



(2) (기본)

$$\frac{\partial \ell^1}{\partial w_{ij}^1} = x_i, \quad \frac{\partial \ell^1}{\partial b_i^1} = 1 \quad \left| \quad \frac{\partial \ell^2}{\partial w_{ij}^2} = h_i', \quad \frac{\partial \ell^2}{\partial b_i^2} = 1, \quad \frac{\partial \ell^2}{\partial h_i^2} = (w_{i1}^2)^*$$

$$\frac{\partial h^1}{\partial \ell^1} = h^1(1-h^1) \quad \left| \quad \frac{\partial h^2}{\partial \ell^2} = h^2(1-h^2)$$

$$\frac{\partial \hat{y}}{\partial w_{ij}^3} = h_i^2, \quad \frac{\partial \hat{y}}{\partial b_i^3} = 1, \quad \frac{\partial \hat{y}}{\partial h_i^2} = (w_{i1}^3)^*, \quad \frac{\partial L}{\partial \hat{y}} = \frac{2}{3} \sum_{k=1}^3 (\hat{y}_k - y_k)$$

$$\frac{\partial L}{\partial w_{11}^1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_1^2} \frac{\partial h_1^2}{\partial \ell^2} \frac{\partial \ell^2}{\partial h_1'} \frac{\partial h_1'}{\partial \ell^1} \frac{\partial \ell^1}{\partial w_{11}^1} \quad \left| \quad \frac{\partial L}{\partial b_1^1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_1^2} \frac{\partial h_1^2}{\partial \ell^2} \frac{\partial \ell^2}{\partial b_1^1}$$

$$\frac{\partial L}{\partial w_{12}^1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_1^2} \frac{\partial h_1^2}{\partial \ell^2} \frac{\partial \ell^2}{\partial h_1'} \frac{\partial h_1'}{\partial \ell^1} \frac{\partial \ell^1}{\partial w_{12}^1} \quad \left| \quad \frac{\partial L}{\partial b_2^1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_1^2} \frac{\partial h_1^2}{\partial \ell^2} \frac{\partial \ell^2}{\partial b_2^1}$$

$$\frac{\partial L}{\partial w_{11}^3} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_{11}^3} \quad \left| \quad \frac{\partial L}{\partial b_1^3} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial b_1^3}$$

$$\hat{y} - y = \begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix}$$

$$\textcircled{1} \frac{\partial L}{\partial w_{11}^3} = \sum \left(\begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix} \odot \begin{bmatrix} 0.75 \\ 0.94 \\ 0.97 \end{bmatrix} \right) \times \frac{2}{3} = \sum \begin{bmatrix} 1.9575 \\ 3.6848 \\ 4.8209 \end{bmatrix} \times \frac{2}{3} = 6.975$$

$$\textcircled{2} \frac{\partial L}{\partial w_{21}^3} = \frac{2}{3} \sum \left(\begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix} \odot \begin{bmatrix} 0.93 \\ 0.99 \\ 1.00 \end{bmatrix} \right) = \sum \begin{bmatrix} 2.4293 \\ 3.8808 \\ 4.97 \end{bmatrix} \times \frac{2}{3} = 7.519$$

$$\textcircled{3} \frac{\partial L}{\partial b_1^3} = \frac{2}{3} \sum \left(\begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix} \right) = 7.667$$

$$h_1^2 (1 - h_1^2) = \begin{bmatrix} 0.19 \\ 0.06 \\ 0.03 \end{bmatrix} \quad h_2 (1 - h_2^2) = \begin{bmatrix} 0.06 \\ 0.01 \\ 0.00 \end{bmatrix}$$

j=1

$$\frac{2}{3} \left(\begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix} \times w_{11}^3 \right) \odot \begin{bmatrix} 0.19 \\ 0.06 \\ 0.03 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0.49 \\ 0.23 \\ 0.13 \end{bmatrix} = B(h_1^2)$$

j=2

$$\frac{2}{3} \left(\begin{bmatrix} 2.61 \\ 3.92 \\ 4.97 \end{bmatrix} \times w_{21}^3 \right) \odot \begin{bmatrix} 0.06 \\ 0.01 \\ 0.00 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0.33 \\ 0.06 \\ 0.02 \end{bmatrix} = B(h_2^2)$$

$$\textcircled{4} \frac{\partial L}{\partial w_{11}^2} = \sum \left(\frac{2}{3} \begin{bmatrix} 0.49 \\ 0.23 \\ 0.13 \end{bmatrix} \odot \begin{bmatrix} 0.27 \\ 0.50 \\ 0.73 \end{bmatrix} \right) = 0.23$$

$$\frac{\partial L}{\partial v_{12}^2} = \sum (B(h_2^2) \odot h_1^1) = \frac{2}{3} \left(\begin{bmatrix} 0.33 \\ 0.06 \\ 0.02 \end{bmatrix} \odot \begin{bmatrix} 0.27 \\ 0.50 \\ 0.73 \end{bmatrix} \right) = 0.09$$

$$\frac{\partial L}{\partial w_{21}^2} = \sum (B(h_1^2) \odot h_2^1) = \frac{2}{3} \sum \left(\begin{bmatrix} 0.49 \\ 0.23 \\ 0.13 \end{bmatrix} \odot \begin{bmatrix} 0.27 \\ 0.73 \\ 0.95 \end{bmatrix} \right) = \frac{2}{3} \left(\begin{bmatrix} 0.1323 \\ 0.1679 \\ 0.1235 \end{bmatrix} \right) = 0.28$$

$$\frac{\partial L}{\partial w_{22}^2} = \sum (B(h_2^2) \odot h_2^1) = \frac{2}{3} \sum \left(\begin{bmatrix} 0.33 \\ 0.06 \\ 0.02 \end{bmatrix} \odot \begin{bmatrix} 0.27 \\ 0.73 \\ 0.95 \end{bmatrix} \right) = 0.10$$

