblings:
 - ((A B) C) — { (0,2), (2,3) }
and hyp. has
(A (B C)) — { (0,1), (1, 3) }



```
/* crossing is counted based on the brackets */
/* in test rather than gold file (by Mike) */
for(j=0;j<bn2;j++){
  for(i=0;i<bn1;i++){
    if(bracket1[i].result != 5 &&
       bracket2[j].result != 5 &&
       ((bracket1[i].start < bracket2[j].start &&
         bracket1[i].end > bracket2[j].start &&
         bracket1[i].end < bracket2[j].end) ||
        (bracket1[i].start > bracket2[j].start &&
         bracket1[i].start < bracket2[j].end &&
         bracket1[i].end > bracket2[j].end))){
```

from evalb.c

State-of-the-Art Parsing

- Parsers trained/tested on Wall Street Journal PTB
 - LR: 90%+;
 - LP: 90%+;
 - Crossing brackets: 1%
- Standard implementation of Parseval:
 - **evalb**

Evaluation Issues

- Only evaluating constituency
- There are other grammar formalisms:
 - LFG (Constraint-based)
 - Dependency Structure
- **Extrinsic** evaluation
 - How well does getting the correct parse match the semantics, etc?

Earley Parsing

Earley vs. CKY

- CKY doesn't capture full original structure
 - Can back-convert binarization, terminal conversion
 - Unit non-terminals require change in CKY

Earley vs. CKY

- CKY doesn't capture full original structure
 - Can back-convert binarization, terminal conversion
 - Unit non-terminals require change in CKY
- Earley algorithm
 - Supports parsing efficiently with arbitrary grammars
 - Top-down search
 - Dynamic programming
 - Tabulated partial solutions
 - Some bottom-up constraints

Earley Algorithm

- Another dynamic programming solution
 - Partial parses stored in “chart”
 - Compactly encodes ambiguity
 - $O(N^3)$
- Chart entries contain:
 - Subtree for a single grammar rule
 - Progress in completing subtree
 - Position of subtree w.r.t. input

Earley Algorithm

- First, left-to-right pass fills out a chart with $N+1$ states
 - Chart entries — sit between words in the input string
 - Keep track of states of the parse at those positions
 - For each word position, chart contains set of states representing all partial parse trees generated so far
 - e.g. `chart[0]` contains all partial parse trees generated at the beginning of sentence

Chart Entries

- Three types of constituents:
 - Predicted constituents
 - In-progress constituents
 - Completed constituents

Parse Progress

- Represented by Dotted Rules
 - Position of • indicates type of constituent
- $_0$ Book $_1$ that $_2$ flight $_3$
 - $S \rightarrow \bullet VP$ [0,0] (predicted)
 - $NP \rightarrow Det \bullet Nom$ [1,2] (in progress)
 - $VP \rightarrow V NP \bullet$ [0,3] (completed)
- [x,y] tells us what portion of the input is spanned so far by rule
- Each state s_i : $\langle dotted\ rule \rangle, [\langle back\ pointer \rangle, \langle current\ position \rangle]$

$_0$ Book $_1$ that $_2$ flight $_3$

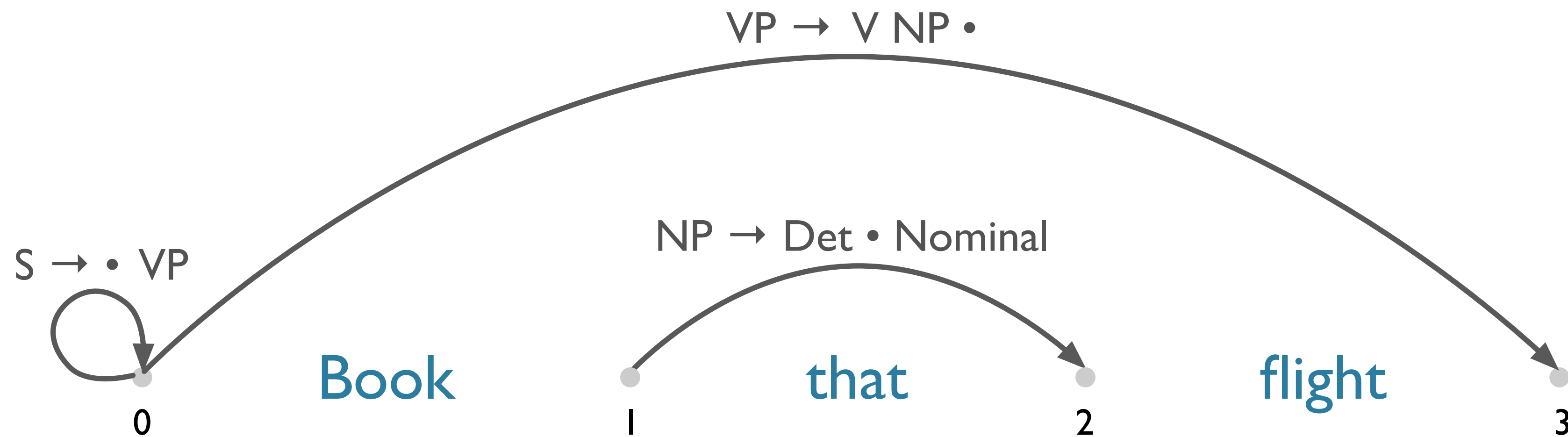
- $S \rightarrow \cdot VP, [0,0]$
 - First 0 means S constituent begins at the start of input
 - Second 0 means the dot is here too
 - So, this is a top-down prediction

0 Book 1 that 2 flight 3

- $S \rightarrow \cdot VP, [0,0]$
 - First 0 means S constituent begins at the start of input
 - Second 0 means the dot is here too
 - So, this is a top-down prediction
- $NP \rightarrow Det \cdot Nom, [1,2]$
 - the NP begins at position 1
 - the dot is at position 2
 - so, Det has been successfully parsed
 - Nom predicted next

0 Book 1 that 2 flight 3 (continued)

- $V \rightarrow V NP \cdot [0,3]$
- Successful VP parse of entire input



Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles $S \rightarrow \alpha \cdot [0,N]$ then input parsed successfully
- Chart will also contain record of all possible parses of input string, given the grammar

Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operations:
 - **predictor**: add predictions to the chart
 - **scanner**: read input and add corresponding state to chart
 - **completer**: move dot to right when new constituent found
- Results (new states) added to current or next set of states in chart
- No backtracking and no states removed: keep complete history of parse

Earley Algorithm

```
function EARLEY-PARSE(words, grammar) returns chart
  ENQUEUE( $(\gamma \rightarrow \bullet S, [0,0])$ , chart[0])
  for  $i \leftarrow$  from 0 to LENGTH(words) do
    for each state in chart[i] do
      if INCOMPLETE?(state) and
        NEXT-CAT(state) is not a part of speech then
        PREDICTOR(state)
      elseif INCOMPLETE?(state) and
        NEXT-CAT(state) is a part of speech then
        SCANNER(state)
      else
        COMPLETER(state)
    end
  end
  return(chart)
```

Earley Algorithm

```
procedure PREDICTOR(( $A \rightarrow a \bullet B \beta$ ,  $[i,j]$ ))  
  for each ( $B \rightarrow \gamma$ ) in GRAMMAR-RULES-FOR( $B, grammar$ ) do  
    ENQUEUE(( $B \rightarrow \bullet \gamma$ ,  $[j,j]$ ),  $chart[j]$ )  
  end
```

```
procedure SCANNER(( $A \rightarrow a \bullet B \beta$ ,  $[i,j]$ ))  
  if  $B \in \text{PARTS-OF-SPEECH}(word[j])$  then  
    ENQUEUE(( $B \rightarrow word[j] \bullet$ ,  $[j,j+1]$ ),  $chart[j+1]$ )
```

```
procedure COMPLETER(( $B \rightarrow \gamma \bullet$ ,  $[j,k]$ ))  
  for each ( $A \rightarrow a \bullet B \beta$ ,  $[i,j]$ ) in  $chart[j]$  do  
    ENQUEUE(( $A \rightarrow a B \bullet \beta$ ,  $[i,k]$ ),  $chart[k]$ )  
  end
```

3 Main Subroutines of Earley

- Predictor
 - Adds predictions into the chart
- Scanner
 - Reads the input words and enters states representing those words into the chart
- Completer
 - Moves the dot to the right when new constituents are found

Predictor

- Intuition:
 - Create new state for top-down prediction of new phrase
- Applied when non part-of-speech non-terminals are to the right of a dot:
 - $S \rightarrow \cdot VP$ [0,0]
- Adds new states to *current* chart
 - One new state for each expansion of the non-terminal in the grammar
 - $VP \rightarrow \cdot V$ [0,0]
 - $VP \rightarrow \cdot V NP$ [0,0]

