Exercise 4

1. Using the natural deduction rules to prove the following statements:
   (1) $\neg\exists x A(x) \vdash\dashv \forall x \neg A(x)$;
   (2) $\exists x A(x) \to B \vdash\dashv \forall x(A(x) \to B)$, where x does not occur in B.

2. Prove the following statements:
   (1) $\vdash \forall x \forall y r(x, y) \to \forall x r(x, x)$, where r is a binary predicate symbol;
   (2) $\vdash \forall x A(x) \to \forall x(A(x) \vee B(x))$;
   (3) $\vdash (\exists x A(x) \to \forall x B(x)) \to \forall x(A(x) \to B(x))$;
   (4) $\vdash \exists y(r(y) \to \forall y r(y))$, where r is a unary predicate symbol.