如何形式证明 $(A \to B) \to B \vdash (B \to A) \to A$ 的问题,证明方法如下:

$$(1) \quad (A \to B) \to B, B \to A, \neg A \vdash B \to A \tag{(e)}$$

$$(2) \quad (A \to B) \to B, B \to A, \neg A \quad \vdash \neg A \tag{(e)}$$

$$(3) A, \neg A, \neg B \vdash A (\in)$$

$$(4) A, \neg A, \neg B \vdash \neg A (\in)$$

$$(5) A, \neg A \vdash B (\neg^{-})$$

$$(7) \quad (A \to B) \to B, B \to A, \neg A \vdash A \to B$$
 (Tr, (2)(6))

$$(8) \quad (A \to B) \to B, B \to A, \neg A \quad \vdash (A \to B) \to B \tag{(e)}$$

$$(9) \quad (A \to B) \to B, B \to A, \neg A \vdash B \qquad (\to^-, (7)(8))$$

$$(10) \quad (A \to B) \to B, B \to A, \neg A \vdash A \qquad (\to^-, (1)(9))$$

$$(11) (A \to B) \to B, B \to A \vdash A (\neg^-, (2)(10))$$

$$(12) (A \to B) \to B \vdash (B \to A) \to A (\to^+)$$