Chart[0]

S0	$\gamma \rightarrow \cdot S$	[0,0]	Dummy start state
S1	$S \rightarrow \cdot NP VP$	[0,0]	Predictor
S2	$S \rightarrow \cdot Aux NP VP$	[0,0]	Predictor
S3	$S \rightarrow \cdot VP$	[0,0]	Predictor
S4	$NP \rightarrow \cdot Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \cdot Proper-Noun$	[0,0]	Predictor
S6	$NP \rightarrow \cdot Det Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \cdot Verb$	[0,0]	Predictor
S8	$VP \rightarrow \cdot Verb NP$	[0,0]	Predictor
S9	$VP \rightarrow \cdot Verb NP PP$	[0,0]	Predictor
S10	$VP \rightarrow \cdot Verb PP$	[0,0]	Predictor
S11	$VP \rightarrow \cdot VP PP$	[0,0]	Predictor

Chart[1]

S12	$Verb \rightarrow book \cdot$	[0,1]	Scanner
S13	$VP \rightarrow Verb \cdot$	[0,1]	Completer
S14	$VP \rightarrow Verb \cdot NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \cdot NP PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \cdot PP$	[0,1]	Completer
S17	$S \rightarrow VP \cdot$	[0,1]	Completer
S18	$VP \rightarrow VP \cdot PP$	[0,1]	Completer
S19	$NP \rightarrow \cdot Pronoun$	[1,1]	Predictor
S20	$NP \rightarrow \cdot Proper-Noun$	[1,1]	Predictor
S21	$NP \rightarrow \cdot Det Nominal$	[1,1]	Predictor
S22	$PP \rightarrow \cdot Prep NP$	[1,1]	Predictor

Book that flight

S0: $\gamma \rightarrow \bullet S[0,0]$

γ
|
 $\bullet S$

Book that flight

S0: $\gamma \rightarrow \cdot S [0,0]$

S3: $S \rightarrow \cdot VP [0,0]$

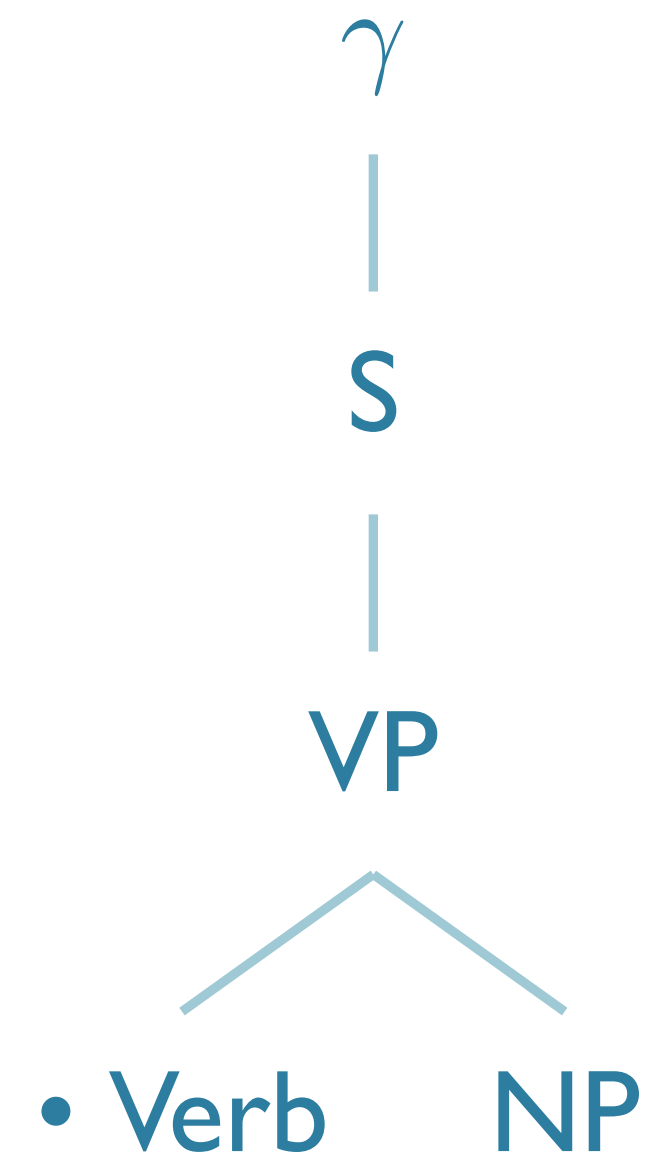
γ
|
S
|
 $\bullet VP$

Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow \cdot VP$ [0,0]

S8: $VP \rightarrow \cdot Verb NP$ [0,0]



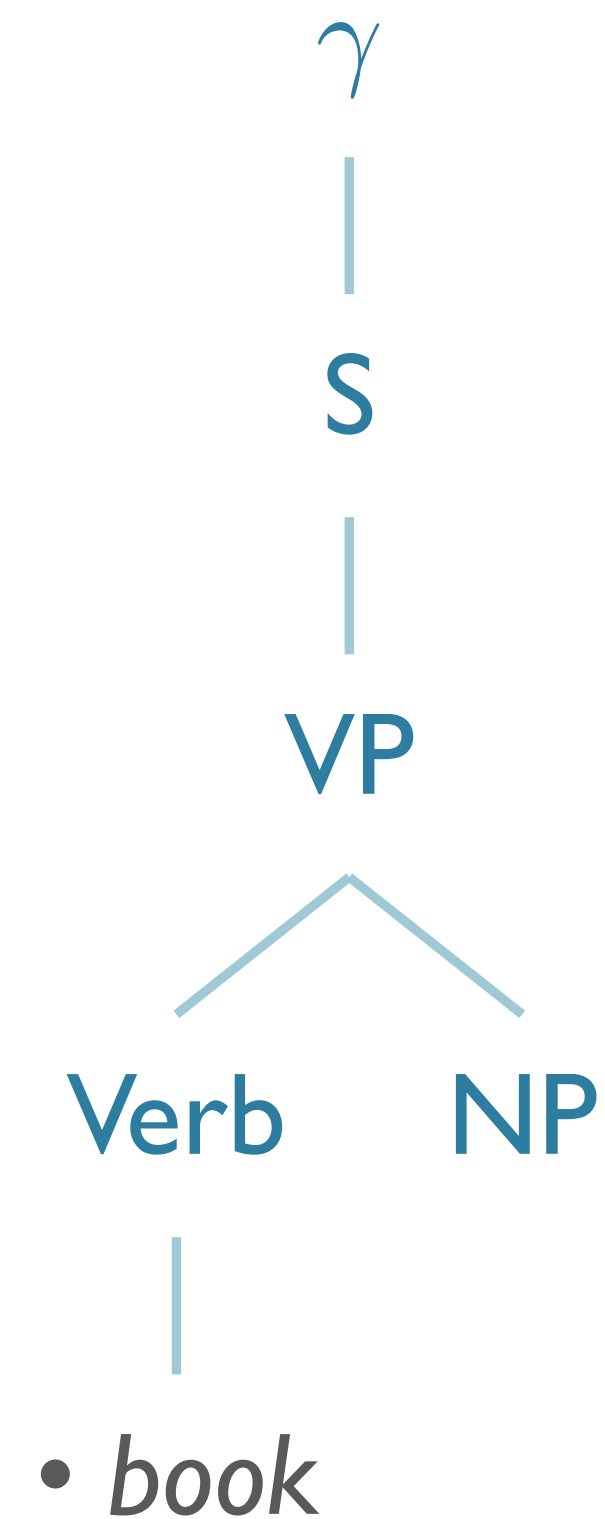
Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow \cdot VP$ [0,0]

S8: $VP \rightarrow \cdot Verb\ NP$ [0,0]

S12: $Verb \rightarrow \cdot book$ [0,0]



