A Formal Boundary Logic for Validity, Authority, and Verification

1 Motivation and Scope

Modern systems fail not primarily due to local error, but due to *boundary failure*: claims, decisions, or validations are transported beyond the region in which they were authorised, specified, or verified. Existing logics presuppose a single ambient context of validity. This assumption is false for real systems.

This document introduces a *Formal Boundary Logic* whose primary purpose is to make boundaries explicit objects of reasoning. The logic is designed to model:

- Region-bounded validity
- Authority-limited inference
- Temporal and procedural closure
- Failure of cross-context transport

The logic is intentionally conservative. It does not replace existing logics. It constrains their application.

2 Regions, Claims, and Judgements

Definition 1 (Regions)**.** A *region* R is an abstract context of validity. Intuitively, a region corresponds to a bounded domain such as a model scope, organisational authority, regulatory mandate, temporal window, or epistemic perspective.

The class of regions is denoted $\mathcal{R}$.

Definition 2 (Claims)**.** Let $\mathcal{P}$ be a set of propositions or claims. These may be logical propositions, empirical assertions, certification outcomes, or policy judgements.

Definition 3 (Regional Judgement)**.** A *regional judgement* is an expression of the form:

$$R \vdash P$$

read as: claim P is valid *within region* R.

Remark 1. The judgement $R \vdash P$ does not assert global truth. It asserts local validity relative to the constraints defining R.

3 Boundary Explicitness

Axiom 1 (No Implicit Universality)**.** There is no inference rule permitting:

$$R \vdash P \quad \Rightarrow \quad P$$

without an explicit boundary discharge.

Remark 2. This axiom blocks the most common source of boundary failure: treating local validation as global fact.

4 Boundary Morphisms

Definition 4 (Boundary Morphism)**.** A *boundary morphism* from region R to region S is a partial function:

$$\phi : R \rightsquigarrow S$$

that specifies how claims valid in R may be transported into S.

Definition 5 (Transport Rule)**.** A boundary morphism $\phi : R \rightsquigarrow S$ *supports* claim P if:

$$R \vdash P \quad \Rightarrow \quad S \vdash \phi(P)$$

Axiom 2 (No Free Transport)**.** If no boundary morphism $\phi : R \rightsquigarrow S$ is defined, then no inference from $R \vdash P$ to $S \vdash P$ is permitted.

5 Authority and Scope

Definition 6 (Authority Function)**.** An authority function $\mathsf{Auth} : \mathcal{R} \to \mathcal{A}$ assigns to each region its authority class.

Axiom 3 (Authority Constraint)**.** A boundary morphism $\phi : R \rightsquigarrow S$ is invalid if:

$$\mathsf{Auth}(R) \not\geq \mathsf{Auth}(S)$$

Remark 3. This formalises authority misalignment. Validation cannot flow upward or laterally without authorisation.

6 Temporal Closure

Definition 7 (Temporal Region)**.** A temporal region is a region R_t indexed by a time interval t.

Axiom 4 (Temporal Non-Extension)**.** If $R_{t_1} \vdash P$ and $t_2 > t_1$, then $R_{t_2} \vdash P$ is invalid unless a temporal morphism is explicitly defined.

7 Grounding

Definition 8 (Grounding)**.** A *grounding* between regions R and S is a pair (ϕ, ψ) where:

$$\phi : R \rightsquigarrow S, \quad \psi : S \rightsquigarrow R$$

such that transport preserves satisfaction conditions.

Theorem 1 (Grounding Necessity Theorem)**.** *Let $R, S \in \mathcal{R}$ and $P \in \mathcal{P}$. If:*

$$R \vdash P \quad \textit{and} \quad S \vdash P$$

then either:

1. *$R = S$, or*
2. *there exists a grounding between R and S.*

Proof. If $R \neq S$ and no grounding exists, then any agreement between $R \vdash P$ and $S \vdash P$ is accidental. By the No Free Transport axiom, validity cannot be shared. Therefore agreement without grounding is formally unlicensed. □

8 Boundary Failure Modes (Formal)

Each of the following is a *provable pathology.*

Proposition 1 (Silent Scope Expansion)**.** *If a claim is transported across regions without an explicit morphism, the resulting judgement is invalid.*

Proposition 2 (Self-Referential Validation)**.** *If a region defines its own authority and grounding without external constraint, all claims become trivially valid and the system collapses.*

Proposition 3 (Human-in-the-Loop Illusion)**.** *If a human judgement is introduced without being modelled as a region with explicit authority, the system falsely appears grounded.*

9 Meta-Logical Status

This logic is:

- Conservative over underlying logics
- Non-classical in its rejection of implicit globality
- Compatible with proof assistants as a meta-layer

It is intentionally incomplete. Boundary logic constrains validity. It does not generate it.

10 Conclusion

Boundary Logic formalises a simple but systematically ignored fact: validity is regional. Claims do not travel for free. Authority, scope, and time are not annotations. They are preconditions of meaning.

This logic provides the formal spine required for verification sidecars, structured audits, and adversarial certification testing. Those constructions are deferred.