Exercise 3

Notation: f_i^j denotes the i-th j-ary function symbol, and p_i^j the i-th j-ary predicate symbol.

1. *Pick out the free and bound occurrences of variables in the following formulas:*

(a) $\forall x_3(\forall x_1 p_1^2(x_1, x_2) \to p_1^2(x_3, a_1))$;
(b) $\forall x_2 p_1^2(x_3, x_2) \to \forall x_3 p_1^2(x_3, x_2)$;
(c) $(\forall x_2 \exists x_1 p_1^3(x_1, x_2, f_1^2(x_1, x_2)) \vee \neg\exists x_1 p_1^2(x_2, f_1^1(x_1))$.

2. *Translate the following sentences into formulas:*

(1) Anyone who is persistent can learn logic.
(2) No politician is honest;
(3) Not all birds can fly, and all birds cannot fly.
(4) If everyone can solve the problem, Hilary can.
(5) Nobody loves a loser.
(6) Everyone loves somebody and no one loves everybody, or somebody loves everybody and someone loves nobody.
(7) You can fool some of the people all of the time, and you can fool all the people some of the time, but you cannot fool all the people all the time.
(8) John hates all people who do not hate themselves.
(9) Any sets that have the same members are equal.
(10) There is no set belonging to precisely those sets that do not belong to themselves.
(11) There is no barber who shaves precisely those men who do not shave themselves.

3. *Show that the following formulas are logically valid.*

(1) $\forall x A(x) \leftrightarrow \neg\exists\neg A(x)$;
(2) $(\forall x A(x) \vee \forall x B(x)) \to \forall x(A(x) \vee B(x))$.

4. *Determine whether the following formulas are logically valid.*

(1) $\neg\exists y \forall x(p_1^2(x, y) \leftrightarrow \neg p_1^2(x, x))$;
(2) $\exists x \exists y(p_1^2(x, y) \to \forall z p_1^2(z, y))$;
(3) $\exists x \exists y(p_1^1(x) \to p_2^1(y)) \to \exists x(p_1^1(x) \to p_2^1(x))$.