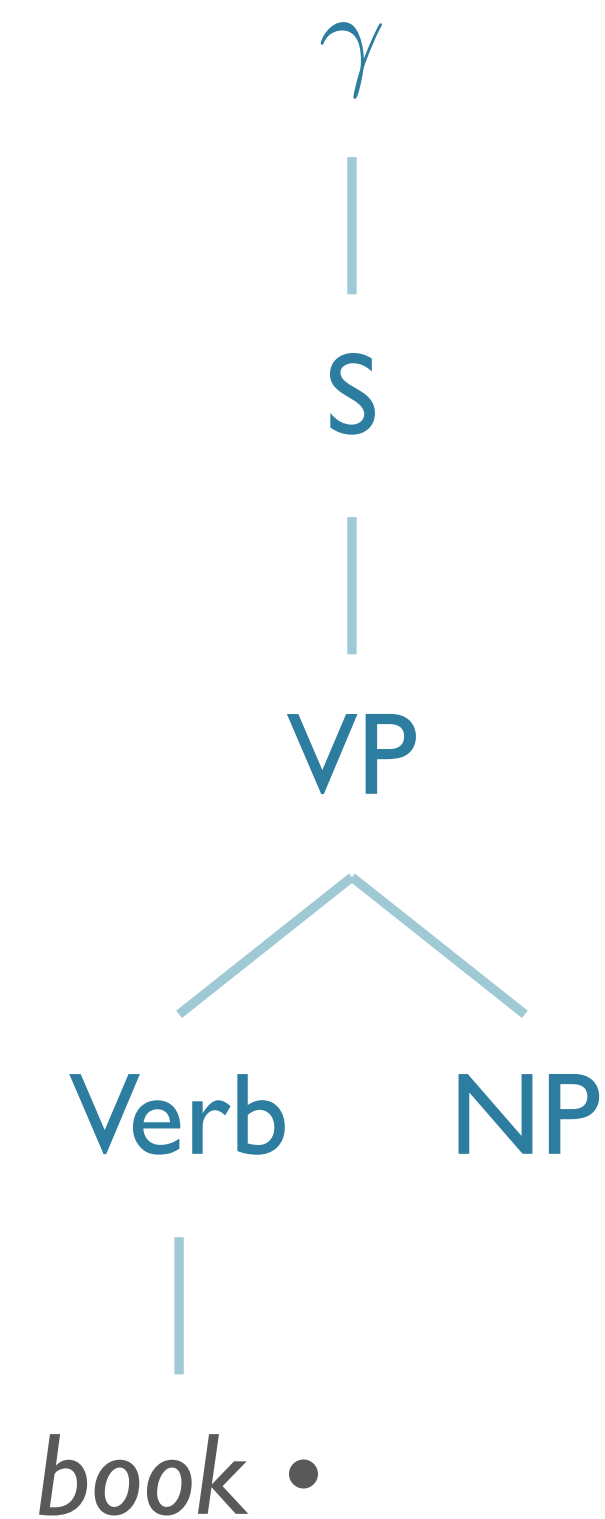
Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow \cdot VP$ [0,0]

S8: $VP \rightarrow \cdot Verb NP$ [0,0]

S12: $Verb \rightarrow book \cdot$ [0,1]

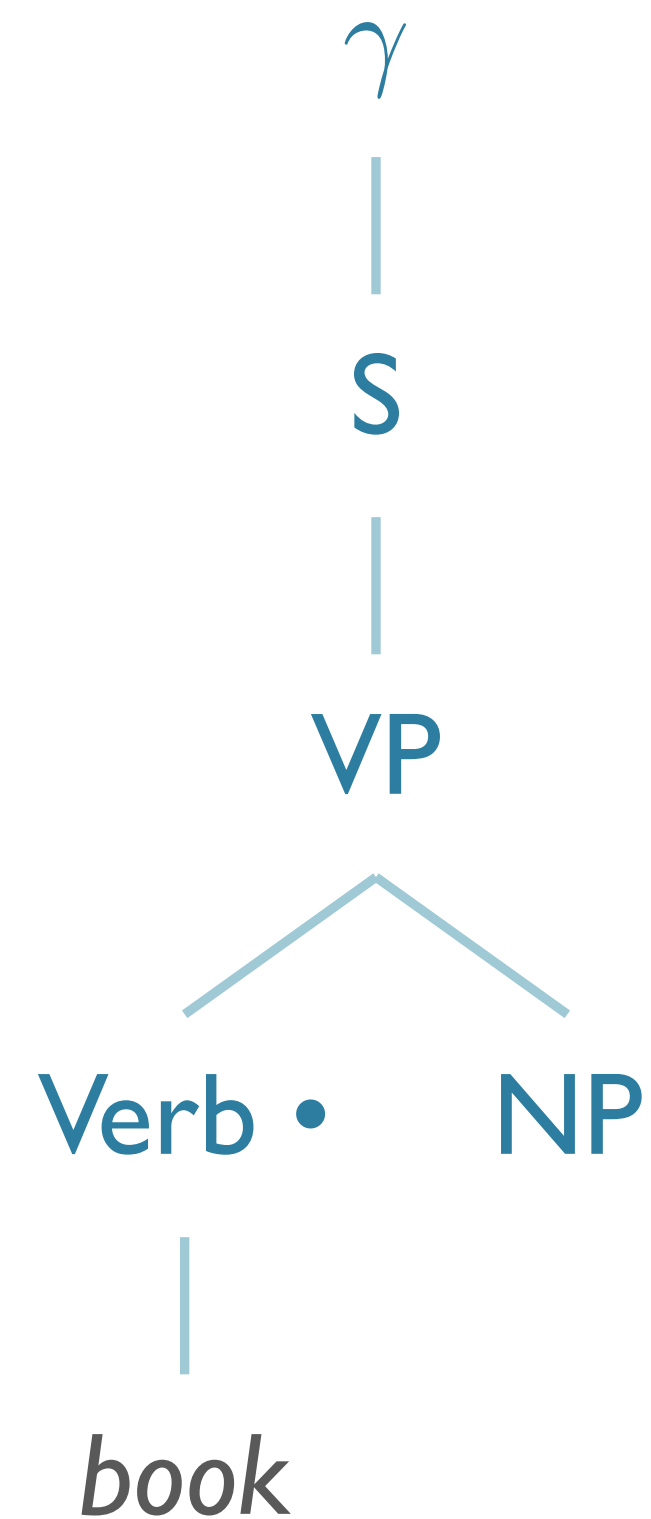


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S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow \cdot VP$ [0,0]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]



Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]



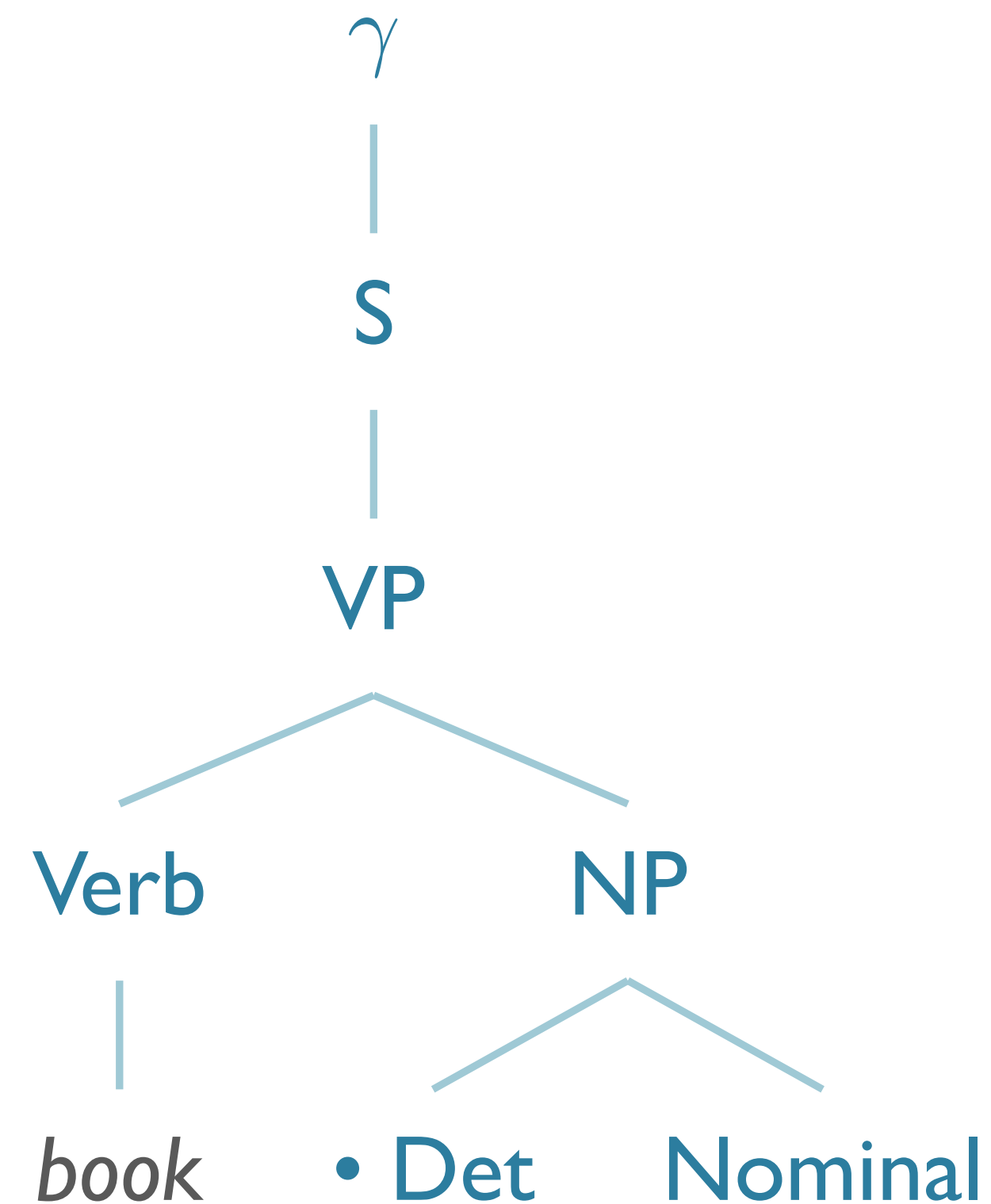
Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow \cdot Det\ Nominal$ [1,1]



