(4.1) $\neg\exists y\exists x(p(x,y) \leftrightarrow \neg p(x,x))$

Assume that there is an assignment v such that

$$(\neg\exists y\exists x(p(x,y) \leftrightarrow \neg p(x,x)))^v = 0.$$

Then, we have

$$\begin{array}{l}(\exists y\exists x(p(x,y) \leftrightarrow \neg p(x,x)))^v = 1\\ \mathbf{E}a\mathbf{E}b((p(x,y))^{v(x/b,y/a)} = 1 \leftrightarrow (\neg p(x,x))^{v(x/b,y/a)} = 1)\\ \mathbf{E}a\mathbf{E}b((p(x,y))^{v(x/b,y/a)} = 1 \leftrightarrow p(x,x)^{v(x/b,y/a)} = 0)).\end{array}$$

Construct a model $M = (U, I)$ such that $U = \{a, b\}$ and $p^I = \{(a,a),(a,b)\}$. Then,

$$\begin{array}{l}p(x,y))^{v(x/a,y/b)} = 1,\\ p(x,x)^{v(x/a,y/b)} = 1.\end{array}$$

□

(4.2) $\exists x\exists y(p(x,y) \to \forall zp(z,y))$

Assume that there is an assignment v such that

$$(\exists x\exists y(p(x,y) \to \forall zp(z,y)))^v = 0.$$

Then,

$$\begin{array}{l}\mathbf{A}a\mathbf{A}b(p(x,y) \to \forall zp(z,y))^{v(x/a,y/b)} = 0)\\ \mathbf{A}a\mathbf{A}b(p(x,y)^{v(x/a,y/b)} = 1\&(\forall zp(z,y))^{v(x/a,y/b)} = 0))\\ \mathbf{A}a\mathbf{A}b(p(x,y)^{v(x/a,y/b)} = 1\&\mathbf{E}c(p(z,y)^{v(y/b,z/c)} = 0)),\end{array}$$

a contradiction, where

$$\begin{array}{l}(\forall zp(z,y))^{v(x/a,y/b)} = 1 \text{ iff } \mathbf{A}c(p(z,y)^{v(z/c,y/b)} = 1)\\ (\forall zp(z,y))^{v(x/a,y/b)} = 0 \text{ iff } \mathbf{E}c(p(z,y)^{v(z/c,y/b)} = 0)\\ (\exists zp(z,y))^{v(x/a,y/b)} = 1 \text{ iff } \mathbf{E}c(p(z,y)^{v(z/c,y/b)} = 1)\\ (\exists zp(z,y))^{v(x/a,y/b)} = 0 \text{ iff } \mathbf{A}c(p(z,y)^{v(z/c,y/b)} = 0).\end{array}$$

□

(4.3) $\exists x\exists y(p(x) \to q(y)) \to \exists x(p(x) \to q(x))$.

Assume that there is an assignment v such that

$$(\exists x\exists y(p(x) \to q(y)) \to \exists x(p(x) \to q(x)))^v = 0.$$

Then,

$$\begin{array}{l}\mathbf{A}a\mathbf{A}b((p(x) \to q(y))^{v(x/a,y/b)} = 1\&(\exists x(p(x) \to q(x))^{v(x/a.y/b)} = 0))\\ \mathbf{A}a\mathbf{A}b((p(x) \to q(y))^{v(x/a,y/b)} = 1\&\mathbf{A}c((p(x) \to q(x))^{v(x/c,y/b)} = 0))\\ \mathbf{A}a\mathbf{A}b((p(x) \to q(y))^{v(x/a,y/b)} = 1\&\mathbf{A}c(p(x)^{v(x/c,y/b)} = 1\&q(x)^{v(x/c,y/b)} = 0))\end{array}$$

Taking $c = a$, we have a contradiction:

$$\begin{array}{l}(p(x) \to q(y))^{v(x/a.y/b)} = 1\\ p(x)^{v(x/a,y/b)} = 1\\ q(x)^{v(x/a,y/b)} = 0.\end{array}$$

□