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$$0.0891 + 0.0418 + 0.019$$

$$\textcircled{5} \quad \frac{\partial L}{\partial b_1^2} = \sum_{j=1}^2 \left(\frac{2}{3} \begin{bmatrix} 0.49 \\ 0.23 \\ 0.13 \end{bmatrix} \right) = 0.57, \quad \frac{\partial L}{\partial b_2^2} = \sum_{j=1}^2 \left(\frac{2}{3} \begin{bmatrix} 0.33 \\ 0.06 \\ 0.62 \end{bmatrix} \right) = 0.27$$

$B(h_1^2) \qquad B(h_2^2)$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h^2} \cdot \frac{\partial h^2}{\partial \ell^2} \cdot \frac{\partial \ell^2}{\partial h'} \cdot \frac{\partial h'}{\partial \ell'} \cdot \frac{\partial \ell'}{\partial w_{ij}}$$

$$\frac{\partial L}{\partial b_1'} = \underbrace{\frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h^2} \cdot \frac{\partial h^2}{\partial \ell^2}}_{B(h)} \cdot \frac{\partial \ell^2}{\partial h'} \cdot \frac{\partial h'}{\partial \ell'} \cdot \frac{\partial \ell'}{\partial b_1'}$$

$$h_1' (1 - h_1') = \begin{bmatrix} 0.20 \\ 0.25 \\ 0.20 \end{bmatrix} \quad h_2' (1 - h_2') = \begin{bmatrix} 0.20 \\ 0.20 \\ 0.05 \end{bmatrix}$$

$$j=1 : \frac{\partial L}{\partial w_{11}'} = \left(w_{11}^2 B(h_1^2) + w_{12}^2 B(h_2^2) \right) \odot \begin{bmatrix} 0.20 \\ 0.25 \\ 0.20 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0.28 \\ 0.09 \\ 0.03 \end{bmatrix} = B(h_1')$$

$$j=2 : \frac{\partial L}{\partial w_{12}'} = \left(w_{21}^2 B(h_1^2) + w_{22}^2 B(h_2^2) \right) \odot \begin{bmatrix} 0.20 \\ 0.20 \\ 0.05 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0.55 \\ 0.18 \\ 0.02 \end{bmatrix} = B(h_2')$$

$$\textcircled{6} \quad \frac{\partial L}{\partial w_{11}'} = \frac{2}{3} \begin{bmatrix} 0.28 \\ 0.09 \\ 0.03 \end{bmatrix} \odot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -0.13 \quad (\text{보이는 것은 계산} \\ -0.17)$$

$$\frac{\partial L}{\partial w_{12}'} = \frac{2}{3} \begin{bmatrix} 0.55 \\ 0.18 \\ 0.02 \end{bmatrix} \odot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -0.35$$

$$\frac{\partial L}{\partial b_1'} = \frac{2}{3} \begin{bmatrix} 0.28 \\ 0.09 \\ 0.03 \end{bmatrix} = 0.23 \quad (\text{보이는 것은 계산} \quad 0.27)$$

$$\frac{\partial L}{\partial b_2'} = \frac{2}{3} \begin{bmatrix} 0.55 \\ 0.18 \\ 0.02 \end{bmatrix} = 0.5$$

(3) Gradient decent (경사하강법)
↳ parameter, weight b change

$$W_{ij}^{k+1} = W_{ij}^k - \alpha \cdot \frac{\partial L}{\partial W_{ij}^k}$$

$$W_{11}^{1+} = W_{11}^1 - 0.1 \times (-0.13) = 1 + 0.013 = 1.013$$

$$W_{12}^{1+} = W_{12}^1 - 0.1 \times (-0.35) = 2 + 0.035 = 2.035$$

$$W_{11}^{2+} = W_{11}^2 - 0.1 \times (0.23) = 1 - 0.023 = 0.977$$

$$W_{12}^{2+} = W_{12}^2 - 0.1 \times (0.09) = 2 - 0.009 = 1.991$$

$$W_{21}^{2+} = W_{21}^2 - 0.1 \times (0.28) = 3 - 0.028 = 2.972$$

$$W_{22}^{2+} = W_{22}^2 - 0.1 \times (0.1) = 4 - 0.01 = 3.99$$

$$W_{11}^{3+} = W_{11}^3 - 0.1 \times 6.975 = 1 - 0.6975 = 0.3025$$

$$W_{21}^{3+} = W_{21}^3 - 0.1 \times 7.519 = 2 - 0.7519 = 1.2481$$

$$b_1^{1+} = b_1^1 - 0.1 \times 0.23 = 0 - 0.023 = -0.023$$

$$b_2^{1+} = b_2^1 - 0.1 \times 0.5 = 1 - 0.05 = 0.95$$

$$b_1^{2+} = b_1^2 - 0.1 \times 0.57 = 0 - 0.057 = -0.057$$

$$b_2^{2+} = b_2^2 - 0.1 \times 0.27 = 1 - 0.027 = 0.973$$

$$b_1^{3+} = b_1^3 - 0.1 \times 7.667 = 1 - 0.7667 = 0.2333$$

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