Book that flight

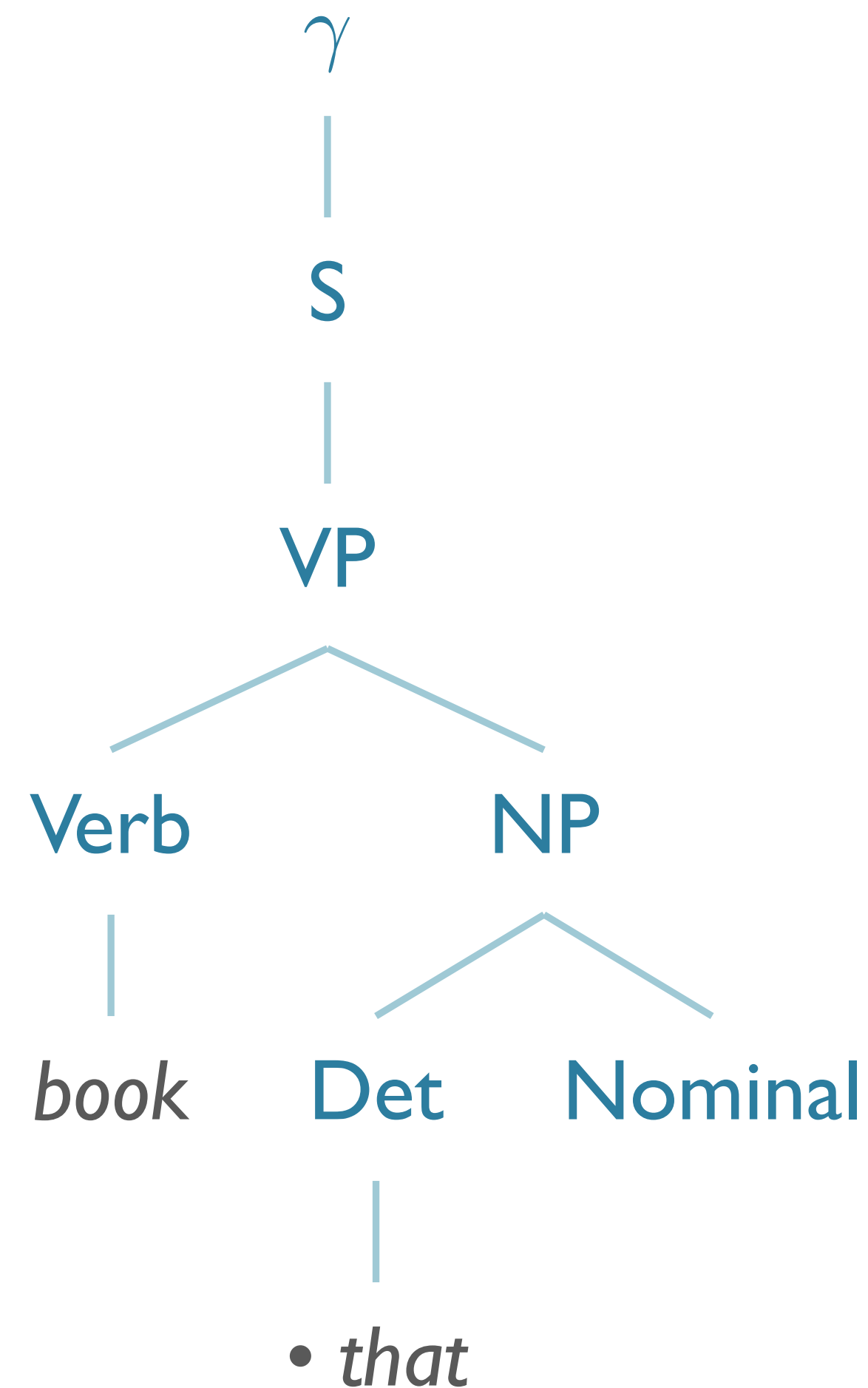
S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow \cdot Det\ Nominal$ [1,1]

S23: $Det \rightarrow \cdot \text{"that"}$ [1,1]



Book that flight

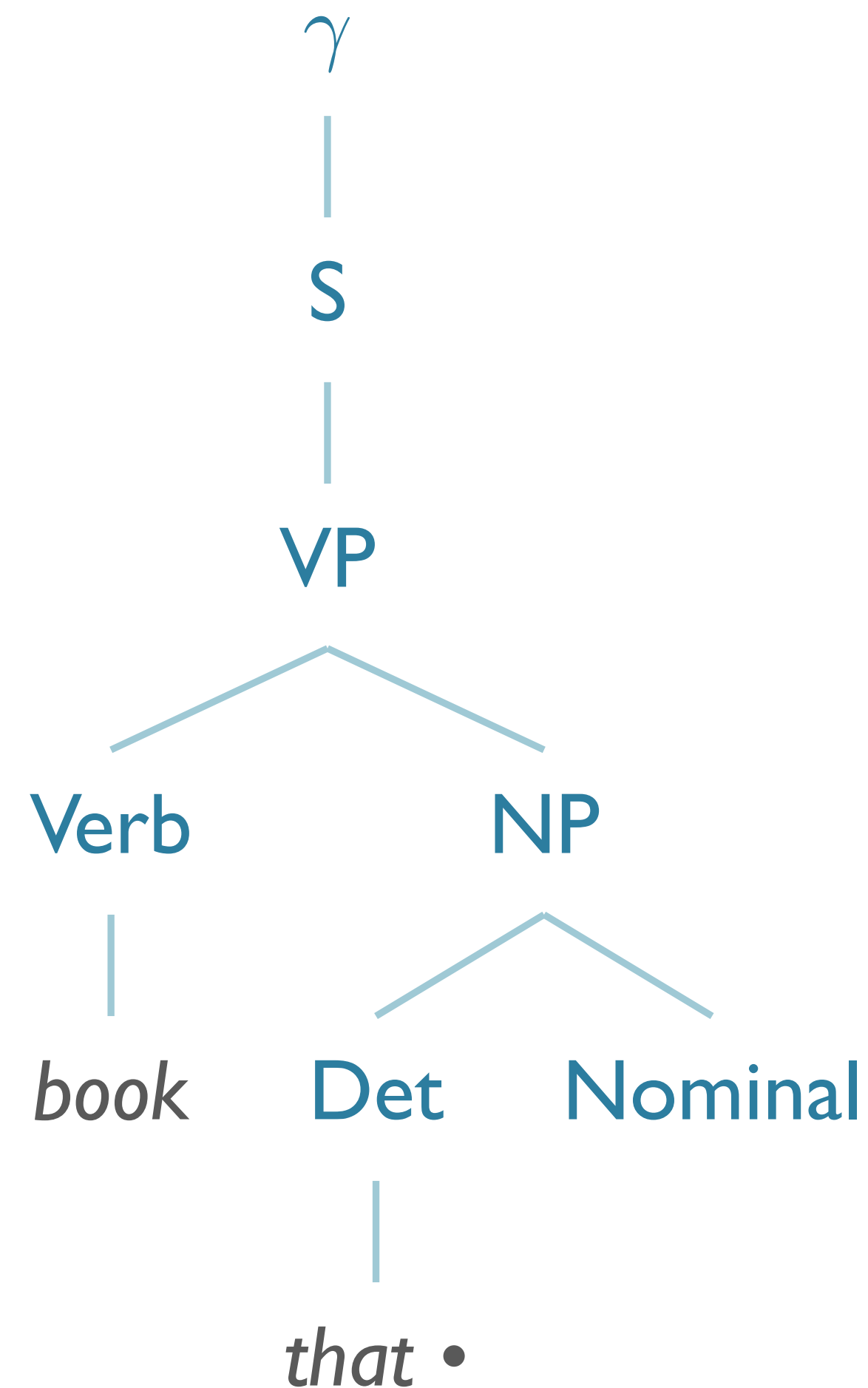
S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow \cdot Det\ Nominal$ [1,1]

