

Theory of Programming: Homework #3

Due date: 2024/01/17

Isabelle part

For this part, submit Isabelle `.thy` files on the class website.

1. (Exercise 4.2) Formalize the following definition of palindromes

- The empty list and a singleton list are palindromes.
- If xs is a palindrome, so is $a\#xs@[a]$.

as an inductive predicate $palindrome :: 'a\ list \Rightarrow bool$ and prove that $rev\ xs = xs$ if xs is a palindrome.

2. (Exercise 4.3) We could also have defined $star$ as follows:

```
inductive star' :: ('a  $\Rightarrow$  'a  $\Rightarrow$  bool)  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  bool for r where  
  refl': star' r x x  
  | step': star' r x y  $\Longrightarrow$  r y z  $\Longrightarrow$  star' r x z
```

The single r step is performed after rather than before the $star'$ steps. Prove $star' r x y \Longrightarrow star\ r\ x\ y$ and $star\ r\ x\ y \Longrightarrow star' r x y$. You may need lemmas. Note that rule induction fails if the assumption about the inductive predicate is not the first assumption.

3. (Exercise 12.5) Verify the program in problem 1 of the second homework in Isabelle (see template). *Hint:* start the proof with `apply (rule strengthen_pre)`.
4. (Exercise 12.7) Verify the program in problem 2 of the second homework in Isabelle (see template). *Hint:* use `algebra_simps` and `power2_eq_square` for algebraic computations.