S23: $Det \rightarrow \textit{“that”} \cdot$ [1,2]



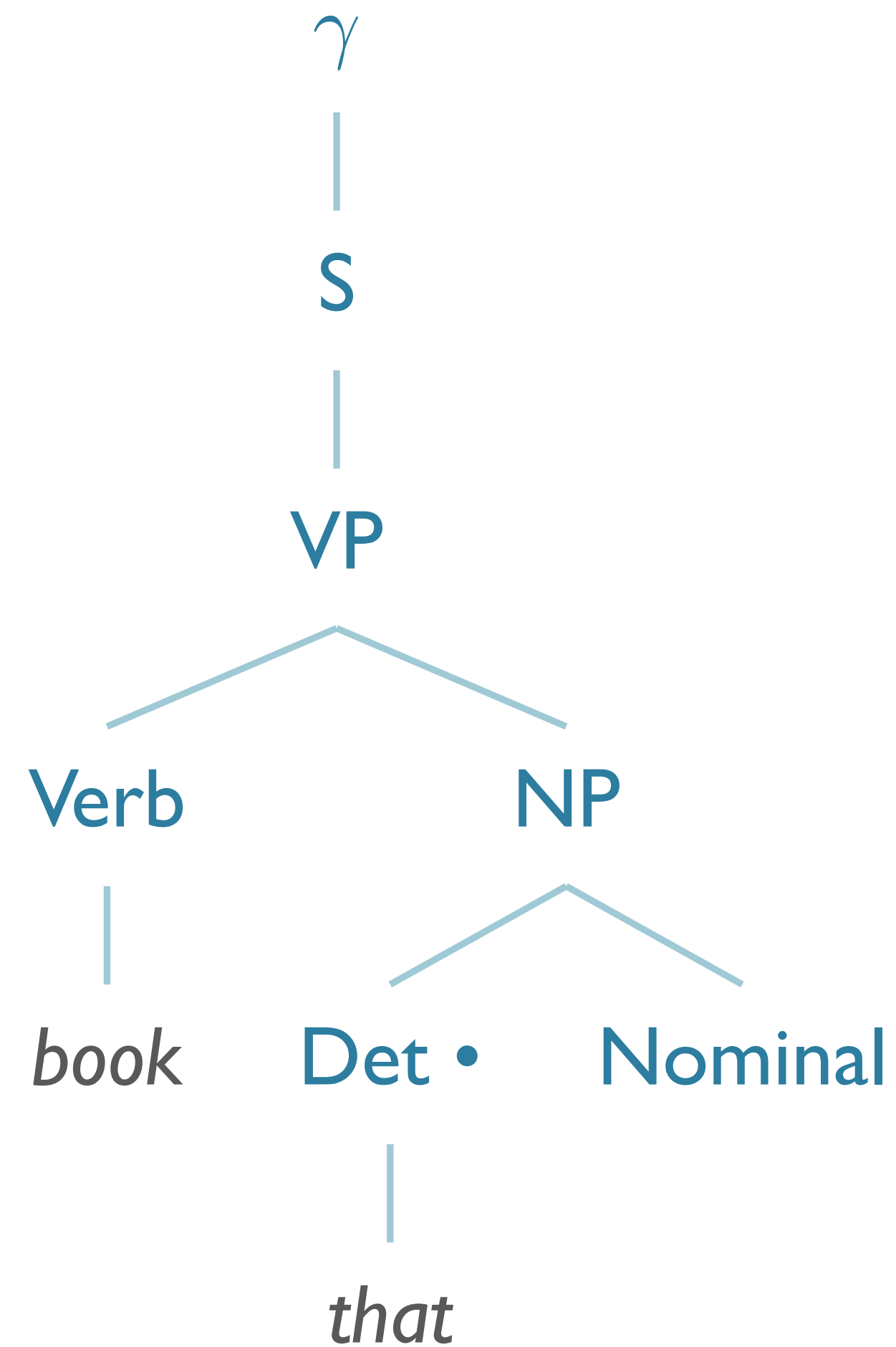
Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow Det \cdot Nominal$ [1,2]



Book that flight

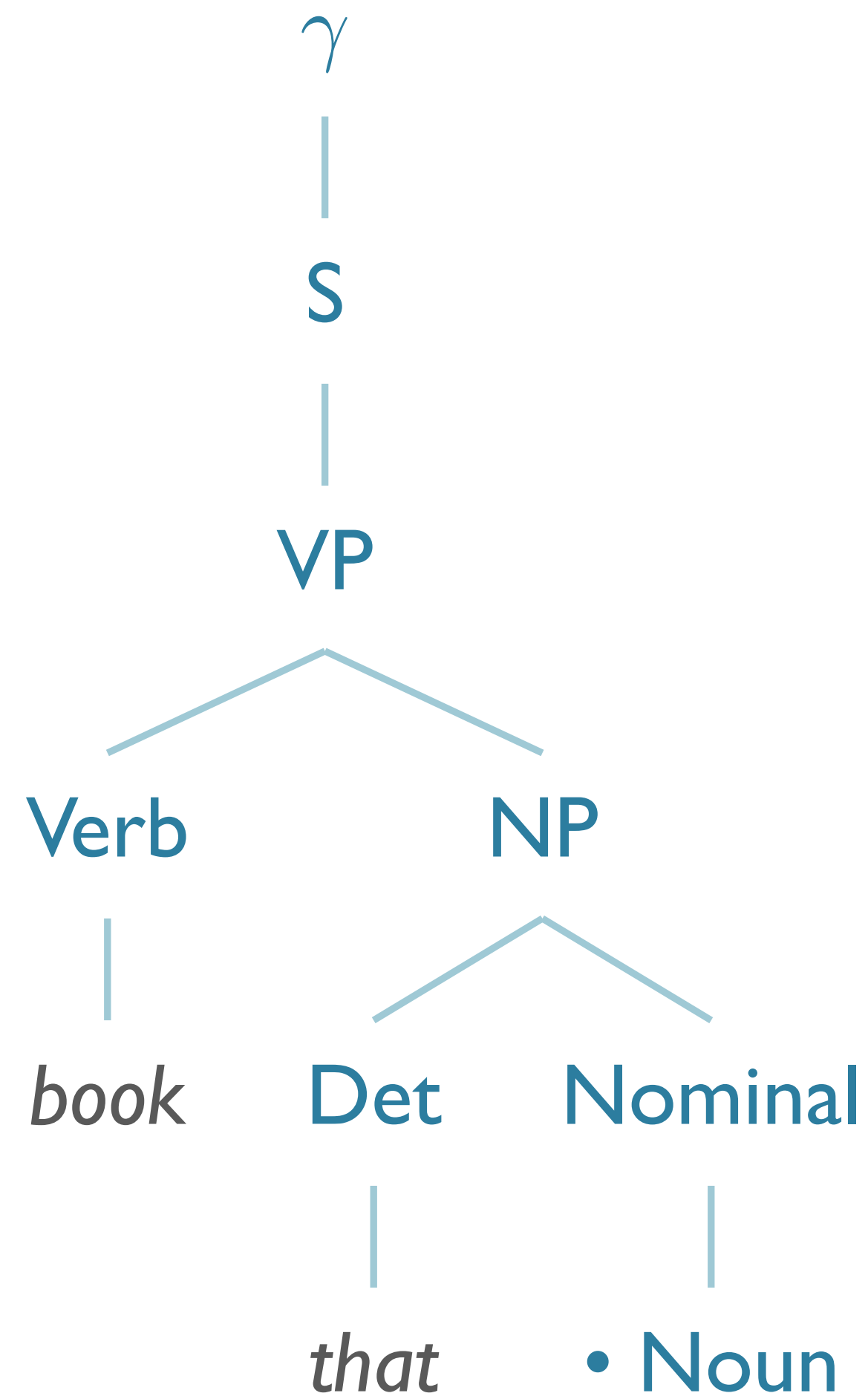
S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow Det \cdot Nominal$ [1,2]

S25: $Nominal \rightarrow \cdot Noun$ [2,2]



Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

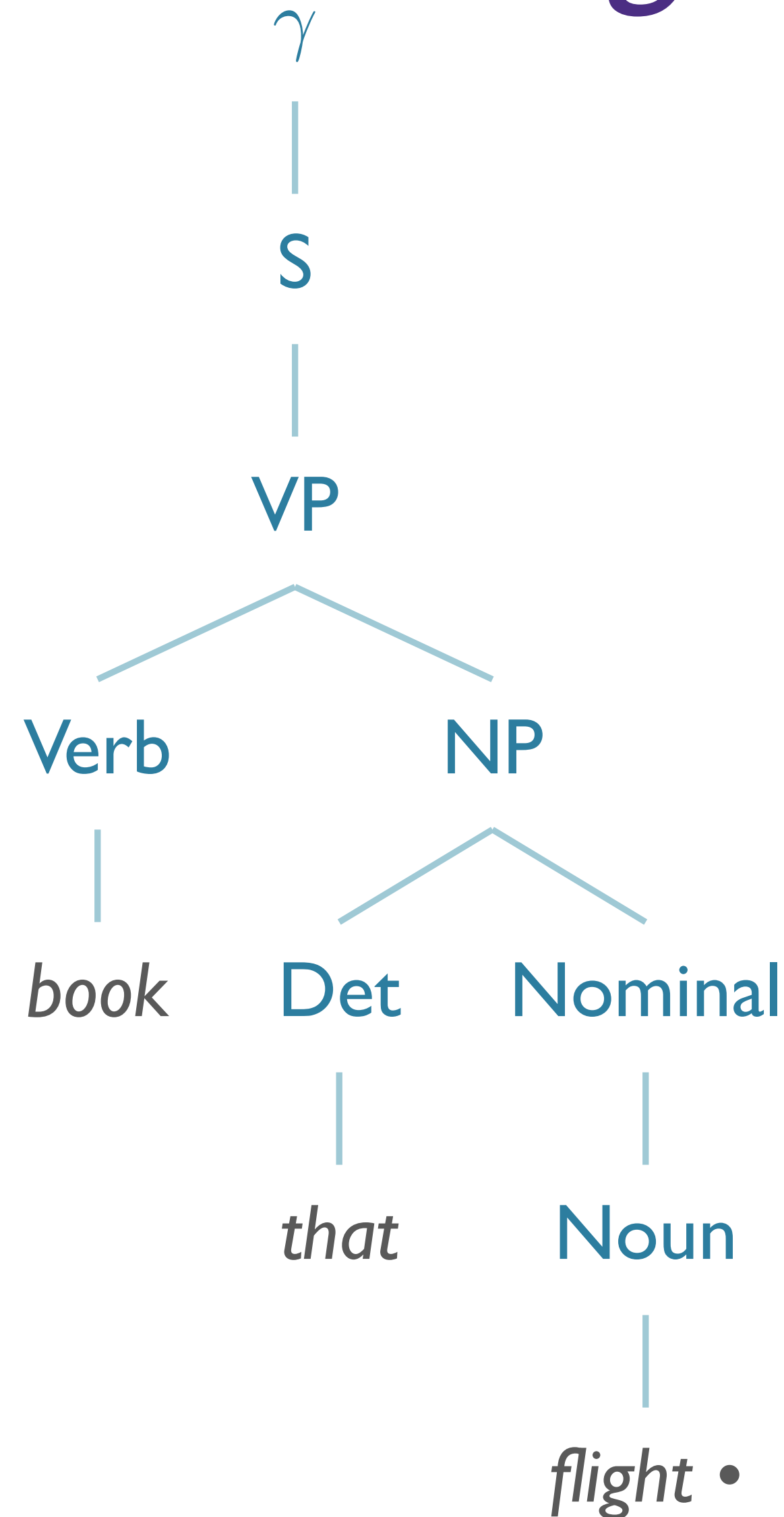
S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow Det \cdot Nominal$ [1,2]

S25: $Nominal \rightarrow \cdot Noun$ [2,2]

S28: $Noun \rightarrow \text{"flight"} \cdot$ [2,3]



Book that flight

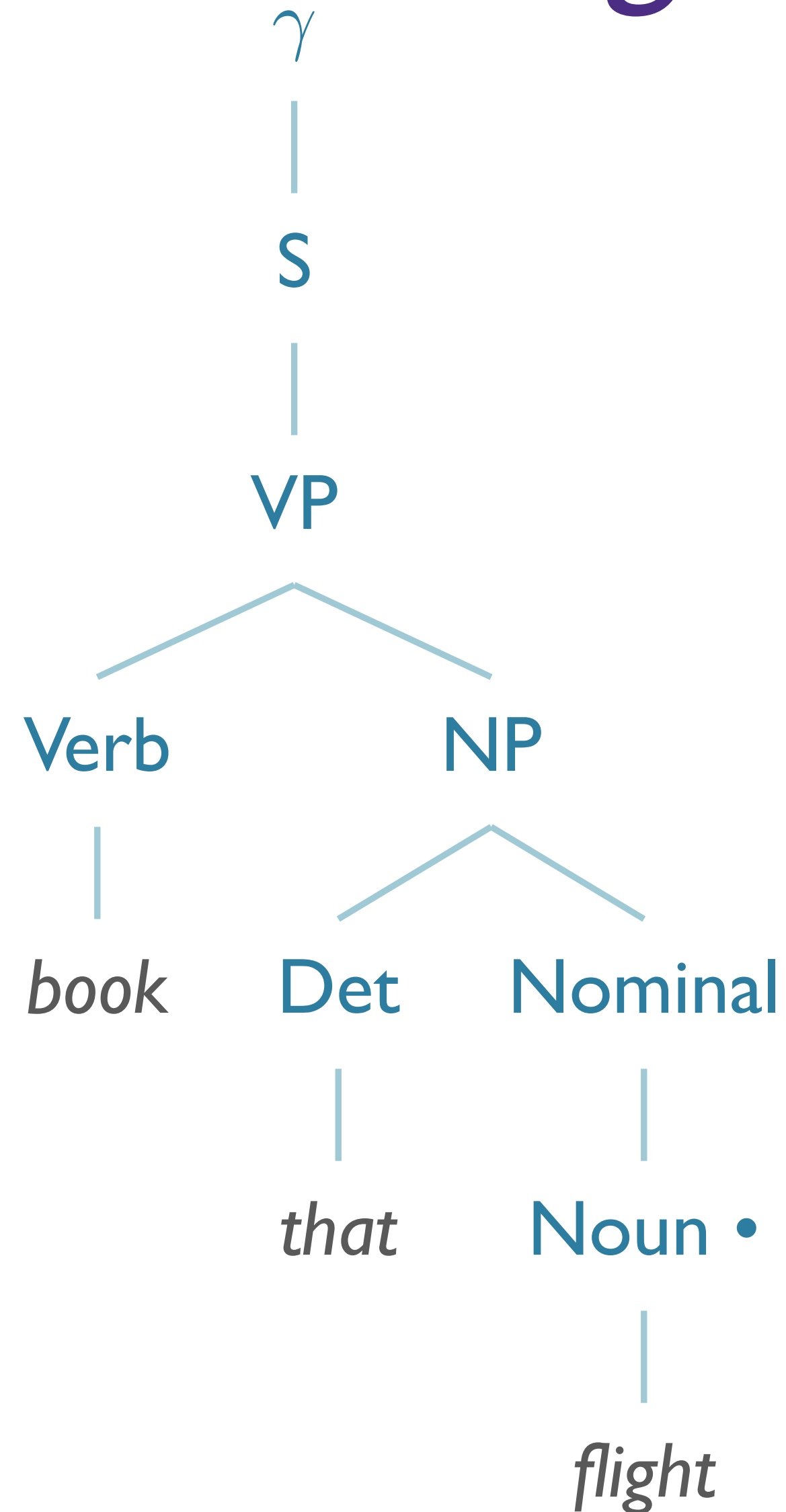
S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow Det \cdot Nominal$ [1,2]

S25: $Nominal \rightarrow Noun \cdot$ [2,3]



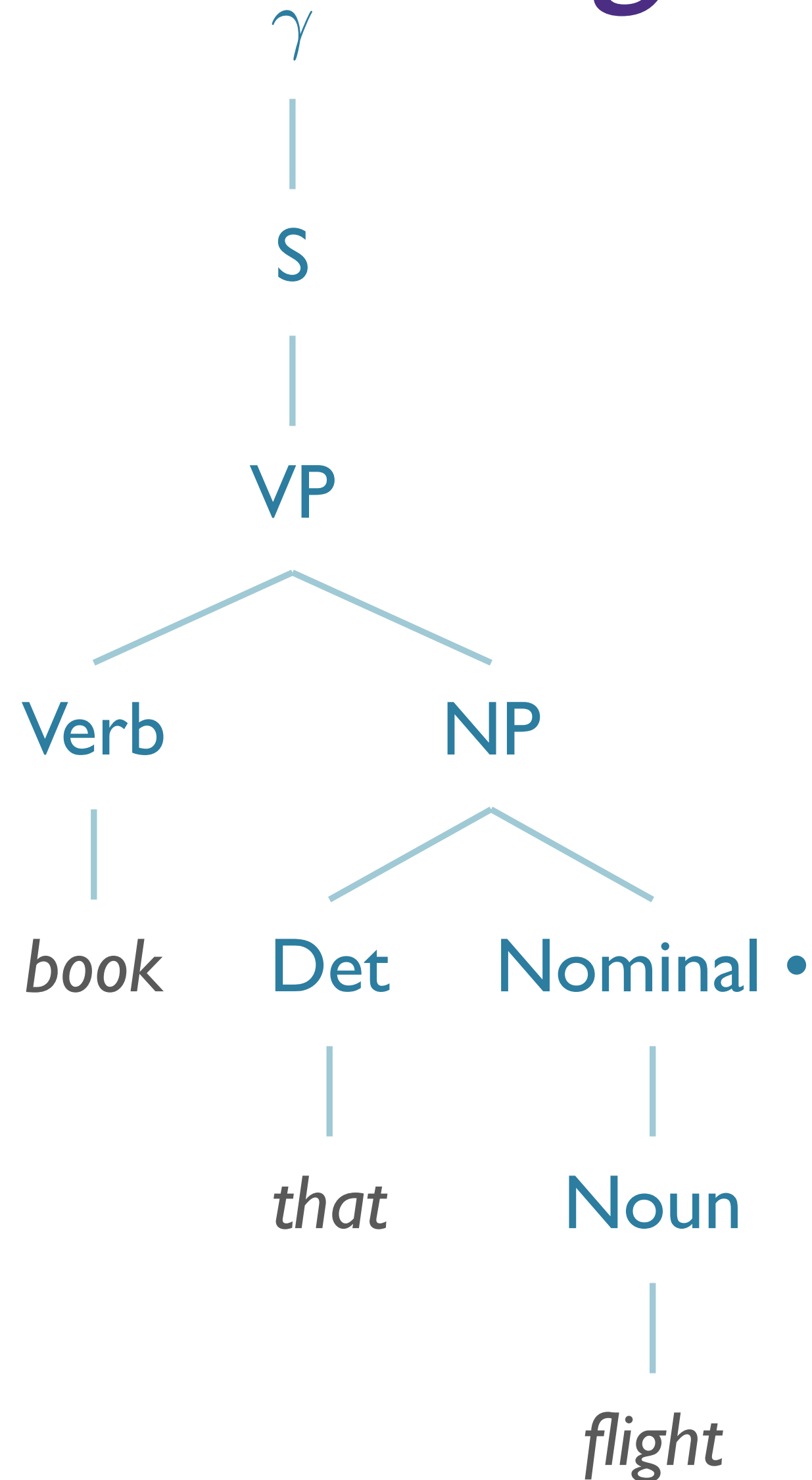
Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb \cdot NP$ [0,1]

S21: $NP \rightarrow Det\ Nominal \cdot$ [1,3]

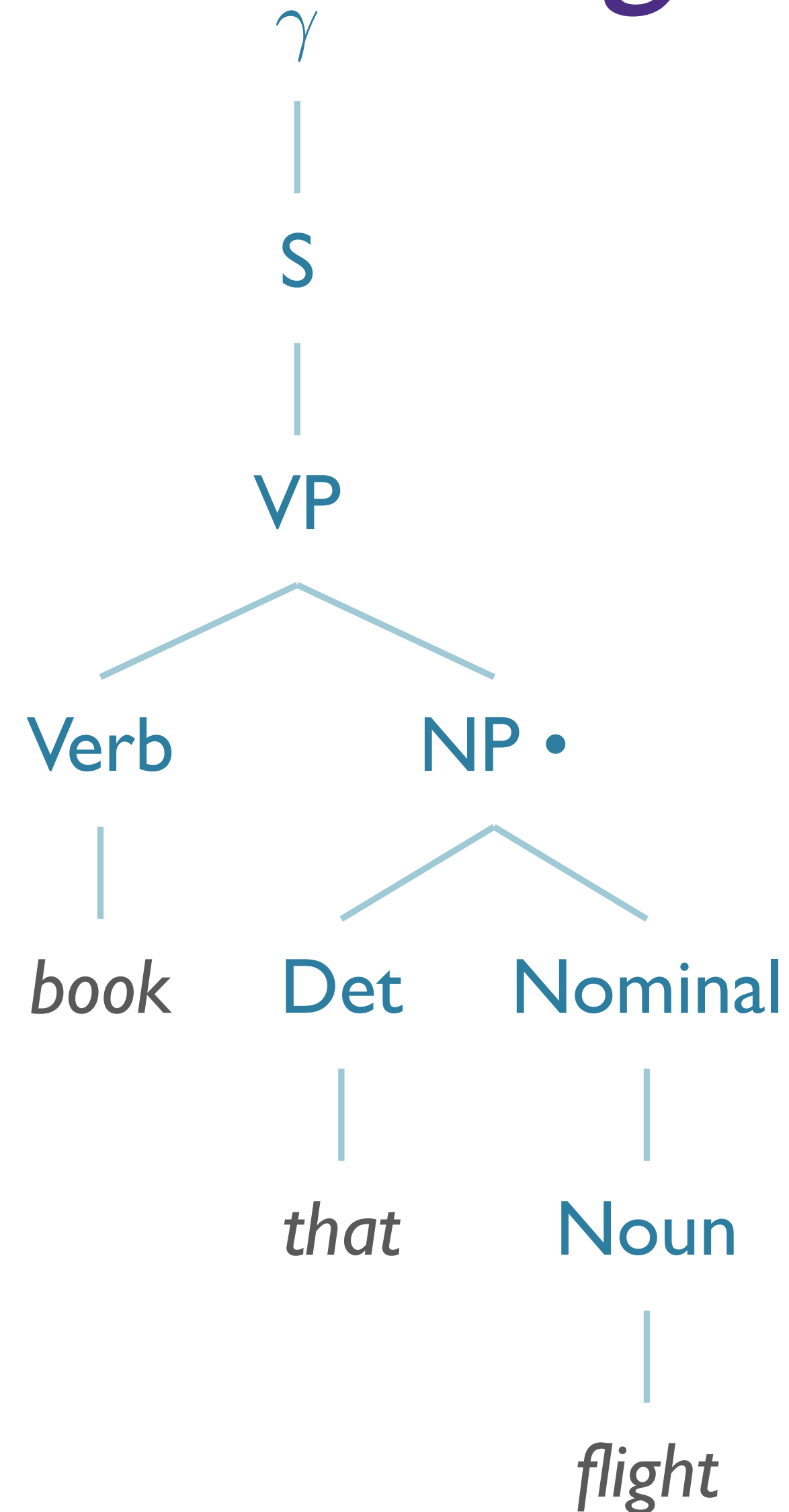


Book that flight

S0: $\gamma \rightarrow \cdot S$ [0,0]

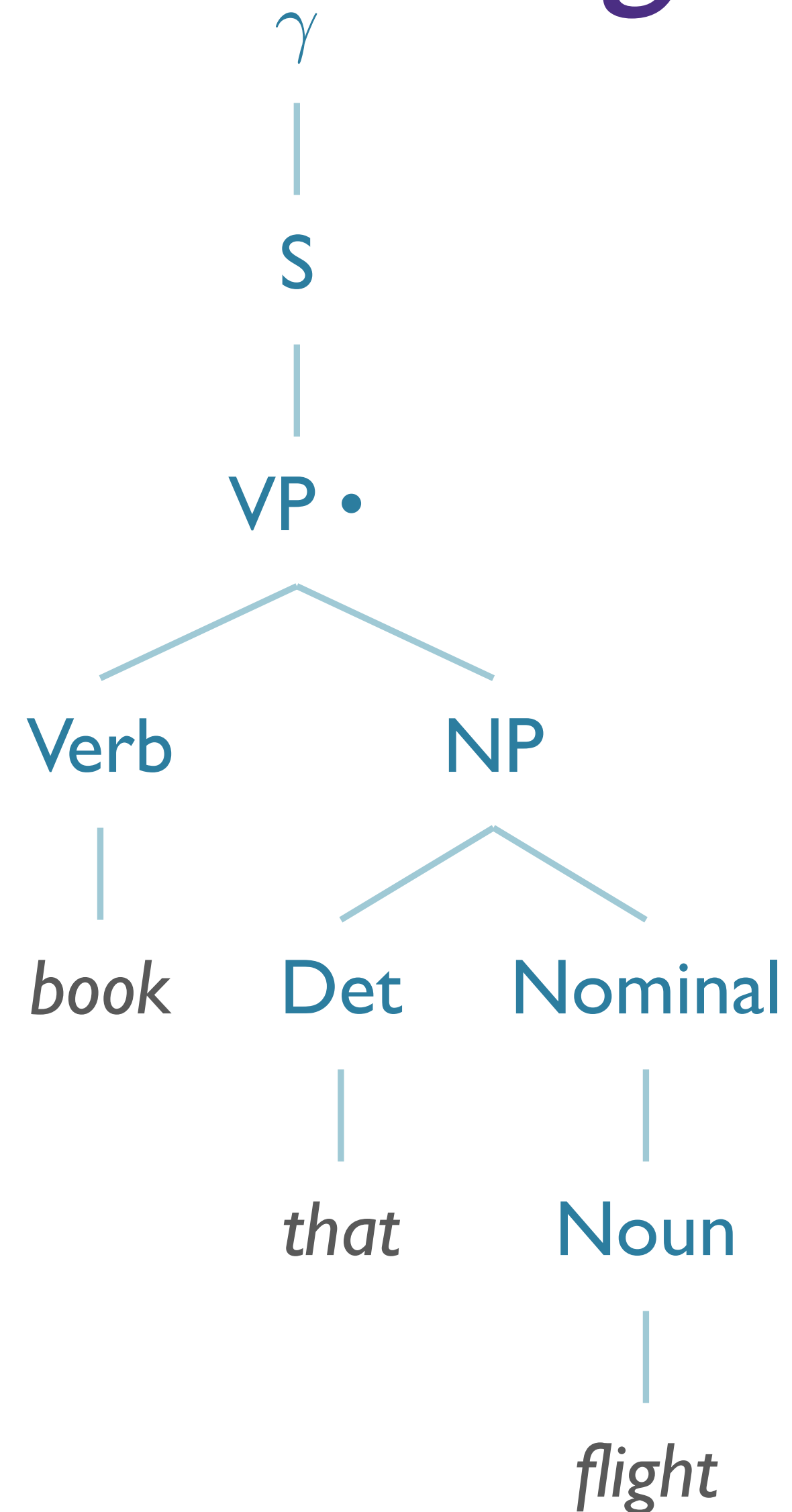
S3: $S \rightarrow VP \cdot$ [0,1]

S8: $VP \rightarrow Verb NP \cdot$ [0,3]



Book that flight

S0: $\gamma \rightarrow \cdot S [0,0]$
S3: $S \rightarrow VP \cdot [0,3]$



What About Dead Ends?

Book that flight

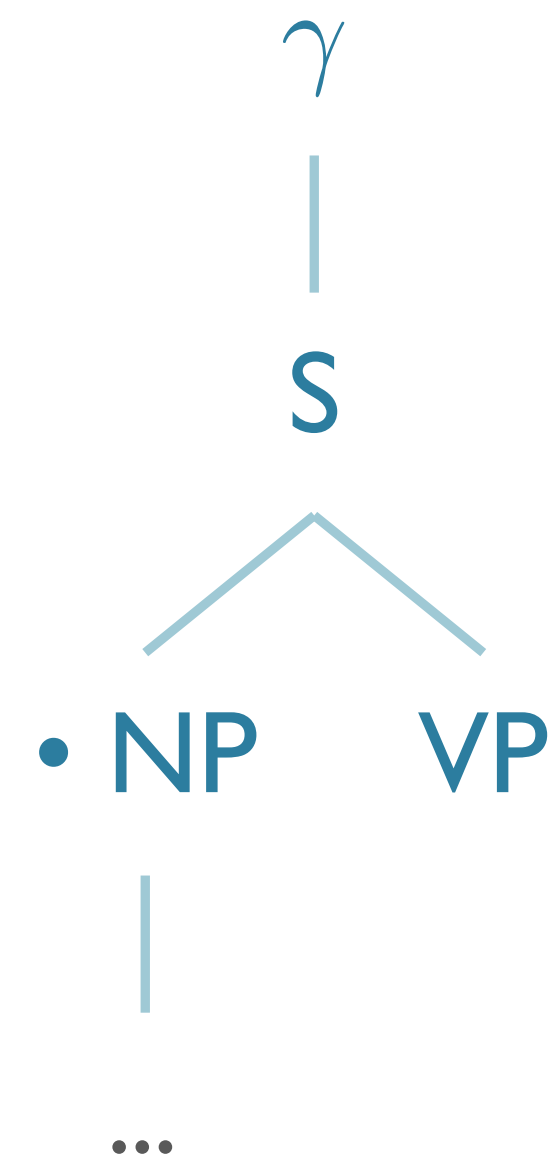
S0: $\gamma \rightarrow \cdot S$ [0,0]

S1: $S \rightarrow \cdot NP VP$ [0,0]

$NP \rightarrow \cdot Pronoun$

$NP \rightarrow \cdot Proper-Noun$

$NP \rightarrow \cdot Det Nominal$



book

Book that flight

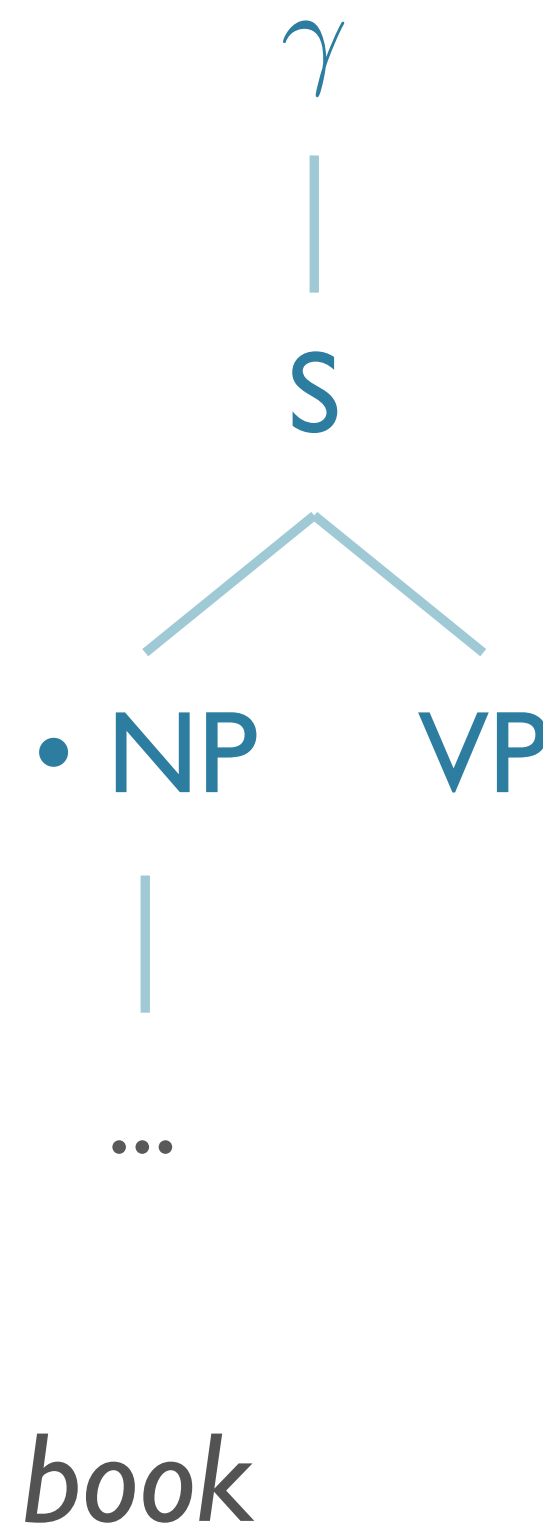
S0: $\gamma \rightarrow \cdot S$ [0,0]

S1: $S \rightarrow \cdot NP VP$ [0,0]

~~$NP \rightarrow \cdot$ Pronoun~~

~~$NP \rightarrow \cdot$ Proper-Noun~~

~~$NP \rightarrow \cdot$ Det Nominal~~



What About Recursion?

What about recursion?

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- No!

```
procedure ENQUEUE(state, chart-entry)  
  if state is not already in chart-entry then  
    PUSH(state, chart-entry)  
  end
```

What about recursion?

- We now have a top-down parser in hand. Does it enter infinite loops on rules like $S \rightarrow S \text{ 'and' } S$?
- No!

```
procedure ENQUEUE(state, chart-entry)  
  if state is not already in chart-entry then  
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  end
```

Exercise: parse 'table and chair' using the very simple grammar
 $\text{Nom} \rightarrow \text{Nom 'and' Nom} \mid \text{'table'} \mid \text{'chair'}$